A 2d random supersymmetric model with disorder

J.M., Douglas Stanford & Edward Witten

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IAS INSTITUTE FOR ADVANCED STUDY

QGaSlab The laboratory for Quantum Gravity & Strings

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Outline

- Some (probably unnecessary) motivation for the SYK model.
- Some technical tools
- Disordered models in 2-dimensions
- Conclusions

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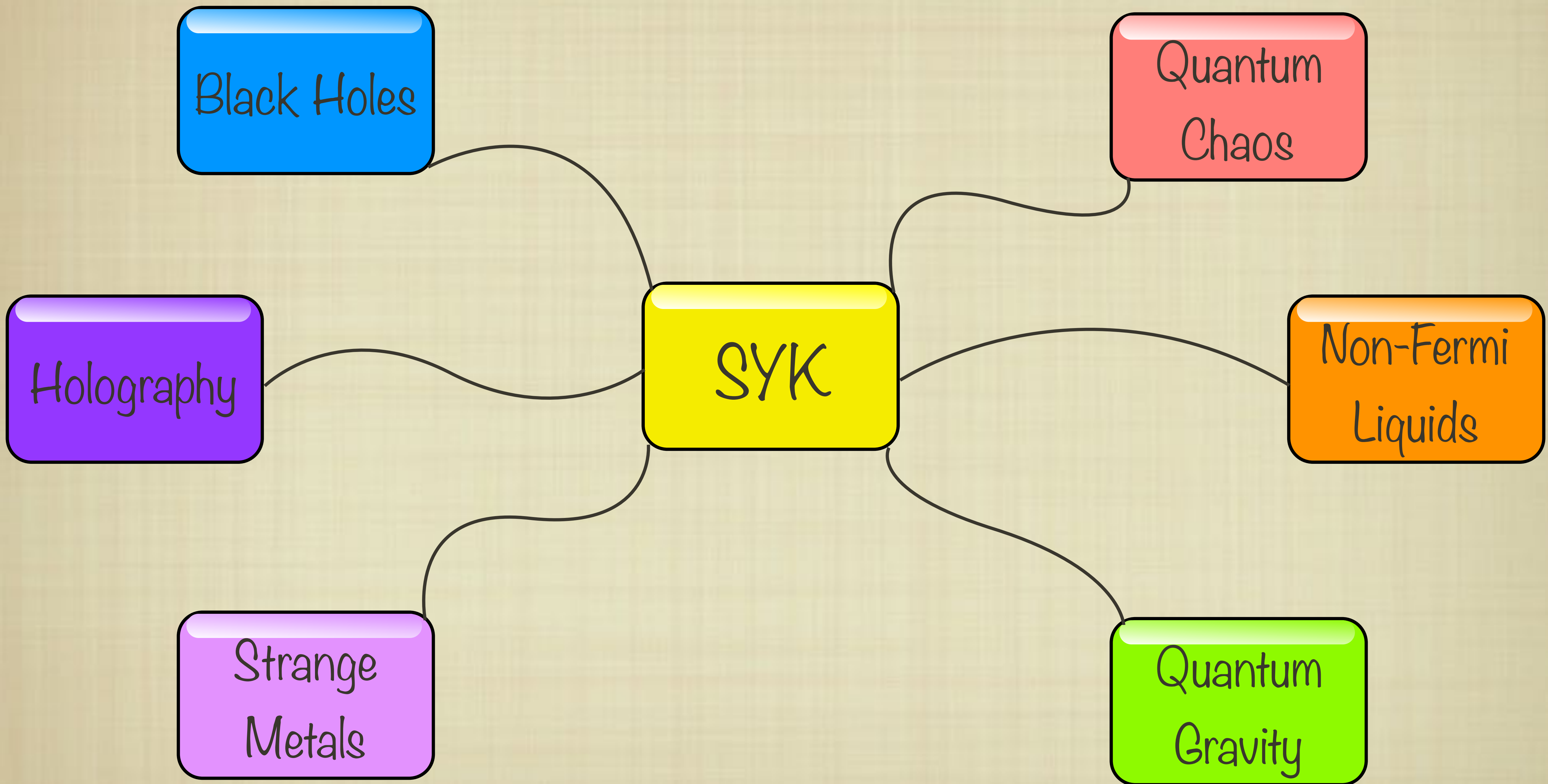
WHY SYK?

