ECHOES OF CHAOS FROM STRING THEORY BLACK HOLES

Gábor Sárosi

1612.04334

with V. Balasubramanian, B. Czech and B. Craps

February 6, 2017

Easy version of the information problem for large AdS black holes

Large BHs in AdS can be in equilibrium with thermal radiation. Other signals of information loss?

Late time behavior of two-point functions in the BH background [Maldacena,'01]

- Gravity calculation: solve for Green's function in black hole background $\rightarrow$ decays to zero for large t exponentially, set by lowest quasinormal mode
- CFT calculation: calculate the thermal (or typical high energy state) two-point function.
  Late time average:

  $$\sum_n |\langle n|O(0)|n\rangle|^2 e^{-\beta E_n},$$

  assuming discrete spectrum (BH has finite entropy!). Typically of order e^{-S}.

Easy version of the information problem for large AdS black holes

Questions:

- What happens after the two-point function hits $\sim e^{-S}$?
- How to understand it from a bulk perspective?

Here we are mainly concerned with the first question.

Example: Sachdev-Ye-Kitaev model

- Model of N interacting Majorana fermions with quenched disorder

$$H = \sum_{ijkl} j_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l$$

  The j_{ijkl} are drawn from a Gaussian distribution with zero mean and $\sigma = \mathcal{J}/N^{3/2}$
- Maximally chaotic (Lyapunov exponent is maximal)
- Emergent reparametrization symmetry in the IR, with effective action agreeing with that of AdS_2-dilaton gravity for the breaking of the symmetry

Strongly indicates that it describes some near extremal black hole

Example: Sachdev-Ye-Kitaev model

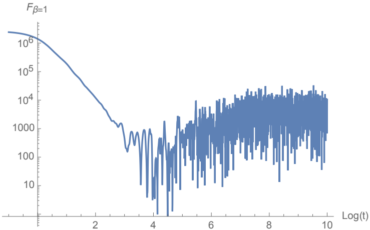
- 2pt-function

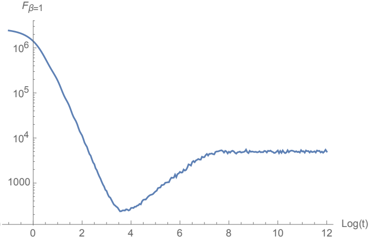
$$\langle O(t)O(0)\rangle_\beta \sim \sum_{m,n} |\langle m|O(0)|n\rangle|^2 e^{-\beta E_m + i(E_m - E_n)t}$$

- Model: spectral form factor (cancellation of phases is the important phenomenon)

$$F_\beta(t) = |Z(\beta - it)|^2 = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$

- In SYK: early time behavior is governed by GR, late time by Random matrix theory [Cotler+many authors, '16]





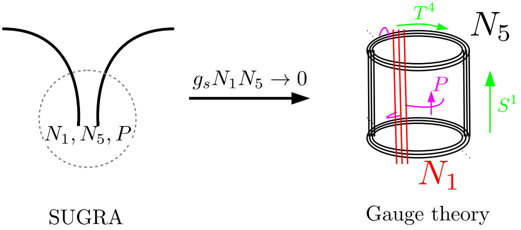
Example: Sachdev-Ye-Kitaev model

- Do black holes really do this? Can we check the validity of the late time prediction in a stringy top-down model?
- Is the dip-ramp phenomena a diagnostic for chaos?

A zeroth order step: D1D5 system at orbifold point

D1D5 CFT

IIB compactified on $S^1 \times T^4$ with S^1 large compared to T^4



Gauge theory flows in the IR (near horizon) to a marginal deformation of the symmetric orbifold

$$\mathsf{CFT}_N = \frac{(T^4)^N}{S_N}, \quad N = N_1 N_5$$

Also dual to IIB on $\mathrm{AdS}_3 \times S^3 \times T^4$

Marginal coupling $\leftrightarrow$ string scale ℓ_s^{-1}

Two charge black holes in D1D5

- **Three charge black hole:** Strominger-Vafa system

  $\frac{1}{4}$-BPS in 5d $\leftrightarrow$ Extremal BTZ in AdS$_3$ $\leftrightarrow$ RR-sector states with only left moving excitations $h_L = P$
- **Two charge black holes:** Ramond ground states

$$|N_{n\mu}, N'_{n\mu}\rangle = \prod_{n,\mu} (\sigma_n^\mu)^{N_{n\mu}} (\tau_n^\mu)^{N'_{n\mu}} |0\rangle$$

$$\sum_{n\mu} n(N_{n\mu} + N'_{n\mu}) = N, \quad N_{n\mu} = 0, 1, 2, \cdots, \qquad N'_{n\mu} = 0, 1$$

