Codimension 2 twist defects in Wilson-Fisher theories

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Based on SY, PTEP 091B01 (2016) [arXiv:1607.05551 [hep-th]]

Wilson-Fisher theory

CFT (conformal field theory)

D>2

recently developing rapidly

[Rattazzi, Rychkov, Tonni, Vichi 08], …

D=2 CFT with Boundary

(eg. [Cardy])

has rich structure and useful

(eg. D-brane [Polchinski])

Generalization

D=2

Boundary

D>2

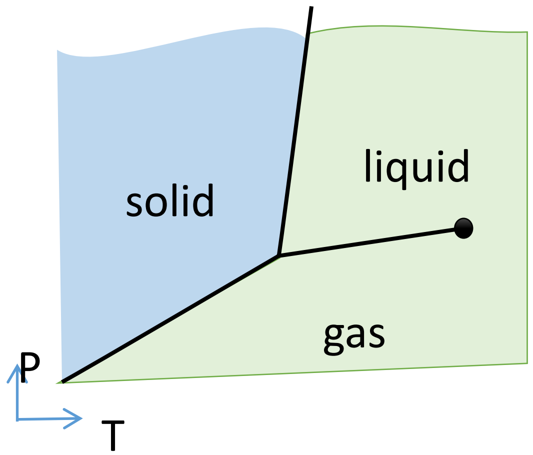
with newly developed technique

Defect

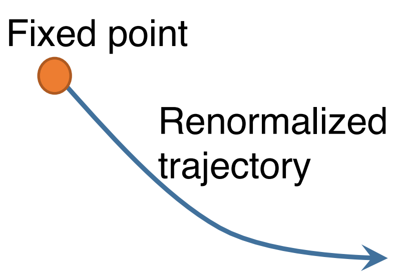
Today's talk

Motivation to study CFT

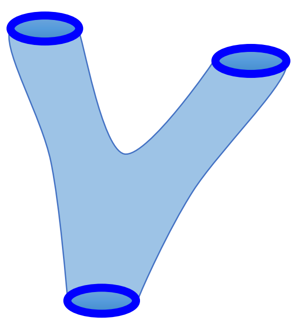
- Critical phenomena



- UV complete QFT



- Worldsheet of string theory



