駒場集中講義2020年後半

⑧ AdS/BCFT [境界を持つCFTへのAdS/CFTの拡張]

Based on
高柳 arXiv:1105.5165 [PRL 107(2011)101602]
藤田-Tonni-高柳 arXiv:1108.5152 [JHEP HEP 11 (2011) 043]
Akal-楠亀-魏-高柳 [arXvi:2007.06800]

内容

[8-1] BCFT

For special choice of boundary conditions, a part of conformal symmetries are preserved. This is called the boundary conformal field theory (BCFT). [Cardy 1984, .., McAvity-Osborn 1995, ….]

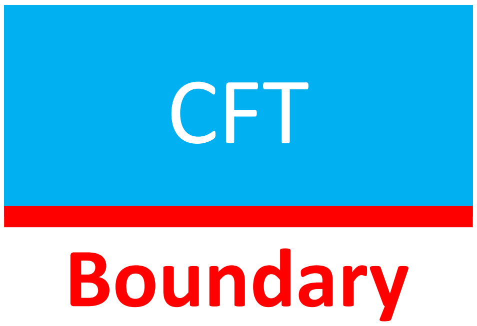
CFTd+1: SO(2,d+1)

U

BCFTd : SO(2,d)

ex. d+1=2 → Boundary State

$$(L_n - \tilde{L}_{-n})|B\rangle = 0$$

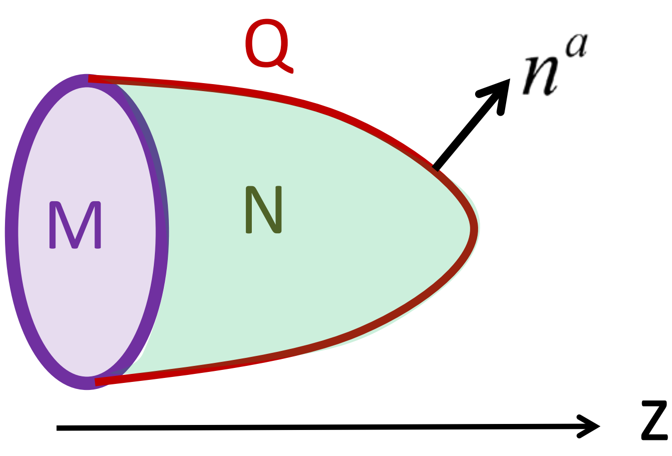


[8-2] Construction of AdS/BCFT

Boundary Condition

A gravity dual of a CFT on a manifold M with a boundary ∂M ?

➡ Generalizing the AdS/CFT, we argue that it is a gravity on an asymptoticaly AdS spacetime N such that ∂N=M∪Q .



The gravity action in Euclidean signature looks like

Gibbons
-Hawking term

$$I = \frac{1}{16\pi G_N}\int_N \sqrt{-g}(R - 2\Lambda + L_{matter}) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}(K + L^Q_{matter}).$$

Bulk matter fields

Matter fields
localized on Q

The coordinate of Q and its induced metric are x^a and h^{ab}.

We define the extrinsic curvature and its trace

$$K_{ab} = \nabla_a n_b, \quad K = h^{ab} K_{ab} \ .$$ (n^a is a unit vector normal to Q.)

e.g. Gaussian normal coordinate: $ds^2 = d\rho^2 + h_{ab}(\rho, x)dx^a dx^b$

$$\Rightarrow \quad K_{ab} = \frac{1}{2}\partial_\rho h_{ab}(\rho, x).$$

Variation: $$\delta I = \frac{1}{16\pi G_N}\int_Q \sqrt{-h}(K_{ab} - K h_{ab} - T^Q_{ab})\, \delta h^{ab}.$$

At the AdS boundary M, we impose the Dirichlet boundary condition $\delta h^{ab} = 0$ following the standard AdS/CFT argument.

On the other hand, at the new boundary Q, we argue to require the Neumann b.c. :

$$\boxed{K_{ab} - K h_{ab} - T^Q_{ab} = 0}$$ `boundary Einstein eq.'

Why Neumann b.c. ?

(1) Keep the boundary dynamical. New data at Q should not be required.

(2) Orientifolds in string theory leads to this condition.

[cf. Randall-Sundrum models]

A Basic Example of AdSd+2/BCFTd+1

To preserve the boundary conformal symmetry, we should have

$$T^{Q}_{ab} \propto h_{ab} \quad \Rightarrow \quad T^{Q}_{ab} = -T\, h_{ab}$$ (T is the tension of Q).

The boundary Einstein eq. looks like $K_{ab} = (K - T)\, h_{ab}$.

In this case, the action takes the following simple form

$$I = \frac{1}{16\pi G_N}\int_N \sqrt{-g}\,(R - 2\Lambda) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}\,(K - T).$$

Now, note that the SO(2,d+1) symmetry of BCFT is the same as that of AdSd+1. Thusthe gravity dual of CFT on a half plane is given by

$$x < 0$$

$$ds^2_{(d+2)} = d\rho^2 + \cosh^2\left(\frac{\rho}{R}\right) ds^2_{AdS(d+1)}.$$

If we assume the range $-\infty < \rho < \infty$, then the metric coincides with that of the pure AdSd+2. If we express the AdSd+1 by

$$ds^2_{AdS(d)} = R^2\left(\frac{-dt^2 + dy^2 + d\vec{w}^2}{y^2}\right),$$

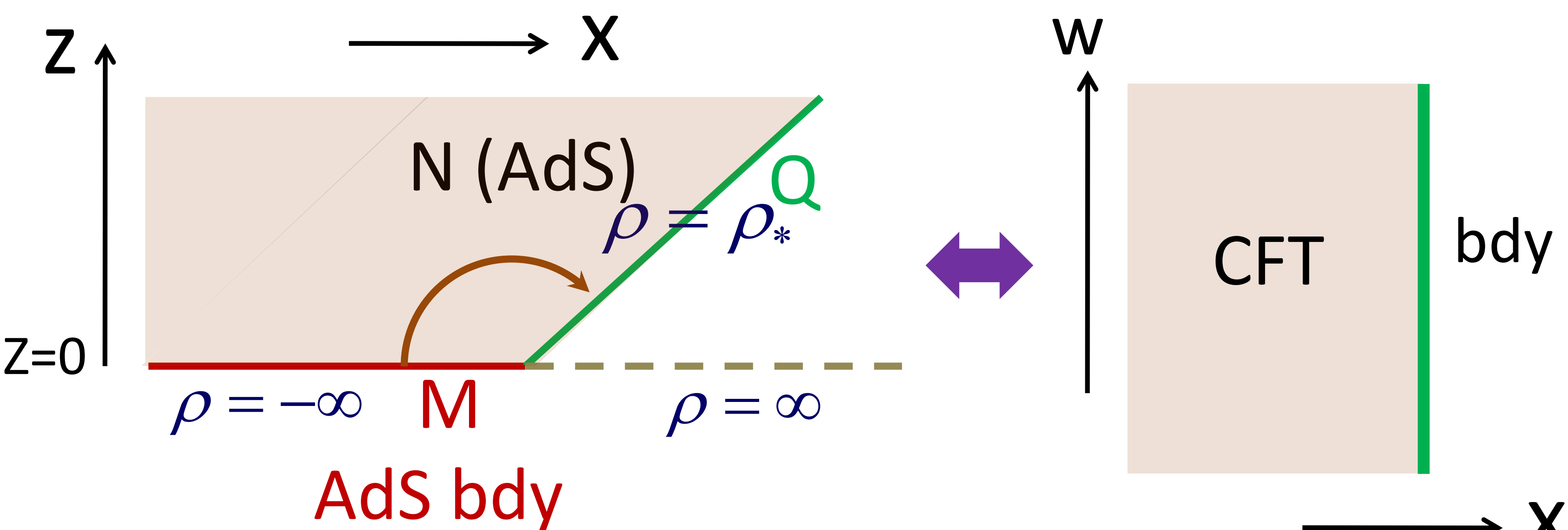
and define $z = y/\cosh(\rho/R), \quad x = y\tanh(\rho/R),$
then we indeed reproduce AdSd+2 metric

$$ds^2 = R^2\left(\frac{-dt^2 + dz^2 + dx^2 + d\vec{w}^2}{z^2}\right),$$

To construct the gravity dual of BCFT, we specify the boundary Q by $\rho = \rho_*$ so that the spacetime N is given by $-\infty < \rho < \rho_*$.

In this metric, we find $K_{ab} = \frac{1}{R}\tanh\left(\frac{\rho_*}{R}\right)h_{ab}$,

and thus the tension of Q is given by $T = \frac{d}{R}\tanh\frac{\rho_*}{R}$.



[8-3] AdS3/BCFT2 and Boundary Entropy

Boundary Entropy

The simplest case of AdS/BCFT is d=2. The BCFT2 has already been studied in detail from the field theory side.

There are several types of boundary conditions with the boundary conformal invariance for a given CFT, labeled by α.

We write the corresponding boundary state as $\left|B_\alpha\right\rangle$.

$$\left\langle O_1 O_2 \cdots O_n \right\rangle_{Disk(\alpha)} = \left\langle 0 \middle| O_1 O_2 \cdots O_n \middle| B_\alpha \right\rangle$$

An interesting quantity in BCFT2 is called the boundary entropy introduced by Affleck and Ludwig in 1991. This quantity measures the degrees of freedom at the boundary.

They conjectured that this quantity decreases under the RG flow like the central charge c in CFT2. This has been proved by Friedan and Konechny in 2004 and is called g-theorem.

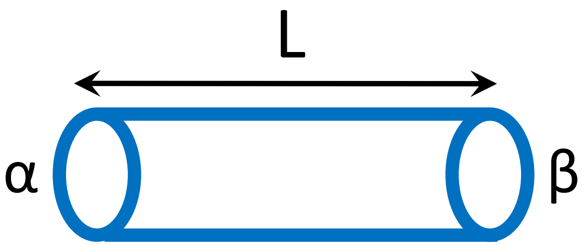
Definition 1 (Disk Amplitude)

It is simply defined from the disk amplitude

$$S_{bdy(\alpha)} = \log g_\alpha \, , \qquad g_\alpha \equiv \langle 0 | B_\alpha \rangle .$$

Definition 2 (Cylinder Amplitude)

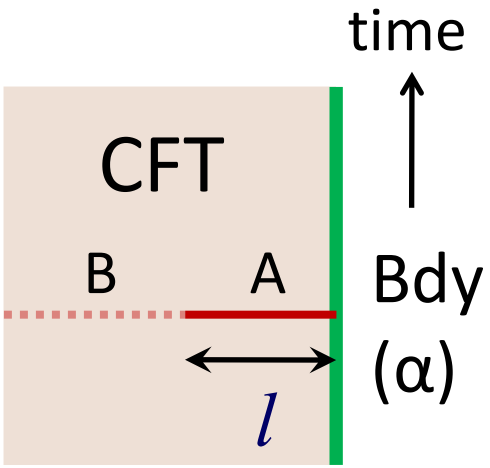
$$Z^{cylinder}_{(\alpha,\beta)} = \langle B_\alpha | e^{-HL} | B_\beta \rangle \underset{L\to\infty}{\approx} \underbrace{g_\alpha g_\beta}_{\text{Boundary Part}} \underbrace{e^{-E_0 L}}_{\text{Bulk Part}} .$$



Definition 3 (Entanglement Entropy)

$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A] ,$$

$$\rho_A = \mathrm{Tr}_B \rho_{tot} \quad .$$



In 2D BCFT, the EE generally behaves like

$$S_A = \underbrace{\frac{c}{6}\log\frac{l}{\varepsilon}}_{\text{Bulk Part}} + \underbrace{\log g_\alpha}_{\text{Boundary Part}} \quad .$$

[Calabrese-Cardy 2004]

Holographic Disk Partition Function(Def.1)

Previously, we found the gravity dual of BCFT on a half space.

