Spinning defects in conformal field theory

Based on the ongoing work with T . Nishioka (Univ. of Tokyo)

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Introduction and Motivation

Conformal Field Theory

Conformal Field Theory (CFT) lives on the fixed point of RG flow.

(HEP)

- world sheet of strings
- world volume of branes
- AdS/CFT correspondence

(CM)

- physics at critical point
  -such as Ising model

CFT describes wide range of phenomena!

Probing CFT

CFT has spectime symmetry $SO(d+1,1)$ in $\mathbb{R}^d$.

- Dynamics is determined by CFT data; the spectrum $\{\Delta_i, l_i\}$ and OPE coefficients c_{ijk}.
- the rest is kinematics.

There are other probes of CFT

$\Rightarrow$ **defects** - extended objects

Defects are ubiquitas in Conformal Field Theory - Boundary and Interface, line operator, etc...

Defects in CFT

Defects play interesting and important roles in CFT.

- Theoretical
  - characterize the phase of theory (ex. Wilson loop)
  - relation to higher-form symmetry
  - BPS defect operators in S(USY)CFT
  - Twist operators to glue multi sheets
- Experimental
  - system at criticality in container
  - introduction of an inpurity in critical system

Conformal defects

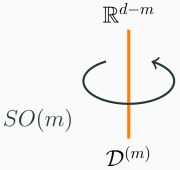
Especially, we consider particular class of defects:

Conformal defects (codimension-m)

defects preserving $\underbrace{SO(d-m+1,1)}_{\text{conformal sym. on defect}} \times \underbrace{SO(m)}_{\text{rotational sym. around defect}}$

- conformal defects allow defect local operators $\hat{\mathcal{O}}(x)$
- additional dynamical information appears

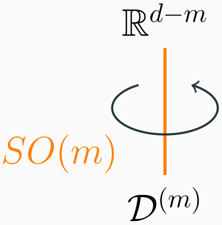
- coupling to defects, defect local operators,...



Questions

- How can we constrain the defect OPE coefficients by bootstrap?

- Can we completly determine kinematic part by residual symmetry?
- Is there any connection with AdS/CFT?
  $\rightarrow$ last talk [Fukuda-NK-Nishioka 17]
- Is there more extention of CFT?
  Spinning defects! $\rightarrow$ this talk

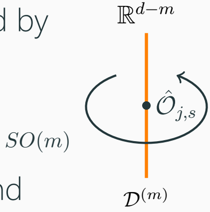


Correlators in defect-CFT

Correlators in defect-CFT

Defect local operators

Defect local operators



- Defect local operators $\hat{\mathcal{O}}_i(x)$ are characterized by
  - dimension $\hat{\Delta}_i$
  - paralell spin j for $SO(d-m)$
  - transverse spin s for $SO(m)$
- The physical coordinate x^μ have parallel x^a and transverse x^i components to the defect,

$$x^\mu = (x^a, x^i) \ , \qquad (a = 1, \cdots, d-m, \quad i = 1, \cdots, m)$$

- To describe $\hat{\mathcal{O}}_{j,s}(x^a)$, introduce a polynomial of two auxiliary vectors z^a and w^i,

$$\hat{\mathcal{O}}_{j,s}(x^a, z^a, w^i) \equiv \hat{\mathcal{O}}_{a_1 \cdots a_j, i_1 \cdots i_s}(x^a) \, z^{a_1} \cdots z^{a_j} \, w^{i_1} \cdots w^{i_s}$$

imposing $z^a z_a = 0, w^i w_i = 0$.

Defect local operator in embedding space

- The physical coordinate x^a on the defect is up lift to X^M in $\mathbb{R}^{d+1,1}$

$$x^a \to X^M = (X^A, x^i = 0)\ , \quad X^A X_A = 0\ ,$$

$(A = 1, \cdots d - m + 2)$

- Similary, the auxiliary vectors z^a and w^i can be up lift to $\hat{Z}^M$ and $\hat{W}^M$ satisfying

$$X \cdot \hat{Z} = 0\ , \quad \hat{Z}^2 = 0,$$
$$X \cdot \hat{W} = 0\ , \quad \hat{W}^2 = 0\ , \quad \hat{Z} \cdot \hat{W} = 0$$

- Defect local operator with spin in $\mathbb{R}^{d+1,1}$ are given in the index-free notation by

$$\hat{\mathcal{O}}_{j,s}(X, \hat{Z}, \hat{W}) \equiv \hat{\mathcal{O}}_{M_1 \cdots M_j, N_1 \cdots N_s}(X)\, \hat{Z}^{M_1} \cdots \hat{Z}^{M_j}\, \hat{W}^{N_1} \cdots \hat{W}^{N_s}$$