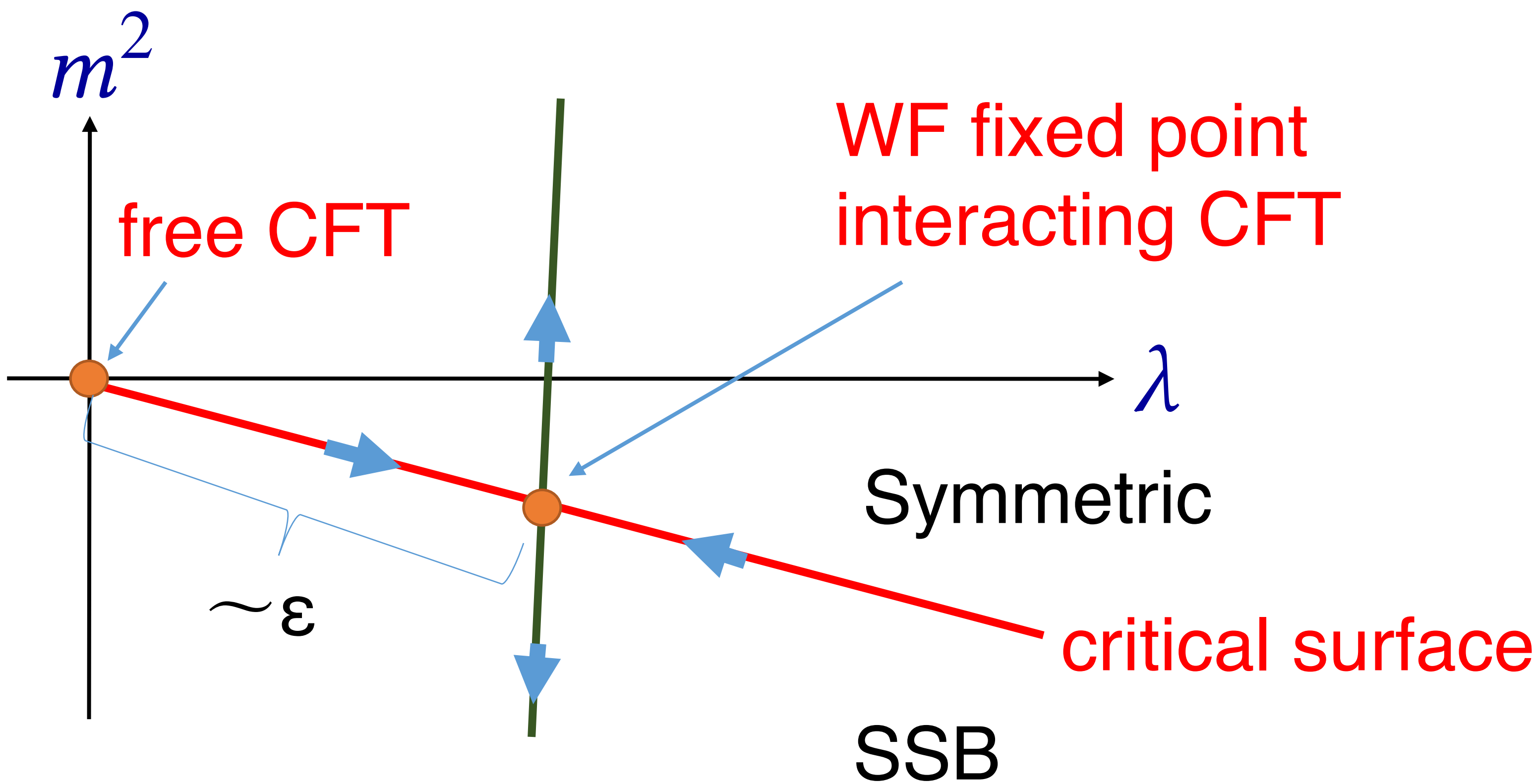
- AdS/CFT correspondence

Wilson-Fisher theory

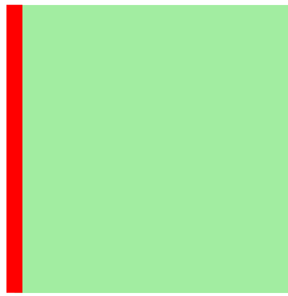
4-ε dim φ^4 theory [Wilson, Fisher]

$$S = \int d^d x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

(Euclidean)



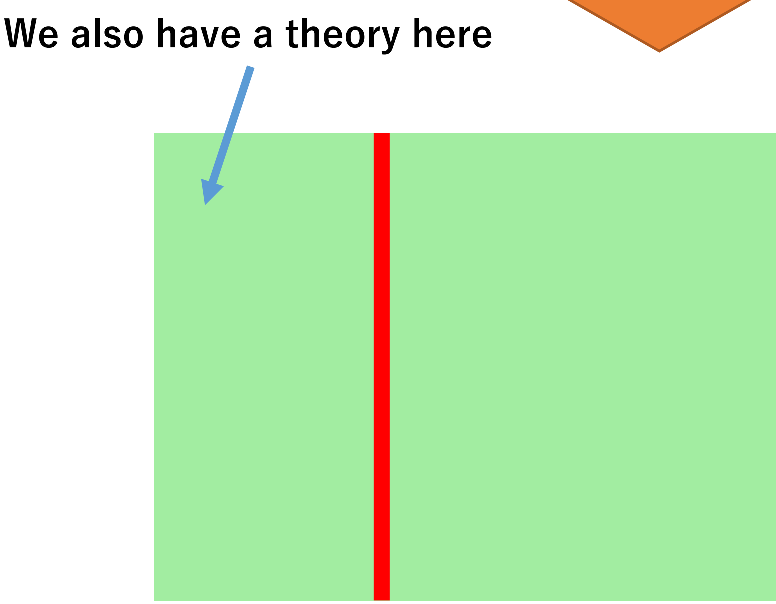
Defect: 1. generalization of boundary



2 dim Boundary CFT

(open string)

Generalization



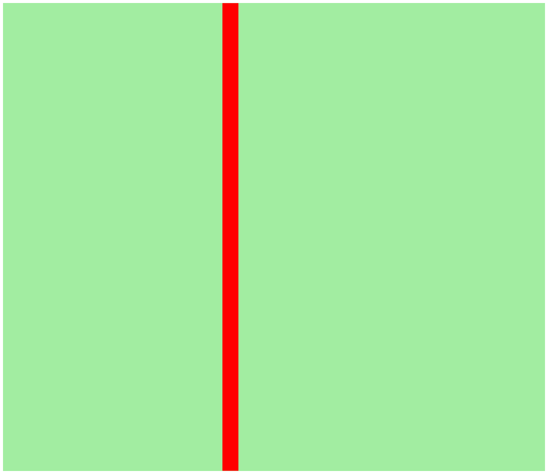
1 dim defect in 2 dim

generalization

D dim defect in d dim

Defect: 2. generalization of Wilson line

Wilson line: Introduce a test particle which couples to the gauge field electorically.



generalization

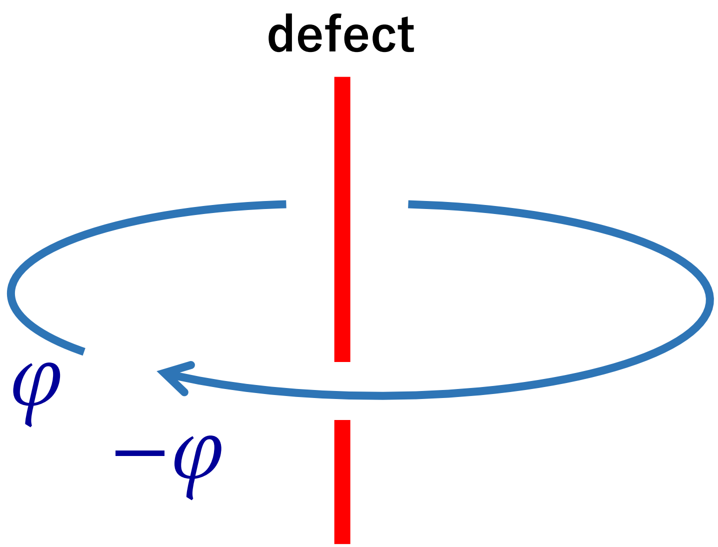
Introduce external test object

Example of defect: “Twist defect”

