CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Conformal blocks from Wilson lines with loop corrections

Yasuaki Hikida (YITP)

Refs: YH-Uetoko, PTEP(2017)113B03; 1801.08549

Mar. 16th, 2018
YITP Workshop “Holography, Quantum Entanglement and Higher Spin Gravity II”

Holography and its use

Quantum gravity $\underset{②}{\overset{①}{\rightleftarrows}}$ Conformal field theory

(3d sl(N) Chern-Simons theory w/ Wilson lines)

(2d conformal model w/ W_N symmetry)

① Quantum gravity from well-defined CFT

- Fix the renormalization prescription for quantum gravity corrections

② CFT data of CFT from gravity

- New way to obtain the expressions of conformal blocks

Previous works

- Formal argument (Verlinde ’90)
  - CS gravity + Wilson line ➔ Liouville conformal blocks
  - A modification of Witten’s work ’89
- A practical computation (Fitzpatrick-Kaplan-Li-Wang ’16)
  - Wilson line + CFT ➔ $1/c$ expansion of conformal blocks
  - Useful for real computations, not purely bulk picture
- A bulk interpretation (Besken-Hedge-Kraus ’17)
  - Wilson line ➔ Conformal weight at $1/c$ order
  - NOT succeeded at $1/c^2$ order, no prescription of regularization

Plan of this talk

- Introduction
- General methods
- Conformal blocks from Wilson lines
- Conclusion

General methods

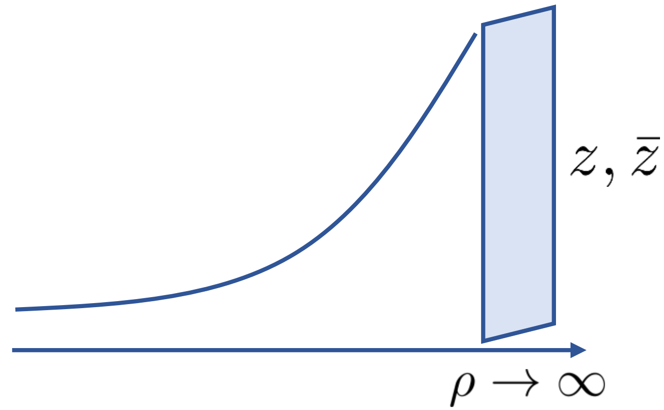
sl(2) CS gravity

- **Solution to EOM (in a gauge)**

$$A = e^{-\rho L_0} a(z) e^{\rho L_0} dz + L_0 d\rho$$

  - Boundary DOM are in *a*(*z*)

    ⇔ sl(2) WZW model (affine sl(2) sym.)



$$\begin{pmatrix} [L_m, L_n] = (m-n) L_{m+n} \\ m, n = 0, \pm 1 \end{pmatrix}$$

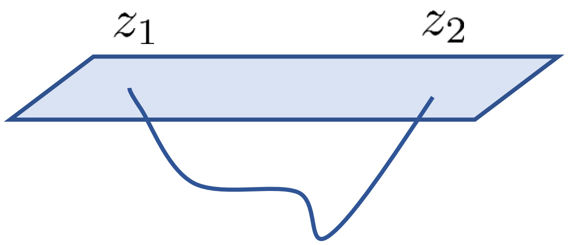
- **Asymptotic AdS condition**

$$(A - A_{\mathrm{AdS}})|_{\rho \to \infty} = \mathcal{O}((e^{\rho})^0) \Longrightarrow a(z) = L_1 + \frac{1}{\hat{k}} T(z) L_{-1}$$

  - AdS background ⇔ $a(z) = L_1$
  - DS reduction of affine sl(2)

    ⇔ Virasoro symmetry by *T*(*z*) with $c = 6\hat{k} = \frac{3\ell_{\mathrm{AdS}}}{2G_N}$

Wilson line in sl(2) CS theory



☐ Wilson line operator

$$W(z_2, z_1) = P\exp\left[\int_{z_1}^{z_2} A_\mu dx^\mu\right]$$
$$\rightarrow P\exp\left[\int_{z_1}^{z_2} a(z)dz\right] = P\exp\left[\int_{z_1}^{z_2} \left(L_1 + \frac{6}{c}T(z)L_{-1}\right)dz\right]$$

