Geodesic Witten diagrams with an external spinning field

[arXiv:1609.04563]
[work in progress]

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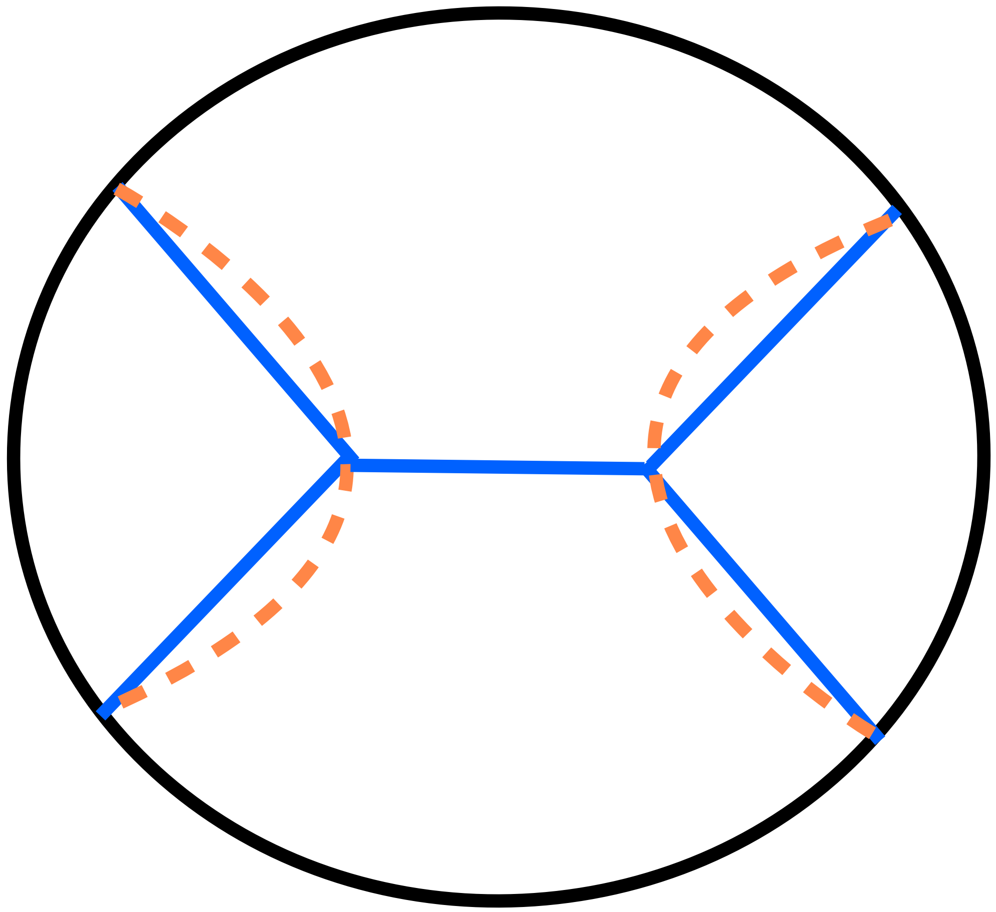
Q What is the gravity dual of conformal block?