[Billo, Caselle, Gaiotto, Gliozzi, Meineri], [Gaiotto, Mazac, Paulos]

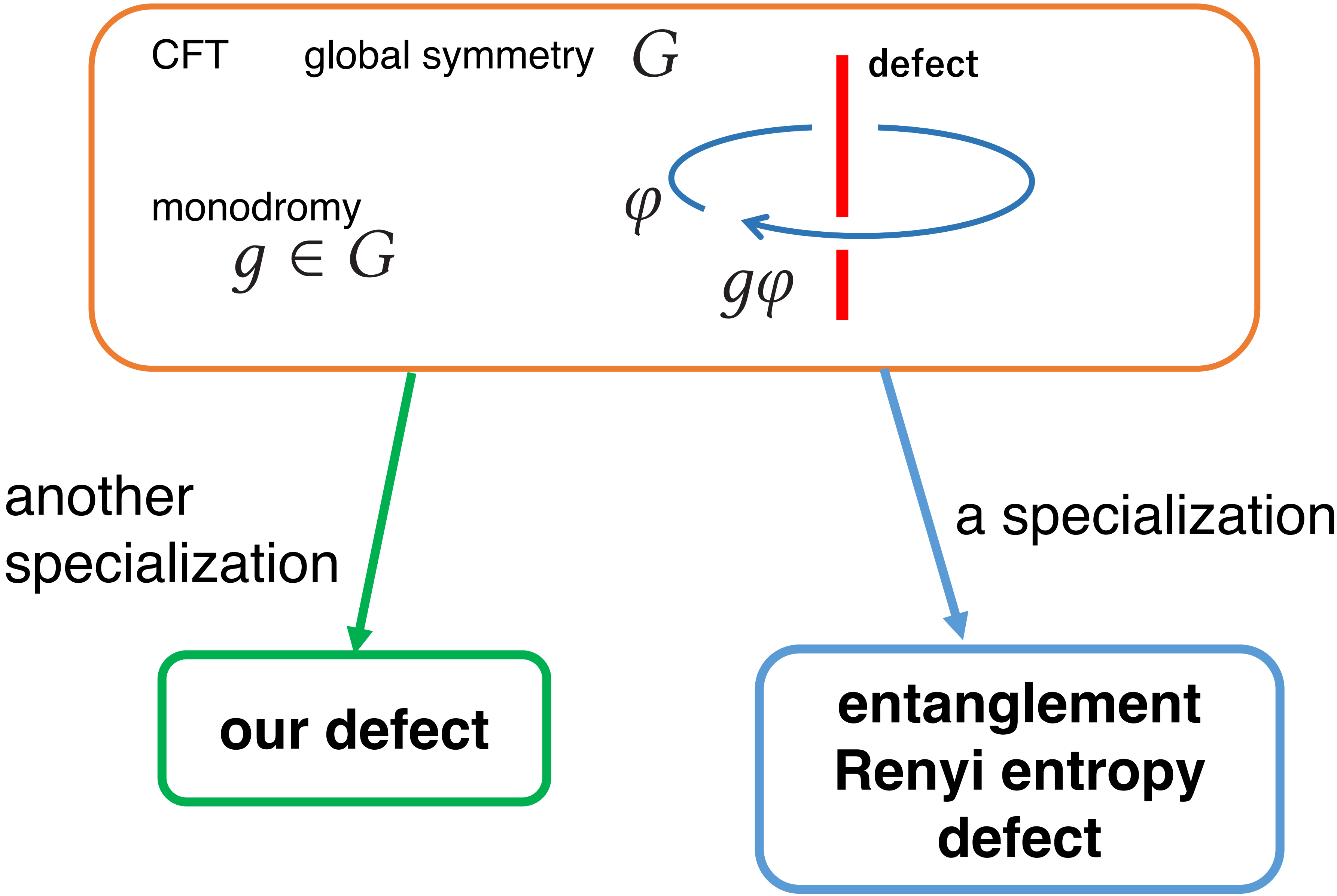
codimension 2, monodromy

Eg. : 3dim Ising
spin operator φ



cf 2 dim orbifold CFT
vertex operators in
the twisted sector.

Relation to entanglement Renyi entropy defect

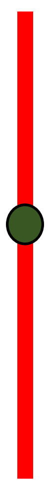


Summary of the result

Analogue of **spectrum of open string**

4- ε dim O(N) model Wilson-Fisher(WF) fixed point (CFT)

Twist defect



local operators on the defect

ψ_s

Obtained the scaling dimensions in Rychkov-Tan' s framework.

