Traversable wormholes in the SYK gravity

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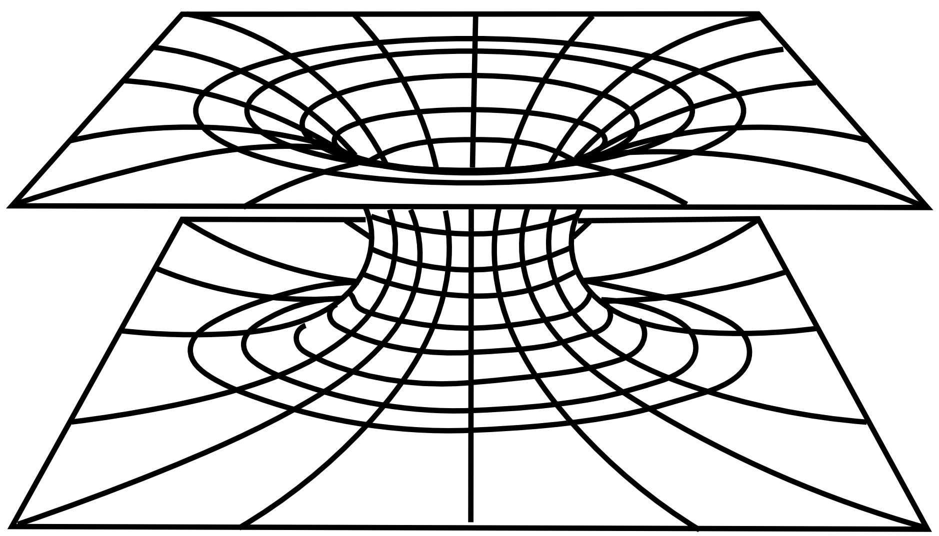
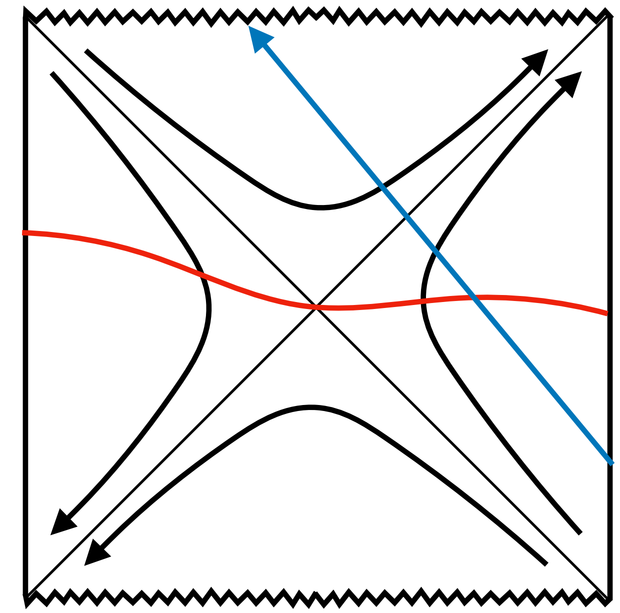
Work in progress
+Joint work in progress with S. Chapman, H.Marrochio, R. Myers, L.Q. Queimada and B.Yoshida

Traversable wormhole?



