RIKEN iTHEMS

iTHMS Colloquium, Apr.17, 2023

Emergence of Extreme Universe from Quantum Information

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics
Kyoto University

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGPQI Center for Gravitational Physics and Quantum Information
Yukawa Institute for Theoretical Physics, Kyoto University

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

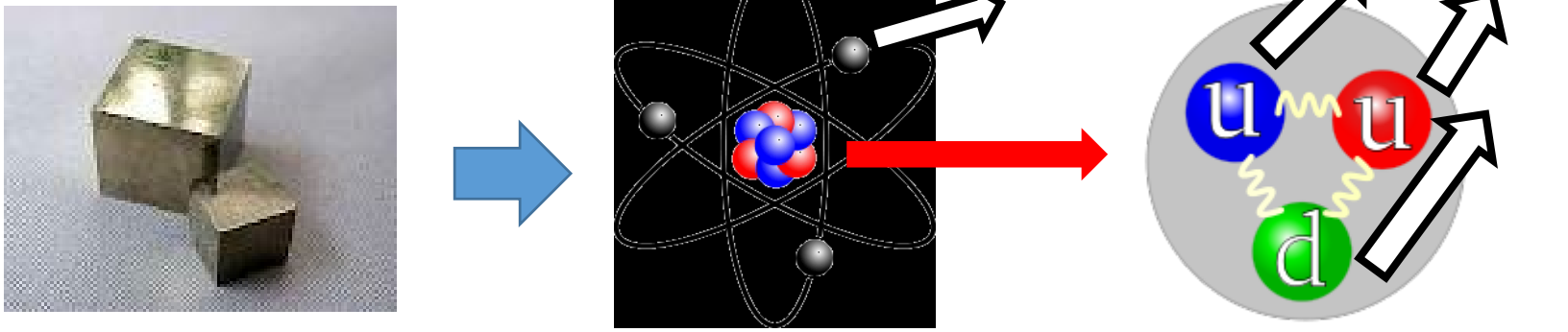
It from Qubit
Simons Collaboration

①Introduction

What is particle physics ?

The study of fundamental particles and forces that constitute matter and radiation.

⇒**Exploring the minimal unit of matter**



Four fundamental forces of Nature and Unification

① Electromagnetic force

②Strong Interaction
(Nuclear force)

③Weak Interaction
(β −decay)

④Gravitational force
Macroscopic theory
➡ Einstein's general relativity

Microscopic theory ?
(Quantum theory)

Unified in terms of
Quantum Field Theory

Should be Unified !

Gauge Theory
(Standard model)

Unification of All forces
＝Quantum Gravity？

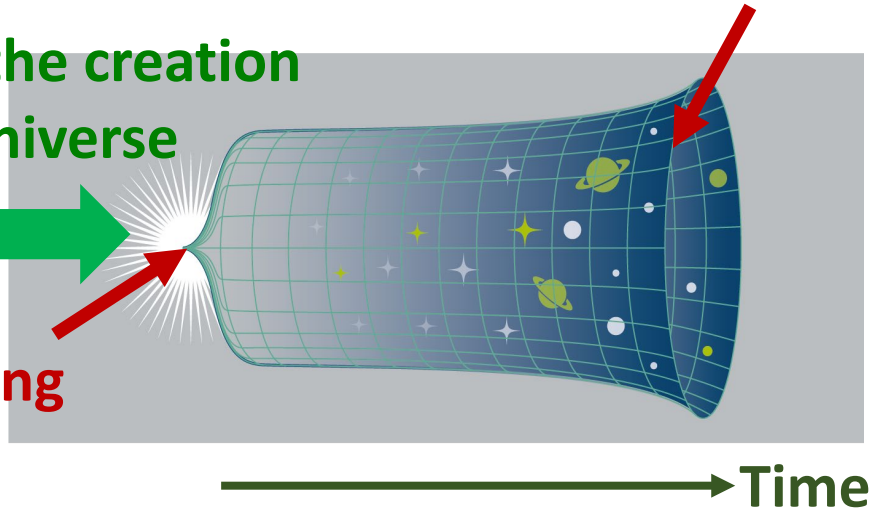
Our Target !

To understand the creation of Universe, we need quantum gravity !

Quantum Gravity

= Microscopic theory of gravity

Explain the creation of the Universe



First of all, we want to magnify the Universe.

➡ We need a “good miscroscope” !

What is the minimal unit of spacetime ?

➔Holography plays a role of microscope in thought experiments !

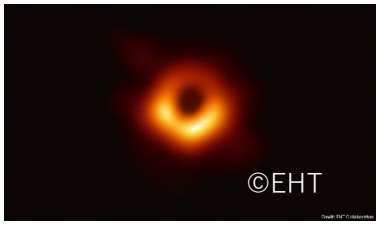
Condensed matter
Biology, Chemistry
➡ Optical,
Electronic, etc
microscope

➡ Cells, Spins,
Electrons,
Crystal structure

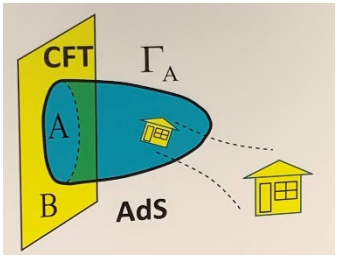
High Energy Physics ➡ Accelerators

➡ Elementary particle
Atomic nucleus

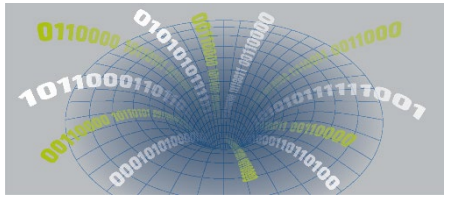
Quantum Gravity ➡ **Holographic Principle**
(Gauge/Gravity duality) ➡ **Qunatum Information**
(Qubits with
Quantum Entanglement)



➡



➡



What is the minimal unit of spacetime ? ➡ Qubit ?!

Qubit ＝ The unit of microscopic information (= a spin)

or

Head 0　　Tail 1

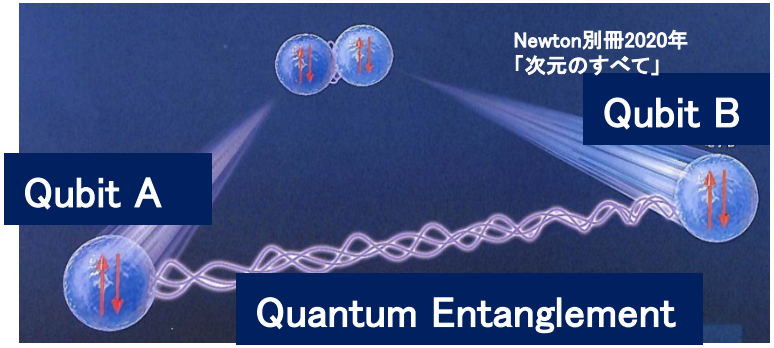
Macroscopic World

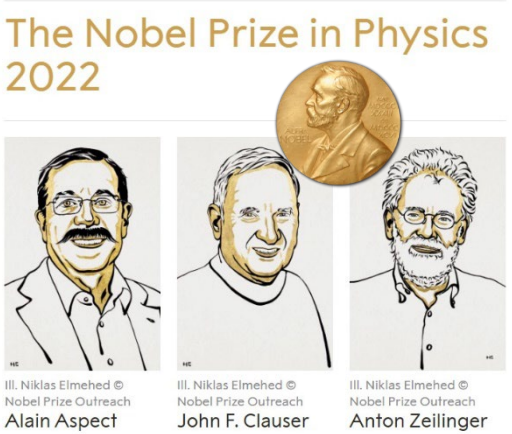
➡

$a|0\rangle+b|1\rangle$

Microscopic World

Quantum Entanglement ＝ Correlation between Qubits





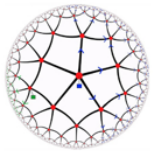
John A. Wheeler, 1990 (known for popularizing the term "Black hole")

"**It from bit** symbolizes the idea that every item of the physical world has at bottom ? a very deep bottom, in most instances ? an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-or-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe."

Holography

In US, "**It from Qubit**: Simons Collaboration on Quantum Fields, Gravity and Information"

[Stanford Univ. ,…Kyoto U., 2015-2022]



It from Qubit
Simons Collaboration

Quantum Many-body dynamics

In Japan : "**Extreme Universe Collaboration**"

[FY 2021-2025, MEXT-KAKENHI
Grant-in-Aid for Transformative Research Areas (A)]

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

Call for Publicly Offered Research (公募研究) for FY 2024-2025 will come soon !

Contents

② Quantum Entanglement

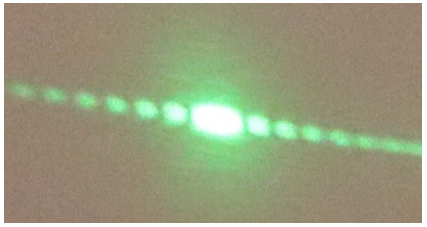
Wave–Particle Duality

What is the nature of light ?

(A) Particle (photon) [Newton,…]
➔Light travels straight.

(B) Wave [Huygens,…]
➔It shows diffraction and interference.

Laser pointer



Vernier calipers

Both are correct !

1905 Einstein's
light quantum hypothesis

Any matter such as electrons also have wave-particle duality !