A Geodesic Witten diagram

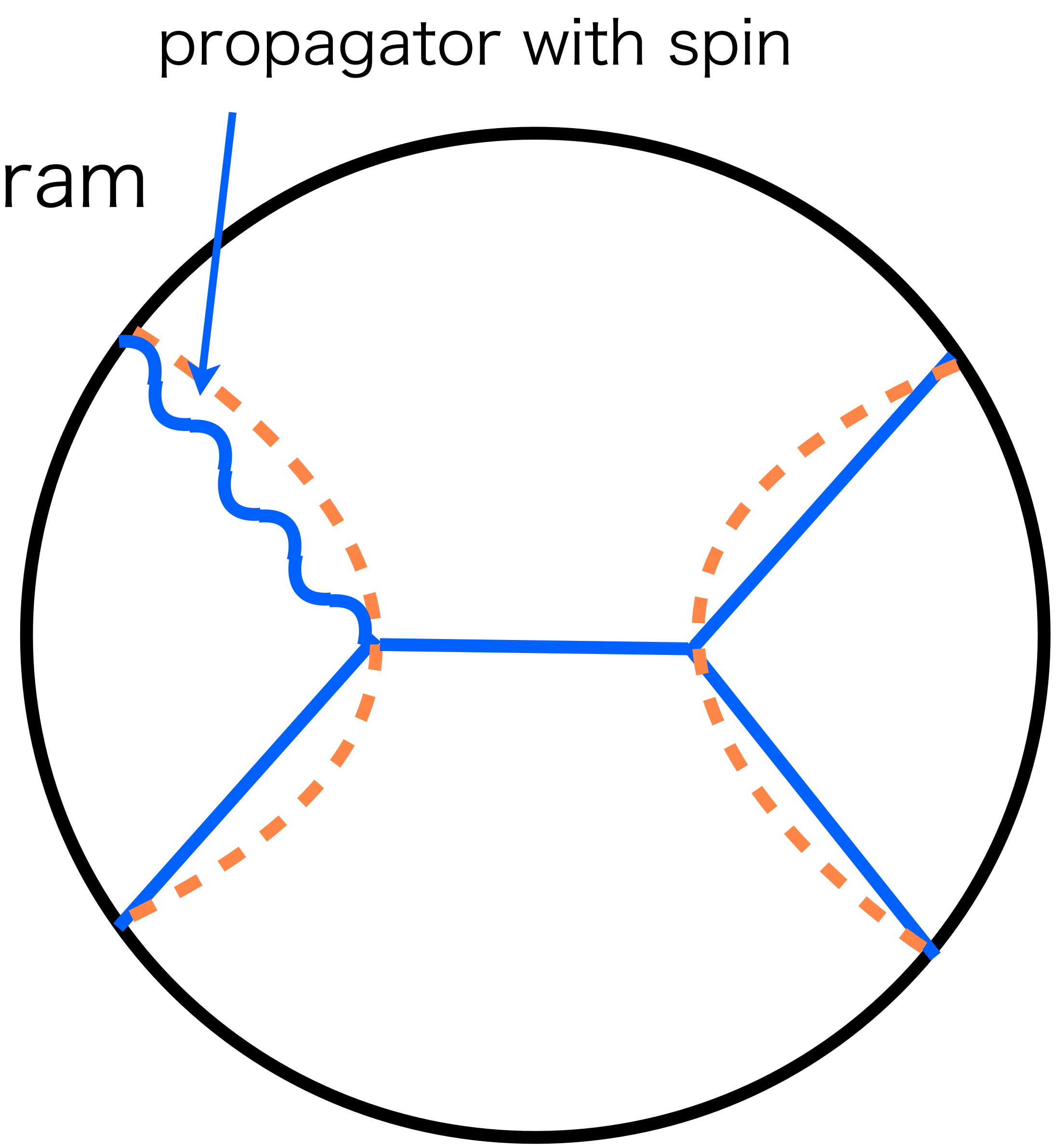


[E. Hijano, P. Kraus, E. Perlmutter, R. Sniverly, 2015]

Our result

Geodesic Witten diagrams
with an external spin-n field

- Construction of geodesic Witten diagram with an external spin-n field and comparing with a known formula of conformal block



Outline

1. Conformal block
   and geodesic Witten diagram

2. Our result with an external
   symmetric traceless tensor field

Conformal block $W_{\Delta,l}(x_i)$

Basis of CFT's correlation function

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

$$= \sum_{\mathcal{O}} \underline{C_{12\mathcal{O}}C_{34\mathcal{O}}}\,\underline{W_{\Delta,l}(x_i)}$$

It depends on the theory.

It doesn't depend on the theory.