Plan

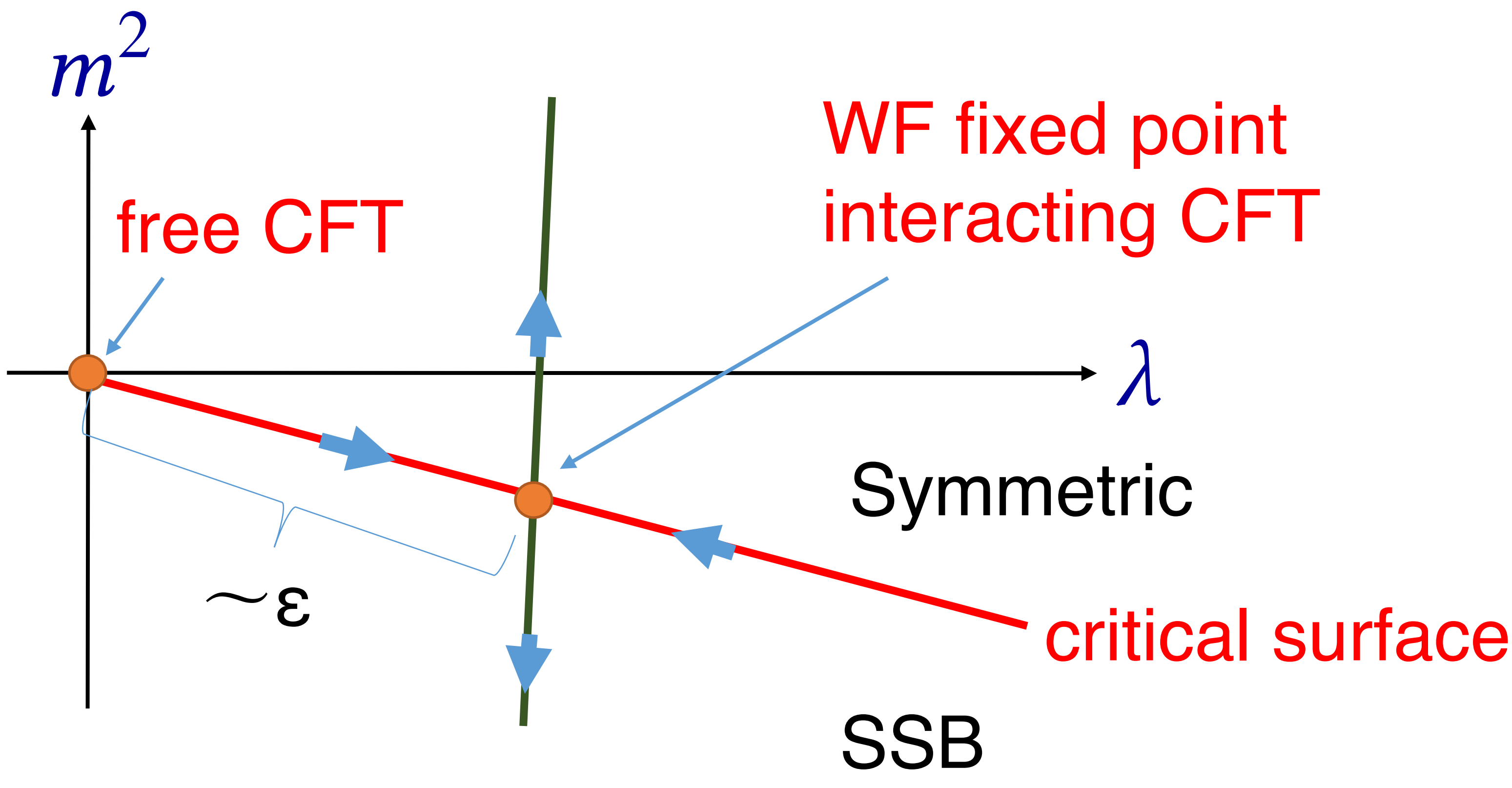
- Review of Rychkov-Tan
- Twist defect
- Discussion