Quantum Theory is based on wave-paticle duality

Particle = Wave (wave function)

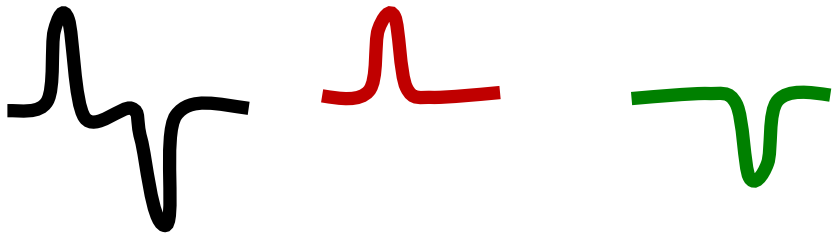
➔ We can superpose waves !

Ket

Quantum State $|\Psi\rangle = a|f\rangle + b|g\rangle$

Sum of functions

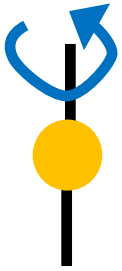
Wave Function $\Psi(x) = a\,f(x) + b\,g(x)$



Qubit

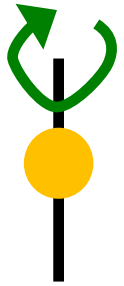
As an example of quantum state consider **electron's spin**.

Right rotation
(Up Spin)



$|0\rangle$

Left rotation
(Down Spin)



$|1\rangle$

One qubit state: $|\Psi\rangle = a|0\rangle + b|1\rangle$

Classical Computer

Classical Information

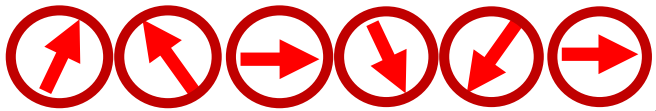
# of C info. = Bits

0 1 0 1 1 0

Quantum Computer

Quantum Information

# of Q info. = Qubits



Quantum Entanglement

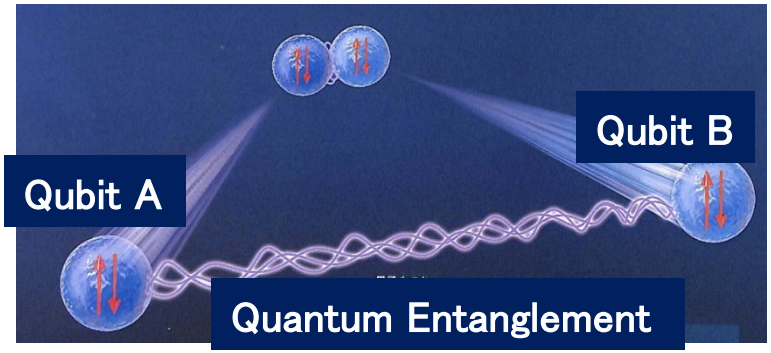
We start with a two qubit system: A and B.

Consider the Bell state :

$$|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$$

If we measure A spin, then that tells us B spin at the same time !

This correlation between A and B is Quantum Entanglement !



Though we know the state for the total system AB, the state for a subsystem is not definite.

Entanglement Entropy (EE)

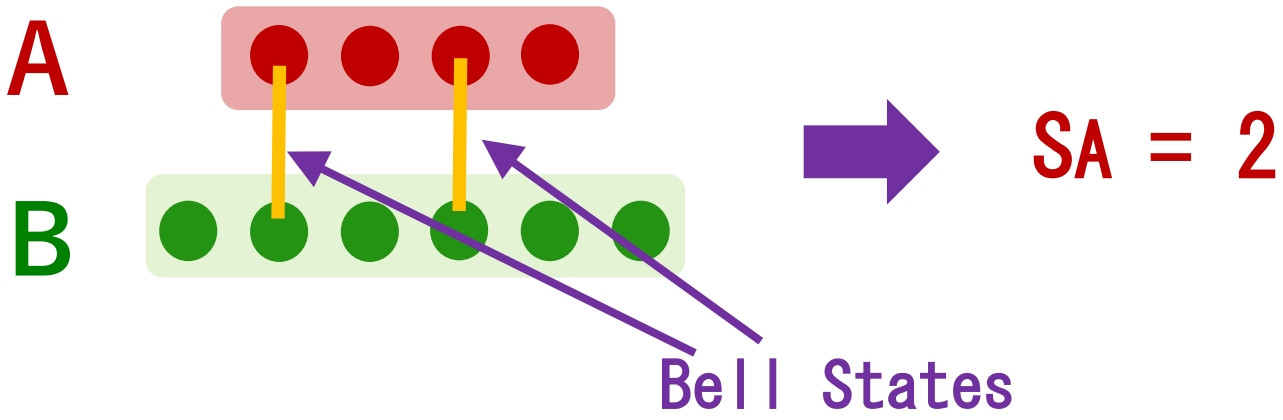
A measure of the amount of quantum entanglement

➡ **Entanglement Entropy (EE)**

Entanglement Entropy between A and B:

SA = # of Bell states between A and B

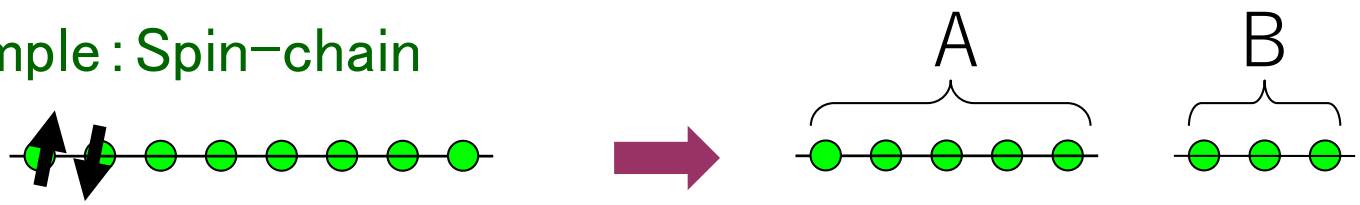
= the amount of "lost information"
if an observer can only be accessible to A.



Precise definition of Entanglement Entropy (EE)

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[\left|\Psi_{tot}\right\rangle\left\langle\Psi_{tot}\right|\right]$.

The entanglement entropy (EE) S_A is defined by

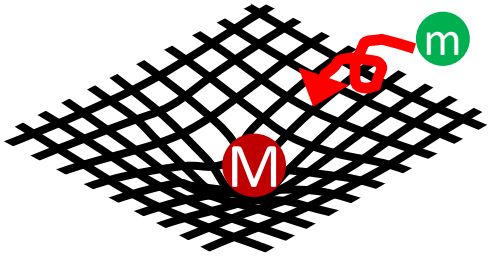
$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]} \propto$$

# of Bell Pairs between A and B

③ Black hole and Entropy

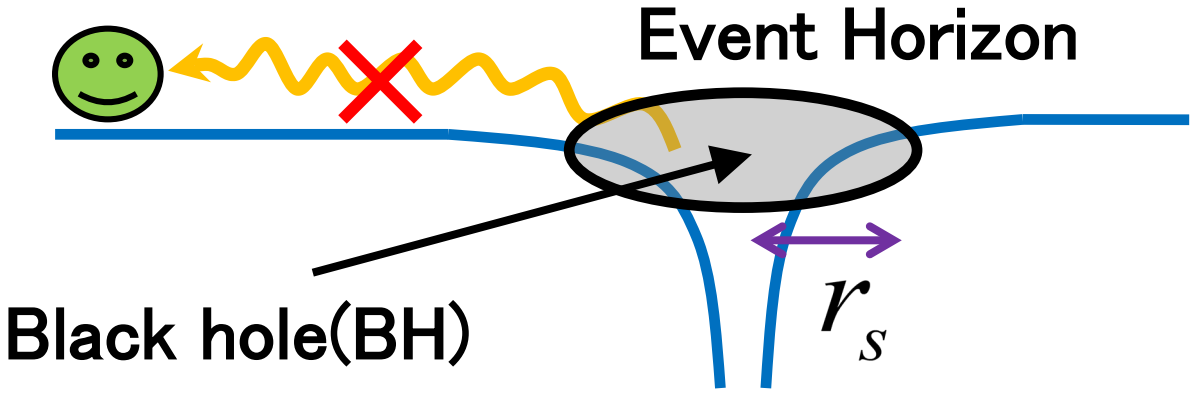
Black hole (BH)

Following general relativity, the spacetime gets distorted !



An extremely dense astrophysical object.
Due to its strong gravitational force, even light rays cannot come out from black hole.

