Gravity Actions from Tensor Networks

[hep-th/1609.04645] with T. Takayanagi and K. Watanabe
and ongoing work with
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Motivation

- We do not know the precise mechanism of AdS/CFT.

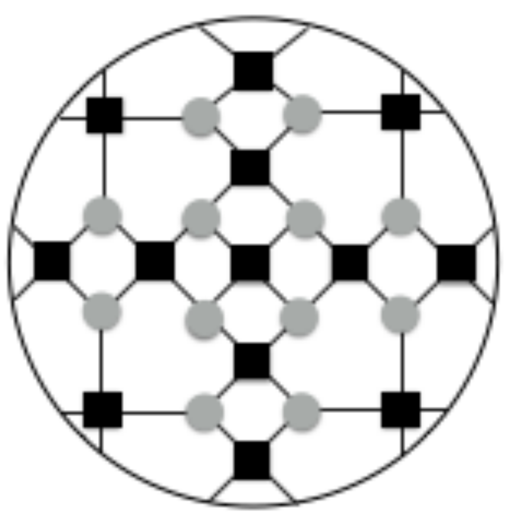
- There is a significant similarity between Tensor network and gravity.

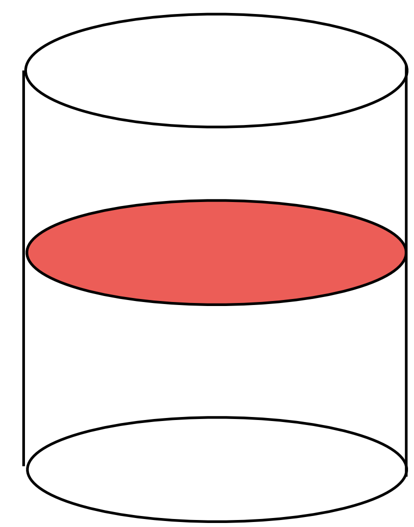
[Swingle]

Tensor Network

Equivalent?

Bulk timeslice in AdS/CFT





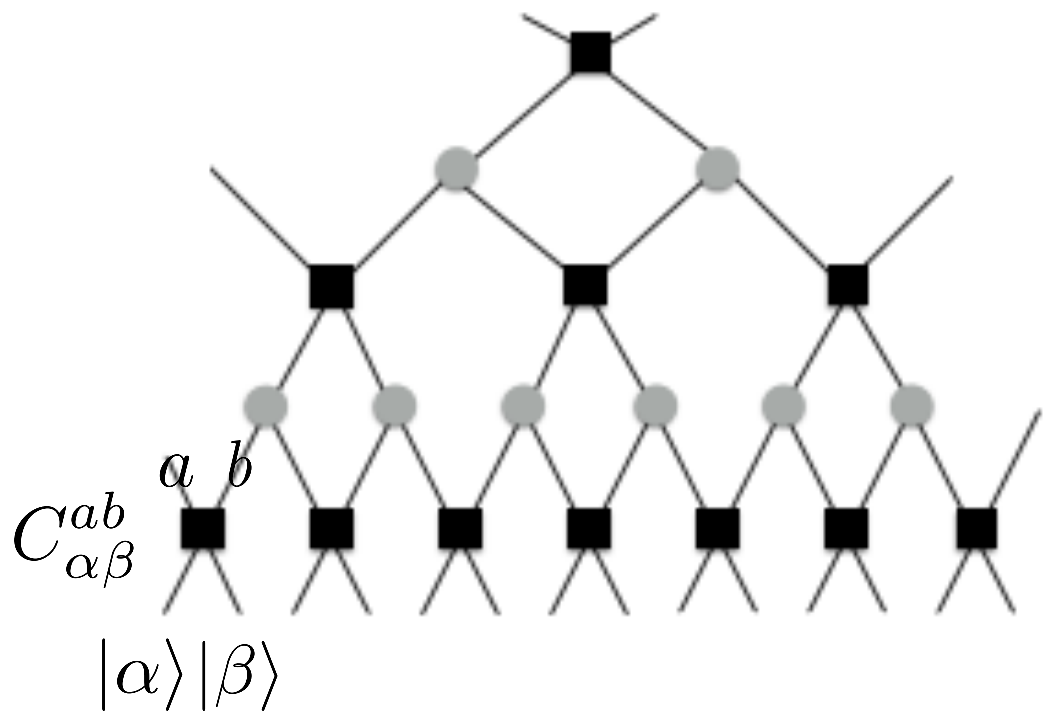
The aim is to find precise relation between Tensor Network and gravity.