$\mu = 1, ..., 8$ indices assoiciated to global symmetries

- Parametrically large, but not classically visible degeneracy $S \sim \sqrt{N}$
- Typical state from grand canonical distribution $e^{-\eta N}$

$$N_{n\mu} = \frac{1}{e^{\eta n} - 1}, \quad N'_{n\mu} = \frac{1}{e^{\eta n} + 1}, \quad N_n = \sum_\mu (N_{n\mu} + N'_{n\mu}) = \frac{8}{\sinh \eta n}$$

$$N = \sum_n n N_n \approx \frac{2\pi^2}{\eta^2} \text{ for small } \eta$$

Two point function in D1D5

Two point function [Balasubramanian,Kraus,Shigemori,'05]

$$\mathcal{O} = \frac{1}{\sqrt{N}} \sum_{a=1}^{N} \mathcal{O}_a \,, \quad h_a = \bar{h}_a = 1.$$

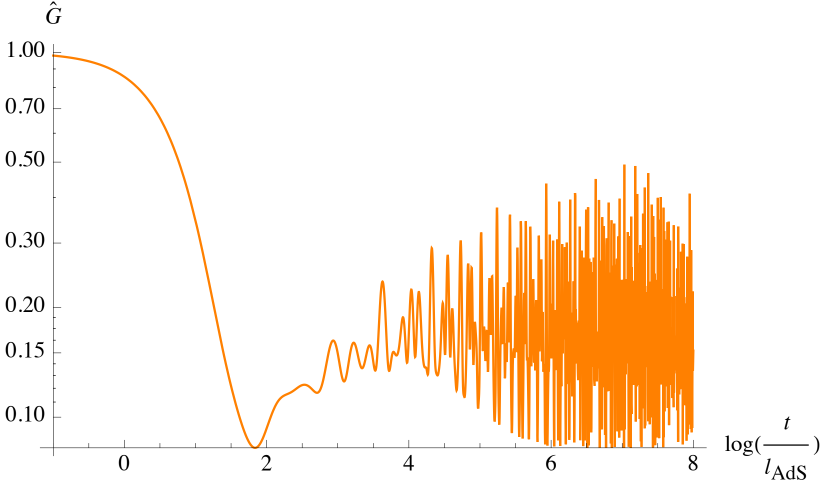
$$\begin{aligned} G(w, \bar{w}) &= \langle N_{n\mu}, N'_{n\mu} | \mathcal{O}^\dagger \mathcal{O} | N_{n\mu}, N'_{n\mu} \rangle \\ &= \frac{1}{N} \sum_{n=1}^{N} n N_n \sum_{k=0}^{n-1} \frac{1}{\left[2n \sin\left(\frac{w-2\pi k}{2n}\right)\right]^2 \left[2n \sin\left(\frac{\bar{w}-2\pi k}{2n}\right)\right]^2} \,. \end{aligned}$$

Remove light crossing singularities: divide by vacuum 2pt function

$$\hat{G}(w, \bar{w}) = \frac{1}{N} \sum_{n=1}^{N} n N_n \sum_{k=0}^{n-1} \left(\frac{4 \sin\frac{w}{2} \sin\frac{\bar{w}}{2}}{2n \sin\left(\frac{w-2\pi k}{2n}\right) 2n \sin\left(\frac{\bar{w}-2\pi k}{2n}\right)} \right)^2$$

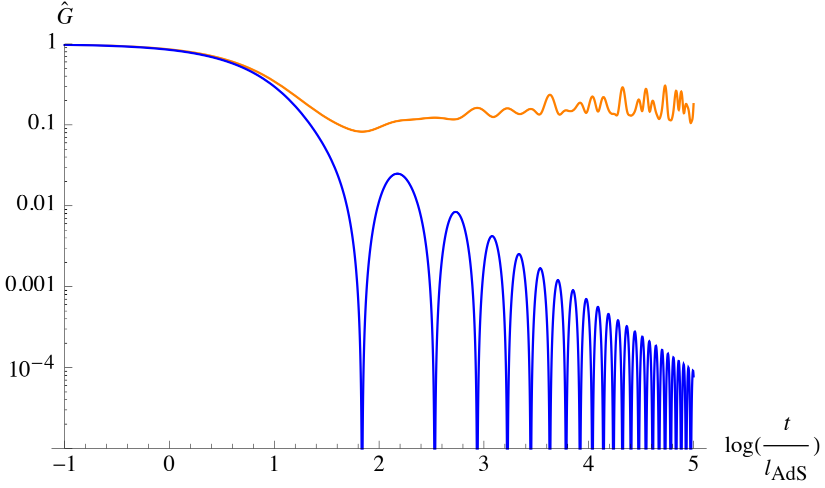
- Not protected quantity, turning on the coupling modifies it!
- Still, for times $t \lesssim \ell_{AdS}$ agrees with $M = 0$ BTZ result
- $M = 0$ implies $\sim t^{-2}$ decay instead of quasinormal ringdown