Correlators in defect-CFT

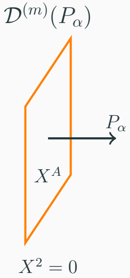
Conformal defects in embedding space

Conformal defects in embedding space

There are two approaches to compute correlators in defect-CFT

$$\langle \mathcal{O}(X_1)\cdots\mathcal{O}(X_k)\rangle_{\mathcal{D}} = \langle \mathcal{D}^{(m)}\,\mathcal{O}(X_1)\cdots\mathcal{O}(X_k)\rangle$$

1. frame vector approach [Gadde 16]
   $\rightarrow$ used in last talk.
2. Index split approach [Billò-Gonçalves-Lauria-Meineri 16]



Index split approach

- Split embedding coordinate X^M into the tangential and transverse coordinates

$$M = (A, I)\ , \qquad A = 1,\, 2,\, \cdots\, ,\, d-m+2\ , \quad I = 1, \cdots, m$$

defect is described as a hypersurface located at $X^I = 0$

- There are two types of scalar product for two embedding space vectors X^M, Y^M,

$$\underbrace{X \bullet Y}_{SO(d-m+1,1)\text{inv.}} \equiv \eta_{AB}\, X^A\, Y^B\ , \qquad \underbrace{X \circ Y}_{SO(m)\text{inv.}} \equiv \delta_{IJ}\, X^I\, Y^J$$

Correlators in defect-CFT

To compute correlators of bulk primaries, we use tensor $C^{MN} = Z^M X^N - X^M Z^N \rightarrow$ decomposed to C^{AB}, C^{AI}, C^{IJ}

However, we only need mixed one C^{AI} to construct invariants!
-Since we can write others in terms of C^{AI}.

Example:One-point function of $\mathcal{O}_{\Delta,l}(X,Z)$ with scalar defect

$$\langle \mathcal{D}^{(m)}\, \mathcal{O}_{\Delta,l}(X,Z)\rangle = a_{\Delta,l}\frac{(C^{AI}C_{AI})^{l/2}}{(X\circ X)^{(\Delta+l)/2}}$$

Correlators in defect-CFT

Spinning conformal defects

Spinning defects

We adapt index-free notation for spinning defects introducing auxiliary transverse vector $\hat{W}$,

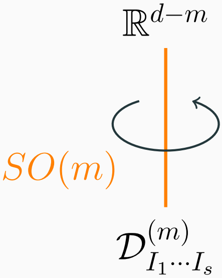
$$\mathcal{D}_s^{(m)}(\hat{W}) \equiv \mathcal{D}^{(m)}_{I_1\cdots I_s}\,\hat{W}^{I_1}\cdots\hat{W}^{I_s}, \quad \hat{W}\circ\hat{W}=0$$

Let us consider one-point function of bulk primary with spinning defect,

$$\langle \mathcal{D}_s^{(m)}(\hat{W})\,\mathcal{O}_{\Delta,l}(X,Z)\rangle$$

which has

- degree $-\Delta$ in X
- degree l in Z
- degree s in $\hat{W}$



One point function of bulk primary

To fix the correlator, we need to find scalar invariants constructed from $\hat{W}^I, X^M$ and Z^M.

Invariants $X \circ X$, $\quad Q_1 = \frac{X \circ \hat{W}}{(X \circ X)^{1/2}}$, $\quad Q_2 = \frac{C^{AI} \, X_A \, \hat{W}_I}{X \circ X}$, $Q_3 = \frac{C^{AI} \, C_{AI}}{X \circ X}$

$$\Rightarrow \langle \mathcal{D}_s^{(m)}(\hat{W}) \, \mathcal{O}_{\Delta,l}(X,Z) \rangle = \sum_h N_h \, \frac{Q_1^{s-l+2h} \, Q_2^{l-2h} \, Q_3^h}{(X \circ X)^{\Delta/2}}$$

where h runs $\frac{l-s}{2} \leq h \leq \frac{l}{2}$. Notice that the correlator is NOT fixed uniquely!

In same manner, we can compute more complicated correlators!

Future Direction

Work in progress

- Correlator of bulk conserved currents J_l, $\Delta = d - 2 + l$
  - conservation law $\partial^M D_M J_l(X, Z) = 0$ fixes the correlator uniquely or not?
- Two point function of defects $\langle \mathcal{D}_s^{(m_1)}(\hat{W})\, \mathcal{D}_{s'}^{(m_2)}(\hat{W}') \rangle$
  - application to the study of the algebra of line and surface operators in CFT
  - pick up the universal contribution from stress-tensor block?
  - relation to mutual information in CFT
- Connection to AdS/CFT correspondence
  - Spinning Defect OPE block has AdS dual field?
  - we just speculate that is related to (projected) Higher Spin fields in AdS