Tensor Network [White][Vidal]

Tensor network: Network of tensors. By contracting tensors, we can express complicated states.

- It can simulate ground state of field theory.

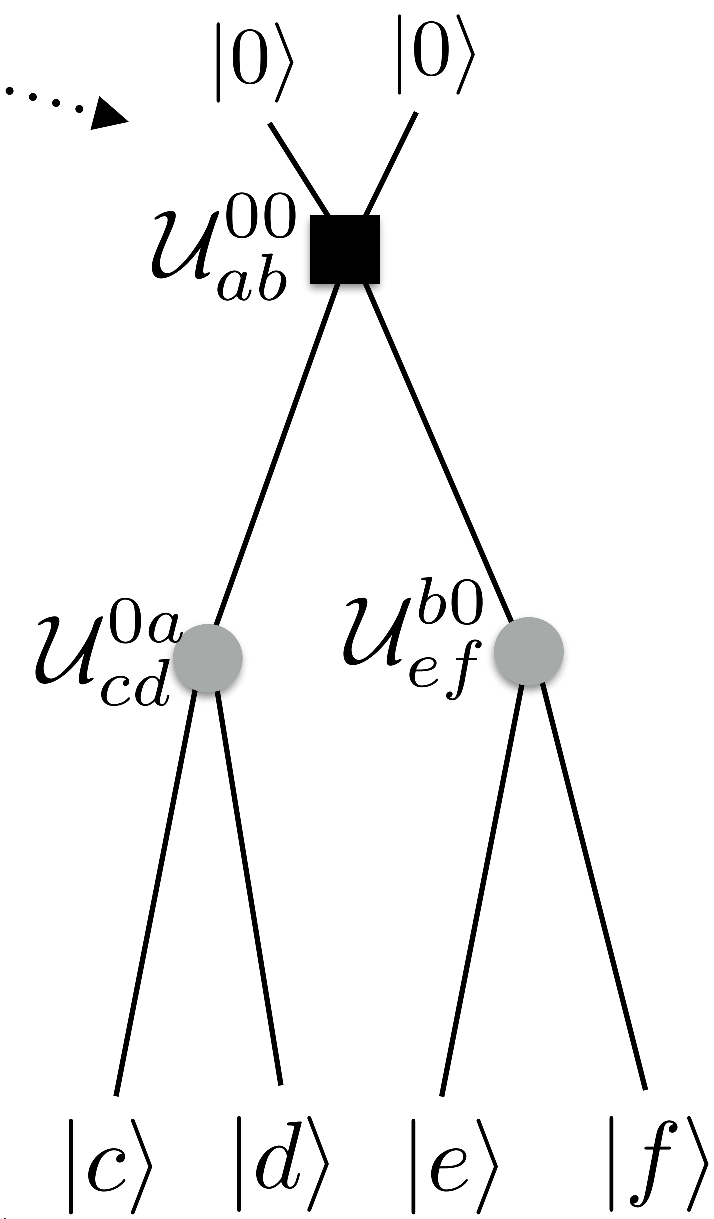
- RG flow of state.



- Grounds state gets simplified and loses short range entanglement gradually.