Two point function in D1D5



$N \approx 2000$

Two point function in D1D5 vs $M = 0$ BTZ

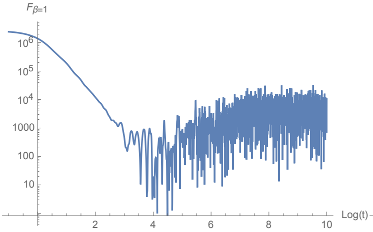


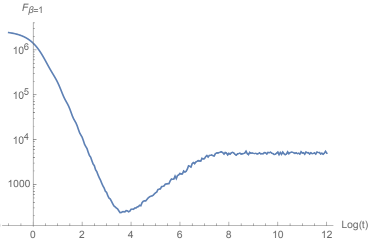
$N \approx 2000$

Late time sporadic behaviour

- To analyse the late time sporadic behaviour, we need smoothing
- Recall: spectral form factor

$$F_\beta(t) = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$





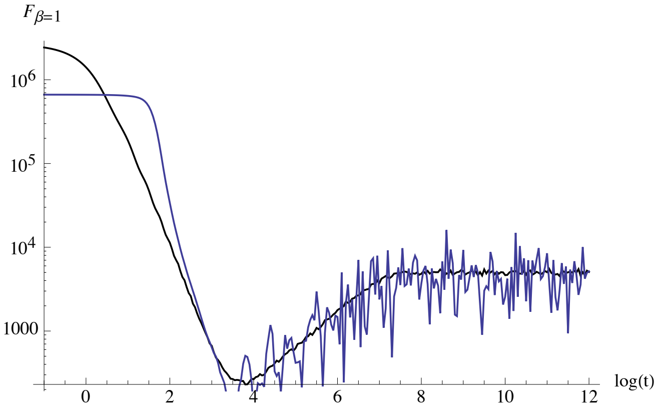
- RMT and SYK: smoothing by ensemble average
- In D1D5 (or other stringy models): no random variables

Try to smooth with a temporal kernel

$$\int dt' g_{\Delta t}(t,t') F_\beta(t')$$

Time average vs ensemble average in RMT

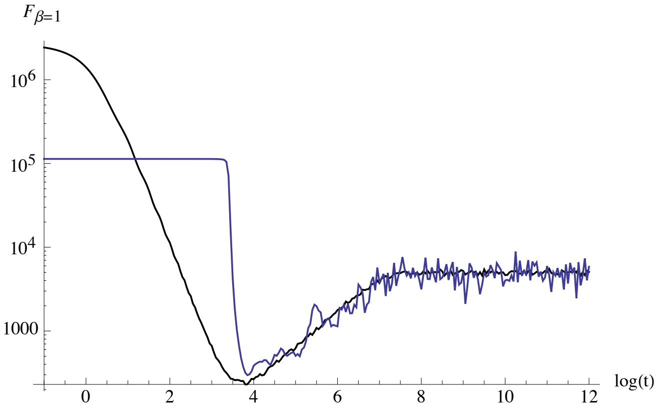
$$F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$



$$\Delta t = 10$$

Time average vs ensemble average in RMT

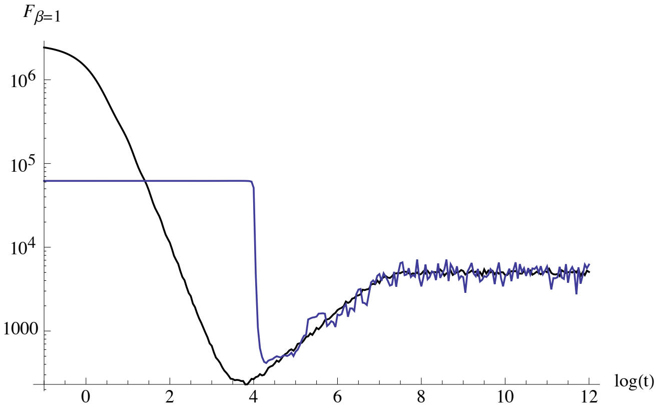
$$F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$