[Sachdev-Ye '92, Kitaev '15, Maldacena-Stanford '16]



ABC's of SYK

[Sachdev-Ye '92, Kitaev '15, Maldacena-Stanford '16, Maldacena et.al '16]

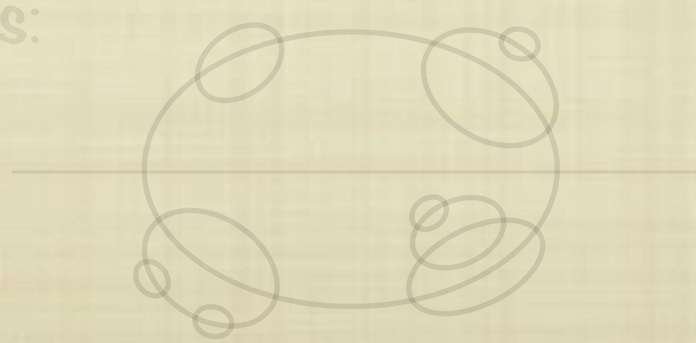
The **Sachdev-Ye-Kitaev** model is a quantum mechanical system of N Majorana fermions with all-to-all random (q)uartic interactions conjectured to be dual to a nearly AdS_2 geometry.

$$H = i^{\frac{q}{2}} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q} .$$

To summarize, among its several remarkable properties:

- It is **solvable** in the large N limit
- It has an **emergent** low-energy conformal symmetry
- It is **maximally chaotic**

The large N limit is dominated by **Feynman diagrams**:



Schwinger-Dyson equation:

$$G(\omega) = \frac{1}{-i\omega - \Sigma(\omega)}$$

$$\Sigma(\omega) = J^2 G(\omega)^3$$

Chaos in a quantum system can be diagnosed through the growth of the **out-of-time-order correlator**

$$F(t) \equiv \langle \psi_j(t)\psi_k(0)\psi_j(t)\psi_k(0)\rangle$$

whose connected part

$$F(t)_{\rm conn} \sim e^{\lambda_L t}$$

defines a Lyapunov/ **chaos exponent** λ_L bounded by the MSS chaos bound $\lambda_L \leq 2\pi/\beta$

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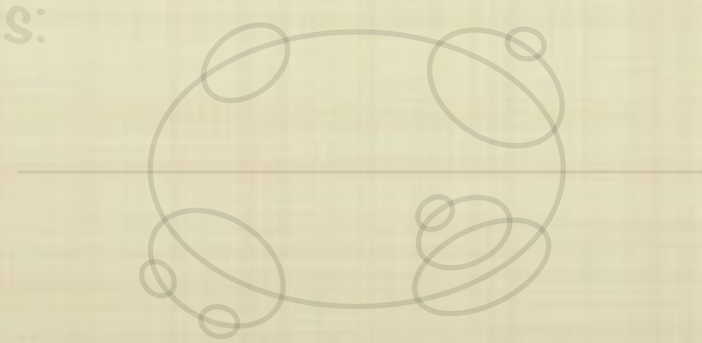

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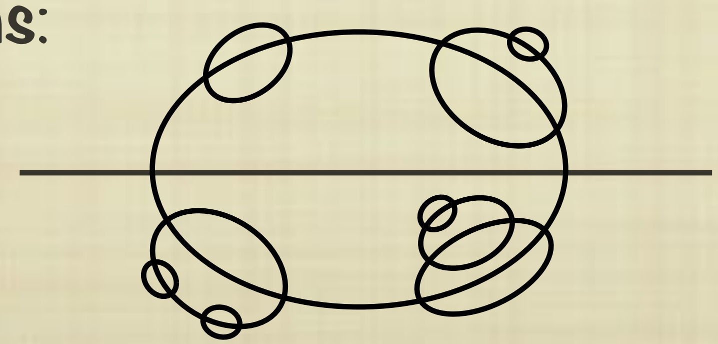