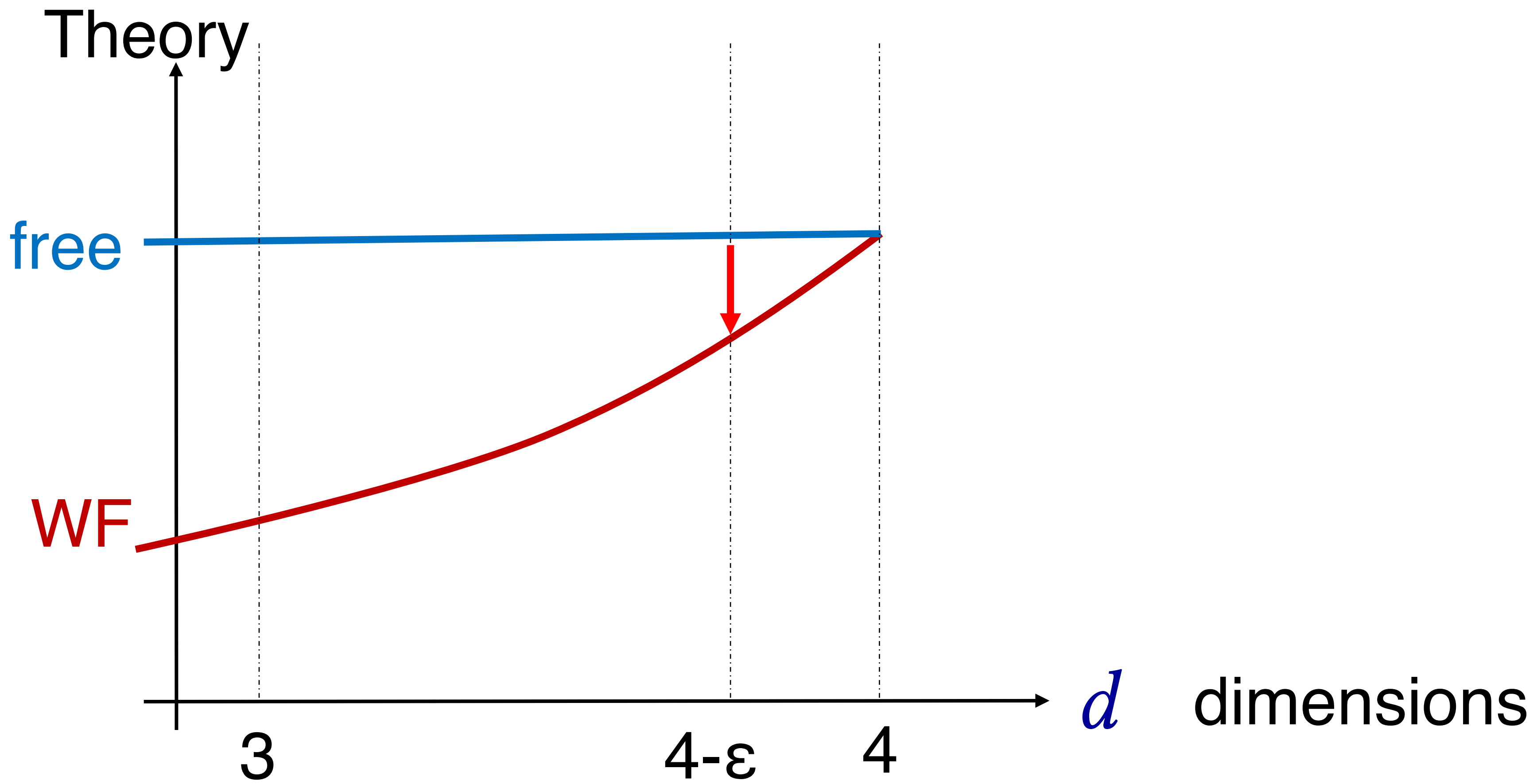
ε-expansion by Rychkov-Tan

4-ε dim φ^4 theory [Wilson, Fisher]

$$S = \int d^d x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

(Euclidean)