$$\Delta t = 60$$

Time average vs ensemble average in RMT

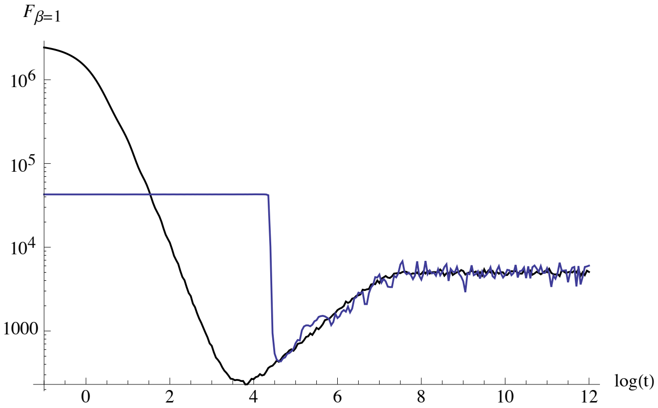
$$F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$



$$\Delta t = 110$$

Time average vs ensemble average in RMT

$$F_\beta(t) = \sum_{m,n=1}^{2^{10}} e^{-\beta(E_m+E_n)} e^{-i(E_m-E_n)t},$$

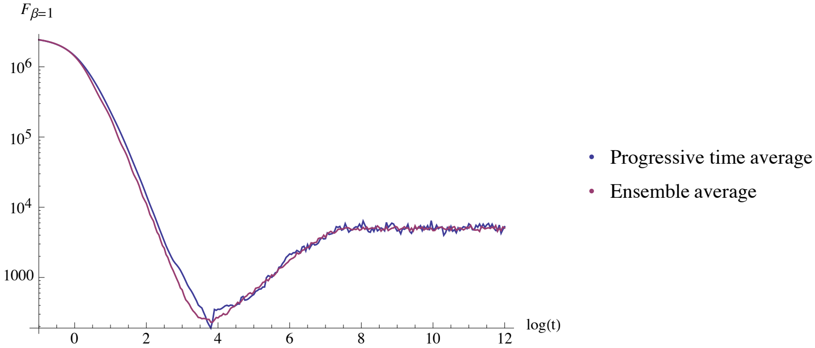


$$\Delta t = 160$$

Time average vs ensemble average in RMT

Progressive average:

$$\Delta t = at$$

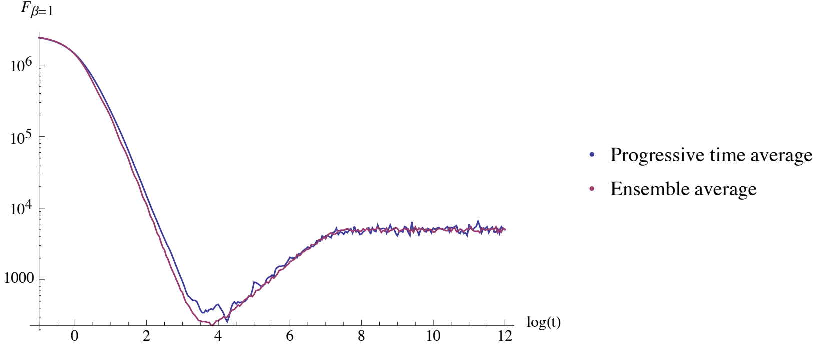


- No a priori knowledge of time scale is required
- Even spacing on log-log plots...

Time average vs ensemble average in RMT

Progressive average:

$$\Delta t = at$$

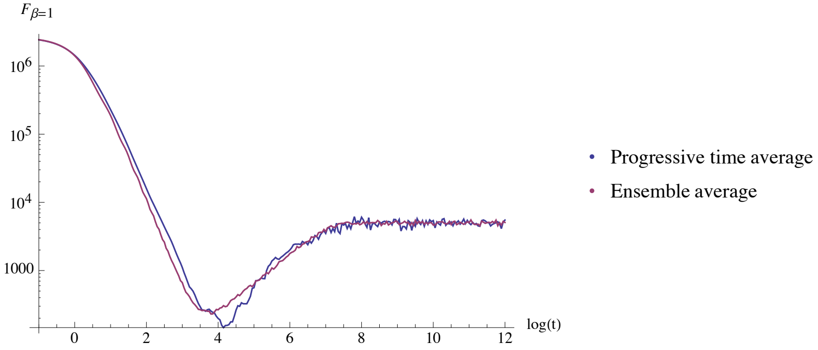


- No a priori knowledge of time scale is required
- Even spacing on log-log plots...