A gauge transformation

1/*c*-expansion

☐ Path integral over gauge fields

- We use correlators of *T*(*z*) in a CFT with center *c*

$$\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^4}$$

Correlators from Wilson lines

- Natural extension of leading order conjecture

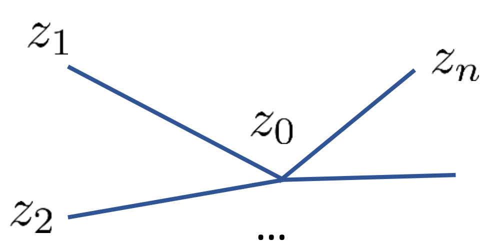
  (Bhatta-Raman-Suryanarayana, Besken-Hegde-Hijano-Kraus ’16, ’17)

$$\langle \mathcal{O}_{h_1}(z_1)\cdots\mathcal{O}_{h_n}(z_n)\rangle = \langle S|\prod_{i=1}^{n} W_{j_i}(z_i,z_0)|j_i,j_i\rangle$$

$(L_1|j,m\rangle = 0 \Rightarrow m = j)$

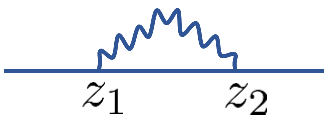
- <S| is a singlet in $\mathcal{R}_{j_1}\otimes\cdots\otimes\mathcal{R}_{j_n}$
- A singlet ⇔ A conformal block
- 2pt function $(z_0 \to z_2)$

$$\langle \mathcal{O}_h(z_1)\mathcal{O}_h(z_2)\rangle = \langle j,-j|W_j(z_1,z_2)|j,j\rangle$$



Renormalization prescription

- Divergences would arise at the coincident points of $T(z)$ in the integral $\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^4}$



- Our proposal for removing divergences

1. Introduce a regulator ϵ

$$\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^{4-2\epsilon}}$$

Scaling invariance is preserved

2. Remove divergences $1/\epsilon^n$ by redefining N_2,c_2

$$W(z_2, z_1) = \mathcal{N}_2 P \exp\left[\int_{z_1}^{z_2} (L_1 + \tfrac{6}{c} c_2 T(z) L_{-1}) dz\right]$$

3. ϵ^0 term in c_2 is fixed by Ward identity at the boundary

$$\langle \mathcal{O}_h(z)\mathcal{O}_h(0)T(\infty)\rangle = h z^2 \langle \mathcal{O}_h(z)\mathcal{O}_h(0)\rangle$$

Conformal blocks from Wilson lines

CFT results

- $\mathcal{O}_{h_j}$:Operator corresponding to |*j*,*j*>
  - Obtained by DS reduction of spin *j* operator in sl(2) WZW model
  - Conformal weight in 1/*c* expansion

$$\begin{aligned} h_j &= -j - \frac{6j(j+1)}{c} - \frac{78j(j+1)}{c^2} + \cdots \\ &= h_0 + \frac{h_1}{c} + \frac{h_2}{c^2} + \cdots \end{aligned}$$

- 2pt function

$$\langle \mathcal{O}_{h_j}(z) \mathcal{O}_{h_j}(0) \rangle = \frac{1}{z^{2h_j}}$$

$$= \frac{1}{z^{2h_0}} \left[1 - \frac{2h_1 \log z}{c} + \frac{2h_1^2 (\log z)^2 - 2h_2 \log z}{c^2} \right] + \cdots$$

Wilson line computations

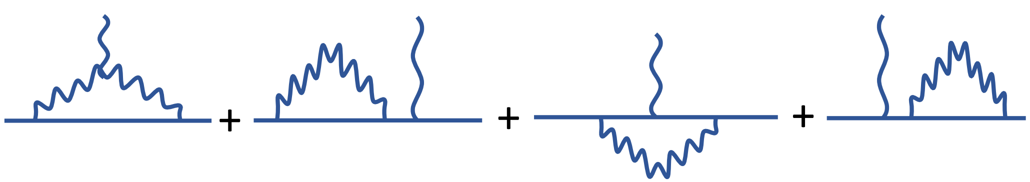
- <O(*z*)O(0)> at 1/*c* order

h_1 reproduced

$$= \frac{1}{cz^{2h_0}}\left[\frac{6(h_0-1)h_0}{\epsilon} + 12h_0(h_0-1)\log z + 2h_0(5h_0-2)\right]$$