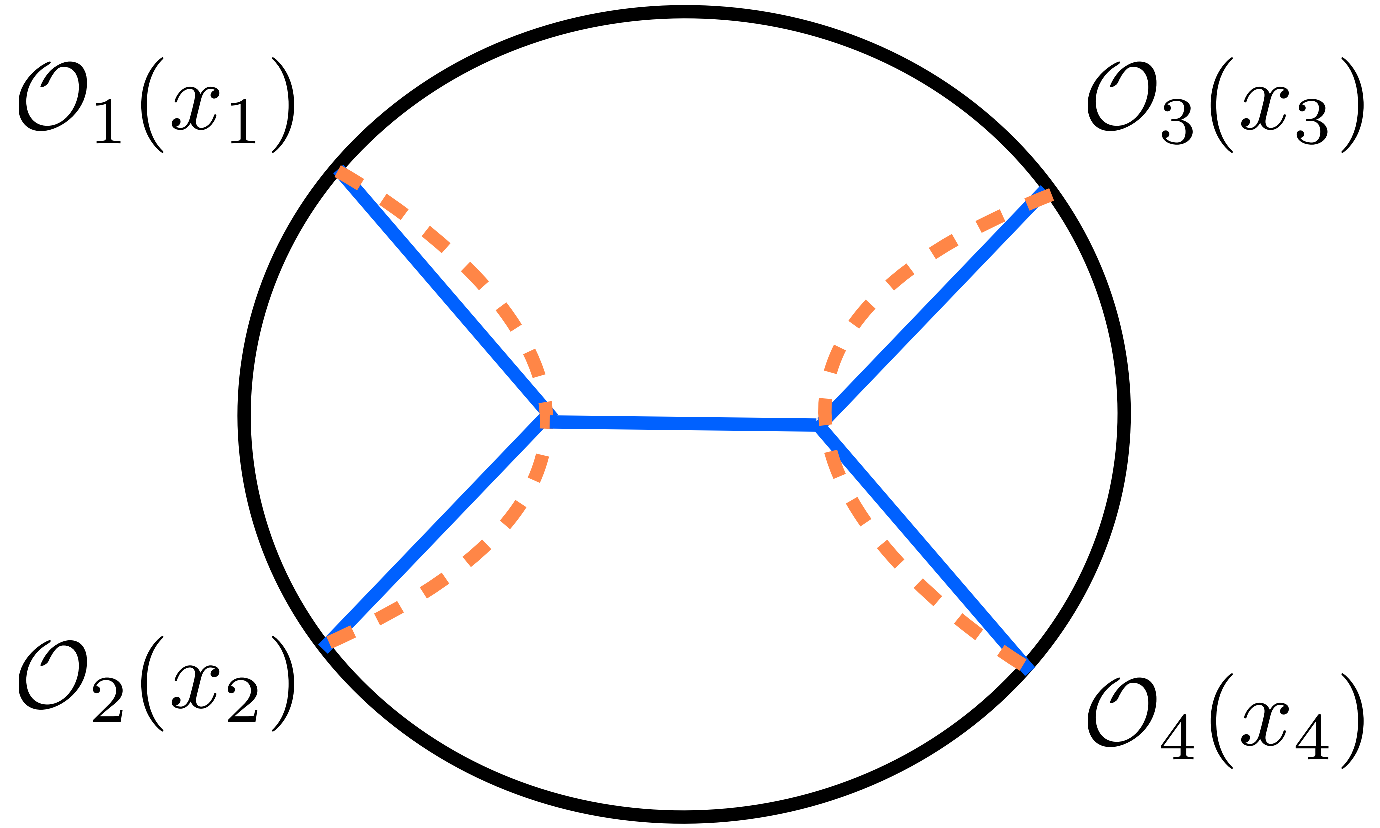
Research topic with conformal block

AGT correspondence

conformal bootstrap

Geodesic Witten diagram

Feynman diagram in AdS spacetime which is integrated over geodesics



amplitude

$$\mathcal{W}_{\Delta,0}(x_i)$$
$$\equiv \int_{\gamma_{12}} d\lambda \int_{\gamma_{34}} d\lambda' G_{b\partial}(y(\lambda), x_1) G_{b\partial}(y(\lambda), x_2)$$
$$\cdot G_{bb}(y(\lambda), y(\lambda'); \Delta)$$
$$\cdot G_{b\partial}(y(\lambda'), x_3) G_{b\partial}(y(\lambda'), x_4)$$

λ, λ' : proper times of the geodesics

Geodesic Witten diagram corresponds to conformal block up to normalization.