➡ **Characteristic object in general relativity**



Schwarzschild Radius

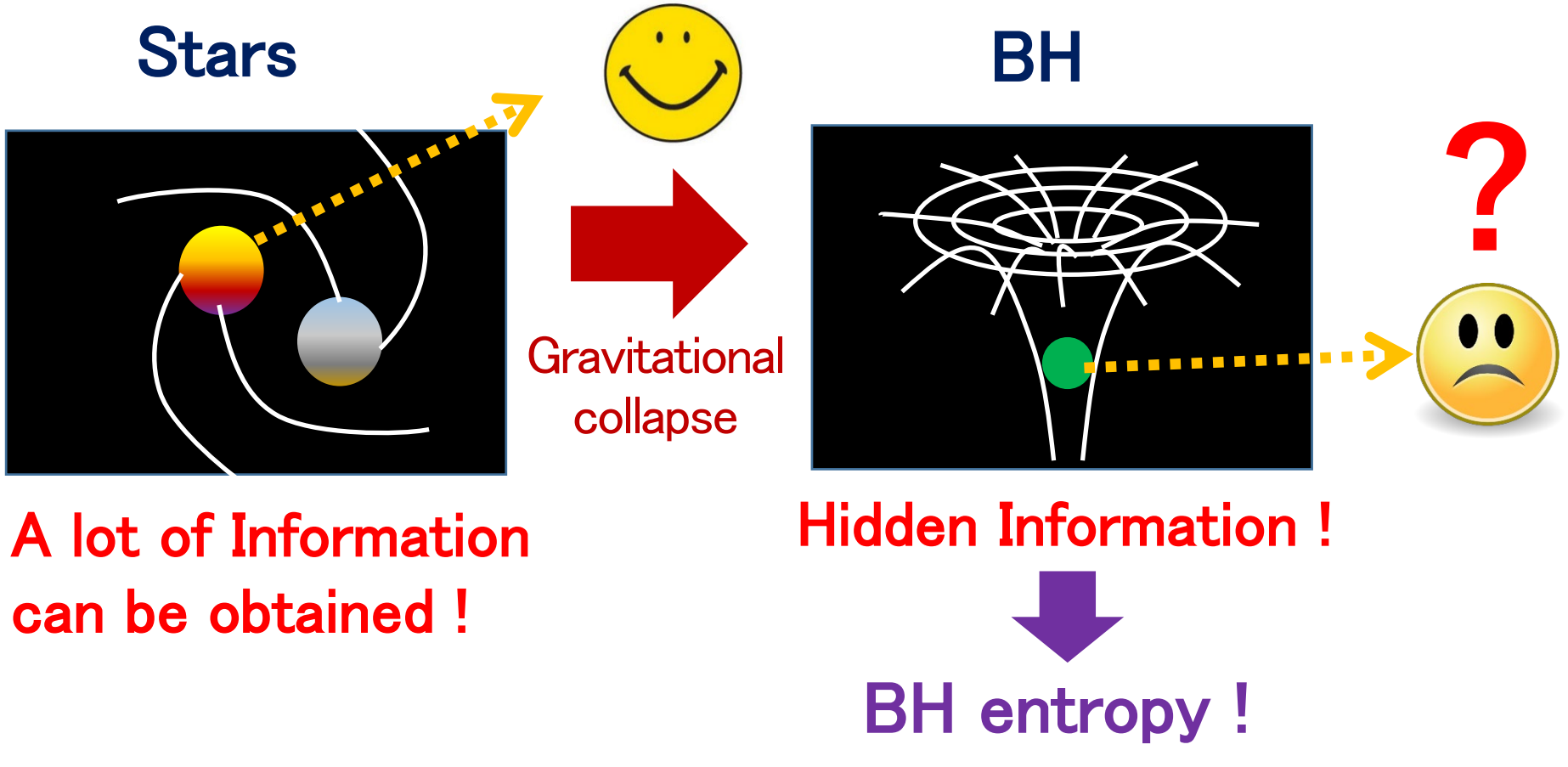
$$r_s = 2G_N M$$

M = mass of the sun
➔ r_s = 3km

M = mass of the earth
➔ r_s = 9mm

BH Entropy

After stars collapsed into a BH, outside observers cannot access the information inside the BH.



Bekentein-Hawking Formula of BH Entropy [1972-1976]

Calculations in general relativity show that a BH has the following entropy:

⇒Still mysterious !

$$S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}$$

⇒ BH thermodynamics !

A_{BH}= Surface Area of Black hole ⇒ **Geometry**

G_N=Newton constant ⇒ **Gravity**

$\hbar$=Planck constant ⇒ **Quantum Mechanics**

k_B=Boltzmann const.⇒ **Stat. Mech. , Quantum Info.**

} **Quantum Gravity!**

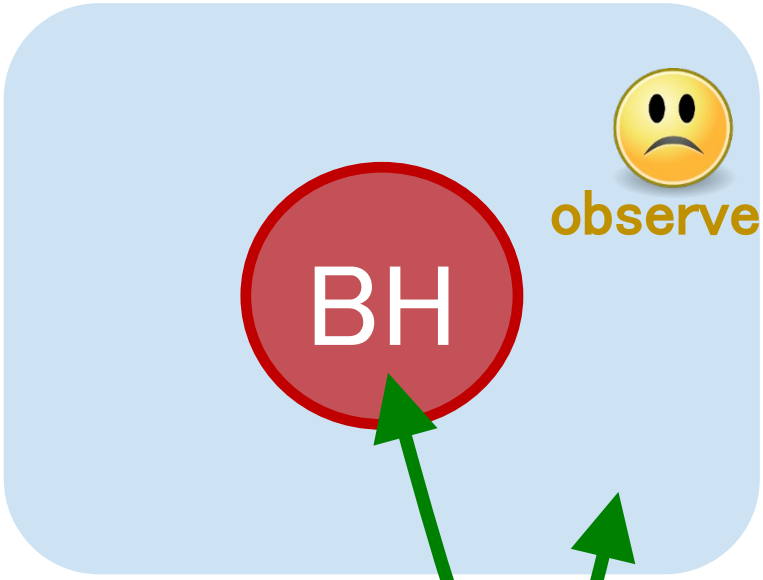
[1] BH Entropy is proportional to the **area**, not to the volume !

[2] BH has the entropy even in the **classical theory** of Gravity !

Analogy between BH and Qubits

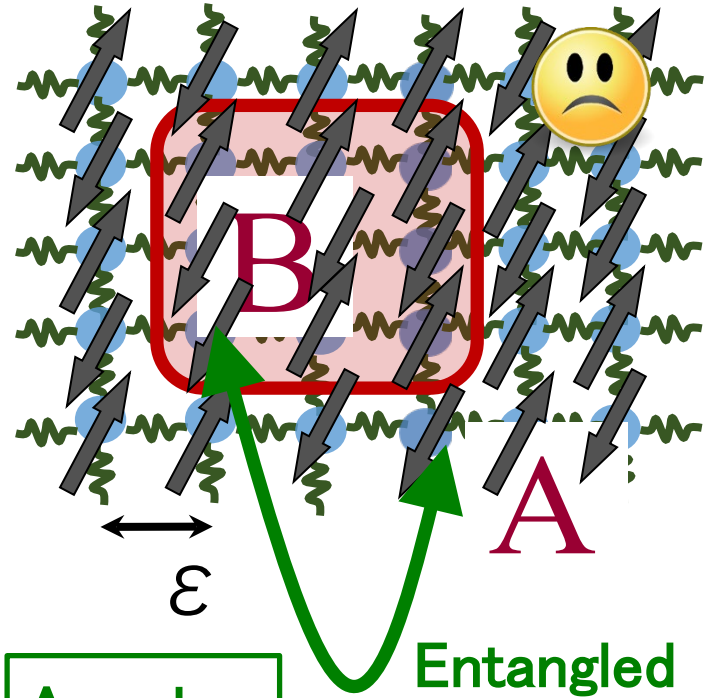
[Original motivation of studying EE in QFTs, Bombelli et.al. 1986, Srednicki 1993]

Blackhole Spacetime



Area law

Quantum Spin System



Area law

?

?

BH entropy SBH

Spacetime

Entanglement entropy SA

Matter

④ Holography and Quantum Entanglement

BH Entropy Formula

$$S_{BH} = \frac{A_{BH}}{4G_N}$$

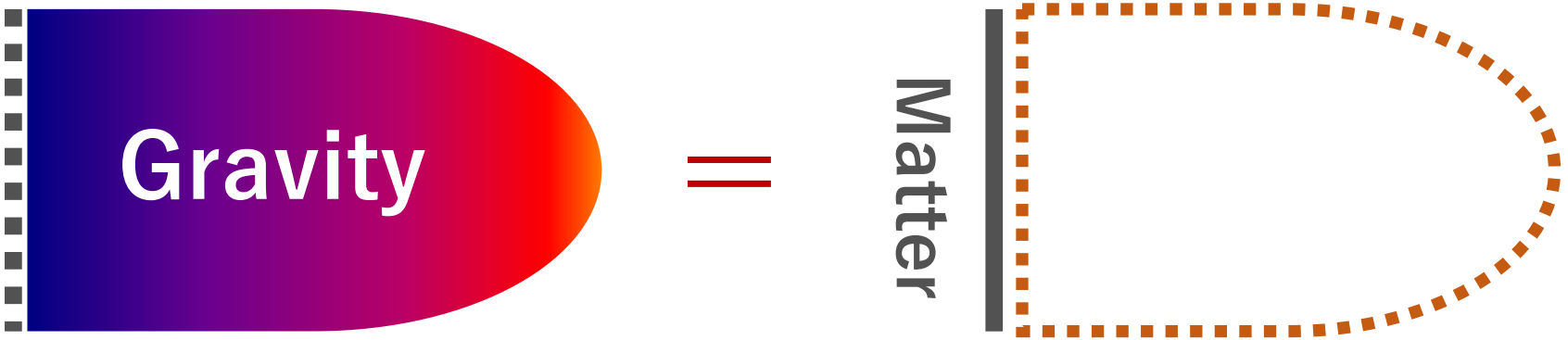
Degrees of freedom in Gravity ∝ Area

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

Gauge/Gravity Duality: best known example of holography

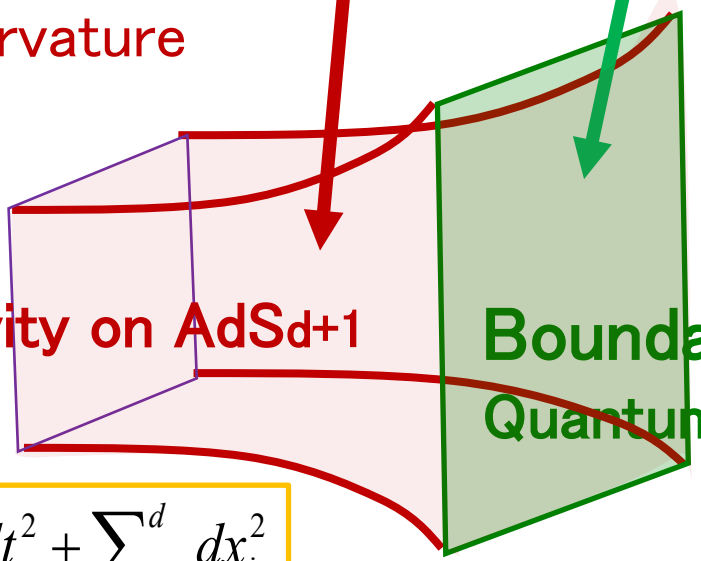
Gauge/Gravity Duality (AdS/CFT) – [Maldacena 1997]