We can map this by the AdS conformal transformation:

$$x_\mu \to \frac{x_\mu + c x^2}{1+2cx+c^2x^2}, \quad z \to \frac{z}{1+2cx+c^2x^2},$$ [Berenstein-Corrado-Fishler - Maldacena 1999]

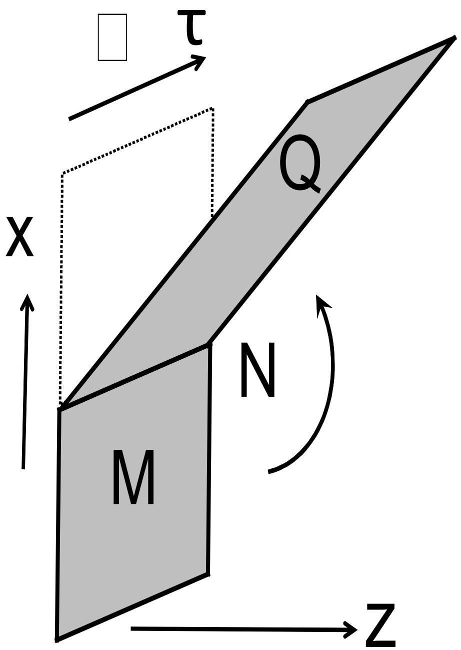
w.r.t the Poincare coordinate: $ds^2 = R^2 \frac{dz^2 + dx^\mu dx_\mu}{z^2}.$

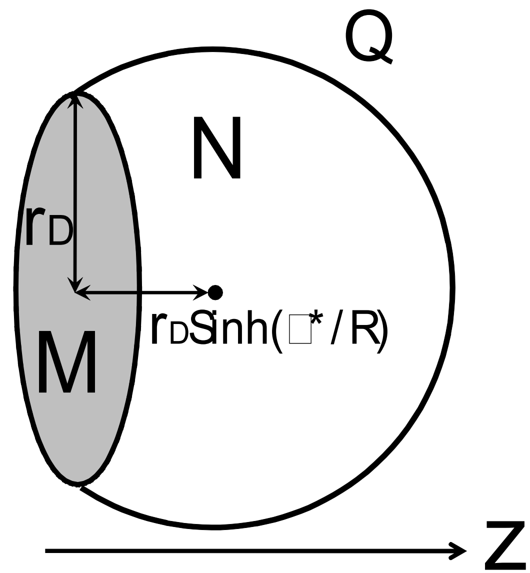
This maps the spacetime N as follows (in Euclidean signature)

$$\frac{x}{z} = \sinh\frac{\rho_*}{R} \Rightarrow \tau^2 + x^2 + \left(z - r_D \sinh\frac{\rho_*}{R}\right)^2 = \left(r_D \cosh\frac{\rho_*}{R}\right)^2$$

2 dim. Plane

A part of round sphere (2 dim. disk)





$$\frac{x}{z} = \sinh\frac{\rho_*}{R}$$

$$\tau^2 + x^2 + \left(z - r_D \sinh\frac{\rho_*}{R}\right)^2 = \left(r_D \cosh\frac{\rho_*}{R}\right)^2$$

By evaluating the Euclidean action,

$$I = -\frac{1}{16\pi G_N}\int_N \sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G_N}\int_Q \sqrt{h}(K-T),$$

we obtain the holographic disk partition function

$$I_{Disk} = \frac{R}{4G_N}\left(\frac{r_D^2}{2\varepsilon^2} + \frac{r_D \sinh(\rho_* / R)}{\varepsilon} + \log\frac{\varepsilon}{r_D} - \frac{\rho_*}{R} - \frac{1}{2}\right).$$

After a holographic renormalization, we finally find

$$S_{bdy} = -I_{Disk}^{ren} = \frac{\rho_*}{4G_N} = \frac{c}{6}\mathrm{Arctanh}(RT).$$

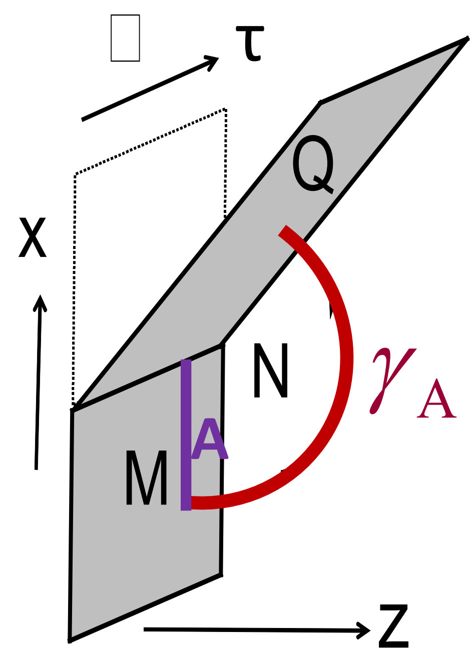
Holographic EE (Def.2)

New Aspect in AdS/BCFT: Minimal Surfaces can end on Q !

In our setup of AdS3/BCFT2, the holographic EE is obtained as

$$S_A = \frac{\text{Length}}{4G_N} = \frac{1}{4G_N}\int_{-\rho_\infty}^{\rho_*} d\rho$$

$$= \frac{\rho_\infty + \rho_*}{4G_N} = \frac{c}{6}\log\frac{l}{\varepsilon} + \frac{\rho_*}{4G_N} \quad .$$



Thus we reproduced the same boundary entropy:

$$S_{bdy} = \frac{\rho_*}{4G_N}.$$

Holographic Dual of Intervals (Def.3)

At a finite temperature T_{BCFT}, there are two solutions (thermal AdS and AdS BH). The interval is defined by $0 < x < \pi z_0 (\equiv L)$ and the Euclidean time is compactified as $\tau \sim \tau + 2\pi z_H$,

$$T_{BCFT} = (2\pi z_H)^{-1}.$$

Low Temperature Phase (Thermal AdS3)

$$ds^2 = R^2\left(\frac{d\tau^2}{z^2} + \frac{dz^2}{h(z)z^2} + \frac{h(z)}{z^2}dx^2\right), \quad h(z) = 1 - \left(\frac{z}{z_0}\right)^2. \quad (x \sim x + 2\pi z_0)$$

High Temperature Phase (BTZ BH)

$$ds^2 = R^2\left(\frac{f(z)d\tau^2}{z^2} + \frac{dz^2}{f(z)z^2} + \frac{dx^2}{z^2}\right), \quad f(z) = 1 - \left(\frac{z}{z_H}\right)^2.$$

The boundary Einstein equation $K_{ab} = (K - T)\, h_{ab}$ leads to

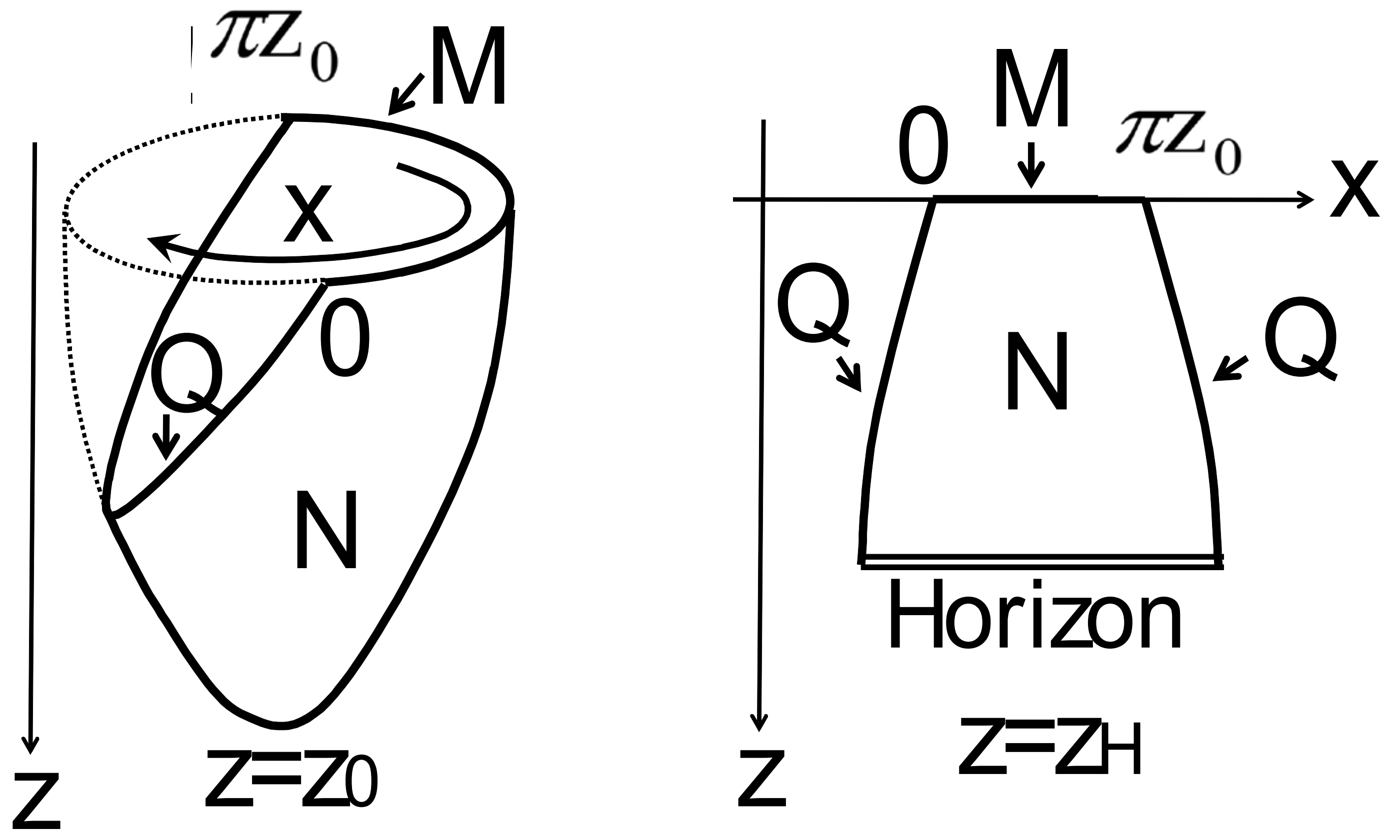
$$\frac{dx}{dz} = \frac{RT}{h(z)\sqrt{h(z) - R^2T^2}} \quad \text{(Low temp.)}$$

$$\frac{dx}{dz} = \frac{RT}{\sqrt{1 - R^2T^2 f(z)}} \quad \text{(High temp.)} \quad .$$

Their solutions are given by

$$x(z) = z_0 \cdot \arctan\left(\frac{RTz}{z_0\sqrt{h(z) - R^2T^2}}\right) \quad \text{(Low temp.)}$$

$$x(z) = z_H \cdot \operatorname{arcsinh}\left(\frac{RTz}{z_H\sqrt{1 - R^2T^2}}\right) \quad \text{(High temp.)}$$



(a) Low temp.

(b) High temp.

The Euclidean partition functions are evaluated as follows

$$I_E = -\frac{\pi}{24}\cdot\frac{c}{L\cdot T_{BCFT}}, \quad \text{(Low temp.)}$$

$$I_E = -\frac{\pi}{6}cLT_{BCFT} - \underbrace{\frac{\rho_*}{2G_N}}_{-2S_{bdy}}. \quad \text{(High temp.)}$$

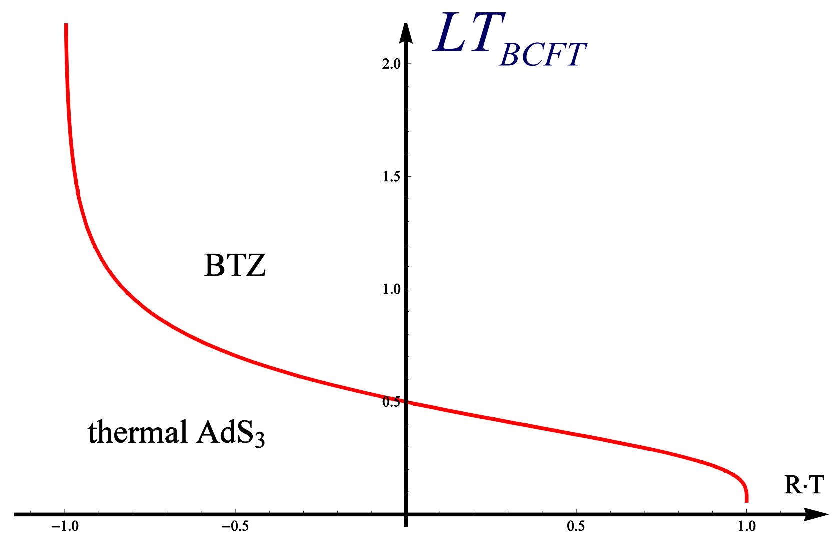
$$\Rightarrow \quad S_{thermal} = \frac{\pi}{3}cLT_{BCFT} + 2S_{bdy}$$

The phase transition occurs

when $I_E(\text{Low}) = I_E(\text{High})$

i.e.

$$T_{BCFT} = -\frac{1}{\pi L}\text{arctanh}(RT) + \frac{1}{L}\sqrt{\frac{1}{4}+\frac{1}{\pi^2}\text{arctanh}^2(RT)}\ .$$



g=0 g=∞

[8-4] Holographic g-Theorem

Consider the surface Q defined by $x = x(z)$ in the Poincare metric

$$ds^2 = R^2\left(\frac{dz^2 - dt^2 + dx^2 + (d\vec{w})^2}{z^2}\right).$$

We impose the null energy condition for the boundary matter

i.e. $T^Q_{ab} N^a N^b \geq 0$ for any null vector N^a.

[cf. Hol. C-theorem: Freedman-Gubser-Pilch-Warner 1999, Myers-Sinha 2010]

For the null vector, $N^t = 1,\ \ N^z = 1/\sqrt{1+(x')^2}\ ,\ N^x = x'/\sqrt{1+(x')^2}\ ,$

we find the constraint

$$(K_{ab} - Kh_{ab})N^a N^b = -\frac{R \cdot x''}{z(1+(x')^2)^{3/2}} \geq 0.$$

Thus we simply get $x''(z) \leq 0$ from the null energy condition.

Let us define the holographic g-function by

$$\log g(z) = \frac{R^{d-1}}{4G_N} \cdot \mathrm{Arcsinh}\left(\frac{x(z)}{z}\right) = \frac{R^{d-2}}{4G_N} \cdot \rho_*(z).$$

Then it is easy to see $\dfrac{\partial \log g(z)}{\partial z} = \dfrac{x'(z)z - x(z)}{\sqrt{z^2 + x(z)^2}} \leq 0$,

because $(x'z - x)' = x''z \leq 0$.

For d=2, at fixed points $\log g(z)$ agrees with the boundary entropy.

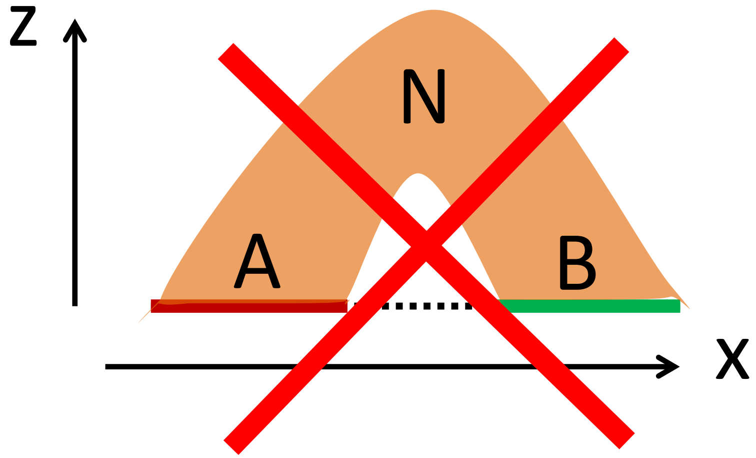
For any dimension d, we find that $\rho_*(z)$ is a monotonically decreasing function of the length scale z.