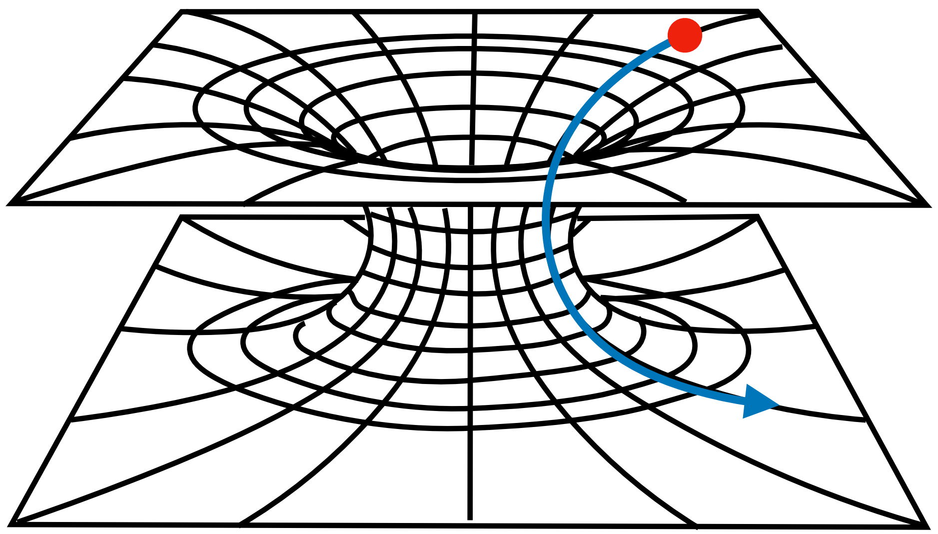


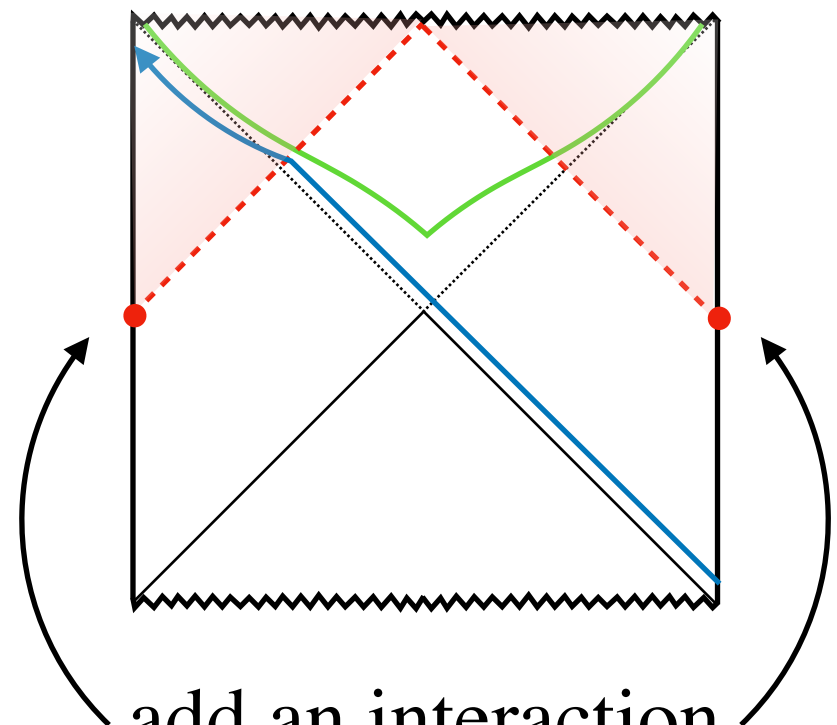
- Wormhole is a vacuum solution of Einstein's equation which on a time-slice, two interiors of black holes are connected via a tube called Einstein-Rosen bridge.

- In AdS/CFT context, Maldacena 03' discovered that a wormhole can be constructed as an entangled state between two CFTs, so called thermofield double state.

- While two asymptotic regions are connected, this type of wormhole is not traversable; any time-like curve starting from one boundary hits the black hole singularity and cannot pass through the Einstein-Rosen bridge.

Traversable wormhole?





- It is well known that to make a wormhole traversable, one need to violate the null energy condition and introduce a negative energy.

$$\int dx^{\pm} T_{\pm\pm} < 0$$

- That seems to be unphysical, but Gao-Jafferis-Wall discovered that adding certain interactions between two boundaries can result in a stress tensor with a negative energy rendering a wormhole traversable by the backreaction.

Traversable wormhole in the two dimensional gravity?

- They calculated the stress tensor in BTZ black hole and checked that it can take negative values by the interaction, but the geometrical structure of the traversable wormhole is yet unclear.

- In this talk, we investigate traversable wormholes in a two dimensional dilaton gravity theory: Jackiw-Teitelboim gravity, where the backreaction can be calculated exactly. *[Almheiri-Polchinski]*
  It effectively describes SYK model, which has been actively studied recently.

  The spacetime structure?

  Why negative energy is necessary?

  Relation between the initial mass and final mass (temperature, entropy)?

- Complexity probes geometric structure behind the horizon and it may be related to the smoothness of the horizon. *[Susskind]*
  Complexity of traversable wormhole?

Jackiw-Teitelboim model

- Many situations where AdS_2 geometry arises by the dimensional reduction from higher dimensional near extremal black holes can be captured by the following two dimensional action

$$I = \frac{\phi_0}{16\pi G_N}\int d^2x\sqrt{-g}R + \frac{1}{16\pi G_N}\int d^2x\sqrt{-g}\phi(R+2) + I_{\text{matter}} + \cdots$$

- Consider the situation where $\phi_0 \gg \phi$ and neglect higher order term in ϕ
- If AdS_2 arises from the near horizon geometry of a four dimensional near extremal black hole, $\Phi^2 \equiv \phi_0 + \phi$ corresponds to the area of two sphere while ϕ_0 is the area of the two sphere of extremal black hole.
- The first term is purely topological and the second term is called Jackiw-Teitelboim action

$$\boxed{I_{JT} = \frac{1}{16\pi G_N}\int d^2x\sqrt{-g}\phi(R+2) + (\text{boundary term})}$$