Time average vs ensemble average in RMT

Progressive average:

$$\Delta t = at$$

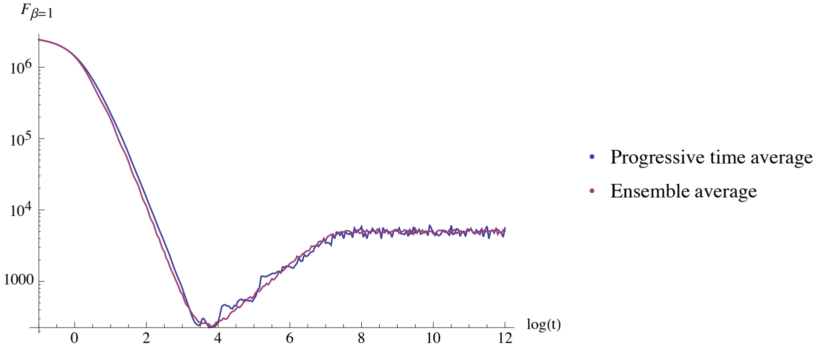


- No a priori knowledge of time scale is required
- Even spacing on log-log plots...

Time average vs ensemble average in RMT

Progressive average:

$$\Delta t = at$$



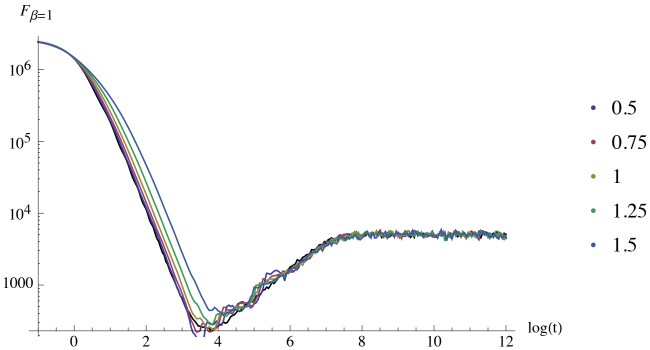
- No a priori knowledge of time scale is required
- Even spacing on log-log plots...

Time average vs ensemble average in RMT

Progressive average:

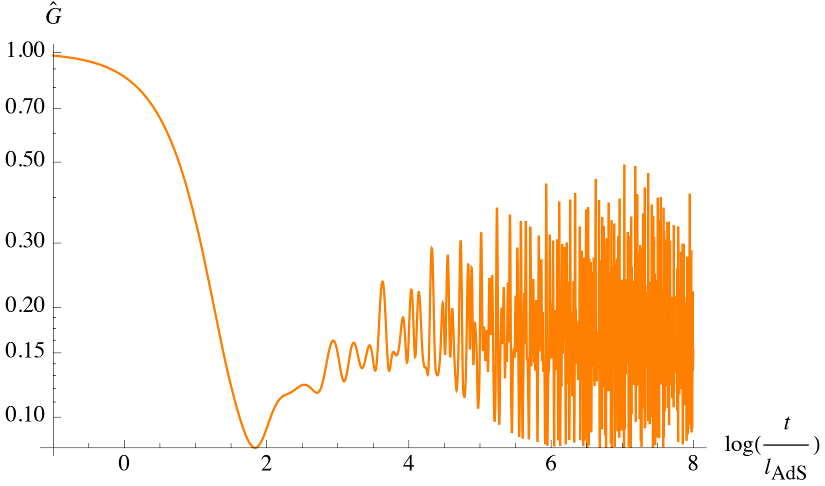
$$\Delta t = at$$

Dependence on a?

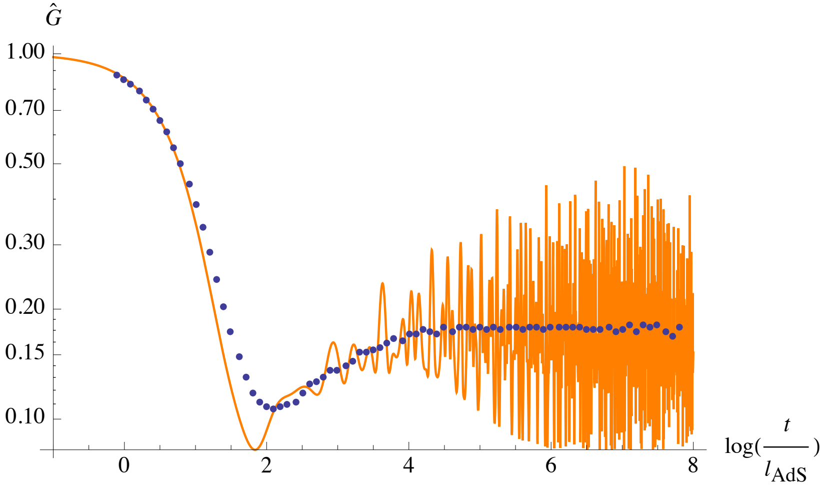


- Late times: does not matter at all (due to Gaussian nearest neighbour spacing)
- Around the dip: pick a such that deviation from decay part is small, while still good smoothing