➡ This is our holographic g-theorem !

Topological Censorship

[cf. without boundary:
Galloway-Schleich-Witt-Woolgar 99]

The g-theorem prohibits the static wormhole-like configuration in our AdS/BCFT:



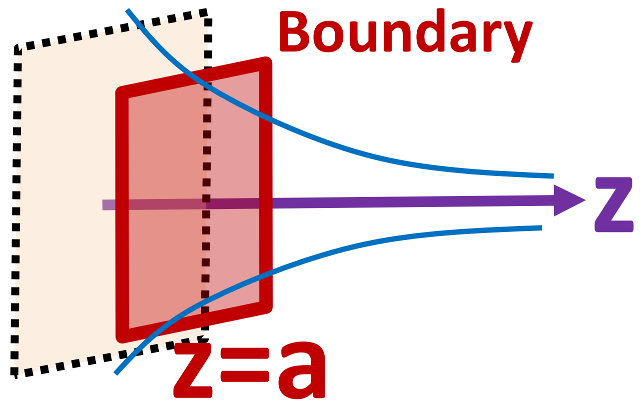
For any time t
(Static)

[8-5] Brane-world Holography

Consider a Poincare AdS$_{d+2}$ and consider a finite cut off surface

$$ds^2 = R^2 \cdot \left(\frac{dz^2 + dx^\mu dx_\mu}{z^2} \right),$$

Boundary : $z = a$.



as its boundary. We impose the Neumann b.c. on the boundary.

In this setup, a (d+1) dimensional gravity is localized on the boundary Q, called brane-world. [Randall-Sundrum 1999(RS2)]

The effective Newton constant in the d+1 dim. gravity can be found via KK reduction:

h = the metric of (d+1) dim. gravity

$$ds^2 = R^2 \cdot \left(\frac{dz^2 + h_{\mu\nu} dx^\mu dx^\nu}{z^2} \right),$$

$$I_G = -\frac{1}{16\pi G_N^{(d+2)}} \int d^{d+1}x\, dz\, \sqrt{g} \left(R^{(d+2)} + \cdots \right)$$

$$= -\frac{R^d}{16\pi G_N^{(d+2)}} \int_a^\infty \frac{dz}{z^d} \int d^{d+1}x\, \sqrt{h} \left(R^{(d+1)} + \cdots \right)$$

$$\equiv -\frac{1}{16\pi G_N^{(d+1)}} \int d^{d+1}x\, \sqrt{h} \left(R^{(d+1)} + \cdots \right).$$

$$\frac{1}{G_N^{(d+1)}} = \frac{1}{G_N^{(d+2)}} \cdot \frac{R^d}{(d-1)\cdot a^{d-1}}.$$

Brane-world Holography

Classical Gravity on AdS$_{d+2}$ with Neumann b.c. on the AdS boundary = **Quantum Gravity on R^{d+1} coupled with CFT$_{d+1}$**

If we consider the holographic EE in this setup,

Area Law of EE in CFT

$$S_A = \frac{\mathrm{A}(\Gamma_A)}{4G_N^{(d+2)}} = \frac{R^d}{4G_N^{(d+2)}}\left(\frac{\mathrm{A}(\partial A)}{(d-1)\cdot a^{d-1}} + O(a^{-(d-3)})\right)$$

$$\approx \frac{\mathrm{A}(\partial A)}{4G_N^{(d+1)}} + O(a^{-(d-3)}).$$

Gravitational Entropy in d+1 dim.

Quantum corrections in d+1 dim.

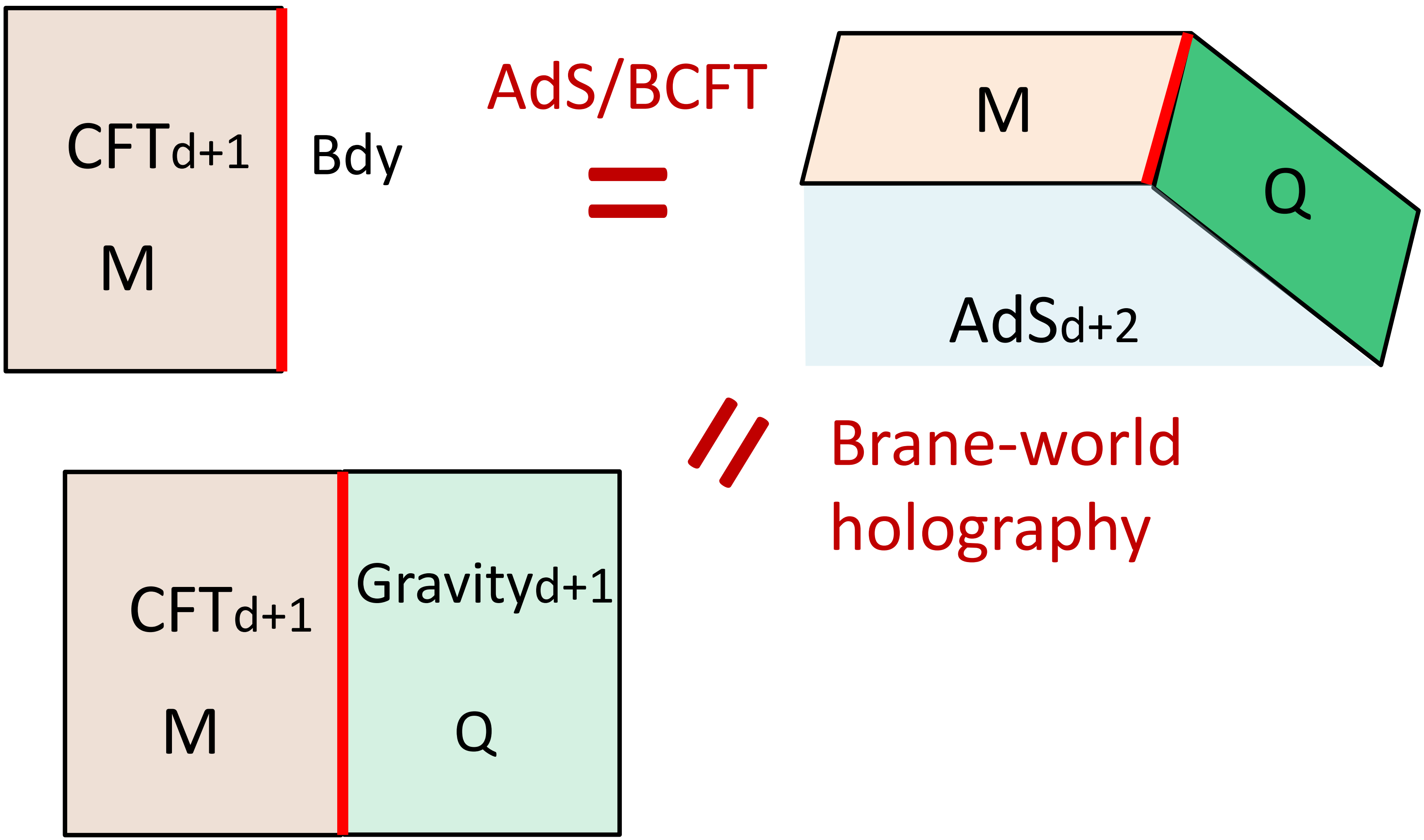
Gravity induced from Matter

We can relate this to a black hole entropy in d+1 dim. gravity by considering a brane-world black hole, where we set ΓA = d+2 dim. BH horizon, ∂A= d+1 dim. BH horizon.

[Hawking-Maldacena-Strominger 2000, Emparan 2006, Iwashita-Kobayashi-Shiromizu-Yoshino 2006,]

AdS/BCFT and Brane-World (Double Holography)

We can apply the brane-world holography to AdS/BCFT as follows.



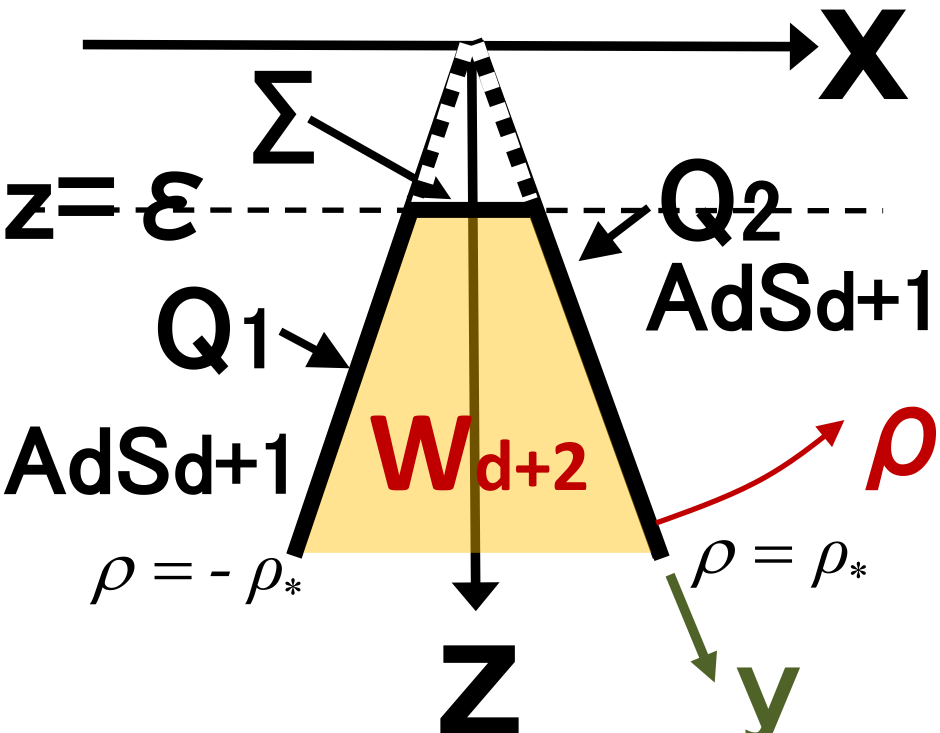
[8-6] Codimension Two Holography

[Akal-Kusuki-Wei-TT 2020, Bousso-Wildenhain 2020]

Setup of Wedge Holography

$$ds^2_{(d+2)} = d\rho^2 + \cosh^2\left(\frac{\rho}{R}\right) ds^2_{AdS(d+1)}, \quad -\rho_* \le \rho \le \rho_*.$$

Wedge W_{d+2}



$$ds^2_{AdS(d)} = R^2\left(\frac{-dt^2 + dy^2 + d\vec{w}^2}{y^2}\right),$$

Poincare AdS_{d+2}

$$ds^2_{(d+2)} = R^2\left(\frac{-dt^2 + dz^2 + dx^2 + d\vec{w}^2}{z^2}\right).$$

Our Claim of Wedge Holography

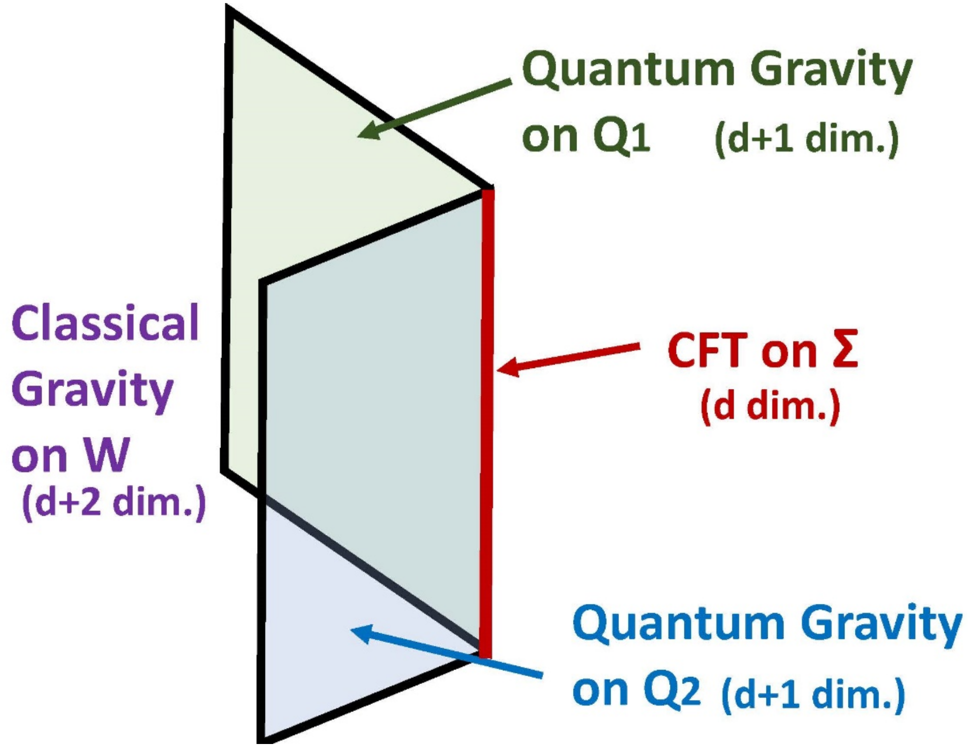
d+2 dim. Classical Gravity on W_{d+2} = d+1 dim. Quantum Gravity on $Q1 \cup Q2$