Solutions of JT model without a matter

- The variation with respect to ϕ yields

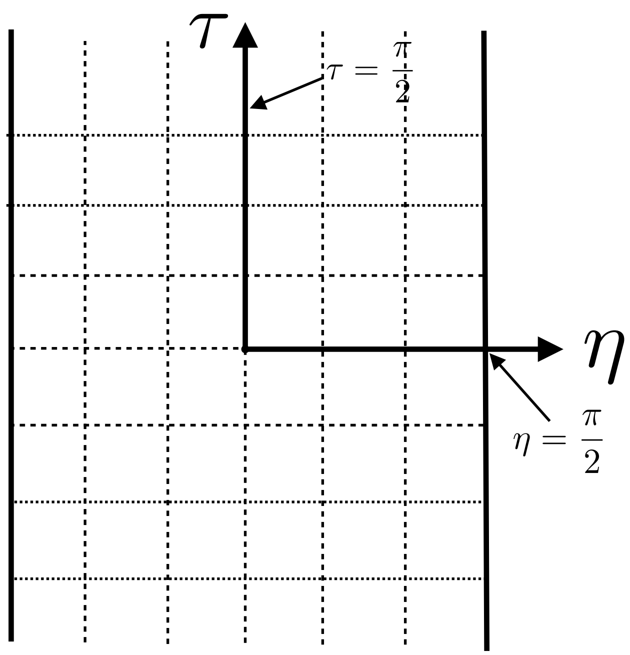
$$R + 2 = 0$$

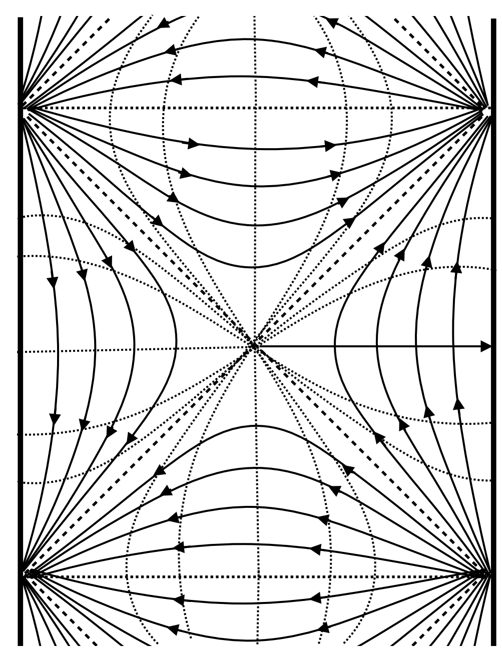
$\rightarrow$ The Einstein-Hilbert action has an AdS$_2$ solution.

$$ds^2 = \frac{-d\tau^2 + d\eta^2}{\cos^2\eta} = -(r^2-\mu)dt^2 + \frac{1}{r^2-\mu}dr^2$$

$$= -\mu \sinh^2\rho dt^2 + d\rho^2 \qquad r = \sqrt{\mu}\cosh\rho$$

The global AdS$_2$ AdS$_2$ Rindler





$$r = \sqrt{\mu}\cosh\rho$$

- The variation with respect to the metric yields the Einstein's equation

$$(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu})\phi = \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \nabla^2 \phi$$

- The following geometric property holds in two dimension

$$R_{\rho\mu\sigma\nu} = \frac{1}{2}(g_{\mu\nu} g_{\rho\sigma} - g_{\rho\nu} g_{\mu\sigma}) R \quad \to \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

- The Einstein's equation reduces to

$$\boxed{\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \nabla^2 \phi + g_{\mu\nu} \phi = 0}$$

- If we fix the AdS_2 metric, this equation determines the dilaton profile

$$\phi = Q \cdot Y$$

[Almheiri-Polchinski]