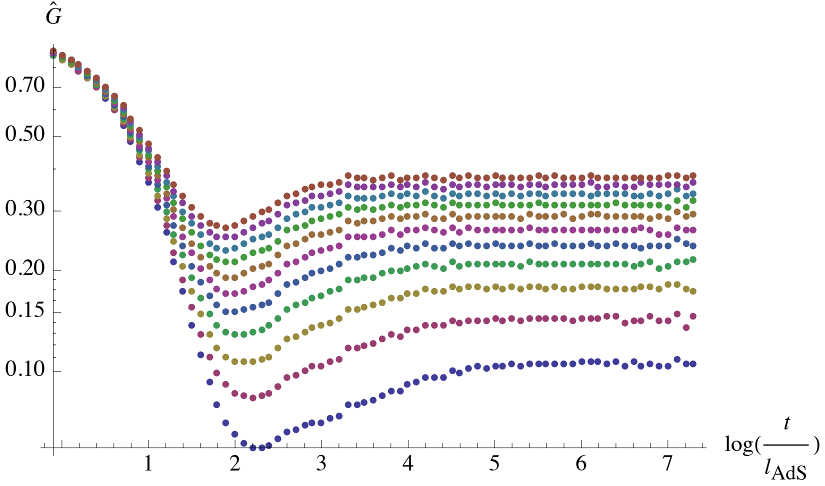
Two point function in D1D5



Two point function in D1D5

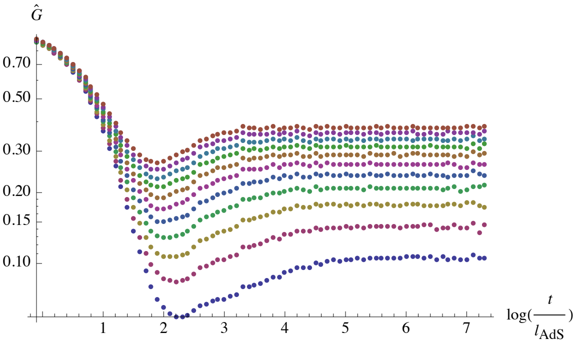


Two point function in D1D5



$$\eta = 0.05 + 0.025j, \quad j = 0, ..., 10$$

Two point function in D1D5



$$\eta = 0.05 + 0.025j, \quad j = 0, ..., 10$$

- Qualitative agreement with RMT and SYK
- What about the scales involved?

Derivation of plateau height

Rewrite the regularized two point function as double Fourier series, to facilitate its spectral decomposition (states propagating)

$$\hat{G}(-t,t) = \frac{1}{N}\sum_{n=1}^{N} nN_n \sum_{k=0}^{n-1} \frac{\sin^4 \frac{t}{2}}{n^4 \sin^2\left(\frac{t+2\pi k}{2n}\right)\sin^2\left(\frac{t-2\pi k}{2n}\right)}$$

Step 1 Notice the geometric sum

$$\begin{aligned}\frac{\sin\frac{t}{2}}{\sin\left(\frac{t-2\pi k}{2n}\right)} &= (-1)^k \frac{q^n - q^{-n}}{q - q^{-1}} = (-1)^k \sum_{\ell=0}^{n-1} q^{2\ell+1-n} \\ &= (-1)^k \sum_{\ell=0}^{n-1} e^{i\frac{t-2\pi k}{2n}(2\ell+1) - i\frac{t-2\pi k}{2}},\end{aligned}$$

with $q = \exp\left(i\frac{t-2\pi k}{2n}\right)$

Derivation of plateau height

Rewrite the regularized two point function as double Fourier series, to facilitate its spectral decomposition (states propagating)

$$\hat{G}(-t,t) = \frac{1}{N}\sum_{n=1}^{N} N_n \frac{1}{n^3} \sum_{\ell_1,\ell_2,\ell_1',\ell_2'=0}^{n-1} e^{i\frac{\ell_1+\ell_1'-\ell_2-\ell_2'}{n}t}\,\mathcal{G}_n(\ell_1+\ell_1'+\ell_2+\ell_2'+2)$$

$$\mathcal{G}_n(x) = \sum_{k=0}^{n-1} e^{2\pi i(2-\frac{x}{n})k} = \sum_{q\in\mathbb{Z}} n\delta_{qn,x}$$