(Quantum) Gravity on d+1 dim. Anti-de Sitter Space = **d dim. Gauge theory (or Conformal field theory)**

Anti-de Sitter space (AdS)
➔ Universe with negative curvature

Conformal Field Theory(CFT)
➔ Gapless matter at quantum critical point

(quantum field theory with only massless fields)



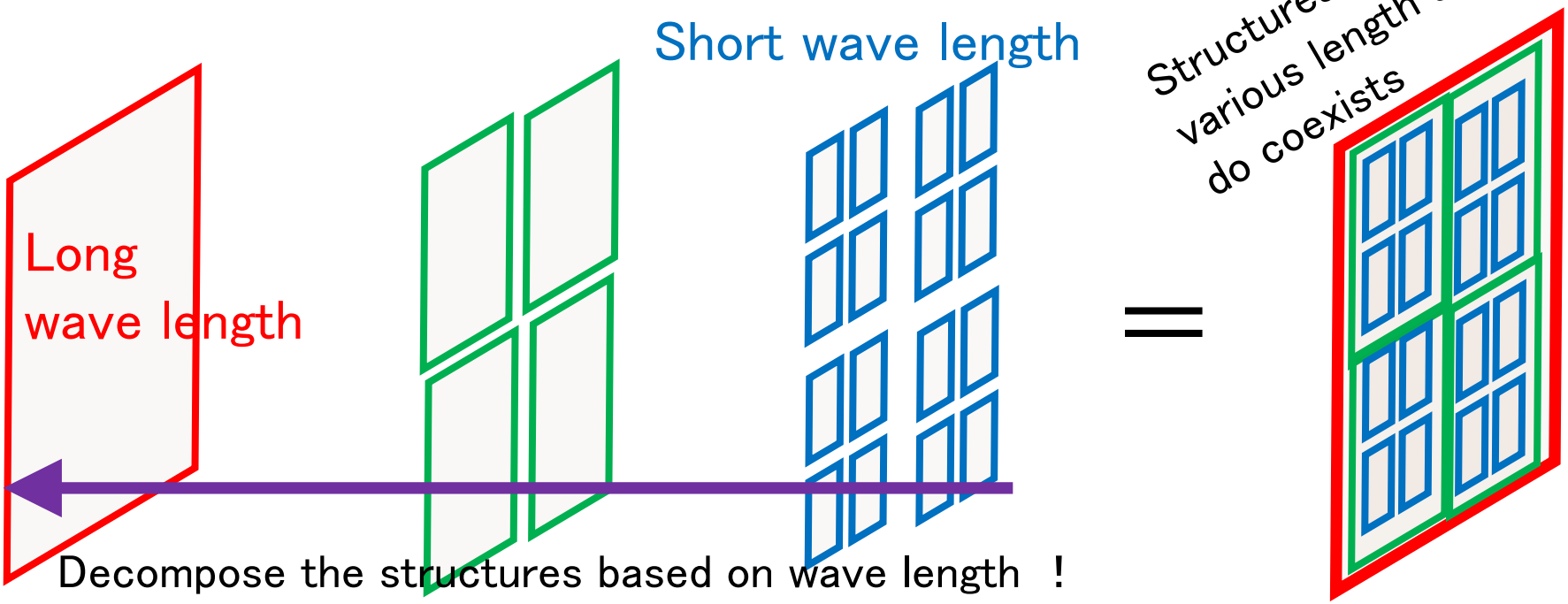
Gravity on AdS$_{d+1}$

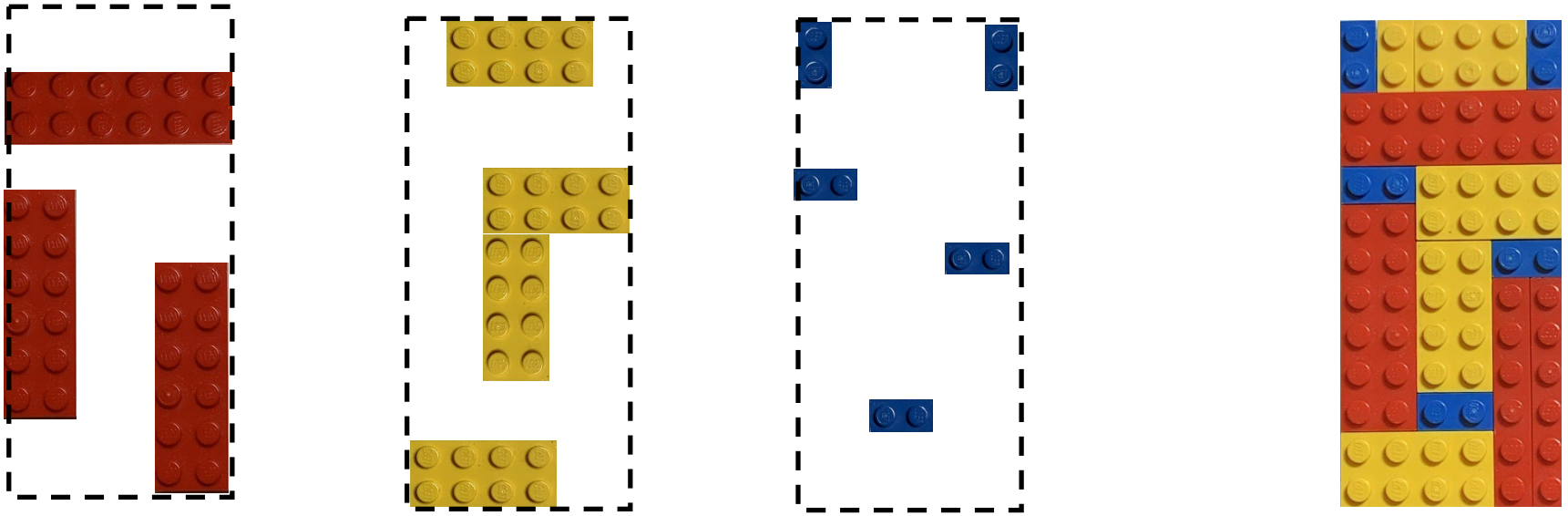
Boundary
Quantum Matter (CFT)

AdS metric

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

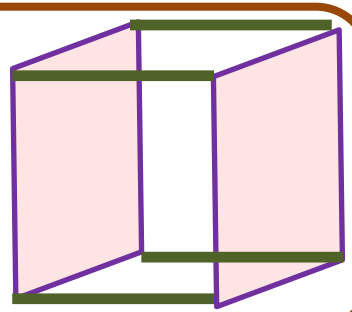
A sketch on how AdS/CFT works





Three Types of Universe　　Λ→cosmological constant

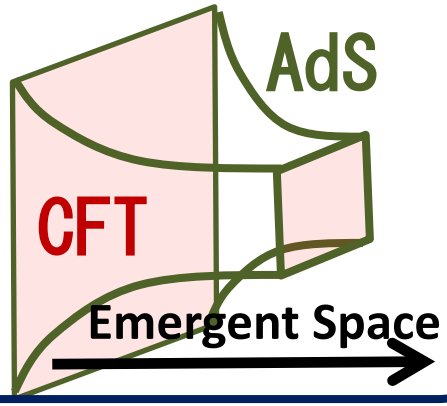
[1] Λ＝0　(Zero curvature)



➔ **Flat Spacetime (**Current Universe is almost flat)

Quantum gravity is described by string theory.

[2]　Λ＜0　（Negative curvature）

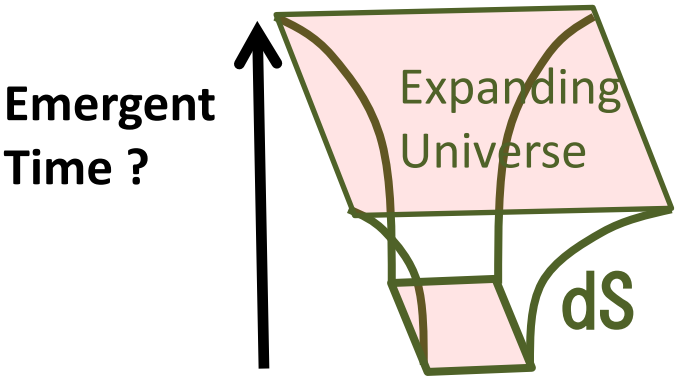


➔　**Anti de-Sitter Space (AdS)**

We can apply the AdS/CFT to describe

quantum gravity in terms of quantum matter.