[E. Hijano, P. Kraus, E. Perlmutter, R. Sniverly, 2015]

Properties of scalar conformal block $W_{\Delta,0}(x_i)$

1. Transformation law
   under conformal transformation

$$W_{\Delta,0}(x'_i) = \prod_{i=1}^{4} \alpha^{-\Delta_i} W_{\Delta,0}(x_i)$$

scale transformation

$$x'^{\mu} = \alpha x^{\mu}$$

2. Solution of conformal Casimir equation

$$\frac{1}{2}(L^{(0)}_{x_1} + L^{(0)}_{x_2})^2 W_{\Delta,0}(x_i) = -\Delta(\Delta - d) W_{\Delta,0}(x_i)$$

$$[L, \mathcal{O}(x_i)] = L^{(0)}_{x_i} \mathcal{O}(x_i)$$

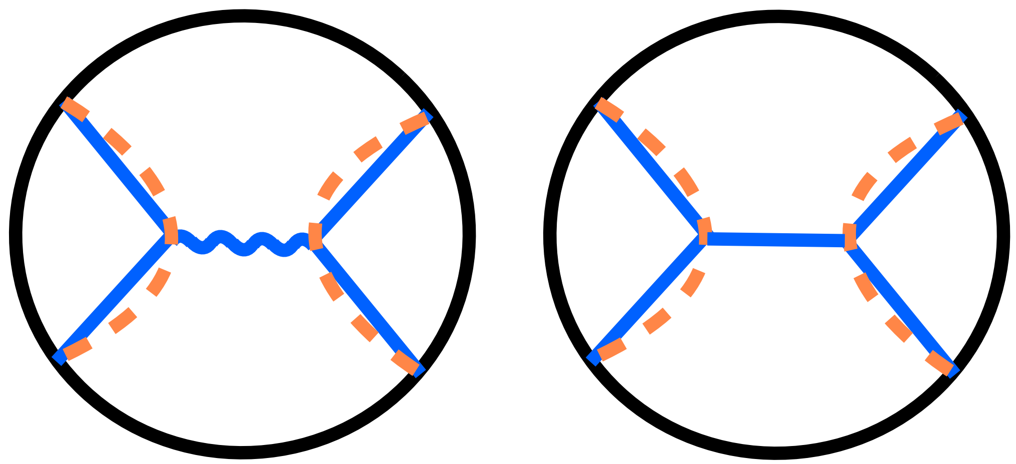
Geodesic Witten diagram
also satisfies these properties.

Our purpose

Generalization to external field with spin

Why?

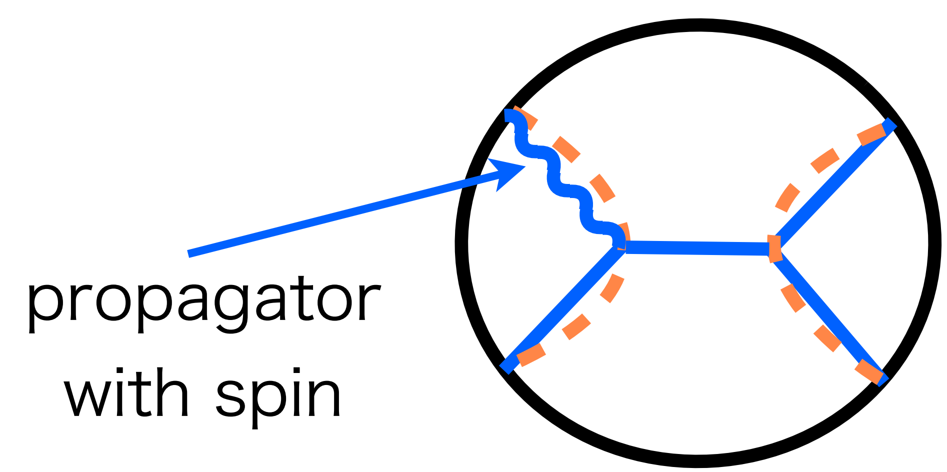
- Conformal block
  for conserved current



[E. Hijano, P. Kraus,
E. Perlmutter, R. Sniverly, 2015]