Step 2 Exchange sums

$$\begin{aligned}\sum_{\ell_1=0}^{n-1}\sum_{m_1=\ell_1}^{\ell_1+n-1} &= \sum_{m_1=0}^{n-1}\sum_{\ell_1=0}^{m_1} + \sum_{m_1=n}^{2n-2}\sum_{\ell_1=m_1-n+1}^{n-1} \\ &= \sum_{m_1=0}^{n-1}(m_1+1) + \sum_{m_1=n}^{2n-2}(2n-1-m_1)\end{aligned}$$

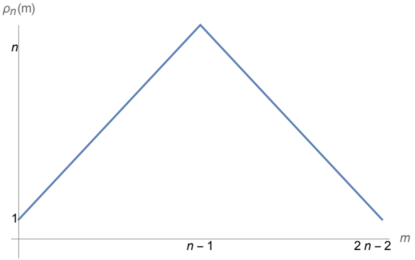
Derivation of plateau height

Rewrite the regularized two point function as double Fourier series, to facilitate its spectral decomposition (states propagating)

$$\hat{G}(-t,t) = \frac{1}{N}\sum_{n=1}^{N} N_n \frac{1}{n^3}\sum_{m_1=0}^{2n-2}\sum_{m_2=0}^{2n-2} \rho_n(m_1)\rho_n(m_2)\, e^{it\frac{(m_1-m_2)}{n}}\, \mathcal{G}_n(m_1+m_2+2)$$

$$\mathcal{G}_n(x) = \sum_{k=0}^{n-1} e^{2\pi i(2-\frac{x}{n})k} = \sum_{q\in\mathbb{Z}} n\delta_{qn,x}$$

Spectral weights follow 'triangle law'



Derivation of plateau height

Infinite time average

$$\bar{G} = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \hat{G}(-t,t) = \frac{1}{N} \sum_{n=1}^{N} N_n \frac{1}{n^3} \sum_{m=0}^{2n-2} \rho_n(m)^2 \, \mathcal{G}_n(2m+2)$$

The function $\mathcal{G}_n(2m+2)$ vanishes unless $2m+2$ is a multiple of n. This requires that

$$m = \frac{n}{2} - 1,\ n-1,\ \frac{3n}{2} - 1 \quad n \text{ even}$$

$$m = n - 1 \quad n \text{ odd}$$

Evaluating $\rho_n(m)$ at these values yields the plateau height

$$\bar{G} = \frac{1}{N} \left(\frac{3}{2} \sum_{n \text{ even}}^{N} N_n + \sum_{n \text{ odd}}^{N} N_n \right)$$

Derivation of plateau height

Large N scaling

Consider ‘typical state’ (grand canonical average)

$$N_n = \frac{8}{\sinh \eta n} \; ; \qquad N \approx \frac{2\pi^2}{\eta^2}. \tag{1}$$

$$\bar{G} \approx \frac{1}{N} \left(\sum_{s=1}^{\infty} \frac{8}{\sinh(\eta s)} + \frac{1}{2} \sum_{s=1}^{\infty} \frac{8}{\sinh(2\eta s)} \right), \tag{2}$$

When N is large, approximate with integrals:

$$\begin{aligned}
\bar{G} &\approx \frac{1}{N} \cdot \frac{8}{\eta} \int_{\delta\eta}^{\infty} \frac{du}{\sinh u} + \frac{1}{N} \cdot \frac{1}{2} \cdot \frac{8}{2\eta} \int_{2\delta\eta}^{\infty} \frac{du}{\sinh u} \\
&\approx \frac{5\eta}{\pi^2} \log\left(\frac{1}{\eta}\right) \\
&\sim \frac{\log S}{S},
\end{aligned}$$

Contrast to an ergodic system with discrete spectrum, where e^{-cS} is expected.