Absorbed by redefining N_2

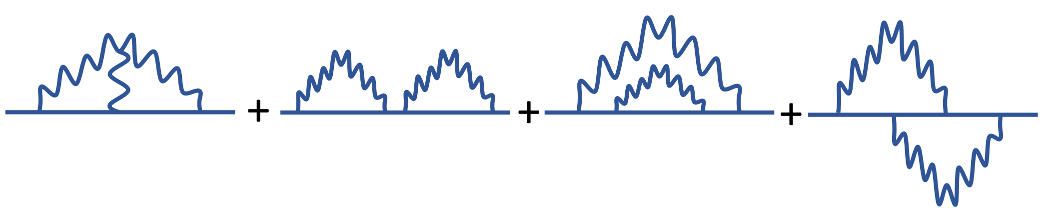
- <O(*z*)O(0)T(∞)> at 1/*c* order



$$\left(h_0 + \frac{h_1}{c}\right) z^2 \langle \mathcal{O}(z)\mathcal{O}(0)\rangle$$

$$c_2 = 1 + \frac{1}{c}\left(\frac{6}{\epsilon} + 3\right) + \cdots$$

- <O(*z*)O(0)> at 1/c^2 order



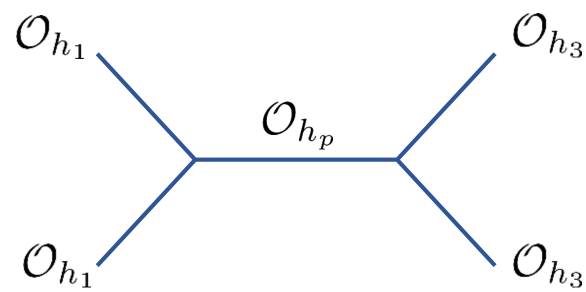
h_2 reproduced with redefined N_2, c_2

Conformal blocks

☐ The expansion of 4pt function

$$\langle \mathcal{O}_{h_1}(z)\mathcal{O}_{h_1}(0)\mathcal{O}_{h_3}(\infty)\mathcal{O}_{h_3}(1)\rangle = \sum_p C^p_{11} C^p_{33} \underbrace{\mathcal{F}(h_1, h_3; h_p; z)}_{\text{Conformal block}}$$

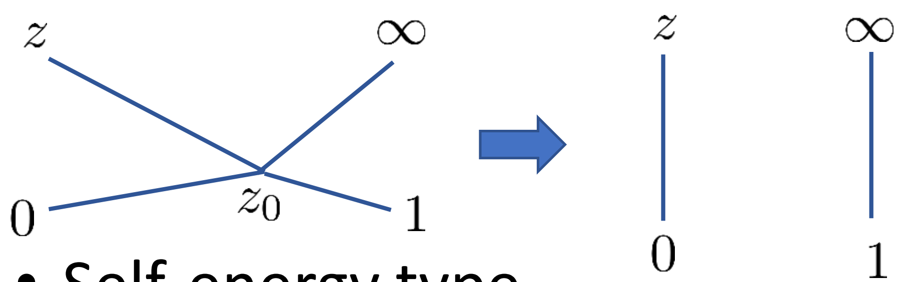
3pt functions



☐ Conformal blocks of $\mathcal{O}_{h_p}$

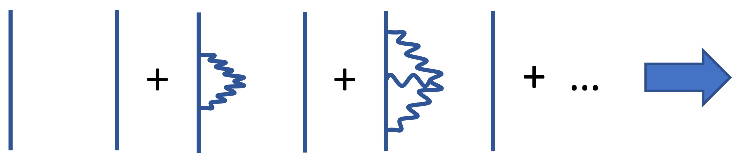
- *p*=0: Identity conformal block
- *p*≠0: General conformal block

Identity block up to $1/c^2$ order



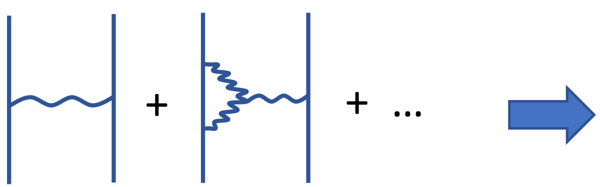
$= \langle W_{j_1}(z,0) W_{j_3}(\infty,1) \rangle$