- To find unknown expressions
  of conformal block

Our target



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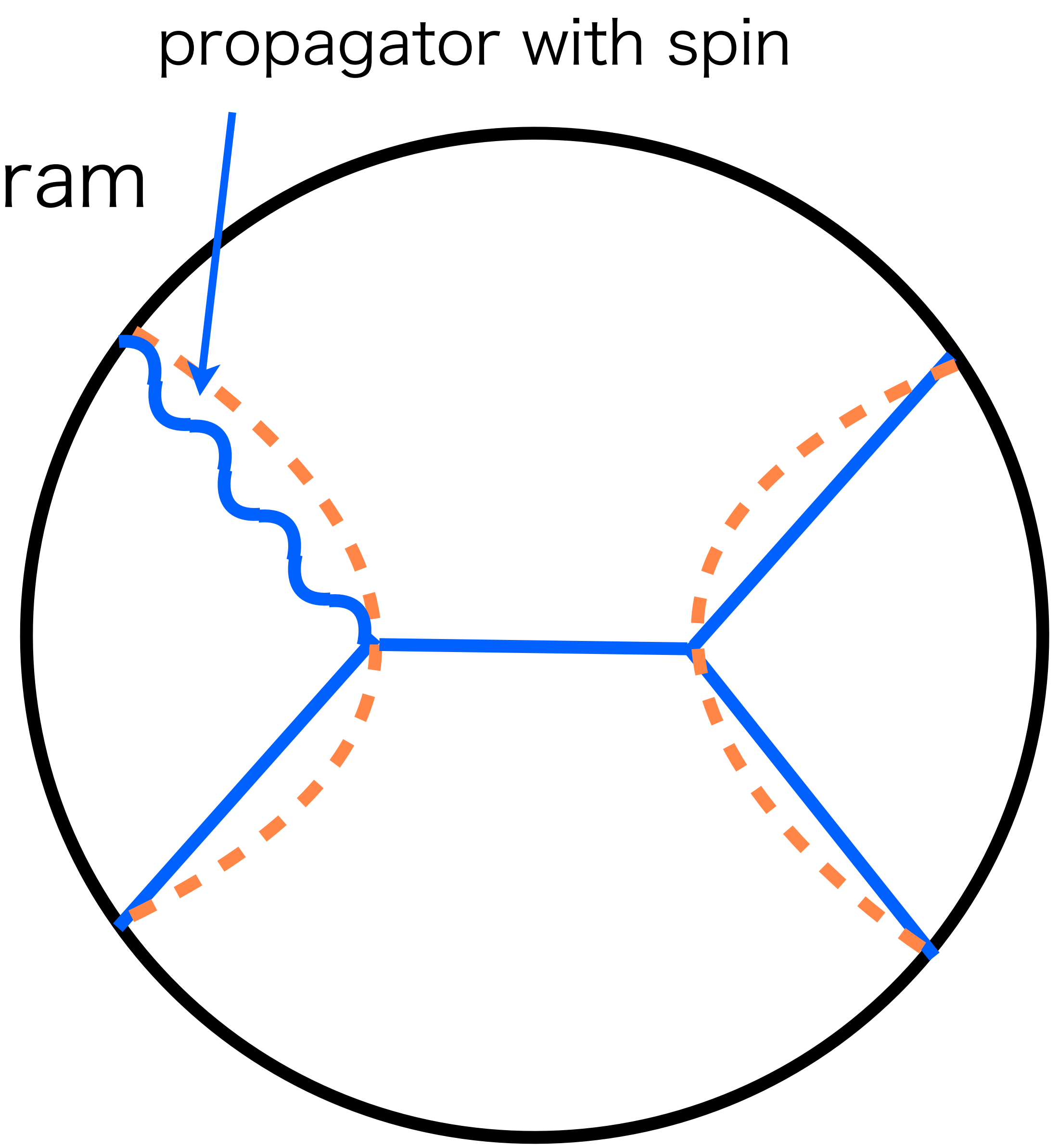
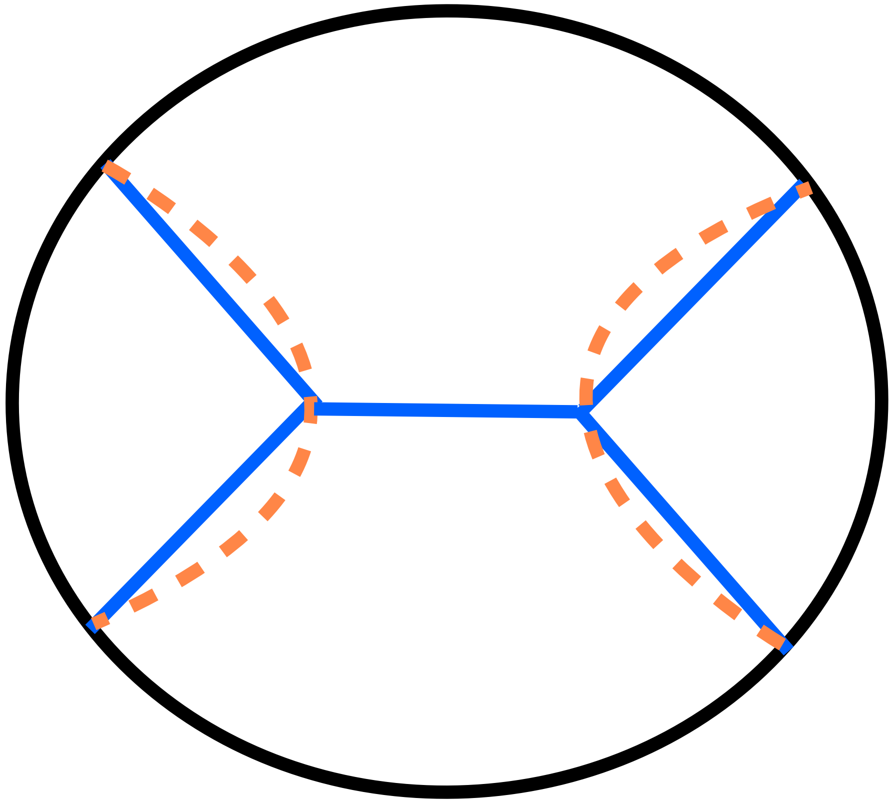