where Y is the embedding coordinate of AdS2 and Q is an arbitrary vector

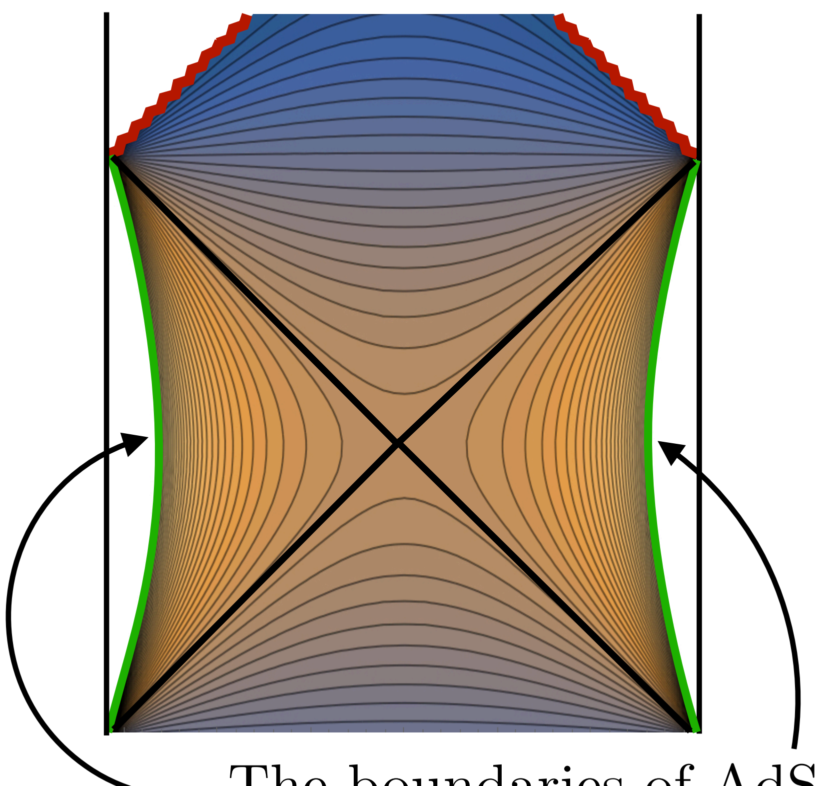
$$Y = \begin{pmatrix} \frac{\cos\tau}{\cos\eta} \\ \frac{\sin\tau}{\cos\eta} \\ \tan\eta \end{pmatrix} = \begin{pmatrix} \cosh\rho \\ \sinh\rho \sinh\sqrt{\mu} t \\ \sinh\rho \cosh\sqrt{\mu} t \end{pmatrix}$$

Black hole solutions of JT model

- For simplicity, we take a solution given by a charge $Q = \phi_0\sqrt{\mu}\,(1,0,0)$

$$\phi = \phi_0\sqrt{\mu}\frac{\cos\tau}{\cos\eta} = \phi_0\sqrt{\mu}\cosh\rho$$

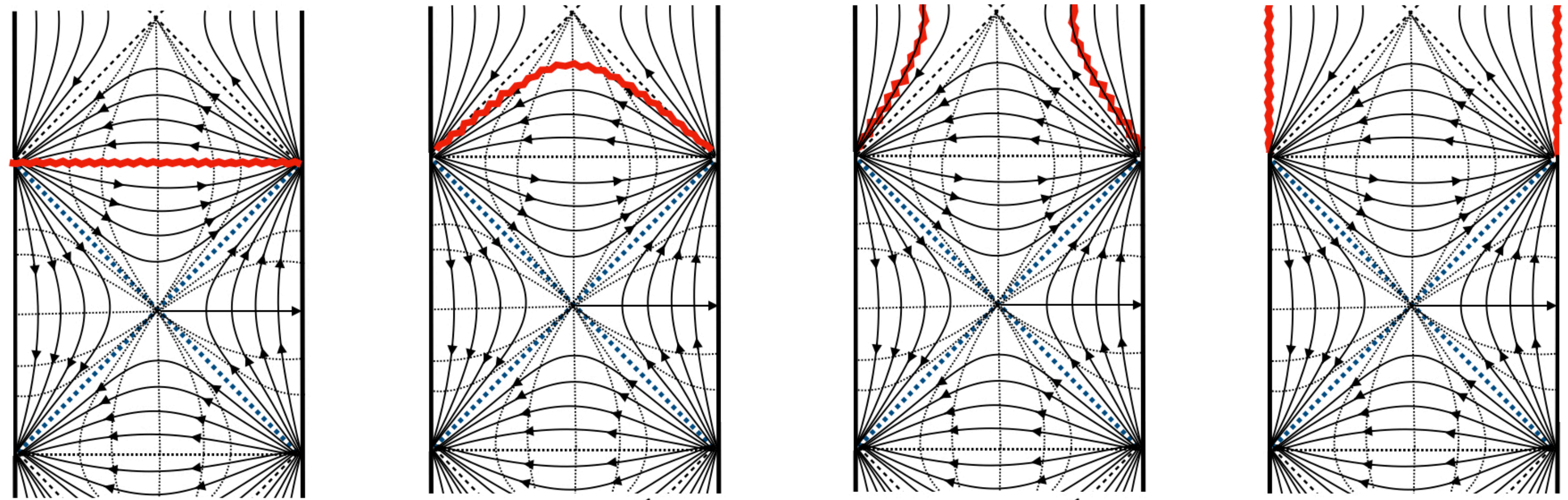
- The dilaton value grows infinitely as it approaches $\rho \to \infty$.
  We consider the near extremal situation where $\phi_0 \gg \phi$, thus we should cut off the spacetime at $\phi = \phi_b \ll \phi_0$ which determines the position of the boundary of AdS_2. *[Maldacena-Stanford-Yang 2016]*