[3] Λ＞0　（Positive curvature)



➔ **de-Sitter Space(dS)**

This describes the creation of Universe.

What is the holographic dual ??

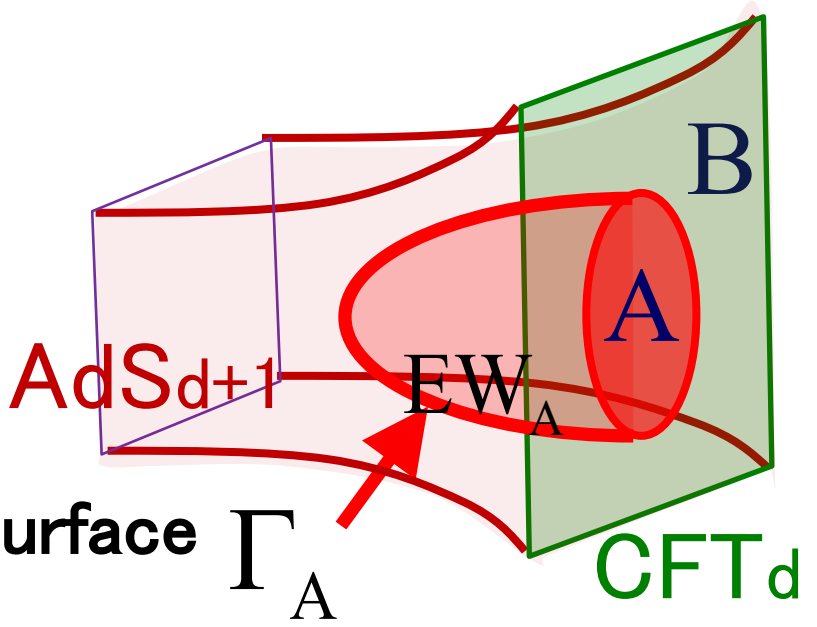
Holographic Entanglement Entropy [Ryu-Takayanagi 2006]

Γ_A = Minimal Area Surface which surrounds A in AdS

Gravity in AdS = Quantum matter on Boundary

$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4G_N}$$

Entanglement Entropy between A and B

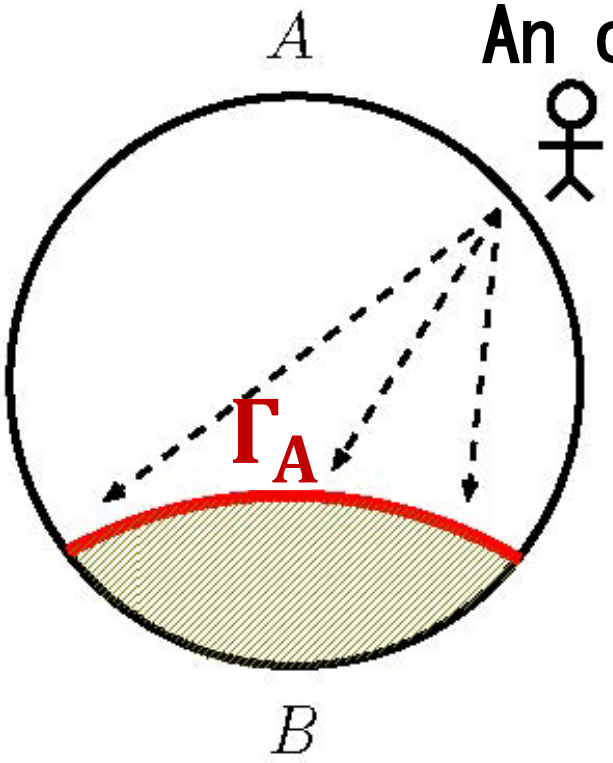


➡ A huge generalization of black hole entropy formula !

➡ Information in A is encoded in the entanglement wedge EWA !
[Czech-Karczmarek-Nogueira-Raamsdonk, Wall 2012, Headrick-Hubeny-Lawrence-Rangamani 2014….]

➡ An extremal surface area gives the time-dependent EE !
[Hubeny-Rangamani-Takayanagi 2007]

Intuitive Understanding this formula



For an observer who is not accessible to B, it looks like there is a "black hole" at Γ_A .

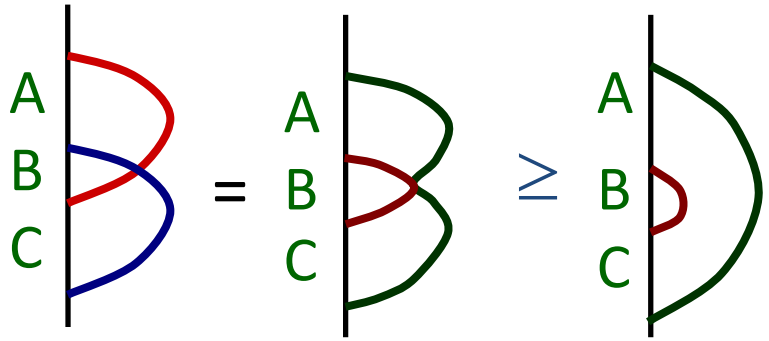
The shaded region is hidden behind the "black hole".

⇒This BH entropy is S_A !

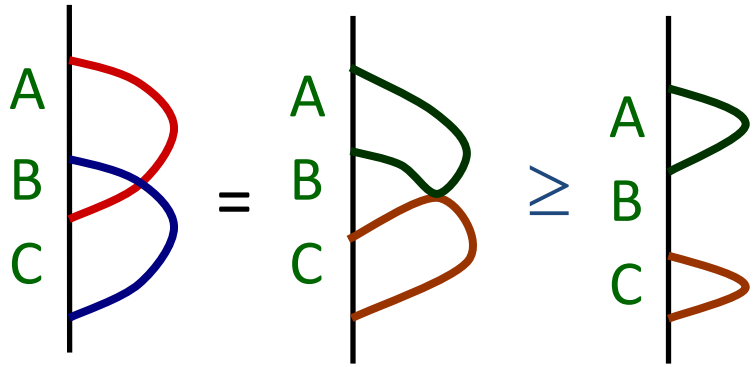
Algebraic properties in Quantum Information
⇔ **Geometric properties** in Gravity

Holographic Proof of Strong Subadditivity(SSA)

[Headrick-TT 07]



$$\Rightarrow S_{AB} + S_{BC} \geq S_{ABC} + S_B$$

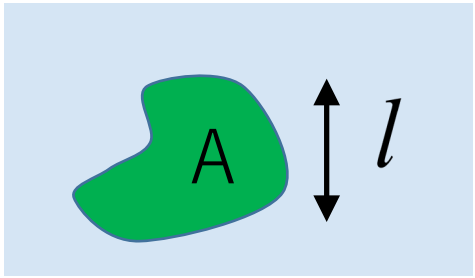


$$\Rightarrow S_{AB} + S_{BC} \geq S_A + S_C$$

$$(\text{Note}: \quad AB \equiv A \cup B)$$

"Triangle inequalities in Geometry = SSA"

General Behavior of HEE (=EE in CFT$_{d+1}$) [Ryu-TT 06,…]



$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[p_1\left(\frac{l}{\varepsilon}\right)^{d-1} + p_3\left(\frac{l}{\varepsilon}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{\varepsilon}\right) + p_d & (\text{if } d+1 = \text{odd}) \\ p_{d-2}\left(\frac{l}{\varepsilon}\right)^2 + q\log\left(\frac{l}{\varepsilon}\right) & (\text{if } d+1 = \text{even}) \end{cases} \right] ,$$

Area law divergence

where $p_1 = (d-1)^{-1}, p_3 = -(d-2)/[2(d-3)], \ldots.$

$\ldots\ldots \quad q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!!$.

A universal quantity (**F**) which characterizes **odd dim. CFT**.

Agrees with **conformal anomaly** (central charge) in **even dim. CFT**

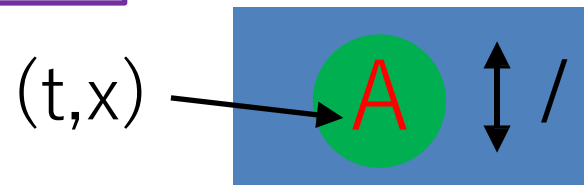
Einstein Equation from Quantum Entanglement

Firstst Law of EE ↔ First law of thermodynamics $T\Delta S = \Delta E$

$$\Delta S_A \cong \Delta E_A$$

[$E_A = -\log \rho_A$: Modular Hamiltonian]

[Casini-Huerta-Myers 13, Bhattachrrya-Nozaki-Ugajin-TT 13]



$$\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,\vec{x},l) = \langle O\rangle\langle O\rangle$$

[Nozaki-Numasawa-Prudenziati-TT 13]

[de Sitter space: de Boer-Haehl-Heller-Myers 16]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Kinetic term **C.C.** **Matter**

➡ The 1st law of EE explains the perturbative Einstein eq.