Tensor Network

State without short range quantum entanglement



$$|0\rangle \otimes |0\rangle$$

Reduce quantum entanglement

$$\sum_{a,b} \mathcal{U}^{00}_{ab} |a\rangle \otimes |b\rangle$$

Coarse graining

Ground state

$$\sum_{c,d,e,f} \mathcal{U}^{cd}_{0a} \mathcal{U}^{b0}_{ef} \sum_{ab} \mathcal{U}^{00}_{ab} |c\rangle \otimes |d\rangle \otimes |e\rangle \otimes |f\rangle$$

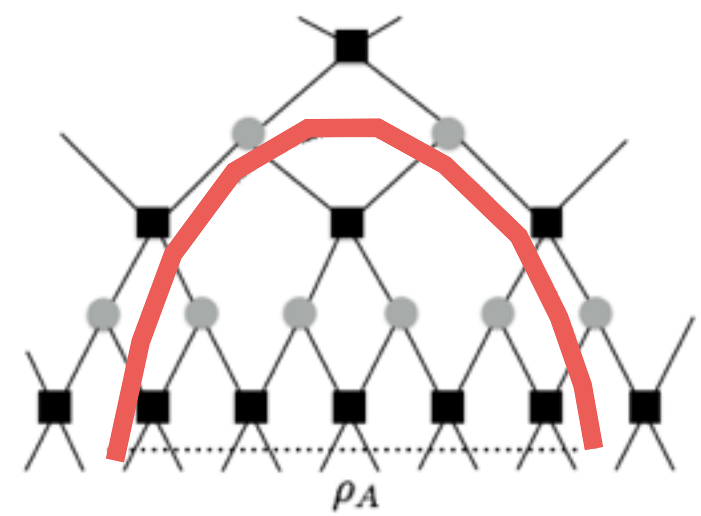
Tensor Network and Ryu Takayanagi Formula

- For certain tensor networks, such as MERA, entanglement entropy of state can be expressed by number of bonds of minimal bond surface.

$$S[\rho_A] \sim \mathcal{O}(1) \text{ constant}$$

$\times$ #(Bonds intersecting the minimal bonds surface)

$\times$ log (dimensions of bond)



- We can identify number of intersecting bonds as area of surface.

$$S[\rho_A] \sim \mathcal{O}(1) \text{ constant} \quad \times \quad \text{(Area of the minimal surface)}$$

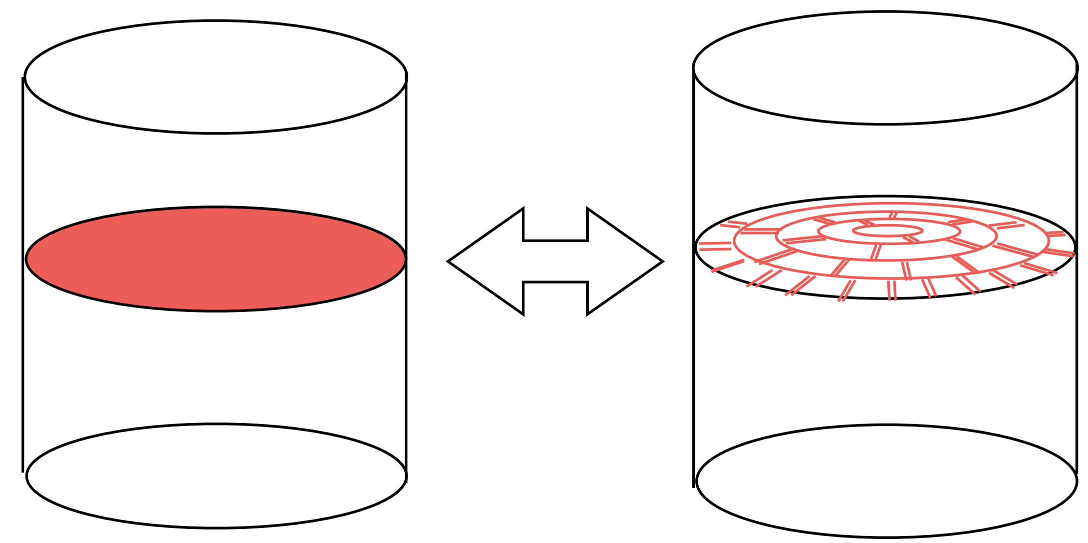
$$\times \quad \log \text{(dimensions of bond)}$$

- Significant similarity to Ryu-Takayanagi formula in AdS/CFT. [Swingle]

Tensor Network = Bulk timeslice?