=

d dim. CFT on Σ

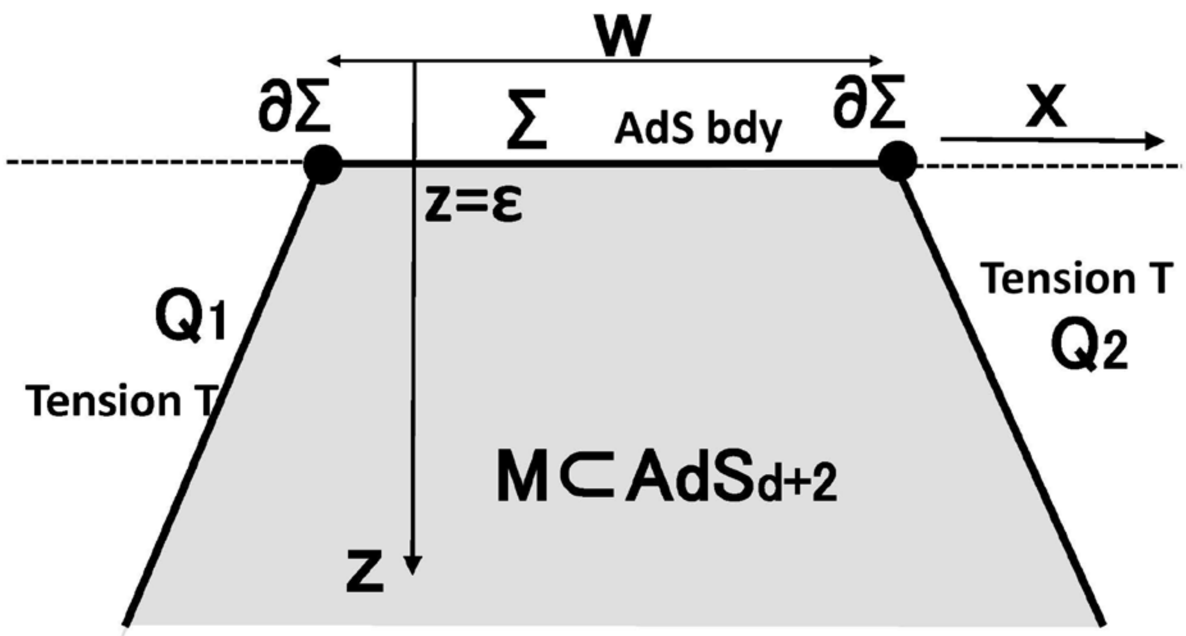
⇓

Codimension Two Holography !



Wedge Holography as A Limit of AdS/BCFT

Our wedge holography can be obtained by the zero width limit $w \to 0$ of the following AdS/BCFT setup:

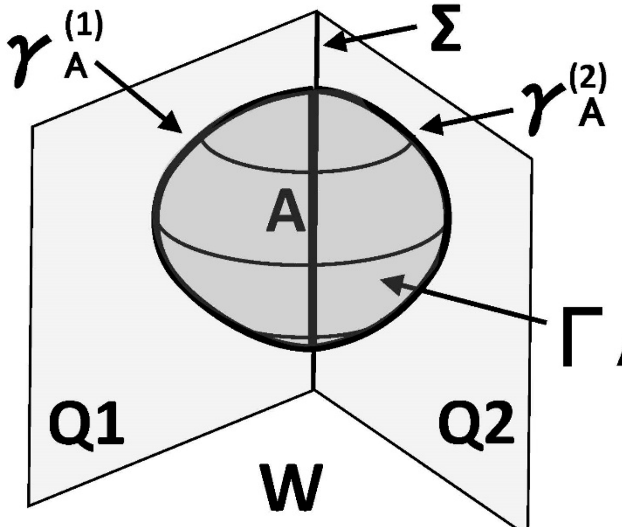


HEE in Wedge Holography

HEE in our wedge holography can be obtained by taking the previous zero width limit of AdS/BCFT.
It is given by a double minimization formula:

$$S_A = \underset{\substack{\gamma_A^{(1)},\gamma_A^{(2)} \\ \partial\gamma_A^{(1,2)}=\partial A}}{\mathrm{Min}} \left[\underset{\substack{\Gamma_A \\ \partial\Gamma_A=\gamma_A^{(1)}\cup\gamma_A^{(2)}}}{\mathrm{Min}} \left[\frac{\mathrm{A}(\Gamma_A)}{4G_N^{(d+2)}} \right] \right]$$

d-1 dim



d-1 dim

d dim

Calculation of HEE

We choose the subsystem A to be a d-1 dim. disk $x^2+\sum_{i=1}^{d-1} w_i^2 \leq l^2.$

In this case, the surface ΓA is found to be a part of sphere:

$$x^2+z^2+\sum_{i=1}^{d-1} w_i^2 = l^2.$$

Thus the HEE is computed as follows:

$$S_A = \frac{R^{d-1}}{2G_N^{(d+2)}}\cdot \mathrm{Vol}(S^{d-2})\cdot \int_0^{\rho_*} d\rho \left(\cosh\frac{\rho}{R}\right)^{d-1}\int_{\varepsilon\cosh(\rho/R)}^{l} d\xi \frac{l(l^2-\xi^2)^{d/2-3/2}}{\xi^{d-1}}$$

$$= p_0\left(\frac{l}{\varepsilon}\right)^{d-2} + p_2\left(\frac{l}{\varepsilon}\right)^{d-4} + \ldots + \begin{cases} p_{d-2}\int_0^{\rho_*/R} dr(\cosh r)^{d-1}, & d=\mathrm{odd} \\ q\int_0^{\rho_*/R} dr(\cosh r)^{d-1}\cdot \log\frac{l}{\varepsilon}+\mathrm{const.}, & d=\mathrm{even}\end{cases}.$$

Area Law

Conformal Anomaly

Agree with the general form for d dim. CFT !

$$p_0=\frac{1}{d-2},\;\; p_2=-\frac{d-3}{2(d-4)},\cdots,$$

$$p_{d-2}=\frac{\Gamma(d/2-1/2)\Gamma(1-d/2)}{2\sqrt{\pi}},\quad q=\frac{\sqrt{\pi}}{\Gamma(3/2-d/2)\Gamma(d/2)}.$$

Example: 2d CFT (d=2)

Our AdS4 wedge holography leads to the HEE (A= an interval):

$$S_A = \frac{R^2}{G_N^{(4)}} \sinh\frac{\rho_*}{R} \cdot \log\frac{l}{\varepsilon} + \text{const.}$$

By comparing with the well-known formula $S_A = \frac{c}{3}\log\frac{l}{\varepsilon}$,

we expect the central charge c is given by

$$c = \frac{3R^2}{G_N^{(4)}} \sinh\frac{\rho_*}{R}.$$

Indeed, we can obtain the same value of central charge by combining the 3dim. Newton constant in brane-world holography:

$$\frac{1}{G_N^{(3)}} = \frac{R}{G_N^{(4)}} \sinh\frac{\rho_*}{R}, \qquad \left[\text{For general d,} \quad \frac{1}{G_N^{(d+1)}} = \frac{1}{G_N^{(d+2)}} \int_0^{\rho_*} d\rho \left(\cosh\frac{\rho}{R}\right)^{d-1}\right]$$

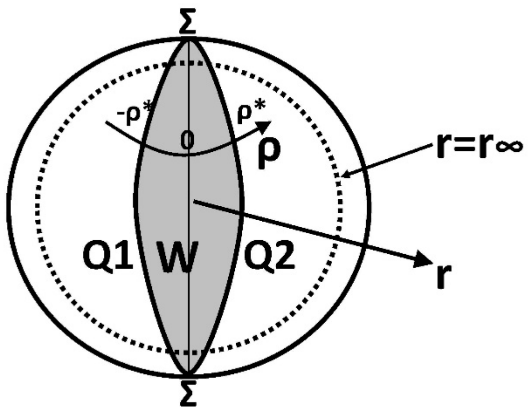
with the Brown-Henneaux relation $c = \frac{3R}{2G_N^{(3)}}$.

Free Energy on Sphere

$$ds^2_{(d+2)} = dr^2 + R^2 \sinh^2\left(\frac{r}{R}\right)(d\theta^2 + \cos^2\theta d\Omega_d^2),$$

$$= d\rho^2 + R^2 \cosh^2\left(\frac{\rho}{R}\right)(d\eta^2 + \sinh^2\eta d\Omega_d^2)$$

$\Rightarrow$ Restrict to $-\rho_* \le \rho \le \rho_*.$



$$I_G = -\frac{1}{16\pi G_N}\int_W \sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G_N}\int_{Q1\cup Q2}\sqrt{h}(K-T) - \frac{1}{8\pi G_N}\int_\Sigma \sqrt{h}K,$$

In the d=2 case (AdS4/CFT2), we find

$$I_G = -\frac{R^2}{2G_N^{(4)}\varepsilon^2}\sinh\frac{\rho_*}{R} + \frac{R^2}{G_N^{(4)}}\sinh\frac{\rho_*}{R}\cdot\log\frac{l}{\varepsilon}.$$

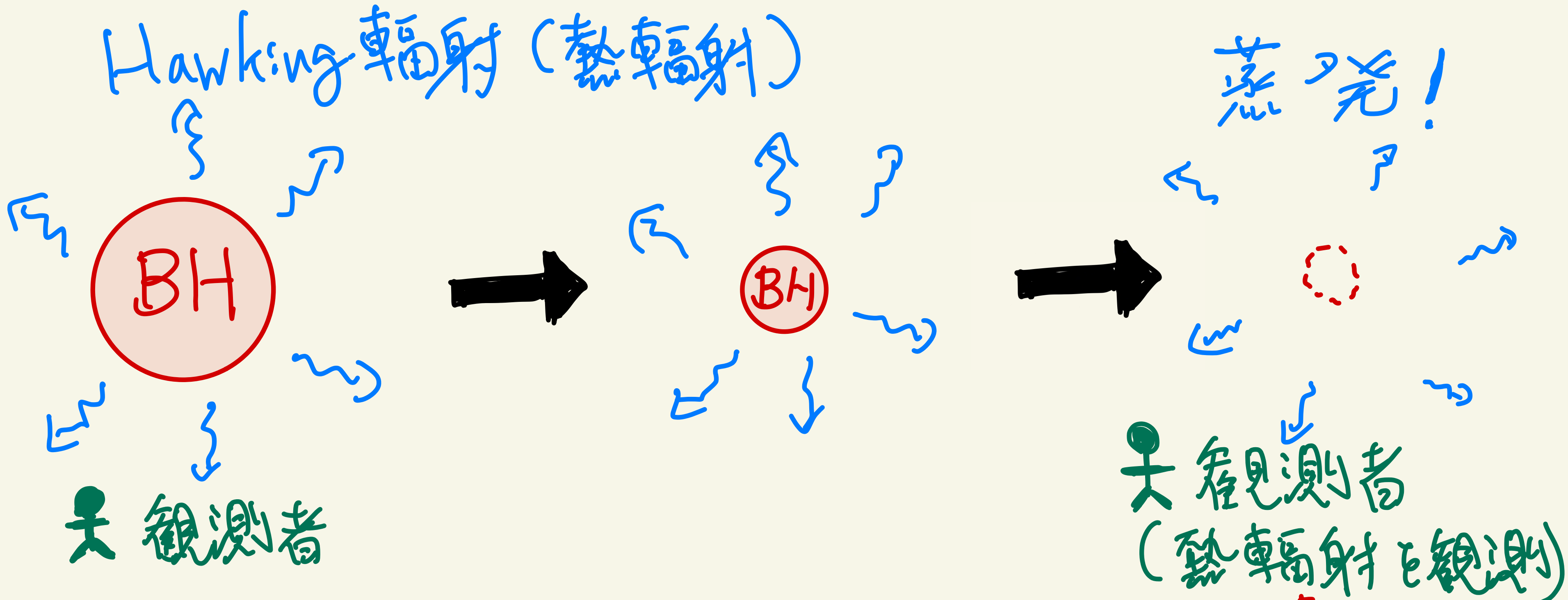
Agree with previous result

$$I_{CFT} = \#\cdot\varepsilon^{-2} + \frac{c}{6}\chi(\Sigma)\log\varepsilon$$

$$c = \frac{3R^2}{G_N^{(4)}}\sinh\frac{\rho_*}{R}.$$

⑨ アイランド公式とBH情報問題

(9-1) BH情報問題とページ曲線



隠れた情報 $= S_{BH} = \dfrac{A_{BH}}{4G_N}$

↓

BHの中の情報が消える!

↓

量子論のユニタリティーが破れる!