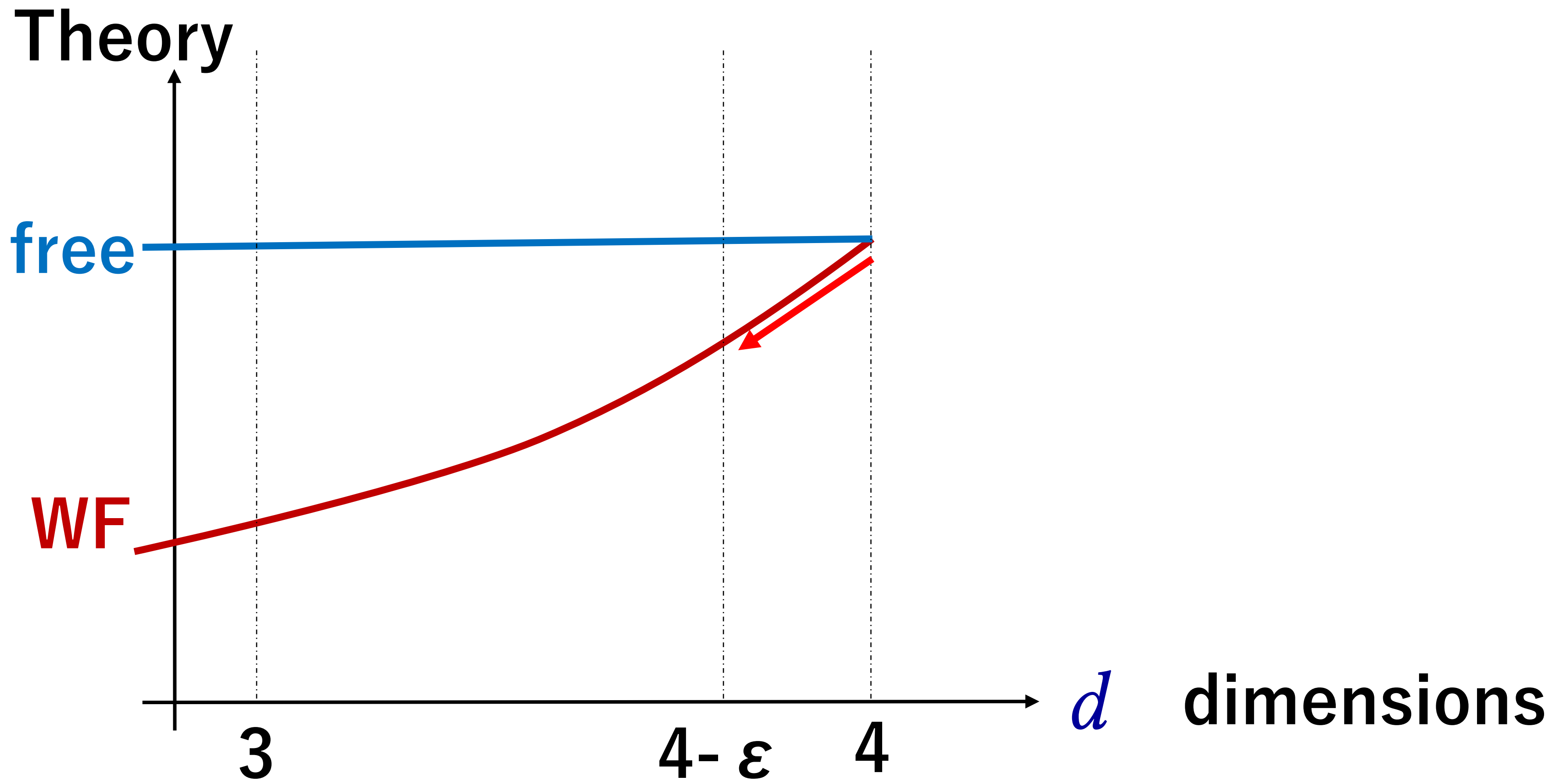




In 4-εdim, WF is close to free

Perturbation is good

Old ε-expansion [Wilson, Fisher]



RT: Do not want to use Lagrangian as far as possible.

Conformal symmetry + α

What you should take

Strategy:

Put a few axioms and find the consequence of them.

We will use conformal symmetry

Axiom I:
Theory at WF fixed point is CFT

$+\alpha$ characterize WF CFT

Axiom II

In $\varepsilon \to 0$

$4 - \varepsilon$ dim WF CFT $\to$ free

Free CFT also satisfy this axiom.
We need more characterization.

notation $V_n(x)$ **local operator in WF**

In $\varepsilon \to 0$ $V_n(x) \to \varphi^n(x)$

（They exists from Axiom II）

To distinguish WF theory from free theory

Axiom III a constant α

$$\Box V_1 = \alpha V_3$$

Idea

Determined by $\Delta_{n+1}, \Delta_n, \Delta_1$

$$\frac{1}{\alpha}\Box V_1(0)$$

$$V_{n+1}(x)V_n(0) = \cdots + \bullet\left(V_1(0) + \bullet\, V_3(0) + \cdots\right) + \cdots$$

Compare

⇩

Relation between

$\Delta_{n+1}, \Delta_n, \Delta_1$

free in $\varepsilon \to 0$

$$\varphi^{n+1}(x)\varphi^n(0) = \cdots + \bullet\left(\varphi(0) + \bullet\, \varphi^3(0) + \cdots\right) + \cdots$$

Calculated by Wick's theorem

Results Δ_n scaling dimension of V_n

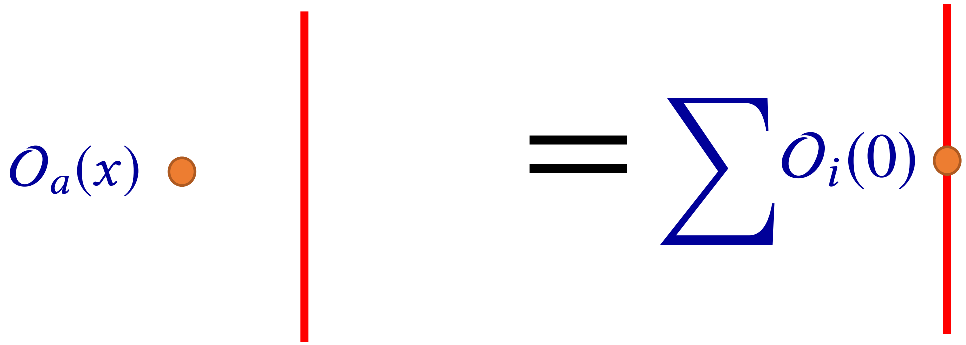
$$\Delta_1 = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} + O(\epsilon^3)$$

$$\Delta_n = n - n\frac{\epsilon}{2} + \frac{1}{6}n(n-1)\epsilon + O(\epsilon^2), \quad n = 2, 3, 4, \ldots$$