Conjecture [Swingle]

Tensor network of CFT ground state= Timeslice of bulk spacetime in AdS/CFT



Timeslice of AdS_{d+1} Tensor network

Problems

- How should we determine correct tensor network?
- Relation between gravity and tensor network is still ambiguous.

Continuous Tensor Network

[Haegeman, Osborne, Verschelde and Verstraete]

[M.M, Ryu, Takayanagi, Wen]

- Interpolation from vacuum state to unentangled trivial state.

$$|\Omega_{vac}\rangle \longrightarrow |\Omega_z\rangle \longrightarrow |\Omega_\infty\rangle$$

Ground state — Coarse grained state — Unentangled trivial state

z: effective lattice spacing

- Boundary states have no short range quantum entanglement, so we can use boundary states as $|\Omega_\infty\rangle$.

- In Euclidean path integral, wave function is

$$\langle\phi_0|\Omega_{vac}\rangle = \int_{\tau<0} \mathcal{D}\phi(x,\tau)\, e^{-\int_{-\infty}^{0} d\tau \int dx \mathcal{L}_E[\phi(\tau,x)]} \Big/ \sqrt{\int \mathcal{D}\phi(x,\tau)\, e^{-S_E[\phi]}}$$

$$\phi(x,0) = \phi_0(x) \qquad \phi(x,\infty) = \phi(x,-\infty)$$

$$\phi(x,\tau=-\infty) = \phi_B(x) \qquad = \phi_B(x)$$

Deforming action

[M.M, Takayanagi, Watanabe]

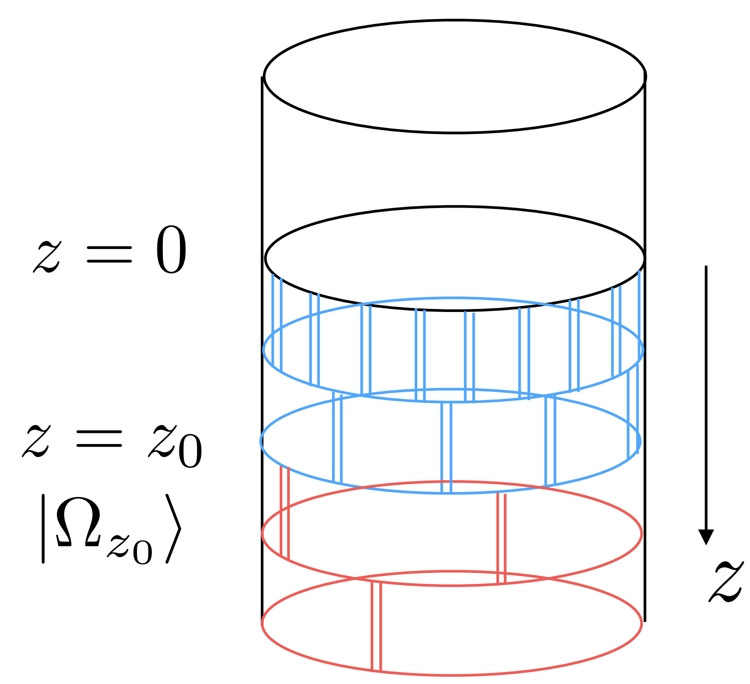
- By deforming this Euclidean path integral, we can simulate tensor network.

- In the deformation, we identify $z = -\tau$ as effective lattice spacing.
  In other word, UV cut off is z dependent, and modes with spatial momentum larger than 1/z are discarded.

$$\langle \phi_0 | \Omega_{vac} \rangle \propto \int_{z>0} \mathcal{D}\phi(x,z) e^{-\int_\epsilon^\infty dz \int dk d\omega \theta(z\sqrt{(k^2+\omega^2)/2})\mathcal{L}_E[\phi]}$$

$$\phi(x,\epsilon) = \phi_0(x)$$

where $\theta(x) = 0 \ (x > 1)$



- By this deformation, we can discard redundant path integral, without changing wavefunction.