[Linear order: Raamsdonk et.al. 13, Non-linear order: Faulkner et.al 17, Sarosi-Ugajin 17]

Pseudo Entropy and Holography [Nakata-Taki-Tamaoka-Wei-TT, 2020]

Q. What is the meaning of the minimal surface area
in an Euclidean time-dependent asymptotically AdS geometry ?

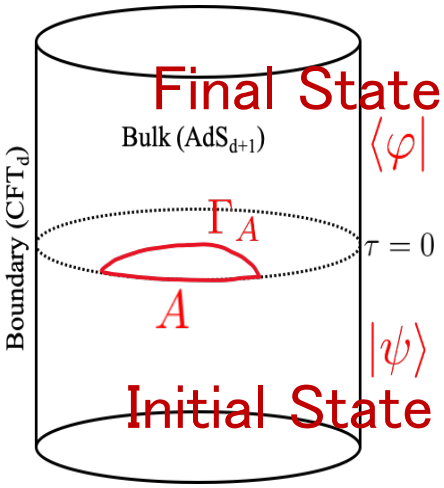
Transition Matrix: $\mathcal{T}^{\psi|\varphi} := \dfrac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}$ $\left(\mathcal{T}_A^{\psi|\varphi} := \mathrm{Tr}_{\bar{A}}\mathcal{T}^{\psi|\varphi}\right)$

Initial State　　Final State

Reduced Transition Matrix⇒**Not Hermitian** in general !

Pseudo Entropy: $S(\mathcal{T}_A^{\psi|\varphi}) = -\mathrm{Tr}\left[\mathcal{T}_A^{\psi|\varphi}\log\mathcal{T}_A^{\psi|\varphi}\right]$

↳ In general, complex valued !



Holographic Pseudo Entropy

$$S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A}\frac{\mathrm{Area}(\Gamma_A)}{4G_N}$$

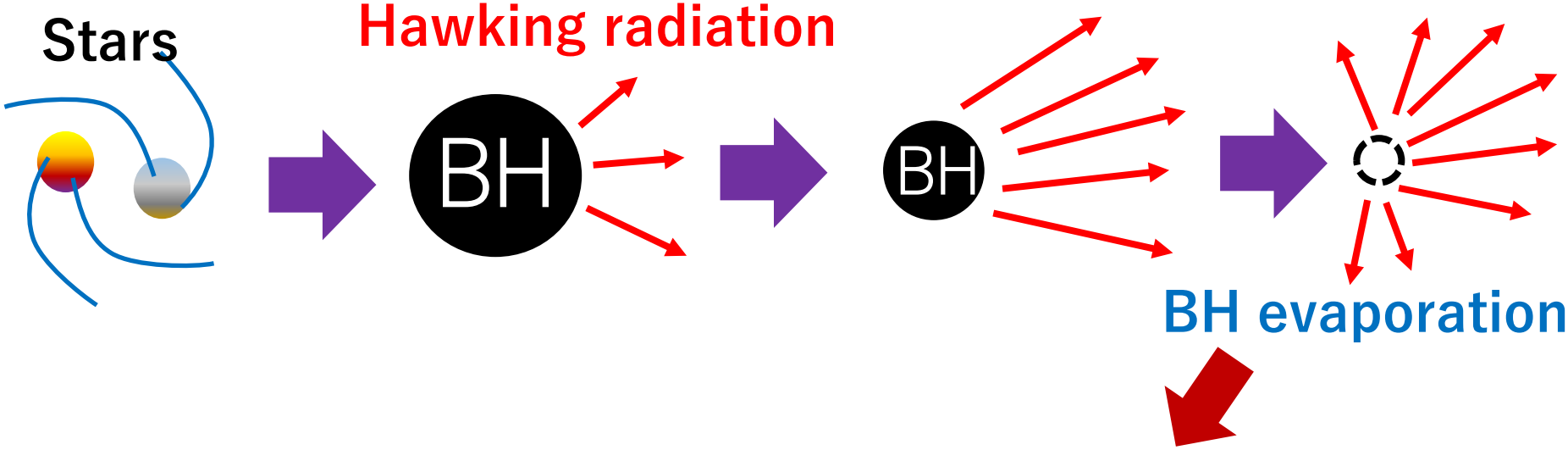
⑤ Applications to Black hole Information Problem

Black hole (BH) information Problem

A BH has a temperature and is a thermal object.

Thus, a BH radiates (Hawking radiation) and loses its energy.

Eventually it evaporates and disappears, called BH evaporation.



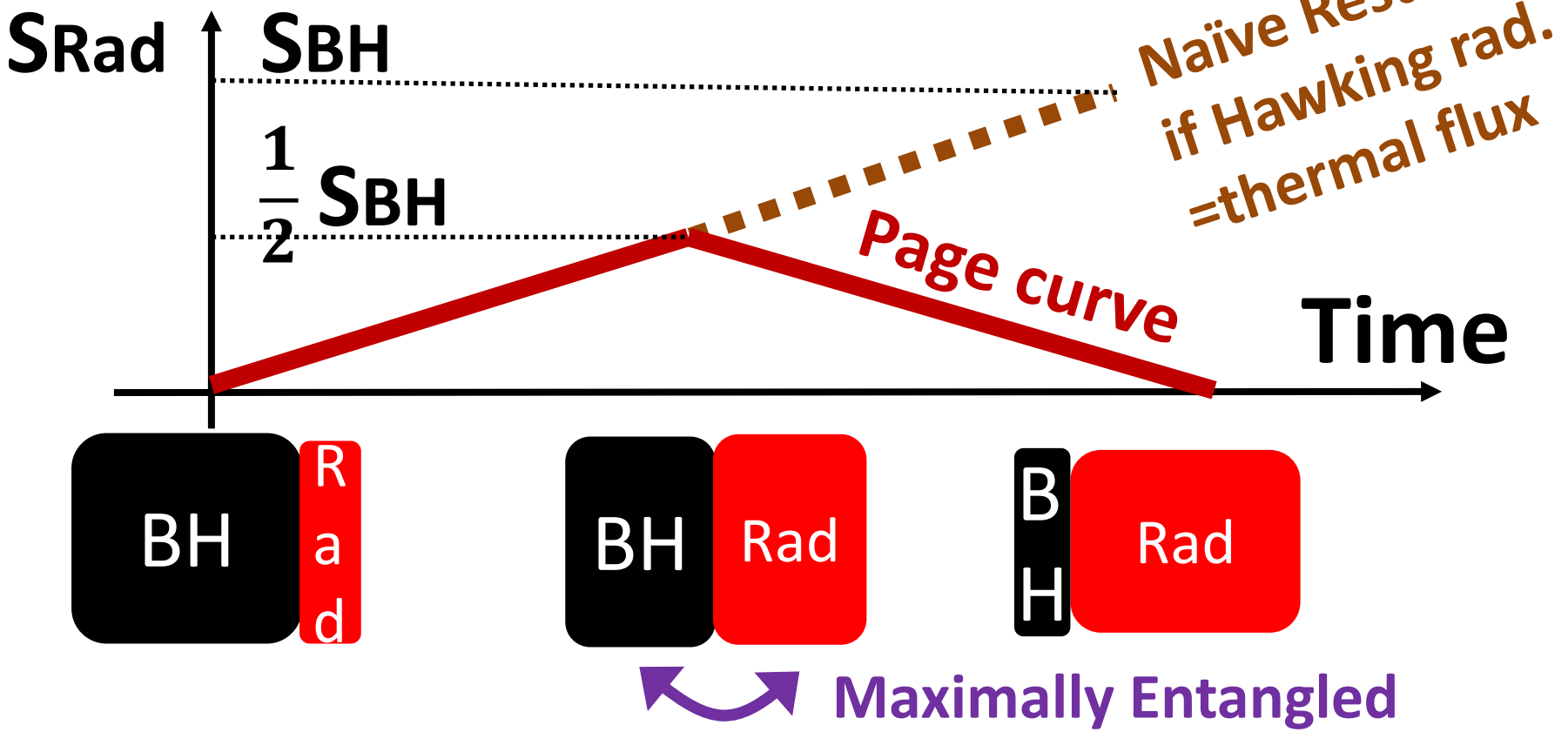
The information inside BH seems to disappear !

➔If so, this contradicts with the unitarity of quantum mechanics !

Page curve [A quantification of BH information problem]

Model: $H_{tot} = H_{BH} \otimes H_{Rad}$ $\quad |\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \in H_{tot}$

$$\rho_{\mathrm{Rad}}(\mathrm{t}) = \mathrm{Tr}_{\mathrm{BH}}\left[|\psi(t)\rangle\langle\psi(t)|\right] \Rightarrow S_{Rad}(t) = -\mathrm{Tr}\left[\rho_{\mathrm{Rad}}(\mathrm{t})\log\rho_{\mathrm{Rad}}(\mathrm{t})\right]$$



Black hole information problem and Island Formula

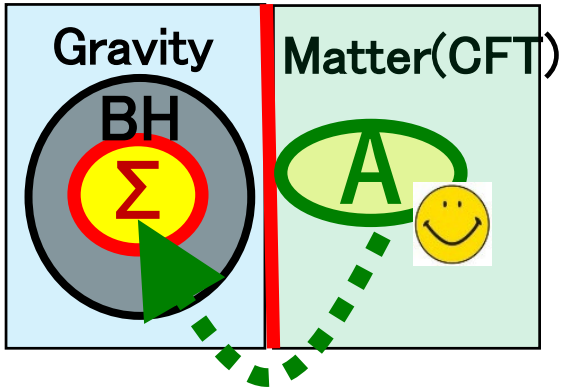
Recently, the Page curved was derive by generalizing the HEE formula to CFTs which is coupled to gravity, so called the Island formula ！

Island Formula:

[Penington 2019, Almheiri et.al. 2019]

$$S_A = \mathrm{Min}\left[\frac{\mathrm{Area}(\Sigma)}{4G_N} + S_{A\cup\Sigma}\right]$$

EE for Radiations　BH entropy　Bulk EE

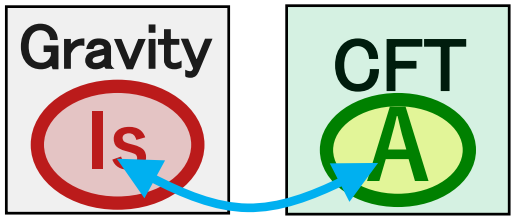


Accessible to the Island Σ

Generalization

This explains the Page curve !