Twist defect

bulk-defect OPE

[Cardy], [McAvity, Osborn]



$$O_a(x) = \sum_i C_{ai}(x) O_i(0)$$

Local operators on the defect

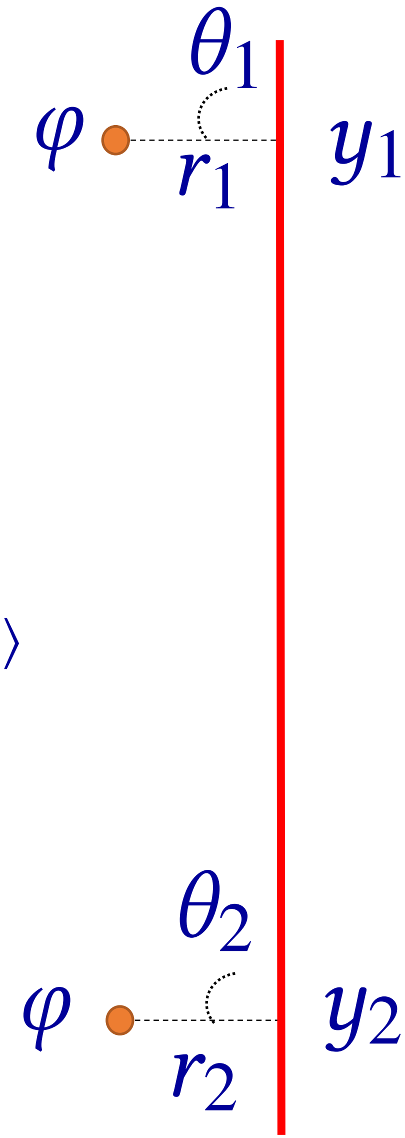
Bulk 2 point function includes some information on the operators on the defect.

codim 2 defect

$$\varphi(x_1) = \sum_i \frac{e^{is_i\theta_1}}{r_1^{\Delta_\varphi-\Delta_i}} O_i(y_1)$$

$$\langle \varphi(x_1)\varphi(x_2)\rangle = \sum_{i,j} C_{\varphi i}\frac{e^{is_i\theta_1}}{r_1^{\Delta_\varphi-\Delta_i}} C_{\varphi j}\frac{e^{is_j\theta_2}}{r_2^{\Delta_\varphi-\Delta_j}}\langle O_i(y_1)O_j(y_2)\rangle$$

$$= \sum_i |C_{\varphi i}|^2 \frac{e^{is_i(\theta_1-\theta_2)}}{r_1^{\Delta_\varphi-\Delta_i} r_2^{\Delta_\varphi-\Delta_j}}\left(\frac{1}{|y_1-y_2|^{2\Delta_i}} + (\text{descendants})\right)$$



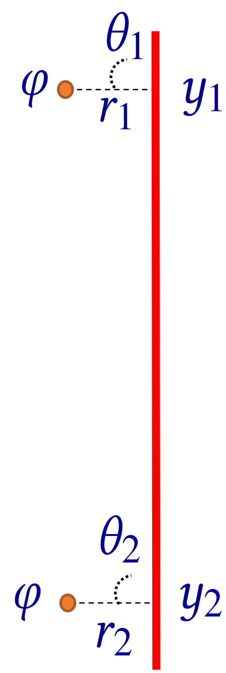
4 dim free theory [Gaiotto, Mazac, Paulos]

Calculate

$$\langle \varphi(x_1)\varphi(x_2)\rangle_{\text{defect}}$$

cf bulk-to-bulk propagator in AdS

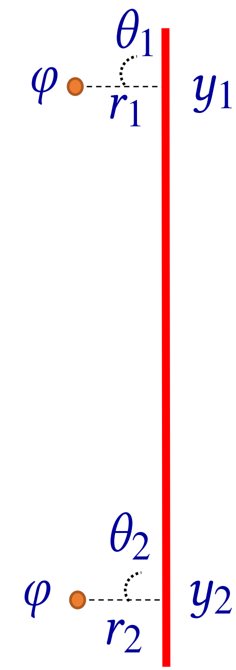
$$\langle \varphi(x_1)\varphi(x_2)\rangle_{\text{defect}} = \sum_{s\in\mathbb{Z}+1/2} G_0(x_1,x_2,s)$$



$$G_0(x_1,x_2,s) = \frac{e^{is(\theta_1-\theta_2)}}{4r_1r_2}\,\frac{\xi^{-\frac{1}{2}}}{\sqrt{1+\xi}\,(\sqrt{\xi}+\sqrt{1+\xi})^{2|s|}}$$

$$\xi = \frac{(y_1-y_2)^2+(r_1-r_2)^2}{4r_1r_2}$$

Read the information of local operators on the defect



$$|y_1 - y_2| \to \infty \qquad (\xi \to \infty)$$

$$G_0(x_1, x_2, s) \to \frac{e^{is(\theta_1 - \theta_2)}}{(r_1 r_2)^{-|s|}} \frac{1}{|y_1 - y_2|^{2(|s|+1)}}$$

local operators on the defect

$\psi_s(y) \quad s \in \mathbb{Z} + 1/2$ exist

scaling dimensions $|s| + 1$

bulk-defect OPE $\varphi(x) = \cdots + \frac{e^{is\theta}}{r^{-|s|}} \psi_s(0) + \cdots$

local operators on the defect

$\psi_s(y)$ $s \in \mathbb{Z} + 1/2$ exist

scaling dimensions $|s| + 1$

How about WF CFT ?