Most efficient path integral proposal

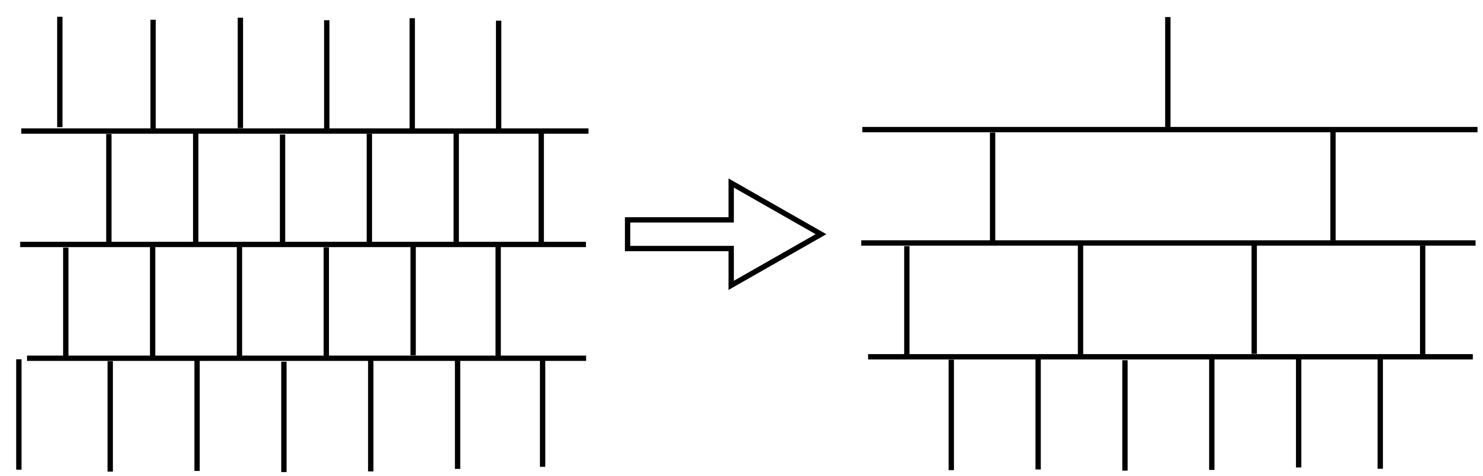
Work in progress [Pawel, Kundu, M.M, Takayanagi, Watanabe]

Questions

- The deformation of the path integral is not unique.
- No criterion relate it to gravity.

Proposal to fix the deformation

We need to eliminate redundant tensors from the path integral, in order to relate tensor network with AdS/CFT and Ryu-Takayanagi formula.



Proposal

The deformation with minimum redundant tensors corresponds to bulk time slice in AdS/CFT.

Most efficient path integral proposal

- We deform the path integral by Weyl transformation to the CFT, without changing the metric at z = 0.

$$ds^2 = dz^2 + dx^2 \quad \Rightarrow \quad ds^2 = e^{\gamma\phi(x,z)}(dz^2 + dx^2)$$

<u>A definition of redundancy</u>

- There is a variety of ways to define redundancy of tensor network.

- Let's consider path integral representation of the vacuum state.

$$\langle\psi_0|\Omega_{vac}\rangle = \int_{\psi(x,\epsilon)=\psi_0(x)} \mathcal{D}\psi \; e^{-S[\psi]} = \boxed{e^{-S_L[\phi]}} \int_{\psi(x,\epsilon)=\psi_0(x)} \mathcal{D}\psi \; e^{-S[\psi,\phi]}$$

$\boxed{e^{-S_L[\phi]}}$ corresponds to amount of redundancy of the original path integral

compared to deformed one.