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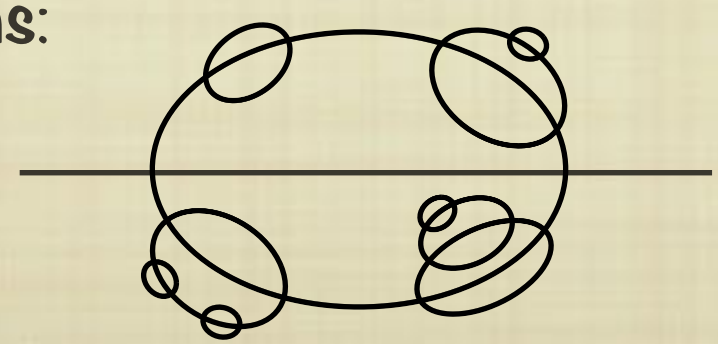

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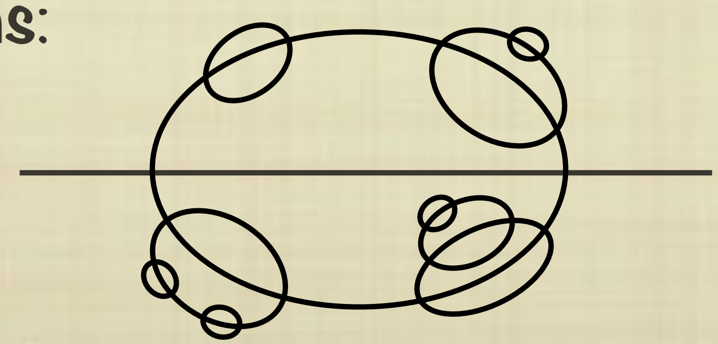

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The SYK's the limit

[Berkooz et.al. '16, Turiaci-Verlinde '17, Berkooz et.al. '17]

A free **boson** with canonical kinetic term in 2-d has dimension zero so that a random q-boson interaction term will be **relevant** but such models either:

- Have potentials with generic negative directions leading to **instabilities** or
- Are stable but do not flow to an SYK-like **fixed point**.

Fermions in 1-d have mass dimension 0 so the 4-fermi interaction is **relevant** and the theory flow to a strongly interacting SYK phase in the IR. In 2-d canonical fermions have dimension 1/2 so 4-fermi interactions are **marginal** (and higher q are irrelevant).

Is there a higher dimensional realization of SYK physics?

Most promising is a **2-d supersymmetric model** with N bosons ϕ_i and N pairs of chiral fermions $(\psi_i, \bar{\psi}_i)$. All fields have canonical kinetic terms and interact in two ways:

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The shadow representation

[Ferarra et.al. '72, Dolan-Osborn '01, Simmons-Duffin '12]

The **shadow formalism** offers a construction of (super)conformal blocks appropriate for constructing **4-point functions** (of two conformal primaries O and O' with dimension Δ) in a D-dimensional CFT with a specified value of the (super)conformal Casimir in a chosen channel.

$$\langle O(x_1)O(x_2)O'(x_3)O'(x_4)\rangle = \varepsilon\int d^Dy\langle O(x_1)O(x_2)\mathcal{V}(y)\rangle\langle\mathcal{V}'(y)O(x_3)O(x_4)\rangle$$

Think of O and O' as conformal primaries in **two decoupled CFTs** in which there also exists primaries $\mathcal{V}$ and $\mathcal{V}'$ of dimension h and $D-h$ respectively. If the product theory is perturbed by $\varepsilon\int d^Dy\,\mathcal{V}(y)\mathcal{V}'(y)$ then the 3-point function $\langle O(x_1)O(x_2)\mathcal{V}(y)\rangle$ describes the coupling of $O(x_1)$ and $O(x_2)$ to a dimension-h primary and is an eigenfunction of C_{12} with eigenvalue $h(h-D)$

The RHS is a conformally invariant, single-valued eigenstate of the **2-particle Casimir** C_{12}

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An example: SYK revisited

The shadow representation manifests various **properties** of the 4-point function:

- **Symmetry** under exchange of $1 \leftrightarrow 2$ or $3 \leftrightarrow 4$ means
$$\Psi_h(\chi) = \Psi_h(\frac{\chi}{\chi - 1})$$
- Since Ψ_h is eigenfunction of the 2-particle **Casimir** with eigenvalue h
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- Ψ_h is **meromorphic** with simple poles of h is a + odd, or - even integer

The SYK model in 1-d with q-fold interactions has fermionic primaries $\psi_i(t)$ at large N, a disorder-averaged 2-point function $\langle\psi_i(t_1)\psi_j(t_2)\rangle = \delta_{ij}\frac{(t_1 - t_2)}{|t_1 - t_2|^{2\Delta}}$ and a **natural 4-pt function**

$$\mathcal{F}(t_1, t_2, t_3, t_4) = N\frac{\langle\psi_i(t_1)\psi_i(t_2)\psi_j(t_3)\psi_j(t_4)\rangle'}{\langle\psi_i(t_1)\psi_i(t_2)\rangle'\,\langle\psi_j(t_3)\psi_j(t_4)\rangle'}$$

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Computing the central charge

Think of a 2-d CFT with **holomorphic stress tensor** $T(z)$ and **spinless operator** O with dimensions $(h,\tilde{h}) = (\Delta/2, \Delta/2)$ and 2-point function $\langle O(x,\bar{x})O(0,0)\rangle = b/|x|^{2\Delta}$. This and conformal invariance, fixes the form of the **OOT 3-point function** so that the OO OPE is

$$O(x_1,\bar{x}_1)O(x_2,\bar{x}_2) \sim \frac{b}{|x_1-x_2|^{2\Delta}}\left(1+\frac{2h}{c}(x_1-x_2)^2 T(x_2)+\dots\right)$$

The **central charge** of a (2-d) CFT can be read off from the **stress-tensor contribution** to the 4-point function

$$\mathcal{F}_T = \frac{N\Delta^2\chi^2}{2c}$$

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Plan

- Some (probably unnecessary) motivation for the SYK model.
- Some technical tools
- Disordered models in 2-dimensions
- Conclusions

A naive 2-dimensional bosonic model

[Klebanov-Tarnopolsky '16, JM-Stanford-Witten '17, Giombi et.al. '17]

Consider N scalars interacting in a **random degree-q potential**. In the large N limit the disorder-averaged Schwinger-Dyson equations are

$$\Sigma = J^2 G^{q-1}, \qquad G = \frac{1}{-\partial^2 - \Sigma}$$

But generic **unboundedness of the action** poses problems for exact solutions (unitarity, non reflection-positive solutions, etc)

$$I = \int d^2x \left[\frac{1}{2}(\nabla\phi_i)^2 + J_{i_1 i_2 \cdots i_q}\phi_{i_1}\phi_{i_2}\cdots\phi_{i_q}\right]$$

Ignoring these issues, in the **IR** we find

$$G = \frac{b}{|x|^{2\Delta}} \qquad \Delta = \frac{2}{q}$$

$$b^q J^2 = \frac{(1-\Delta)^2}{\pi^2}$$

The **ladder kernel** K is constructed from diagrams

and has **eigenvalue** $k(h,\tilde{h})$

Properties of the model:

- $k(2,0) = k(0,2) = 1$ implies the existence of a **holomorphic stress tensor**
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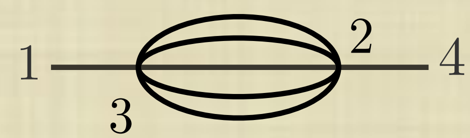

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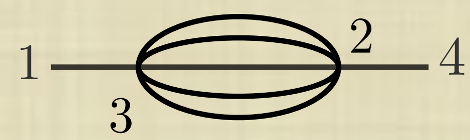

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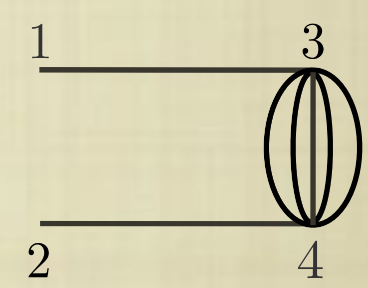