Employ Rychkov-Tan's framework.

4 dim free CFT bulk-defect OPE

$$\varphi^3(x) = \cdots - \frac{3}{8}\frac{e^{is\theta}}{r^{2-|s|}}\psi_s(0) + \cdots$$

4- ε dim WF theoy, bulk-defect OPE

$$V_1(x) = \cdots + C_{1s}\frac{e^{is\theta}}{r^{\Delta_1-\Delta_s}}\psi_s(0) + \cdots$$

$$\begin{aligned} V_3(x) =& \frac{1}{\alpha}\Box V_1(x) \\ =& \cdots + \frac{1}{\alpha}C_{1s}\Box\frac{e^{is\theta}}{r^{\Delta_1-\Delta_s}}\psi_s(0) + \cdots\ , \\ =& \cdots + \frac{-s^2+(\Delta_1-\Delta_s)^2}{\alpha}C_{1s}\frac{e^{is\theta}}{r^{\Delta_1+2-\Delta_s}}\psi_s(0) + \cdots\ . \end{aligned}$$

Compare this in ε →0 with free CFT

Δ_s

Result

$$\Delta_s = |s| + 1 + \left(-\frac{1}{2} - \frac{1}{24|s|}\right)\epsilon + O(\epsilon^2)$$

※Agree with Feynman diagrammatic calculation

[Gaiotto, Mazac, Paulos]

O(N) model

$$\Delta_s = |s| + 1 + \left(-\frac{1}{2} - \frac{N+2}{8(N+8)|s|} \right) \epsilon + O(\epsilon^2)$$

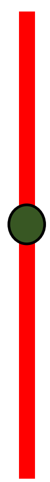
※**Agree with Feynman diagrammatic calculation**

[SY, work in progress]

Summary of the result

4- ε dim O(N) model Wilson-Fisher(WF) fixed point (CFT)

Twist defect



local operators on the defect

ψ_s

Obtained the scaling dimensions in Rychkov-Tan’ s framework.

Prospects: validity of various methods

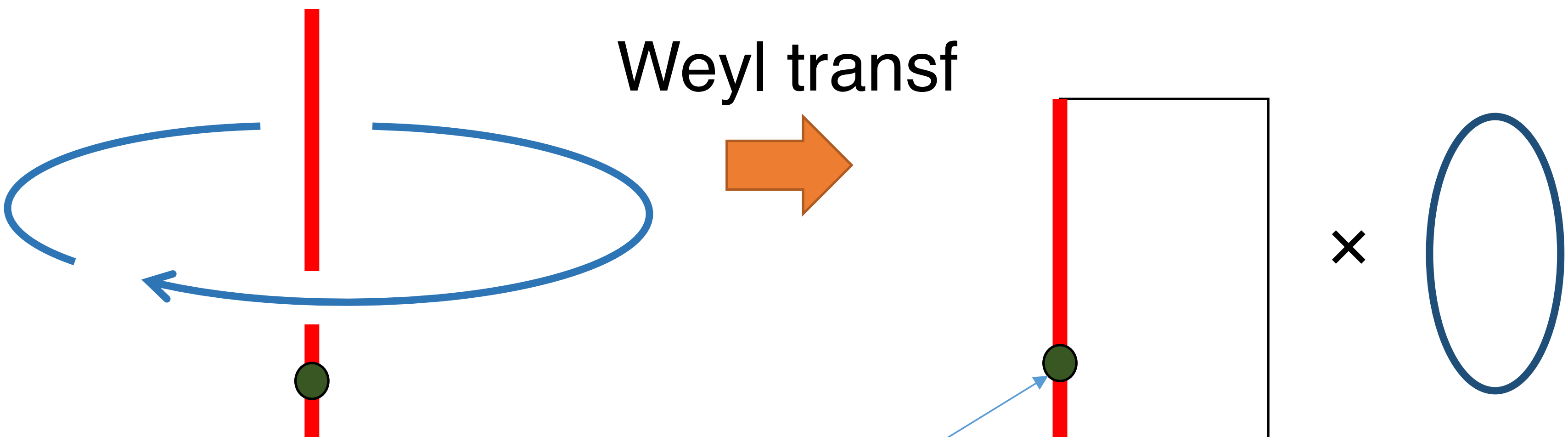
Study O(N) model by

- **Large N**
- **Numerical bootstrap**
- **Monte Carlo**
- **Large s ?**
- **Experiments ?**

Is the defect CFT a miniature of AdS/CFT?

flat space + defect

$$AdS_{d-1} \times S^1$$



Local operator on the defect ψ_s

φ_s Each KK mode

$$\Delta_s = \frac{d-2}{2} + \sqrt{\frac{d-2}{2} + m_s^2}$$

In the large N limit

Same as AdS/CFT correspondence