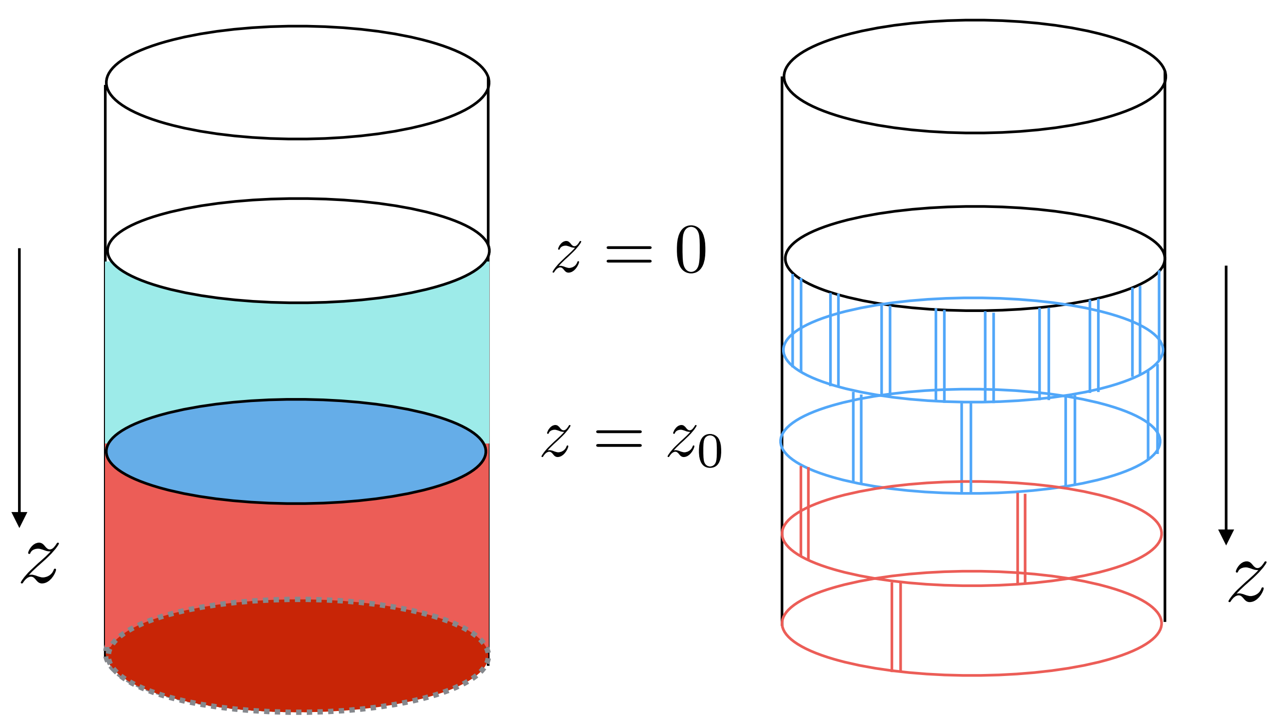
Most efficient path integral proposal

Emergent metric

"The most efficient path integral" $\Longleftrightarrow$ $e^{-S_L[\phi]}$ takes maximal value.

We identify the metric $ds^2 = e^{\gamma\phi(x,z)}(dz^2 + dx^2)$ as metric on the bulk timeslice.

$\Rightarrow$ $\phi(x, z)$ is determined by minima of $S_L[\phi]$



Example: 2d CFT

For Weyl transformation,

$$\left[\mathcal{D}\phi_{ds^2=e^{\gamma\phi}(dz^2+dx^2)}\right] = e^{S_L}\left[\mathcal{D}\phi_{ds^2=dz^2+dx^2}\right]$$

where S_L is the Liouville action

$$S_L = \frac{c}{24\pi}\int_\epsilon^\infty dz \int_{-\infty}^\infty dx \left[(\partial_x\phi)^2 + (\partial_z\phi)^2 + \frac{\mu}{\gamma^2}(e^{\gamma\phi} - 1)\right] \qquad \mu > 0$$

is minimized when

$$e^{\gamma\phi} = \frac{4}{\mu} z^{-2} \qquad ds^2 = \frac{4}{\mu}\frac{dz^2 + dx^2}{z^2}$$

So we get AdS solution as true minimum.

In contrast to Einstein-Hilbert action, Liouville action is bounded from below.