Estimate the ramp

$$\hat{G}(-t,t) = \frac{1}{N}\sum_{n=1}^{N} N_n \frac{1}{n^3} \sum_{m_1=0}^{2n-2}\sum_{m_2=0}^{2n-2} \rho_n(m_1)\rho_n(m_2)\, e^{it\frac{(m_1-m_2)}{n}}\, \mathcal{G}_n(m_1+m_2+2)$$

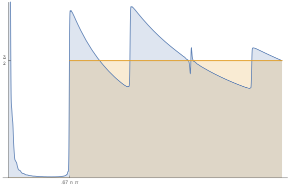
- Regularized two point function has the form

$$\hat{G}(-t,t) = \frac{1}{N}\sum_{n=1}^{N} N_n C_n(t)$$

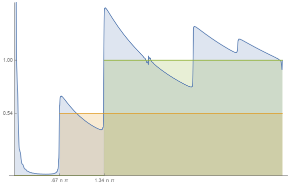
  with $C_n(t)$ containing frequencies $\frac{k}{n}$, $k \in \mathbb{Z}$.
- The dip and plateau time for $C_n(t)$ must be close at $t_p \sim \gamma n$ since there is a single time scale $\frac{1}{\mathrm{gap}} = n$ present.

Estimate the ramp

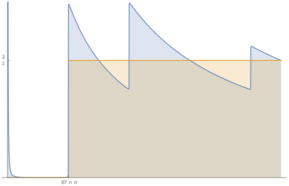
$$\int_{t/2}^{3t/2} C_n(t')dt'$$



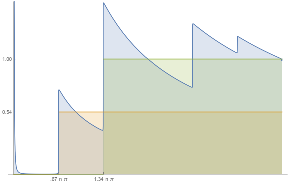
$n = 100$



$n = 101$



$n = 1500$

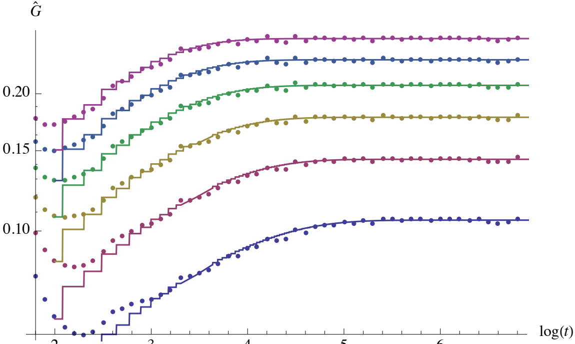


$n = 1501$

Estimate the ramp

Replace $\int_{t/2}^{3t/2} C_n(t')dt' \to C_{\mathrm{plateau}}\Theta(t-\gamma n)$, $\gamma \approx 2$

$$\int_{t/2}^{3t/2} \hat{G}(-t',t')dt' \approx \frac{1}{N}\sum_{n=1}^{t/\gamma} \frac{8}{\sinh(\eta n)} + \frac{1}{N}\sum_{n=1}^{t/(2\gamma)} \frac{1}{2}\frac{8}{\sinh(2\eta n)}$$
$$\approx \frac{5\sqrt{2}}{\pi\sqrt{N}} \log\left[\frac{\sqrt{N}}{\sqrt{2}\pi\delta} \tanh\left(\frac{t\pi}{\sqrt{2N}\gamma}\right)\right] + \frac{8\eta}{\sqrt{2N}\pi}\log 2$$



Two point function in D1D5

Qualitative behaviour is similar but scales naturally differ from RMT

$$\begin{array}{rcc} & \text{D1D5} & \text{RMT} \\ t_p \sim & S & e^{aS} \\ H_{\text{plateau}} \sim & \dfrac{\log S}{S} & e^{-bS} \end{array}$$

- Ramp is not linear but still seems to be parametrically long (persist for $N \to \infty$). Definition of the dip time is not straightforward.
- This is for a grand canonical ‘typical’ microstate. Variances among random superpositions of RR ground states are entropy surpressed ($e^{-a\sqrt{N}}$) [Balasubramanian,Czech,Hubeny,Larjo,Rangamani,Simon,’08]

Key questions

- Can we turn on the **marginal coupling** (at least perturbatively)? Intuition: the **plateau time** is set by the **smallest spacing** in the spectrum. At the orbifold point, there is no mixing between different long string sectors leading to a large spacing. Interactions introduce mixing, bringing down the spacing to e^{-S}. The corresponding plateau time is consistent with RMT. Do we get an RMT-curve at finite coupling?
- Examined case: zero coupling, zero temperature (but large entropy). Consistent with holographic regime of SYK $1 << \beta J << N$. Tractable question: what happens at zero coupling and finite temperature? Do we still have a dip-ramp-plateau? Or is it really an 'echo of chaos'?
- Bulk understanding of late time behaviour: orbifold $D1 - D5$ is dual to zero slope limit of string theory in the bulk, contains Vasiliev as a subsector [Gaberdiel,Gopakumar,2014]. Can we understand the ramp and plateau from a bulk higher spin perspective?

Questions?