➔ Unitarity of BH evaporation!



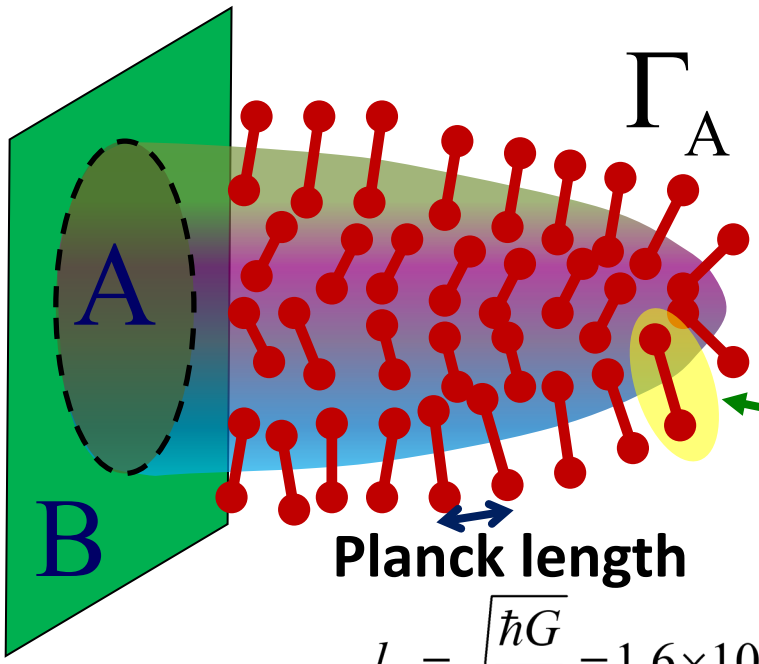
Entangled !

[Balasubramanian –Kar–Ugajin 2020–2021]

As the BH evaporation proceeds,
a secret hole (called Island) appears inside BH.
Via this secret tunnel, we recover BH information.

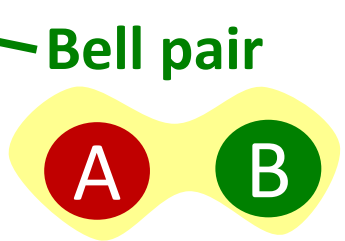
⑥ Emergence of Universe from Quantum Entanglement

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathbf{Area}(\Gamma_A)}{4l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm^2 !



Bell pair

= Planck scale mini Universe

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\times10^{-35}\,m$$

Spacetime may emerge from entangled Qubits !

➔ **Tensor Network (TN) realizes this idea !**

Tensor Network (TN)

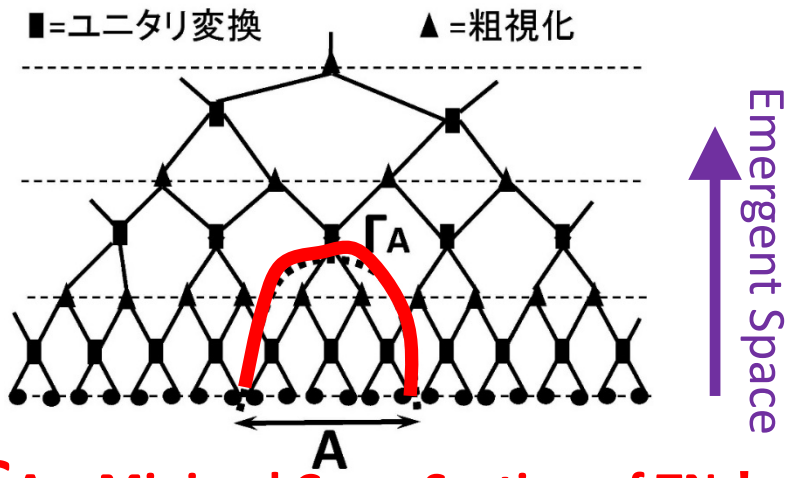
[DMRG: White 92,.. CTM: Nishino-Okunishi 96, PEPS: Verstraete-Cirac 04, ….]

TN = Graphical description of quantum states

Quantum State = Network of quantum entanglement

[Ex. 1] MERA TN [Vidal 2005]

➔ **Describe CFT vacuum**

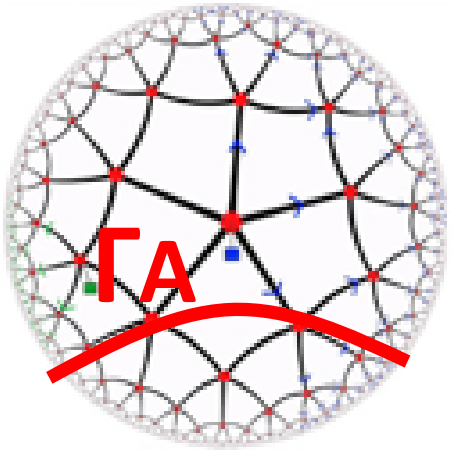


S_A = Minimal Cross Section of TN !

[Ex. 2] HaPPY model

[Patawski-Yoshida-Harlow-Preskill 2015]

➔ Use quantum error correcting code



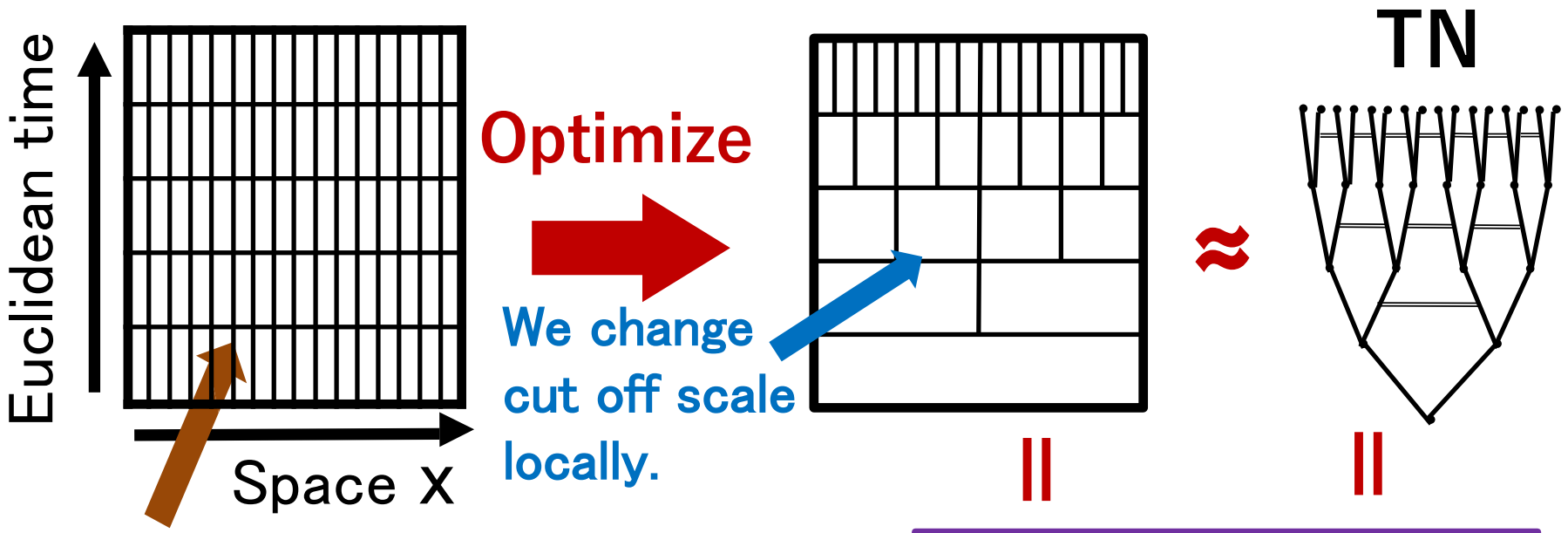
Geometric Structure of Qubits = AdS [Swingle 2009]

[Ex.3 Path-integral Optimization] [Caputa-Kundu-Miyaji-Watanabe-TT 2017]

Q. Can we describe CFT as a tensor network ? ⇒ **Path-integral**

Basic Principle

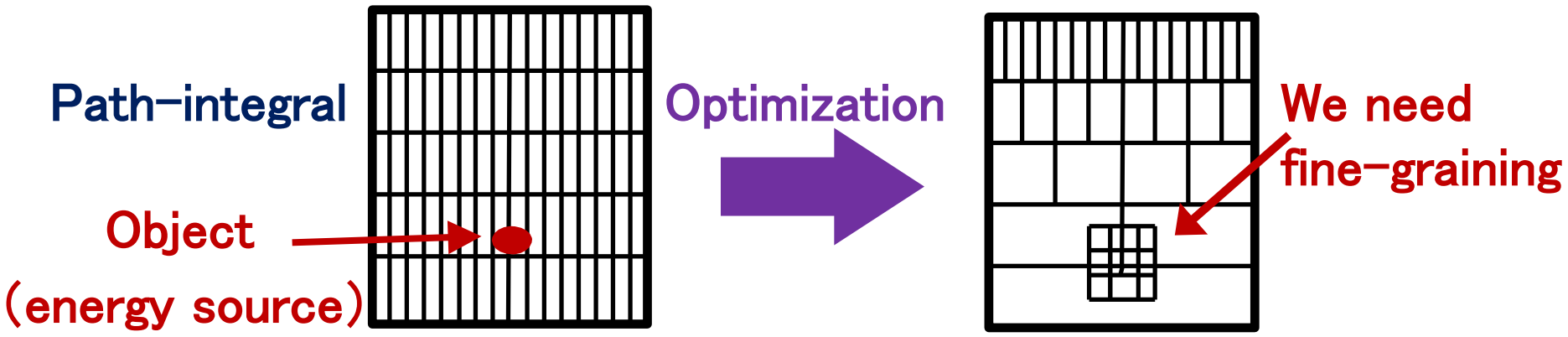
Minimize the computational cost of (discretized) path-integral.