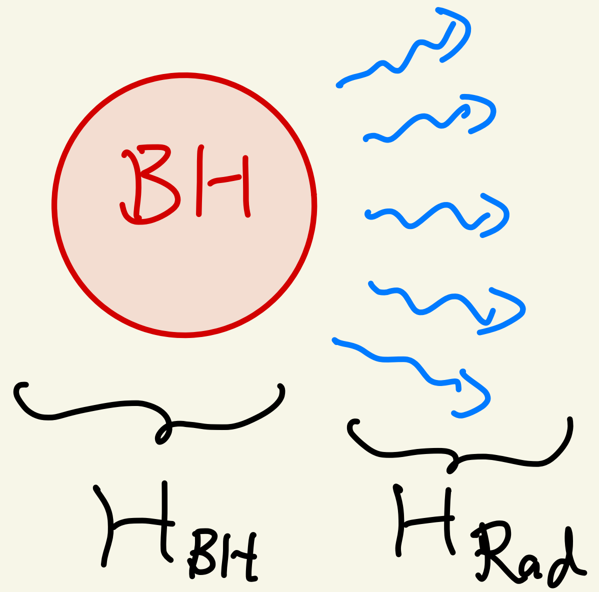
逆に量子論が正しいとすると、何が期待されるか？

➡「BH + 輻射」はPure Stateで、全体として、ユニタリー時間発展する。

$$H_{tot} = H_{BH} \otimes H_{Rad}, \quad |\Psi(t)\rangle_{tot} = e^{-i\hat{H}t}|\Psi(0)\rangle_{tot}$$

そこで、両者のEEを定義する。

$$\rho_{Rad}(t) = \mathrm{Tr}_{BH} |\Psi(t)\rangle\langle\Psi(t)|$$

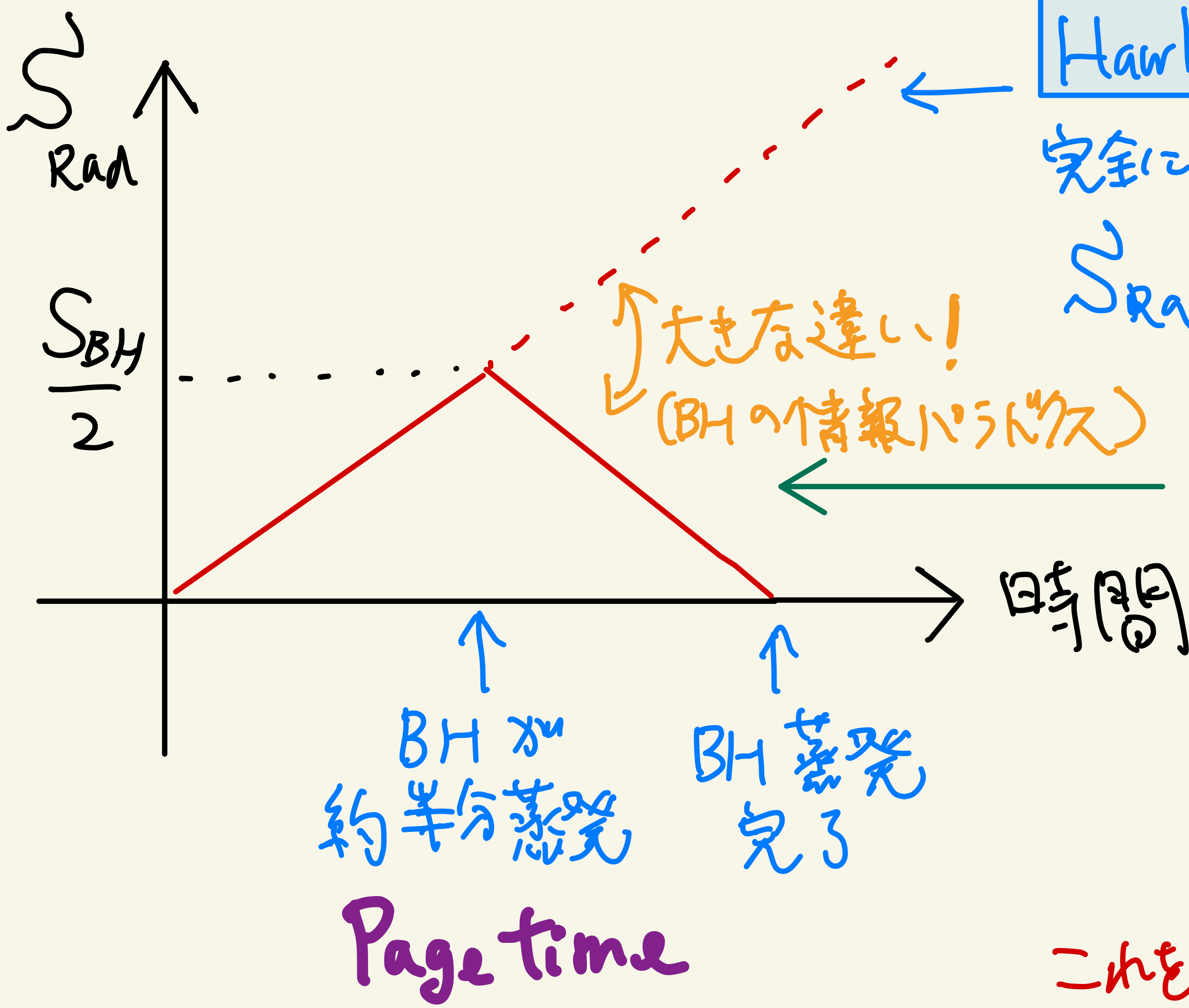


➡ $$S_{Rad}(t) = -\mathrm{Tr}\,\rho_{Rad}\log\rho_{Rad}$$

を考える。

ページ曲線 (Page curve)

$S_{Rad} = S_{BH}$ なので、{BH蒸発開始時 / BH蒸発後は、ともに 0。



Hawking Paradox

完全に熱輻射と思うと、S_{Rad} は単調増加

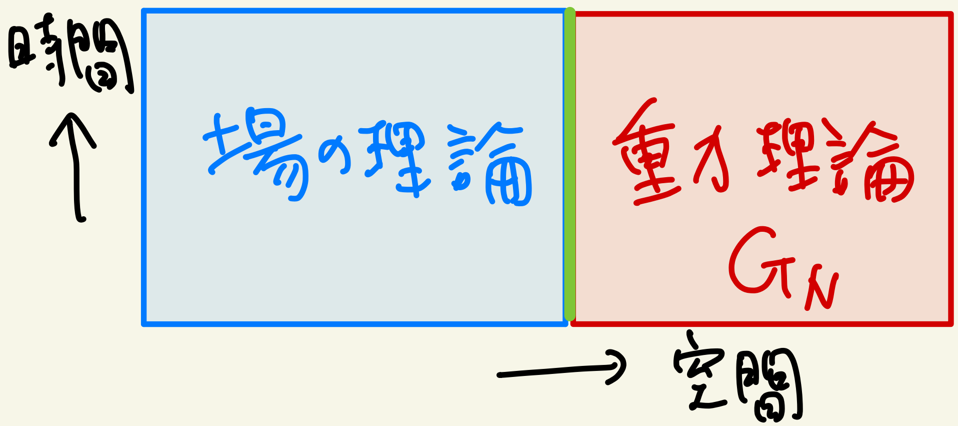
しかし、量子力学に従うと、S_{Rad} は途中から減少して、ゼロになる!
(ページ曲線と呼ぶ)

↓

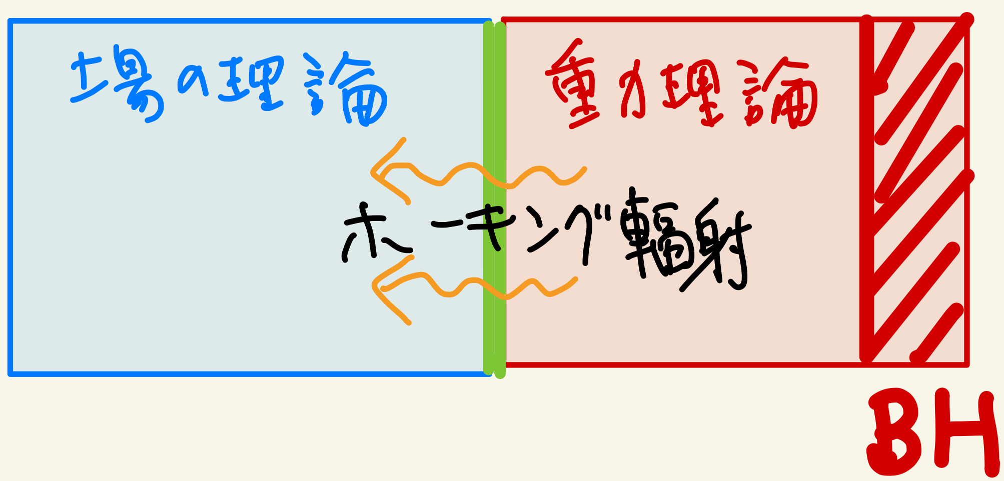
これをどう説明するのか?

(9-2) アイランド公式

重力理論と場の理論が結合する時空を考える：

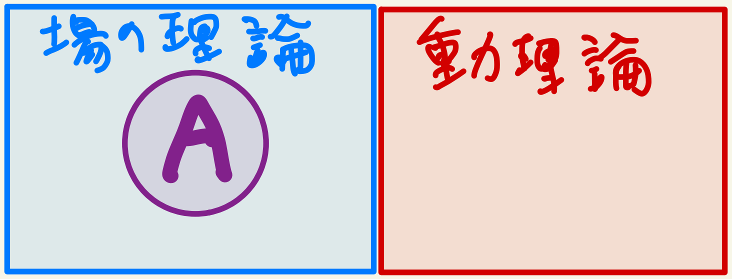


我々の考えたいSetup



アイランド公式

[Penington, Almheiri-Engelhardt-Marolf-Maxfield 2019]

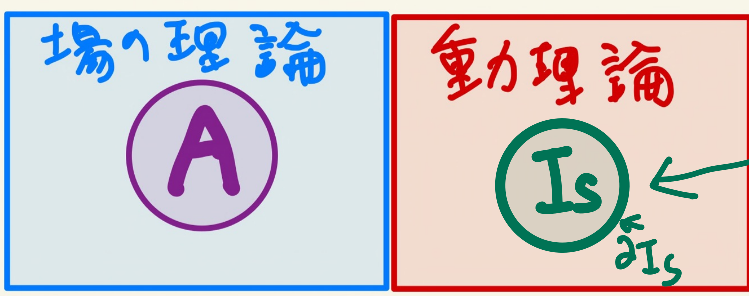


の Setup での EE $S_A^{重力}$ は、

次の公式で計算される。

HRT公式と同じ

$$S_A^{重力} = \underset{I_s}{\mathrm{Min}}\ \underset{I_s}{\mathrm{Ext}}\left[\frac{A(\partial I_s)}{4G_N} + S_{A\cup I_s}\right]$$



I_s : アイランド in 重力理論

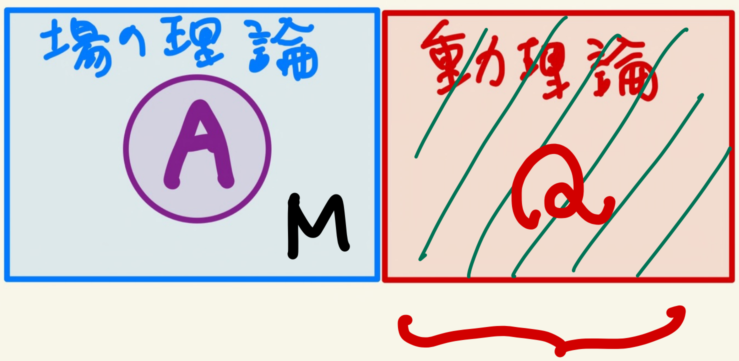
↓

∂I_s が Quantum Extremal Surface in general spacetimes

アイランド公式の導出

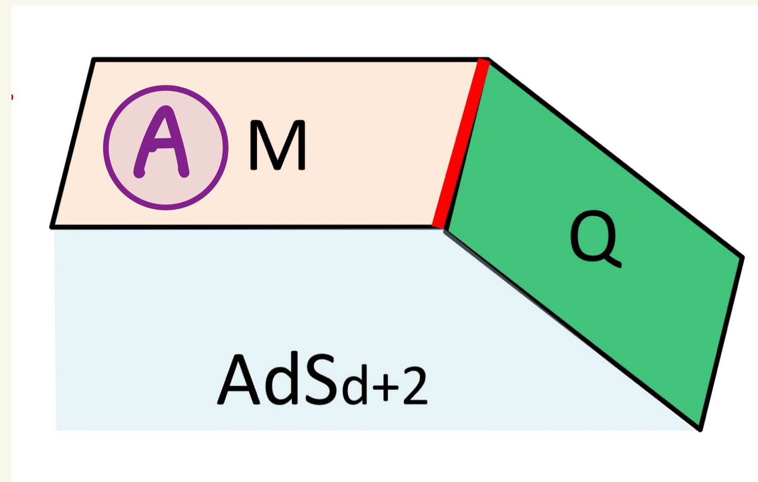
(a) AdS/BCFT + Brane-World Holography

[Almheiri - Mahajan - Maldacena - Zhao 2019]



AdSとする.

Brane-World Holography =

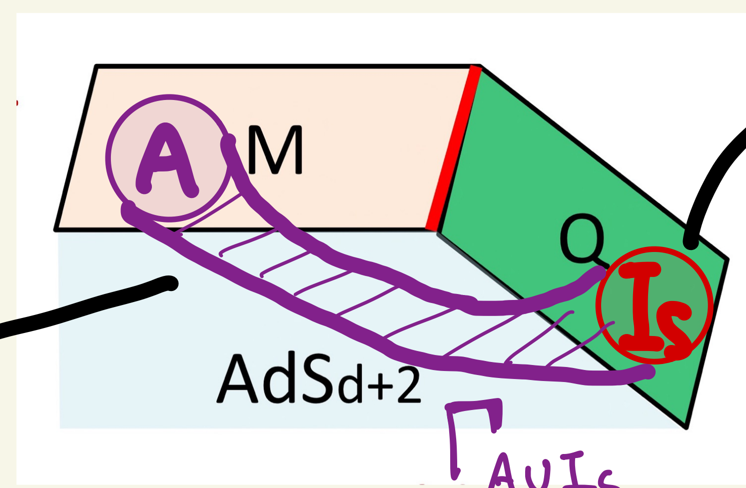


AdS/CFT ||

AdS/BCFT =

↓ HEEの計算

$S_A^{BCFT} = S_A^{重力}$



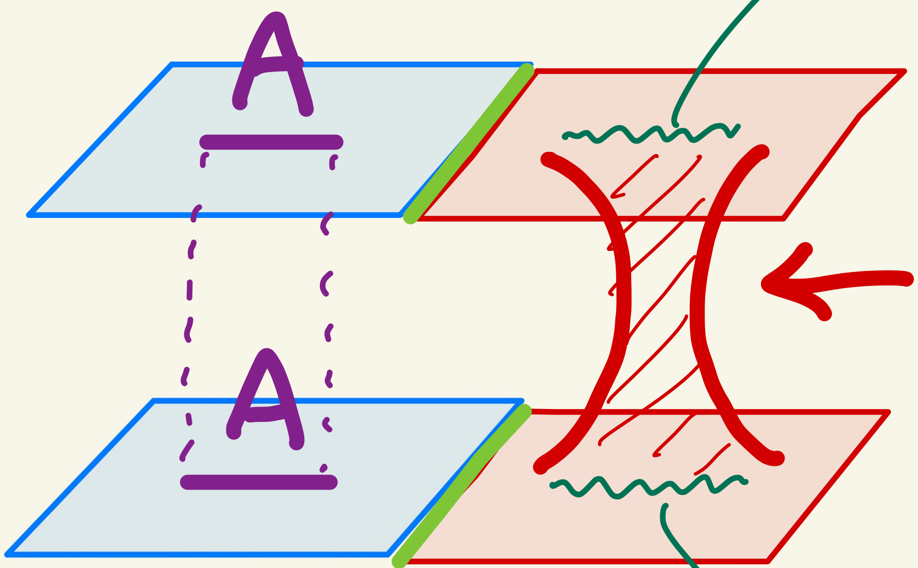
この $\Gamma_{A\cup I_s}$ が $S_{A\cup I_s}$ の寄与となる!