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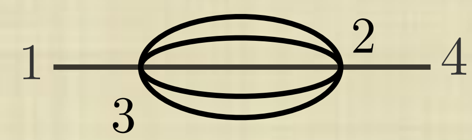

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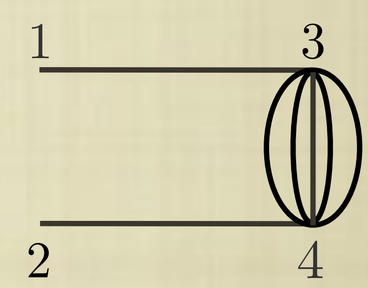

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A better 2-d bosonic model

[JM-Stanford-Witten '17]

The **positive-definite** square of a random potential $\sim \frac{1}{4}(B^a)^2 + iB^aC^a \cdot \phi \cdots \phi$ giving a model with two sets of fields ϕ and B and Schwinger-Dyson equations

$$G_\phi = \frac{1}{-\partial^2 - \Sigma_\phi}, \qquad G_B = \frac{1}{\frac{1}{2} - \Sigma_B}$$

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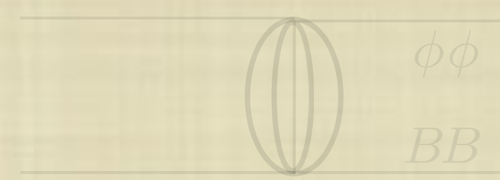

This model has **two parameters** q and r=M/N and solutions that are compatible with unitarity and reflection positivity.

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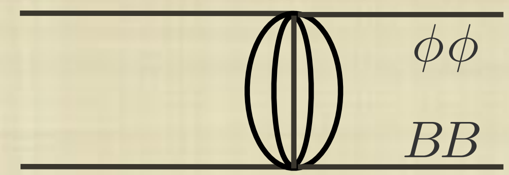

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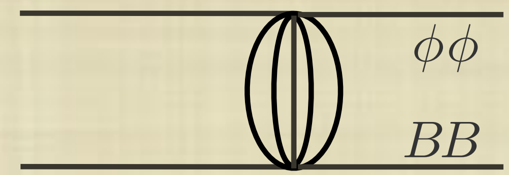

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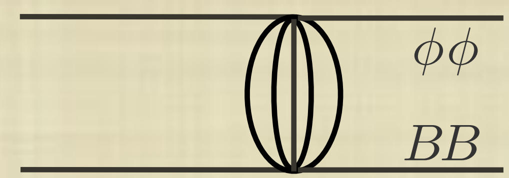

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An N=1, 2-dimensional supersymmetric model

[JM-Stanford-Witten '17]

Note the following:

- Unlike the bosonic models, the self-energy of this model is **not UV divergent**.
- **We do not need to tune** any counter-term to reach the conformal solution
- Consistent with **numerical** solution

Consider an $\mathcal{N}=1$ theory with a **random superpotential.**

- Scalar superfields: $\Phi(X)=\phi(x)+i\theta\psi(x)+i\bar{\theta}\,\bar{\psi}(x)+\theta\bar{\theta}F(x)$
- Supercharge generators: $Q=\partial_\theta-\theta\partial_x,\quad \bar{Q}=\partial_{\bar{\theta}}-\bar{\theta}\partial_{\bar{x}}$
- Superspace derivatives: $D_\theta=\partial_\theta+\theta\partial_x,\quad D_{\bar{\theta}}=\partial_{\bar{\theta}}+\bar{\theta}\partial_{\bar{x}}$
- Large N discrete R-symmetry: $\theta\to-\theta,\ \bar{\theta}\to+\bar{\theta},\ \Phi_i\to-\Phi_i$

$$I=\int d^2d^2\theta\left[\frac{1}{2}D_{\bar{\theta}}\Phi_i D_\theta\Phi_i+iC_{i_1i_2\cdots i_q}\Phi_{i_1}\Phi_{i_2}\cdots\Phi_{i_q}\right]$$