Initially, short wave length modes can be neglected.

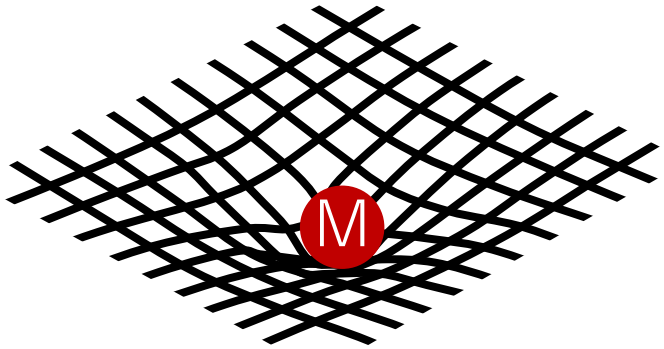
A time slice of AdS emerges !

Upshot：Minimizing computational costs leads to gravity ！



Energetic source (=information source)
distorts the spacetime

➔ The essence of general relativity !



Recent Update [Boruch-Caputa-Ge-TT 2021]

Path-integral Optimization
= Maximization of Hartle-Hawking wave function for *AdS Universe*

Gravity might be an ideal “quantum computer” ?

How to optimize path-integral (in 2 dim. CFT)

Idea: Local change of UV cut off scale = Metric change

$$ds^2 = e^{2\omega(x,z)}(dx^2 + dz^2).$$

Owing to conformal symmetry, the wave function behaves as

$$\Psi[\phi, \omega] = e^{C[\omega]} \cdot \Psi[\phi, \omega = 0]$$

Optimization ⇒ Minimize the cost C[ω] !

[C[ω]≈Computational Complexity]

In two dim. CFT, C[ω] is given by Liouville action:

$$C_{2D}[\omega] = \frac{c}{24\pi}\int dxdz\left[(\partial_x\omega)^2 + (\partial_z\omega)^2 + e^{2\omega}\right]$$

Minimization leads to AdS metric !

How our universe emerges from quantum information ?

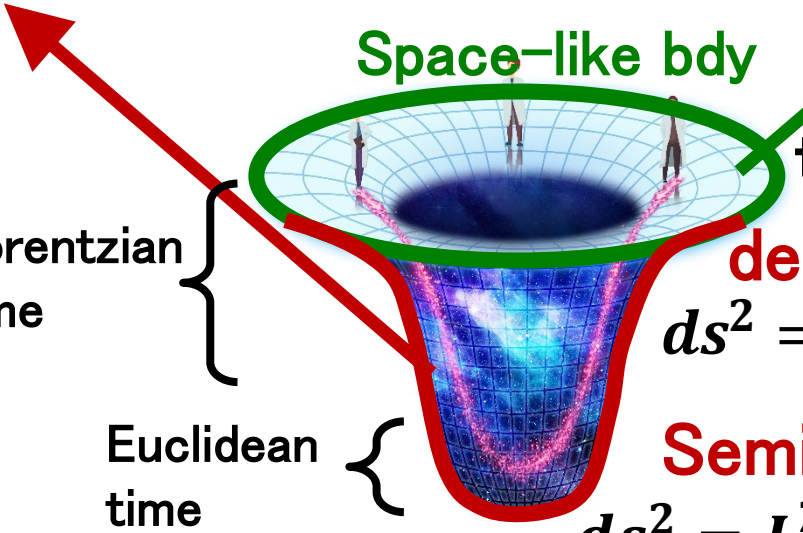
➡ How about de Sitter space instead of AdS ?

A sketch of dS/CFT [Strominger 2001, Maldacena 2002,⋯.]

Gravity in d+1 dim.
de-Sitter spacetime

Dual

Euclidean d. dim
non-unitary CFT on S^d



de Sitter

$$ds^2 = L_{ds}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{ds}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$$

Time emerges from Euclidean CFT !

Ψ[dS gravity]=Z [CFT]

Hartle-Hawking wave function

First Example for CFT dual of dS in Einstein gravity

[Hikida-Nishioka-Taki-TT, PRL 129 (2022) 4, 041601 (Editor's suggestion)]

Large c limit of SU(2)k WZW model (a 2dim. CFT)

= **Einstein Gravity on 3 dim. de Sitter** (radius L_{ds})

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}}$$

$$\Delta \approx iL_{dS} \cdot E_{dS}$$

Conformal dim.

Energy in dS

Central charge

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E}}$$

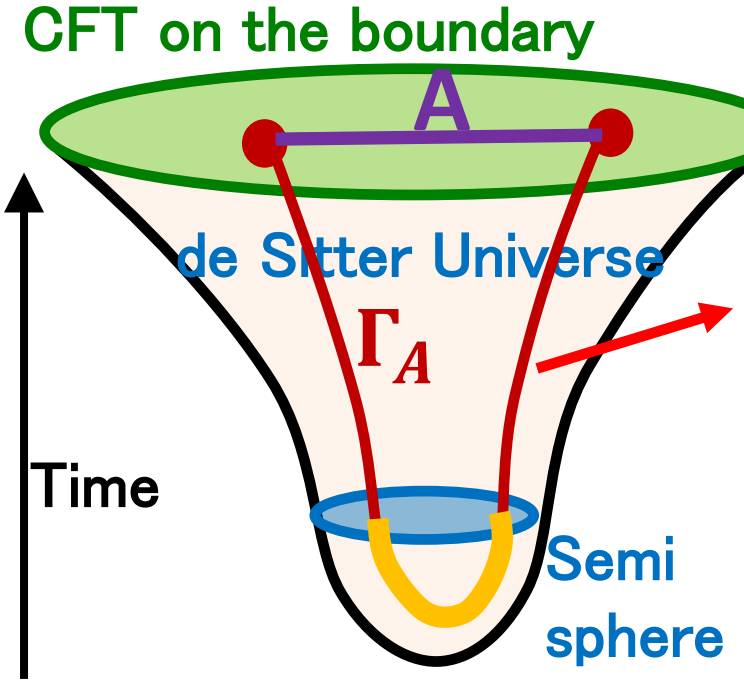
CFT partition function

De Sitter Entropy

Introduced as Viewpoint in "Physics", published by APS [2022, July]

How does TIME emerge from quantum information ?

[Doi-Harper-Mollabashi-Taki-TT 2022]



The geodesic Γ_A which gives holographic entanglement entropy S_A includes the **time-like part** !

S_A becomes complex valued !

[This is more properly regared as the **pseudo entropy** !]

Real part of S_A ➔ Emergence of space coordinate

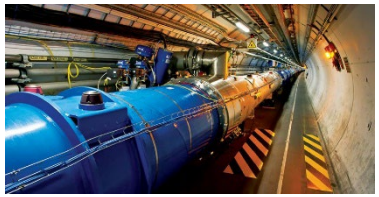
Imaginary part of S_A ➔ Emergence of time coordinate

⑦ Conclusions

Our contributions
and recent developments

Traditional Physics

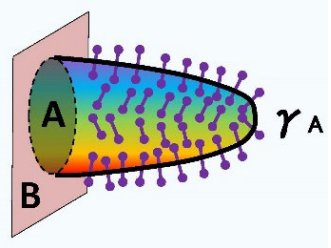
Microscopes
Accelerators



Matter = Collection of Particles

Crystallization

The formula
"Entropy＝Area"
is robust for general
spacetimes



A New direction in Physics

Holography

Is gravity the fastest "quantum computer" ?

➔New insights into quantum matter, quantum computation and quantum cryptography

Universe = Collection of Qubits

Emergence

Does gravitational spacetime emerge from qubits？

➔New approach to quantum gravity

Future problems

- An explicit proof of AdS/CFT [⇐Quantum info. ?]
- Generalization of AdS/CFT to other spacetimes
  (e.g. cosmological spacetimes such as de Sitter spaces)
- Understanding the microscopic origin of BH entropy
  purely from a gravitational (closed string) viewpoint.
- Solve BH information problem completely

  :

  :
- Quantum Gravity Origin of Our Universe

Thank you very much !