$\frac{A(\partial I_s)}{4G_N}$ の寄与!

(b) Replica Wormhole

[Penington-Shenker-Stanford-Yang 2019
Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini]

$\mathrm{Tr}[\rho_A^2]$ の計算



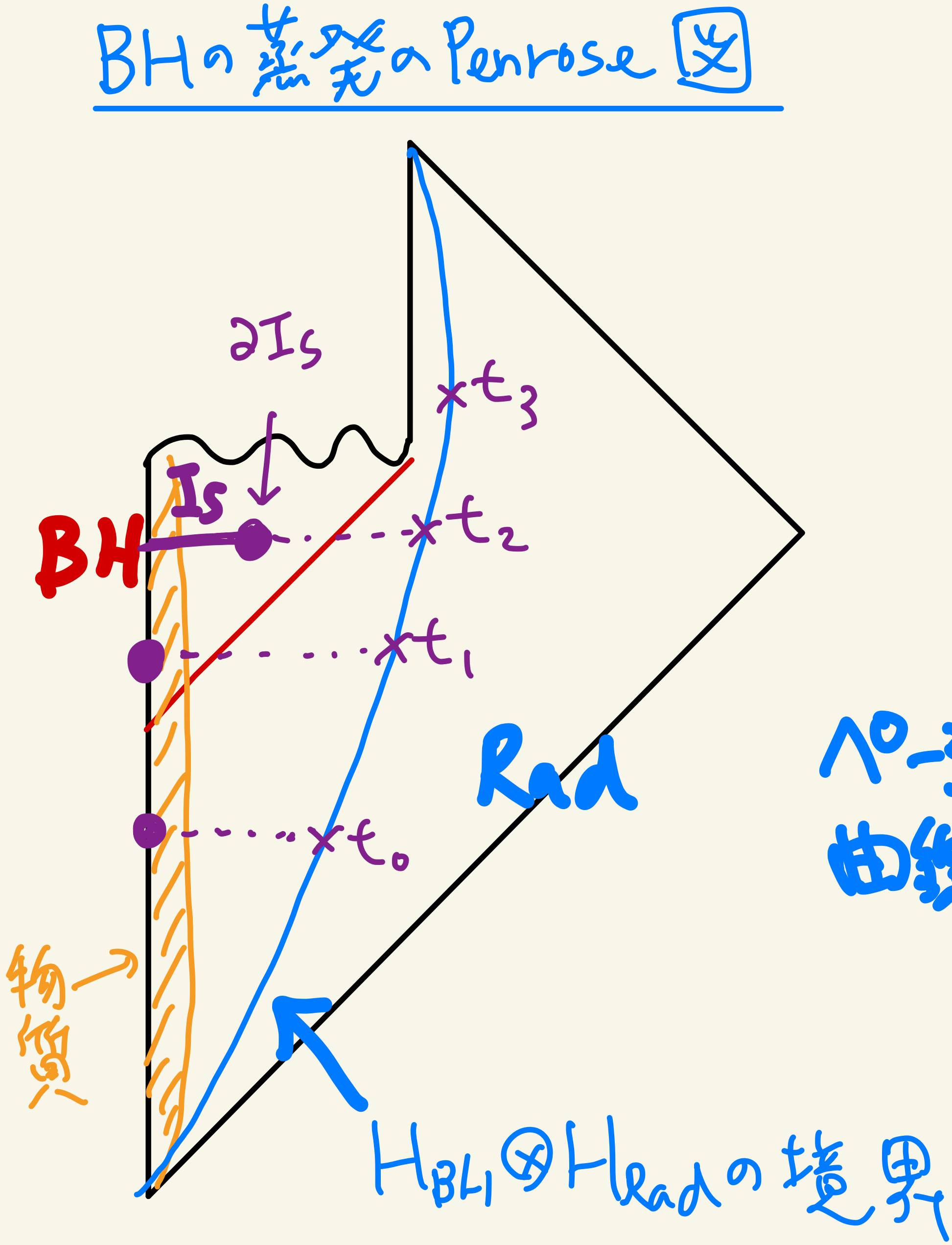
同様に、$\mathrm{Tr}\,\rho_A^n$ を計算して、$S_A = -\frac{\partial}{\partial n} \log \mathrm{Tr}\,\rho_A^n \Big|_{n=1}$

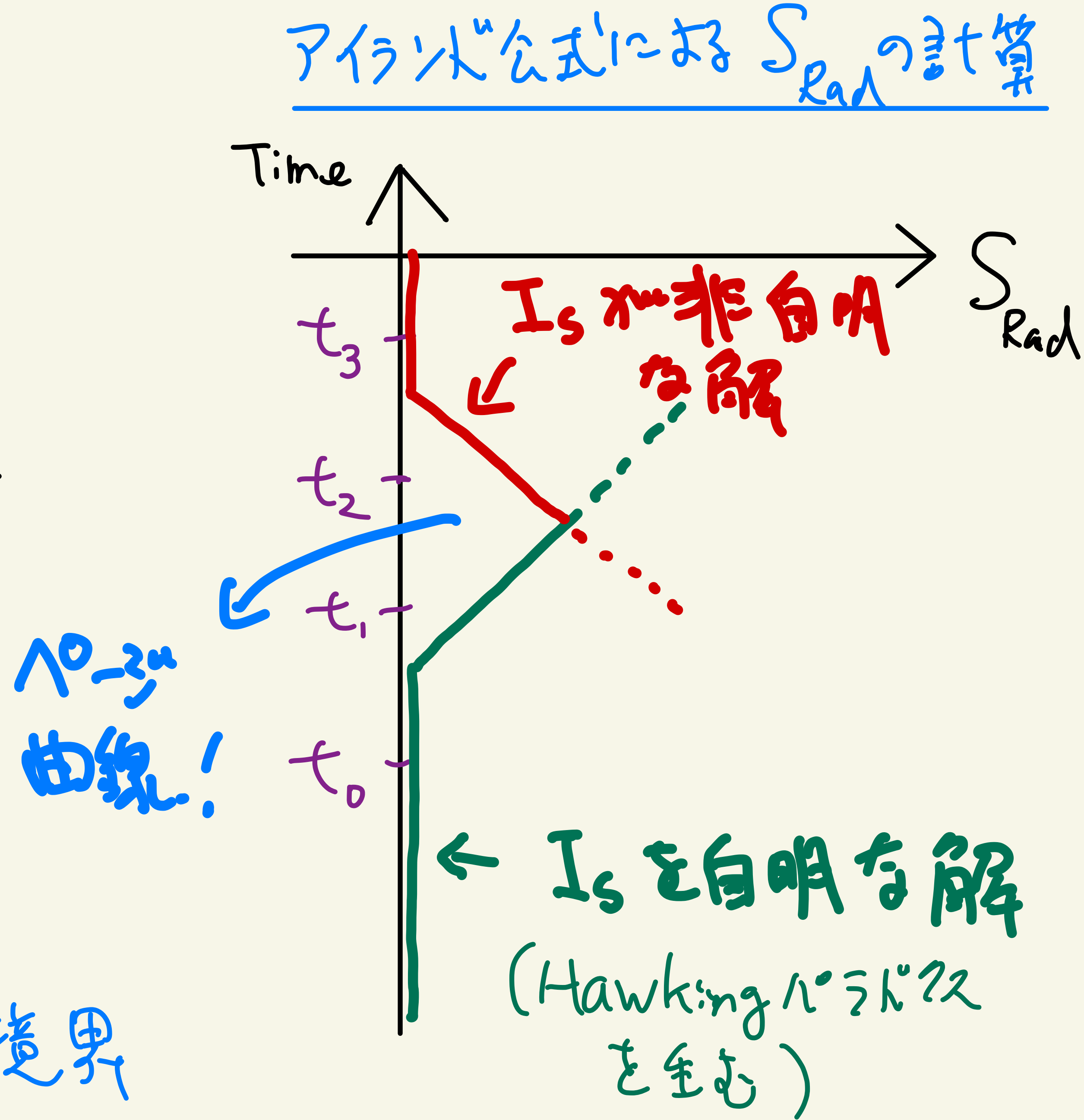
を計算すると、アイランド公式が得られる。

[宇賀神さんのセミナー参照]

(9-3) ページ曲線の導出 [上記の文献 +Review 2006.06872]

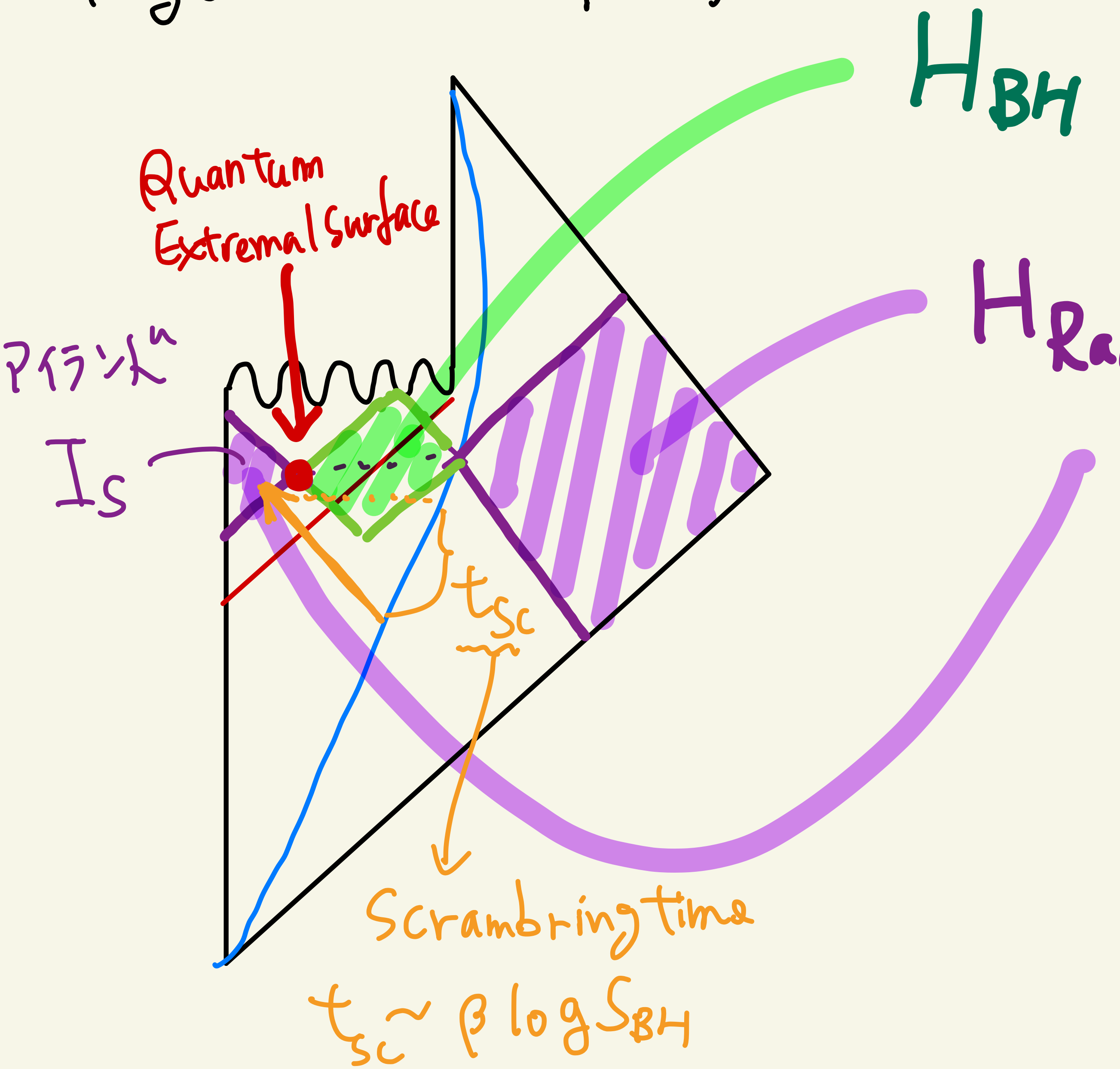
(i) アイランド公式を直接適用





エンタングルメント・ウェッジ (EW)

Page time 以降を考えると、



↓

つまり、輻射に、

BHの内部(アイランド)