At large-N the 2-point function is determined by the **S-D equations**

$$G(p)=\frac{4}{p^2(1-\Sigma(p))},\qquad \Sigma(x)=-J^2G(x)^{q-1}$$

which has the IR solution

$$G(X_1,X_2)=\frac{b}{|x_1-x_2-\theta_1\theta_2|^{2\Delta}},\qquad \Delta=\frac{1}{q},\qquad b^qJ^2=\frac{1}{\pi^2}$$

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- Supercharge generators: $Q = \partial_\theta - \theta\partial_x, \quad \bar{Q} = \partial_{\bar{\theta}} - \bar{\theta}\partial_{\bar{x}}$
- Superspace derivatives: $D_\theta = \partial_\theta + \theta\partial_x, \quad D_{\bar{\theta}} = \partial_{\bar{\theta}} + \bar{\theta}\partial_{\bar{x}}$
- Large N discrete R-symmetry: $\theta \to -\theta, \ \bar{\theta} \to +\bar{\theta}, \ \Phi_i \to -\Phi_i$

$$I = \int d^2 d^2\theta \left[\frac{1}{2} D_{\bar{\theta}}\Phi_i D_\theta \Phi_i + iC_{i_1 i_2 \cdots i_q}\Phi_{i_1}\Phi_{i_2}\cdots\Phi_{i_q}\right]$$

At large-N the 2-point function is determined by the **S-D equations**

$$G(p) = \frac{4}{p^2(1-\Sigma(p))}, \quad \Sigma(x) = -J^2 G(x)^{q-1}$$

which has the IR solution

$$G(X_1, X_2) = \frac{b}{|x_1 - x_2 - \theta_1\theta_2|^{2\Delta}}, \quad \Delta = \frac{1}{q}, \quad b^q J^2 = \frac{1}{\pi^2}$$

An N=1, 2-dimensional supersymmetric model

[JM-Stanford-Witten '17]

Note the following:

- Unlike the bosonic models, the self-energy of this model is **not UV divergent**.
- We **do not need to tune** any counter-term to reach the conformal solution
- Consistent with **numerical** solution

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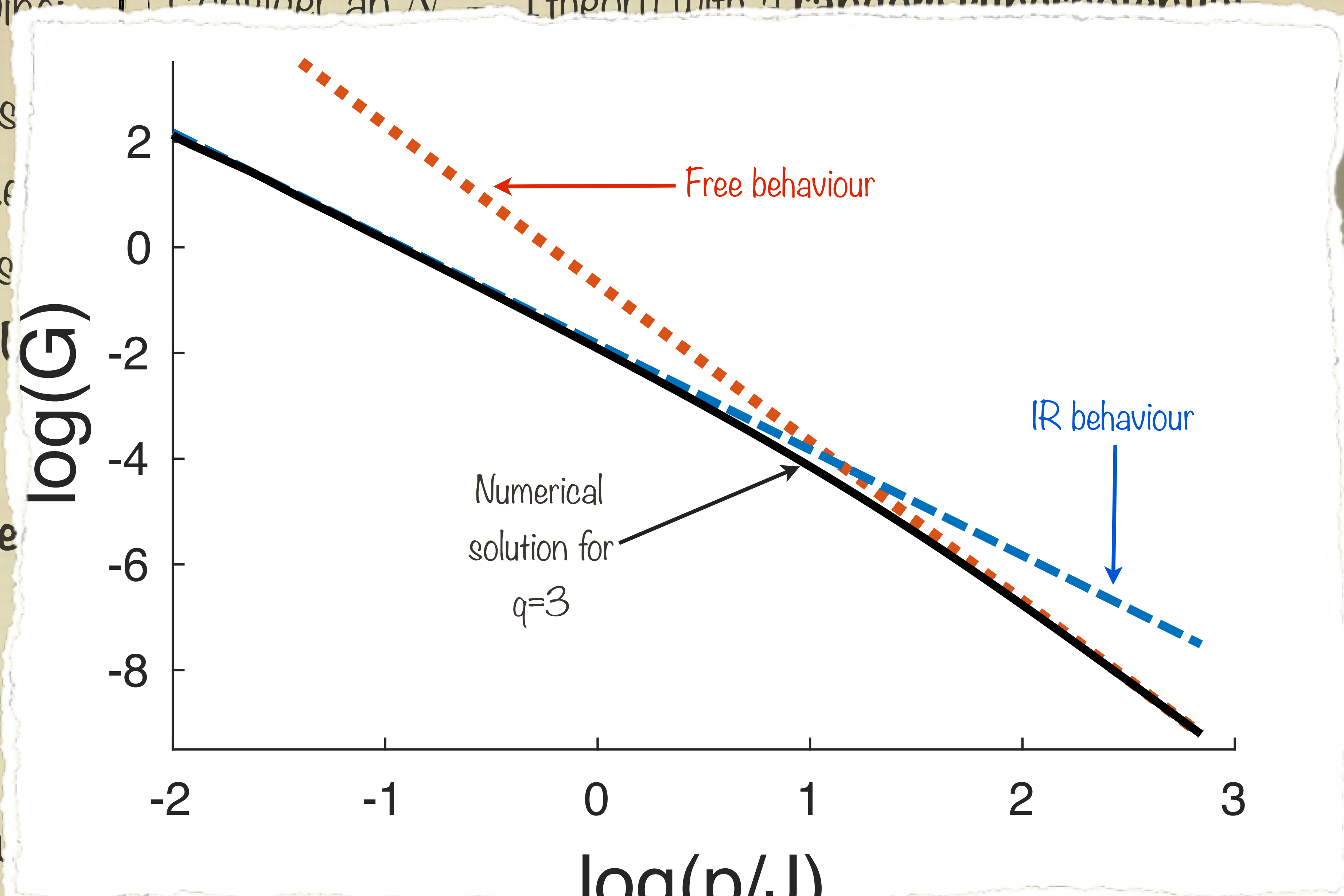