- This dilaton profile gives a black hole solution of JT model with the mass, temperature and entropy

$$M = \frac{\mu}{8\pi G_N}, \quad T = \frac{\sqrt{\mu}}{2\pi}, \quad S = \frac{\phi_0}{4G_N}(1+\sqrt{\mu})$$

where $S_0 = \dfrac{\phi_0}{4G_N}$ is the extremal entropy and the linear dependence on the temperature of S may be interpreted as an entropy of near extremal BHs.



- If the dilaton value goes to zero, the effective Newton coupling G_N/Φ^2 becomes infinitely large and the classical approximation will break down. At this point, the radius of S^2 in four dimensional black hole shrinks to zero.

  $\rightarrow$ This point corresponds to the black hole singularity

$$\Phi^2 = \phi_0 \left(1 + \sqrt{\mu} \frac{\cos\tau}{\cos\eta} \right) = 0 \quad \Leftrightarrow \quad \frac{\cos(\tau - \pi)}{\cos\eta} = \frac{1}{\sqrt{\mu}}$$

- It can be either timelike or spacelike according to the temperature (If we take $\phi_b \ll \phi_0$, we should consider black holes with $T \ll \frac{1}{2\pi}$)
- The black hole horizon is determined by the following condition

$$\partial_\pm \Phi^2 = 0 \Leftrightarrow \tau \pm \eta = 0$$

Jackiw-Teitelboim model with a matter

- Next we consider the matter contribution to the JT action

$$I_{\mathrm{JT}} = \frac{1}{16\pi G_N}\int d^2x\sqrt{-g}\phi(R+2) + I_{\mathrm{matter}}[g,\chi]$$

- We expect that the shockwave of the energy momentum tensor of the matter $T^{\chi}_{\mu\nu}(x)$ modifies the dilaton profile and the spacetime structure.

$$\boxed{\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\nabla^2\phi + g_{\mu\nu}\phi + 8\pi G_N T^{\chi}_{\mu\nu}(x) = 0}$$

- Solve this equation with respect to ϕ in the fixed global AdS_2 metric
- The equation can be written with a light-cone coordinate $x^\pm = \tau \pm \eta$

$$\partial_\pm\partial_\pm f + \frac{1}{4}f = -8\pi G_N \cos\left(\frac{x^+ - x^-}{2}\right) T^{\chi}_{\pm\pm}(x^+, x^-). \quad \phi = \frac{f(x^+, x^-)}{\cos\eta}$$

- The heat kernel is expressed as

$$G(x^\pm) = 2\sin\left(\frac{x^\pm - x'^\pm}{2}\right)\theta(x^\pm - x'^\pm)$$

- The general solution to the equation can be written as a sum

$$\phi = \phi^{(0)} + \phi^{+} + \phi^{-}$$

where $\phi^{(0)}$ is the original dilaton profile and $\phi^{\pm}$ are written as the integral

$$\phi^{\pm} = \frac{16\pi G_N}{\cos\eta} \int_{-\infty}^{x^{\pm}} \sin\left(\frac{x'^{\pm} - x^{\pm}}{2}\right) \cos\left(\frac{x'^{\pm} - x^{\mp}}{2}\right) T^{\chi}_{\pm\pm}(x'^{\pm})$$

- For simplicity, we take a matter energy-momentum tensor of the form

$$T^{\chi}_{\pm\pm}(x'^{\pm}) = E\delta(x'^{\pm} - x_0^{\pm})$$

which represents shock waves localized at $x^{\pm} = x_0^{\pm}$

- The solution can be written as

$$\boxed{\phi^{\pm} = \frac{8\pi G_N E}{\cos\eta} \theta(x^{\pm} - x_0^{\pm})[\cos(\tau - \theta) \mp \sin\eta]}$$

where $\theta = x_0^{\pm} - \dfrac{\pi}{2}$

Continuity across the shock wave

- Notice that on the shock wave $x^{\pm} = x_0^{\pm} \Leftrightarrow \tau \pm \eta - \frac{\pi}{2} = \theta$

$$\phi^{\pm} = \frac{8\pi G_N E}{\cos\eta} \theta(x^{\pm} - x_0^{\pm})[\cos(\mp\eta + \frac{\pi}{2}) \mp \sin\eta] = 0$$

$\rightarrow$ Across the shock wave, the dilaton profile varies continuously.
For the four dimensional black holes, it corresponds to the fact that the radius of the two sphere is continuous across the shock wave.
[c.f. Vaidya geometry in higher dimensions]