- Self-energy type



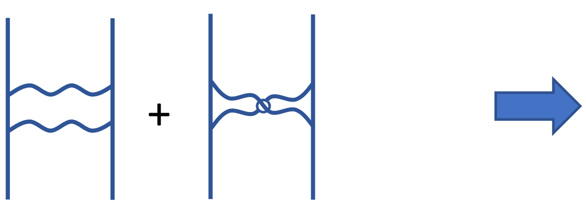
$$h_0 \to h_0 + \frac{1}{c} h_1 + \frac{1}{c^2} h_2$$

- *T* exchange



Coefficient is shifted as $h_0 \to h_0 + \frac{1}{c} h_1$

- *TT* exchange



Reduces to the analysis of Fitzpatrick et al

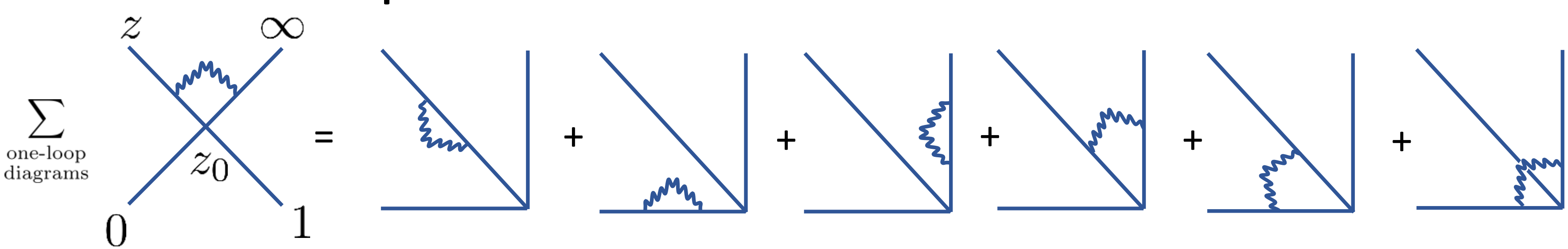
General blocks at 1/c order

☐ Known results (e.g. Fitzpatrick et al ‘16)

$$\mathcal{F}(h_1, h_3; h_p; z) = z^{-2h_3+h_p}\,{}_2F_1(h_p, h_p; 2h_p; z)$$
$$+\frac{1}{c}\left[h_1 h_3 f_a(h_p, z) + (h_1 + h_3) f_b(h_p, z) + f_c(h_p, z)\right] + \cdots$$

Known in all orders of z Known in first few orders of z

☐ Our computations



Obtain the all order expressions of f_a,f_b,f_c in z

Conclusion

Summary

Quantum gravity $\underset{②}{\overset{①}{\rightleftarrows}}$ Conformal field theory

[Wilson lines in 3d sl(N) Chern-Simons theory]

[Conformal blocks in 2d W_N CFT w/ large c]

① Quantum gravity from well-defined CFT

- Fix the renormalization prescription from the boundary theory

  ➔ Reproduced conformal weight at $1/c^2$ order

② CFT data of CFT from gravity

- A new systematic way for $1/c$ expansion of conformal blocks

  ➔ Obtained new expressions of conformal blocks

(The analysis has been generalized from N=2 to N=3)

Future directions

□ Treatment of heavy operators

Operators with $h_{1,s} \propto \mathcal{O}(c)$ ⟺ Conical spaces

- Quantization of conical spaces (cf. Raeymaekers '14)
- Application to heavy-light correlators (YH-Uetoko '17)
- Extension to W_3 case is an open problem

□ Introduce supersymmetry (YH-Uetoko in progress)

- Important to see relation to superstring theory
  - *N*=3 SUSY ⇔ AdS3 x SU(3)/U(1) (x SO(5)/SO(3)) (Creutzig-YH-Rønne'13,'14)
  - *N*=4 SUSY ⇔ AdS3 x S3 x T4 (x S3 x S1) (Gaberdiel-Gopakumar '13,'14)