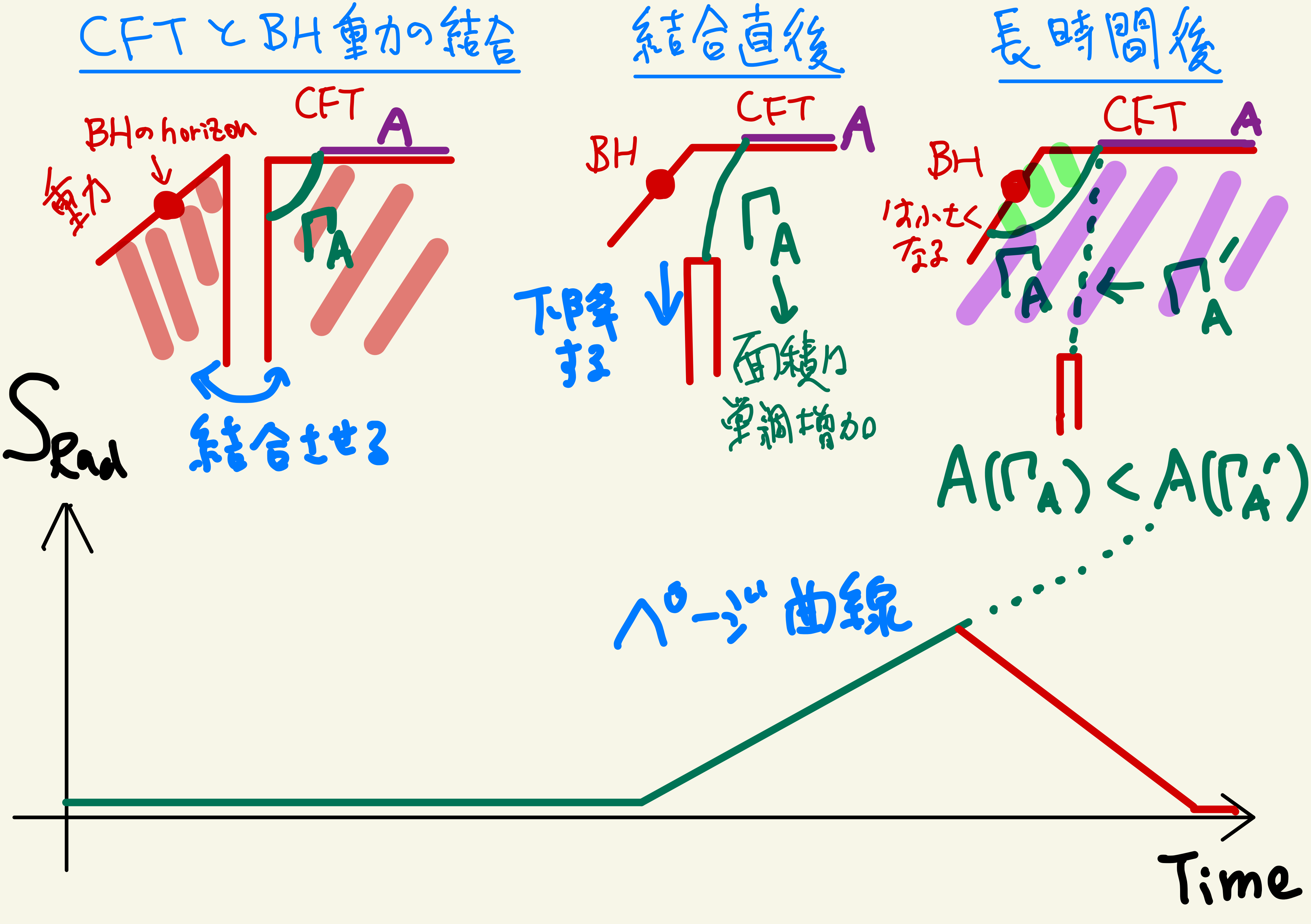
の情報が含まれている!

↓

情報問題の解決

のシナリオ!

(ii) AdS/BCFT + Brane-World Holographyの適用



(9-4) アイランド公式を用いた計算例

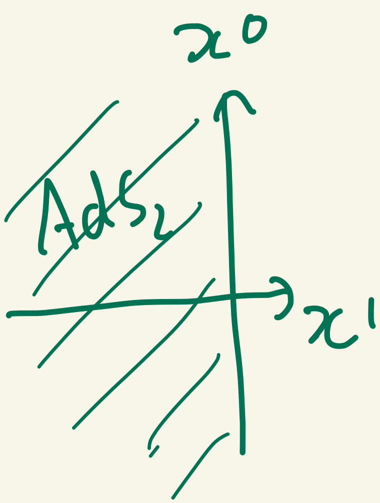
AdS_2 Eternal BH の Radiation [1910.11077 Almheiri-Mahajan-Maldacena]

JT-gravity (2次元重力の1つ)

$$I_G = \frac{1}{4\pi}\int dx^2 \sqrt{-g}\left(\phi R + 2(\phi - \phi_0)\right) + I_{CFT}$$

ϕ ↑ dilaton　　ϕR ⟷ $S_{BH} = \phi$

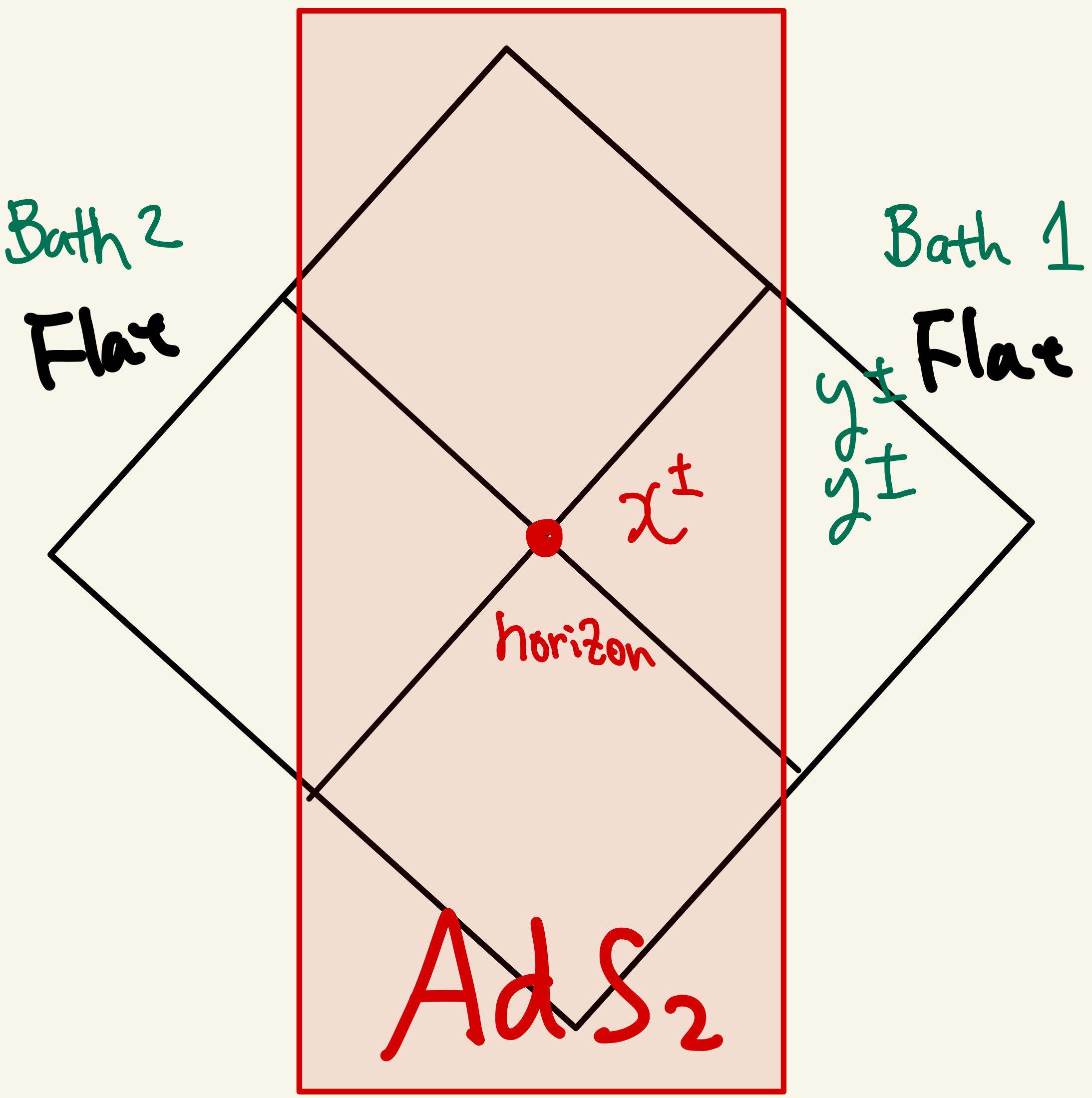
$$\frac{\delta I_G}{\delta \phi} = 0 \rightarrow R + 2 = 0 \rightarrow AdS_2$$



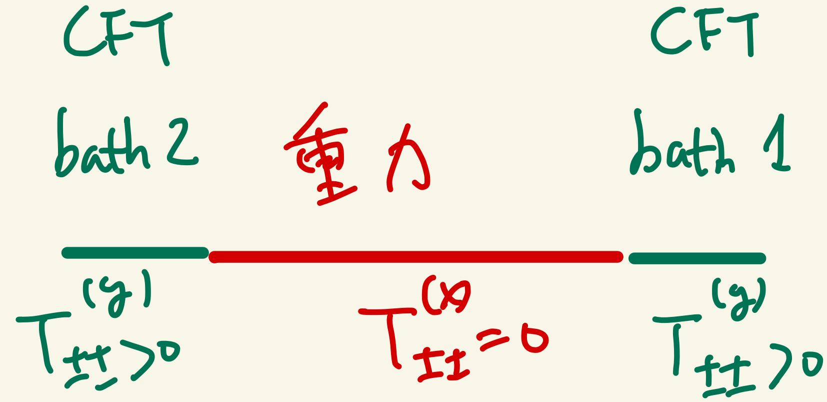
$$ds^2 = \frac{-4dx^+dx^-}{(x^- - x^+)^2}, \quad \phi = \phi_0 + \frac{2\phi_r}{(x^- - x^+)^2}$$

$$x^{\pm} = x^0 \pm x^1$$

AdS$_2$ Eternal BH + 熱浴



Hartle-Hawking State

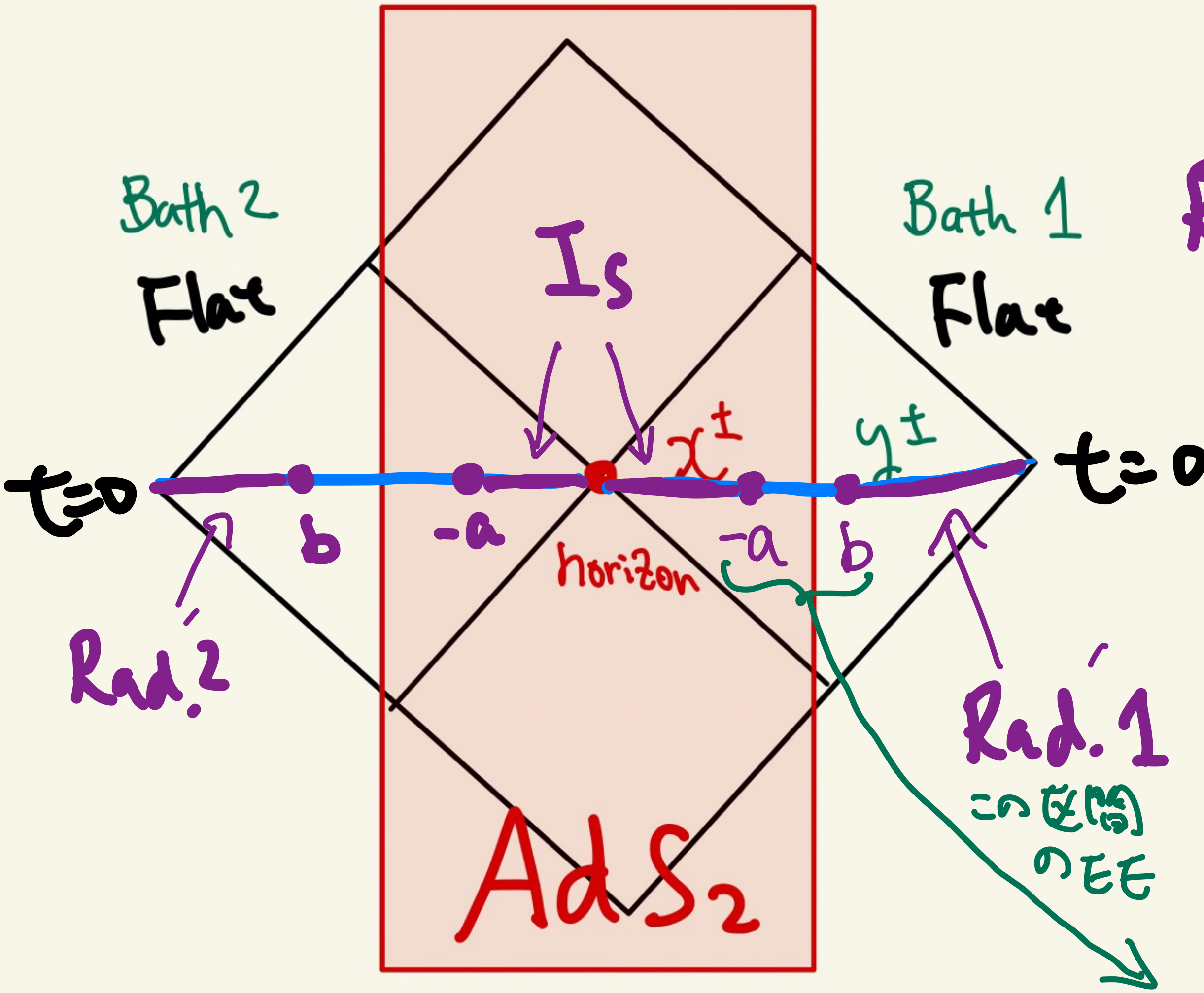


$$x^\pm = \tanh \frac{\pi y^\pm}{\beta}$$

$$\left(\frac{\partial x^+}{\partial y^+}\right)^2 \underset{\overset{\|}{0}}{T^{(x)}_{++}} = \underset{\overset{\nparallel}{0}}{T^{(y)}_{++}} + \frac{c}{24}\{x^+, y^+\}$$