The solution for a single shock wave

- We first analyze the solution for a single shock wave along $x^+ = \frac{\pi}{2}$ emitted from the right boundary.

$$\boxed{\Phi^2 = \phi_0 \left(1 + \sqrt{\mu}\frac{\cos\tau}{\cos\eta}\right) + 8\pi G_N E \theta(x^+ - \frac{\pi}{2}) \left(\frac{\cos\tau}{\cos\eta} - \tan\eta\right)}$$

- The horizon which is originally located at $x^- = 0 \Leftrightarrow \eta = \tau$ is shifted according to the following equation

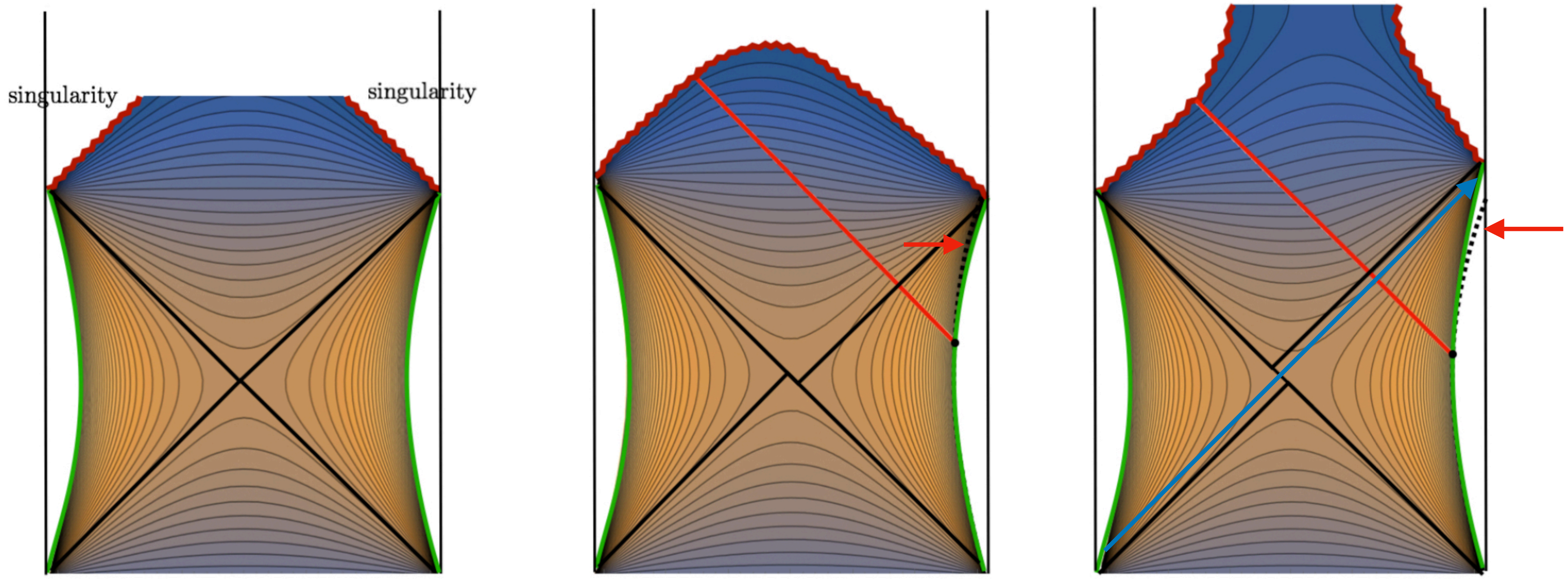
$$\partial_+ \Phi^2 = \frac{\phi_0\sqrt{\mu}}{\cos^2\eta}[\sin(\eta - \tau) + \mathcal{E}(\sin(\eta-\tau) - 1)] = 0$$

$$\Leftrightarrow \sin(\tau - \eta) = -\frac{\mathcal{E}}{1+\mathcal{E}} \qquad \mathcal{E} \equiv \frac{8\pi G_N E}{\phi_0\sqrt{\mu}} = \frac{4G_N}{\phi_0}\frac{E}{T}$$

- When $\mathcal{E} > 0$, the horizon shifts in the negative x^- direction

  When $\mathcal{E} < 0$, the horizon shifts in the positive x^- direction

  (If $\mathcal{E} < -1/2$, the horizon vanishes from the region $0 < \tau < \frac{\pi}{2}$)