Example: 2d CFT

Excited state

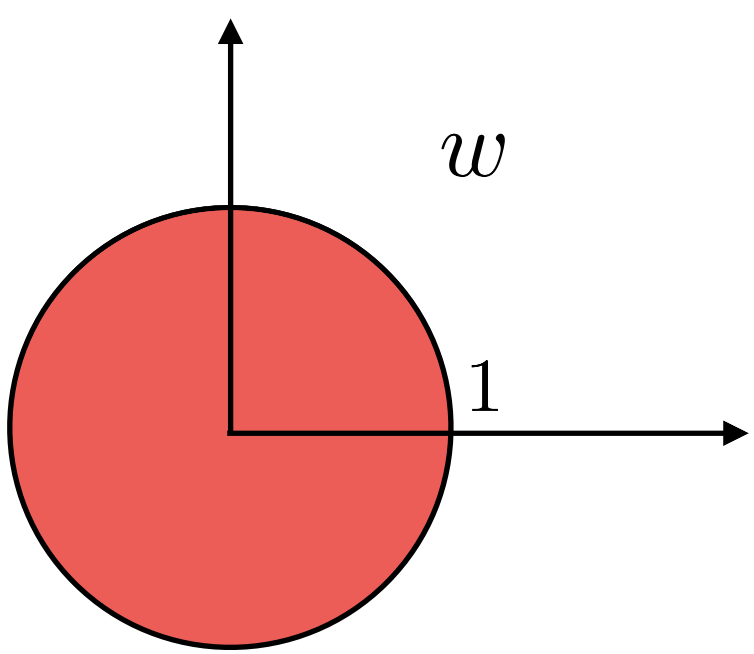
For state excited by primary operator with dimension h at $w = 0 \ (|w| < 1)$

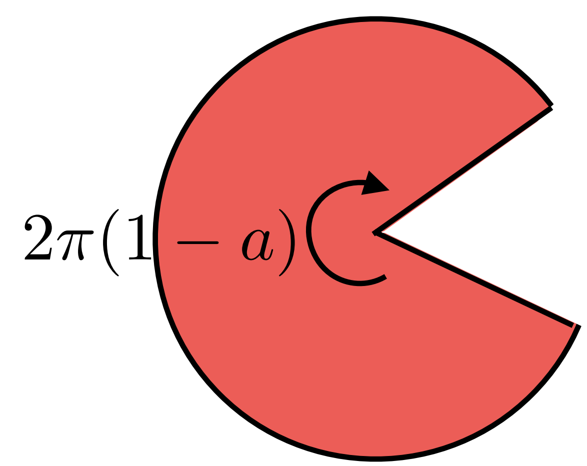
assuming large c, corresponding solution is

$$ds^2 = \frac{4}{\mu} \cdot \frac{a^2}{|w|^{2(1-a)}(1-|w|^{2a})^2} dw d\bar{w} \qquad \left(a = 1 - \frac{12h}{c}\right)$$

deficit angle geometry with angle $2\pi(1-a)$.

In AdS_3/CFT_2, the relation is $a = \sqrt{1 - \frac{24h}{c}}$, consistent when $h << c$.





Example: 2d CFT

BTZ black hole

Similarly, by considering wave function of Thermo field double state,

$$|\Psi_{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\beta E_i} |E_i\rangle \otimes |E_i\rangle$$

$$\langle \tilde{\psi}_1, \tilde{\psi}_2 | \Psi_{TFD} \rangle = \int_{-\beta/4 < z < \beta/4} \mathcal{D}\psi(x,z) \; e^{-S_{CFT}}$$

$$\psi(x, -\beta/4) = \tilde{\psi}_1(x)$$

$$\psi(x, \beta/4) = \tilde{\psi}_2(x)$$

we get Liouville action.

As the minimum of the action, we obtain BTZ solution.

$$ds^2 = \frac{4}{\mu} \cdot \frac{4\pi^2}{\beta^2} \cdot \frac{dz^2 + dx^2}{cos^2(\frac{2\pi z}{\beta})}$$

Summary

- We expressed flow of tensor network states by path integral with deformed action.

- We fixed one tensor network flow by assuming "most efficient path integral".

- We identified bulk metric with background metric, which is consistent with several examples.

<u>Questions</u>

- Entanglement entropy? Quantum corrections? Higher dimensions?