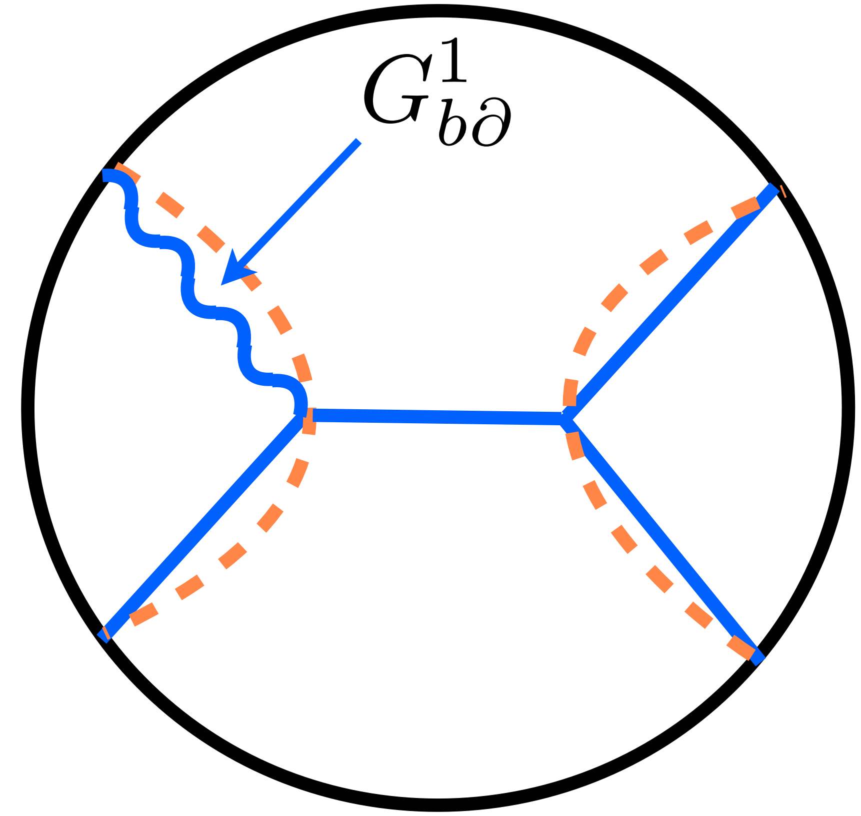
Geodesic Witten diagram with an external spin-1 field