$E > 0$ $E < 0$

- The AdS_2 boundary is also backreacted by the shock wave

The boundary is defined as a spacetime cut off at $\phi = \phi_b \equiv \phi_0\sqrt{\mu}\alpha$

$$\Phi^2 = \phi_0\sqrt{\mu}\left(\frac{1}{\sqrt{\mu}} + \frac{\cos\tau}{\cos\eta} + \mathcal{E}\left(\frac{\cos\tau}{\cos\eta} - \tan\eta\right)\right) = \phi_0\sqrt{\mu}\left(\frac{1}{\sqrt{\mu}} + \alpha\right)$$

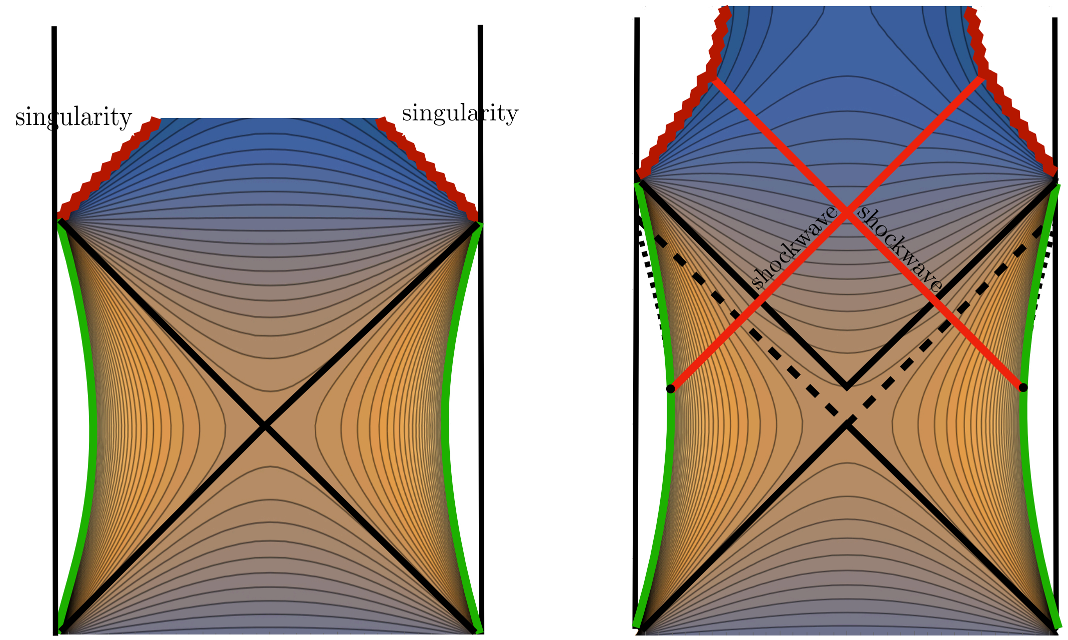
$$\Leftrightarrow \frac{\cos\tau}{\cos(\eta+\eta_s)} = \frac{\sqrt{\mathcal{E}^2+\alpha^2}}{\mathcal{E}+1} \equiv \alpha_s$$

where we defined the shift in η

$$\cos\eta_s = \frac{\alpha}{\sqrt{\mathcal{E}^2+\alpha^2}}, \quad \sin\eta_s = -\frac{\mathcal{E}}{\sqrt{\mathcal{E}^2+\alpha^2}}$$

- When $\mathcal{E} > 0$, the boundary shifts outwards, when $\mathcal{E} < 0$ it shifts inwards

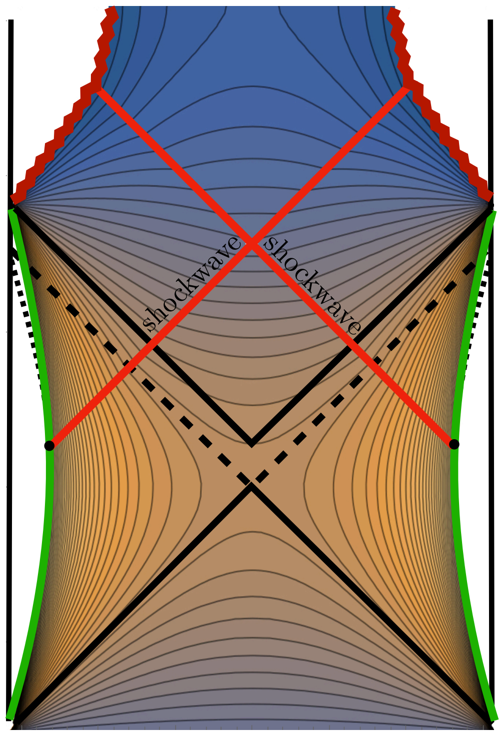
Traversable wormhole created by double shock waves