Thermal Flux

$$y^\pm = t \pm \sigma$$



BH → $-a \leq \sigma \leq b$

Rad → $\begin{cases} \sigma \geq b \\ \sigma \leq -a \ (I_S) \end{cases}$

$$ds^2 = -\frac{4\pi^2}{\beta^2} \times \frac{dy^+ dy^-}{\sinh^2 \frac{\pi}{\beta}(y^- - y^+)}$$

$$\phi = \phi_0 + \frac{2\pi\phi_r}{\beta} \cdot \frac{1}{\tanh \frac{\pi}{\beta}(y^- - y^+)}$$

$$S_{Rad1} = \phi_0 + \frac{2\pi\phi_r}{\beta} \cdot \frac{1}{\tanh \frac{2\pi a}{\beta}} + \frac{c}{6} \log \left[\frac{\sinh^2 \frac{\pi(a+b)}{\beta}}{\sinh \frac{2\pi a}{\beta} \times \varepsilon^2} \right]$$

$$\frac{A(\sigma=-a)}{4G_N} = \frac{A(\partial I_S)}{4G_N}$$

$$\frac{c}{3} \log \frac{|x_b' - x_a'|}{\varepsilon} + \frac{c}{6} \varphi(\sigma=-a)$$

Weyl変換

$$\frac{\partial S_{Rad}}{\partial a} = 0 \rightarrow \frac{\sinh \frac{\pi(a-b)}{\beta}}{\sinh \frac{\pi(a+b)}{\beta}} = \frac{12\pi\phi_r}{c\beta} \cdot \frac{1}{\sinh \frac{2\pi a}{\beta}}$$

特に、$\frac{\phi_r}{c\beta} \gg 1$ $(S_{BH} \gg c)$ の場合に、

$$a \approx b + \frac{\beta}{2\pi} \log\left(\frac{24\pi\phi_r}{c\beta}\right) \gg 1$$ となる。

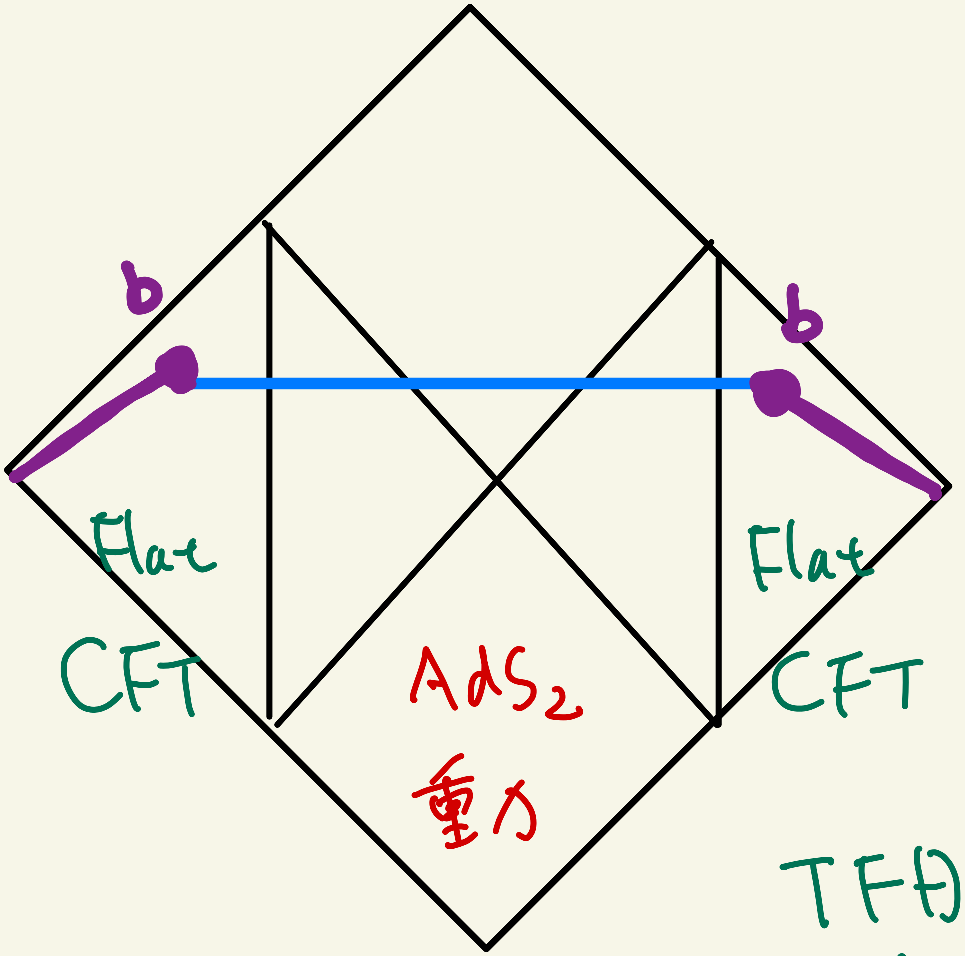
この時、$$\tilde{S}_{Rad1} \approx \underbrace{\left(\phi_0 + \frac{c}{6}\log\frac{1}{\varepsilon^2}\right)}_{} + \frac{2\pi\phi_r}{\beta} = \tilde{S}_{BH}$$

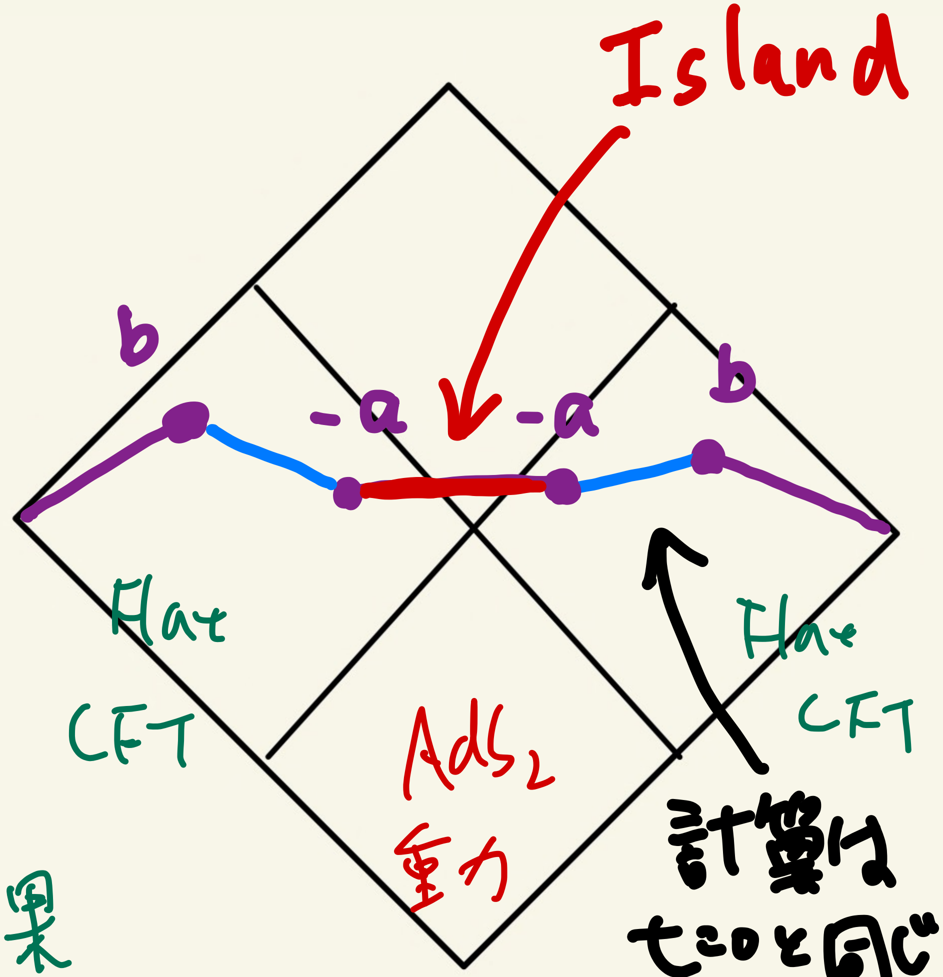
G_N のくり込み $\rightarrow \phi_0^{(ren)}$

(注)

$$\tilde{S}_{Rad} = \tilde{S}_{Rad1} + \tilde{S}_{Rad2} = 2 \cdot S_{BH}$$

時間発展（両側を上方へ）を考えると、以下の2通りの可能性





TFDの結果
↓

アイランドなし.

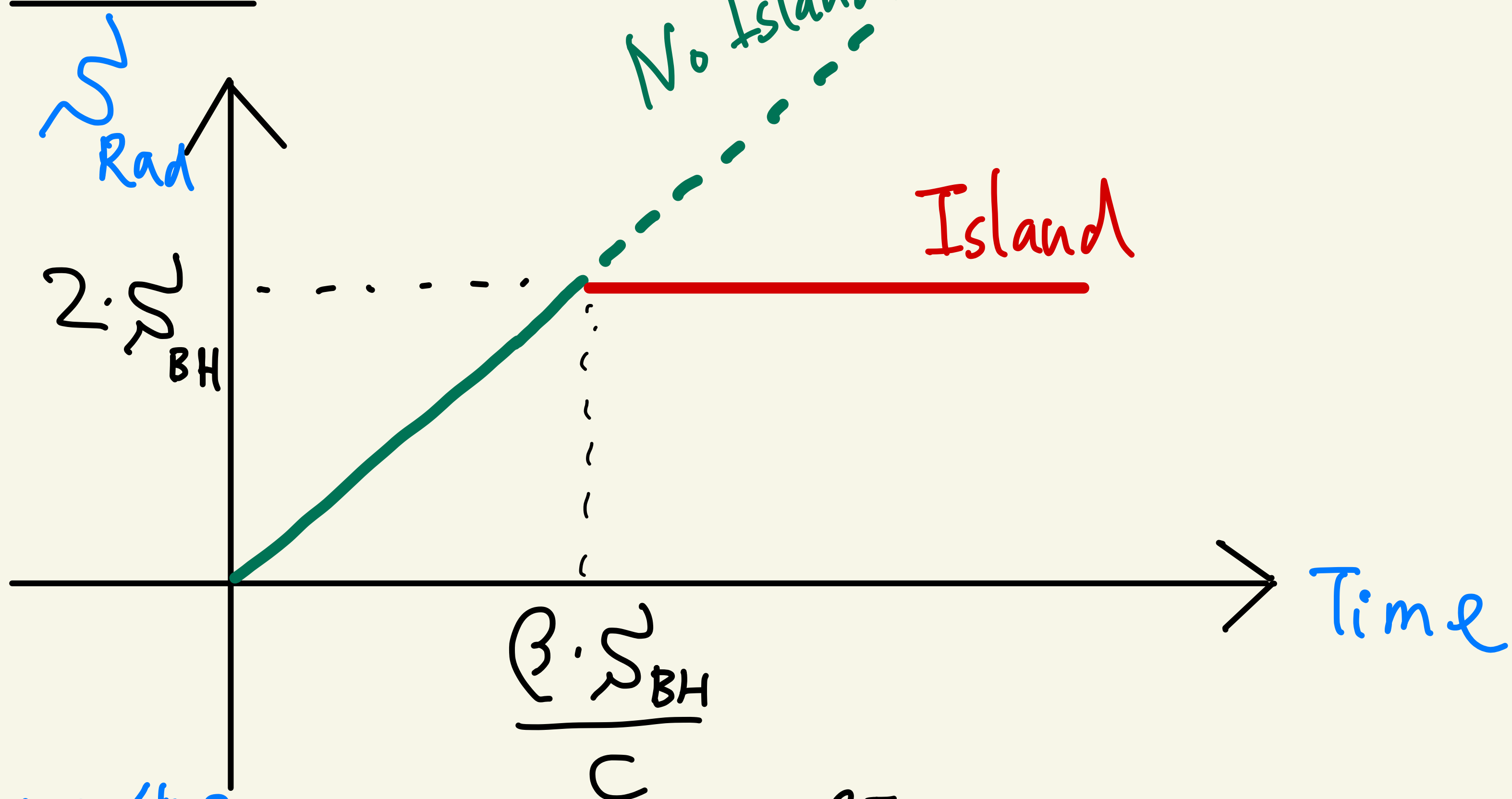
$$S_{Rad} \approx \frac{c}{3}\log\left[\frac{\pi}{\beta}\cosh\left(\frac{2\pi t}{\beta}\right)\right]$$

$$\approx \frac{2\pi c}{3\beta}\cdot t \qquad (t \gg \beta)$$

アイランド有り

$$S_{Rad} \approx 2\cdot S_{BH}$$

まとめ



何故2倍?

$$\rho_{t=0} = \frac{1}{Z}\underbrace{\left(\sum_n e^{-\frac{\beta E_n}{2}} |n\rangle_1 |n\rangle_2\right)}_{\text{TFD state}}\left(\sum_m e^{-\frac{\beta E_m}{2}} \langle m|_1 \langle m|_2\right)$$

$$\downarrow$$

$$\rho_{t=\infty} = \left(\frac{1}{Z}\sum_n e^{-\beta E_n} \underline{|n\rangle_1\langle n|}\right) \otimes \left(\frac{1}{Z}\sum_m e^{-\beta E_m} \underline{|m\rangle_2\langle m|}\right)$$

↳ Purified by Radiation ↩