[MN, K. Tamaoka, 2016]

$$\left(\frac{\partial}{\partial x_1^a} + \frac{2\Delta_1 (x_{12})_a}{|x_{12}|^2}\right)$$



$$\propto$$



$$= \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\lambda' \left(G^1_{b\partial}(y(\lambda), x_1; \Delta_1 + 1)\right)^{\mu}_{a} G_{b\partial}(y(\lambda), x_2; \Delta_2)$$

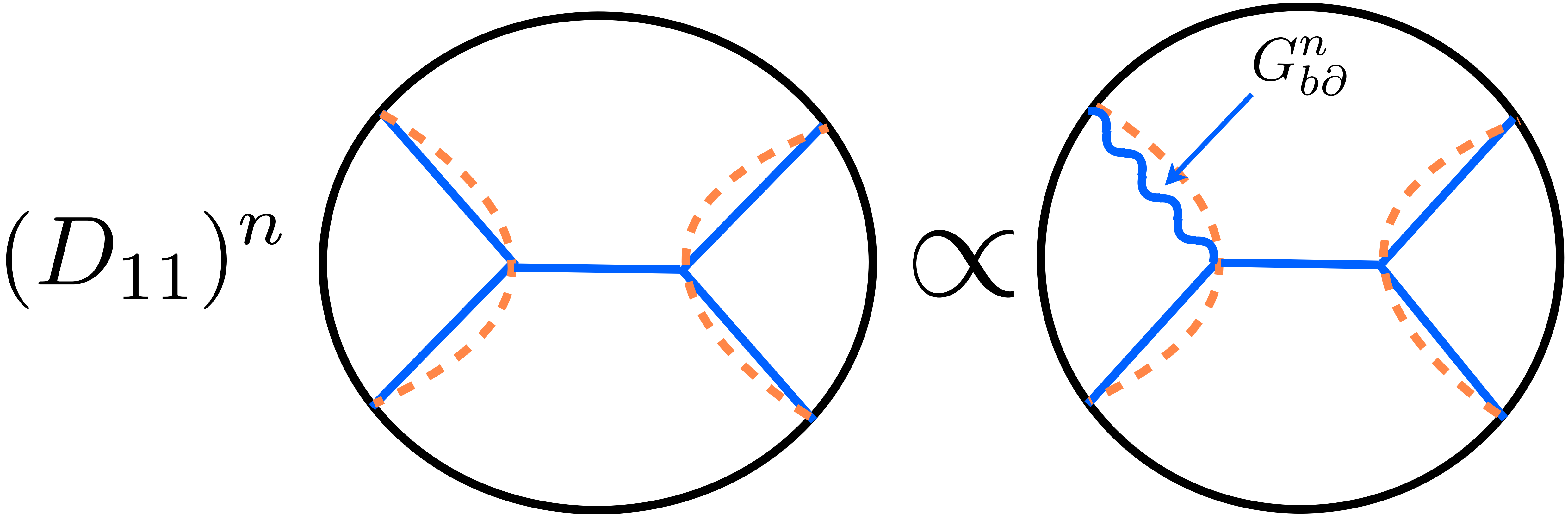
$$\cdot \underline{u(\lambda)^2 \frac{\partial}{\partial y^{\mu}(\lambda)}} \left(G_{bb}(y(\lambda), y(\lambda'); \Delta)\right)$$

$$A_\mu \phi g^{\mu\nu} \partial_\nu \phi$$

$$\cdot G_{b\partial}(y(\lambda'), x_3; \Delta_3) G_{b\partial}(y(\lambda'), x_4; \Delta_4)$$

Geodesic Witten diagram with an external spin-n field

[MN, K. Tamaoka, 2016]



$T_{A_1 \cdots A_n} \phi \nabla^{A_1} \cdots \nabla^{A_n} \phi$: three point coupling

D_{11} : a differential operator

This is consistent with the result of conformal block in [M. S. Costa, J. Penedones, D. Poland, S. Rychkov, 2011] .

Summary

conformal block $\longleftrightarrow$ geodesic Witten diagram

We construct geodesic Witten diagrams
with an external spin-n field
and check that they correspond to conformal blocks.