- We consider the traversable wormholes created by two negative energy shock waves emitted from the both boundaries, which are similar to ones created by the double trace deformation of the thermofield double state

$$\Phi^2 = \phi_0\sqrt{\mu}\left(\frac{1}{\sqrt{\mu}} + \frac{\cos\tau}{\cos\eta}\right)$$
$$+\phi_0\sqrt{\mu}\left(\mathcal{E}\theta(x^+ - \frac{\pi}{2})\left(\frac{\cos\tau}{\cos\eta} - \tan\eta\right) + \mathcal{E}\theta(x^- - \frac{\pi}{2})\left(\frac{\cos\tau}{\cos\eta} + \tan\eta\right)\right)$$

- The region before two shock waves collide, the situation is the same as the single shock wave case



- After the two shock waves collide, the dilaton profile becomes

$$\Phi^2 = \phi_0 \left(1 + \sqrt{\mu}(1 + 2\mathcal{E}) \frac{\cos \tau}{\cos \eta} \right)$$

This is the same as the dilaton profile of the original black hole, but whose temperature becomes lower!

$$T = \frac{\sqrt{\mu}}{2\pi} \to T' = \frac{\sqrt{\mu}}{2\pi}(1 + 2\mathcal{E}) < T$$

If $E \sim \mathcal{O}(1)$, the temperature decreases $\mathcal{O}(G_N)$ and mass & entropy decrease $\mathcal{O}(1)$

$$0 > \Delta T \sim \mathcal{O}(G_N)$$

$$0 > \Delta M \sim \mathcal{O}(1)$$

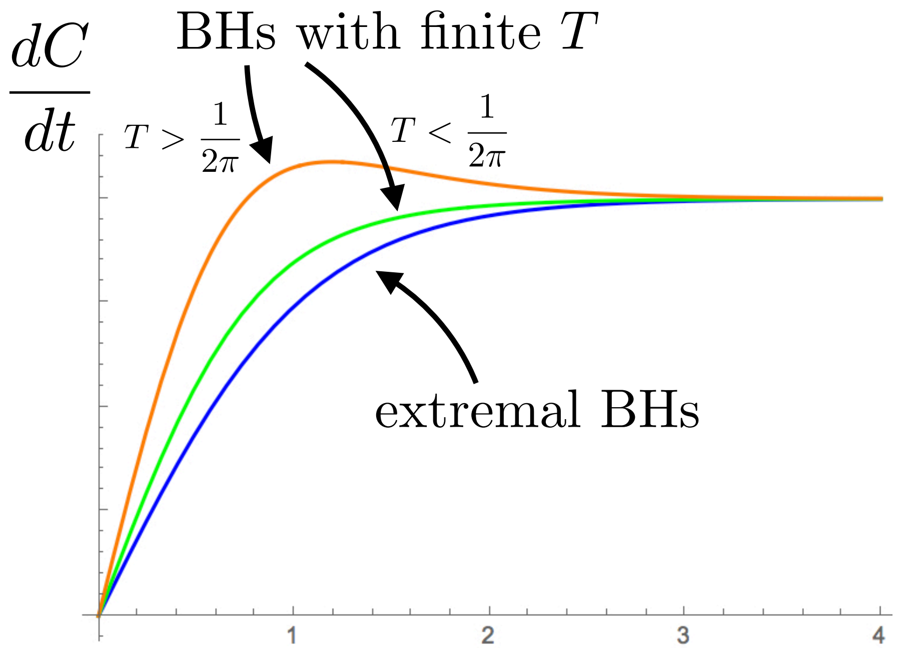
$$0 > \Delta S \sim \frac{\Delta E}{T} \sim \mathcal{O}(1)$$

For the wormhole to be traversable, the spacetime should be elongated in the future direction and the horizon should be lifted so that infalling particle won't enter there.

→the black hole singularity curve is also elongated in the future direction and the temperature should decrease if the BH becomes traversable.

Complexity of traversable wormholes?

- One of the interesting applications is to calculate the complexity of traversable wormholes. *[work in progress with Beni, Hugo, Leonel, Rob and Shira]*



The rate of complexity growth for a non-traversable wormhole

- Holographic complexity is related to the volume of the wormhole.

$$C \sim \frac{1}{G_N} \int_{\text{geodesic}} \Phi^2 ds$$

For non-traversable wormhole, it grows linearly at late times and the rate of growth approaches

$$\frac{dC}{dt} \sim S_0 T \qquad S_0 = \frac{\phi_0}{4G_N}$$

$\rightarrow$dominated by the extremal entropy!

- For the traversable wormholes, the rate of complexity growth is expected to become slow down since the horizon shifts in the future direction.

- The relation to the entropy decrease? The second law of entropy & the rate of complexity growth

- It's also interesting to explore complexity= JT action *[work in progress]*

Thank you