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Additional U(1) symmetry

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Since $k^{FB}(1/2,0) = q-1$, we find indeed that $k_A^{FB}(1/2,0) = 1$ as expected for the (1,0) part of the U(1) current. **BUT** the function $1/\left(1-k_A^{FB}(1/2,0)\right)$ has a **double pole at (1/2,0)** that lies on the integration contour of the 4-point function. We therefore expect the model to not find a true IR fixed point.

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The holomorphic and anti-holomorphic parts of the current associated to a **2-d (S)CFT** with a continuous symmetry must be separately conserved. This means that in a 2-d supersymmetric model with **SYK-like ladder structure** we must expect to find a dimension (1/2,0) primary in the FB channel. A candidate theory has **N/2 complex superfields** and their complex conjugates interacting in a random superpotential.

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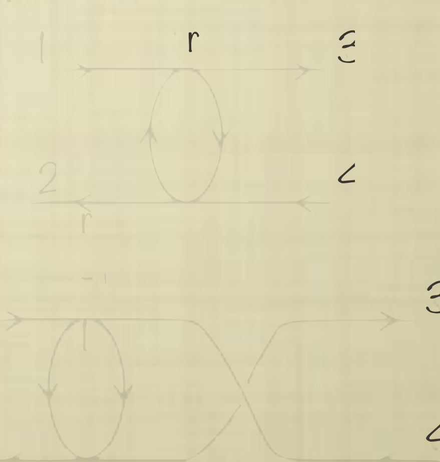

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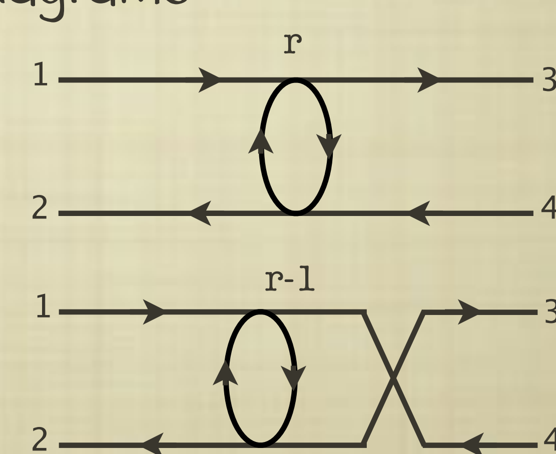

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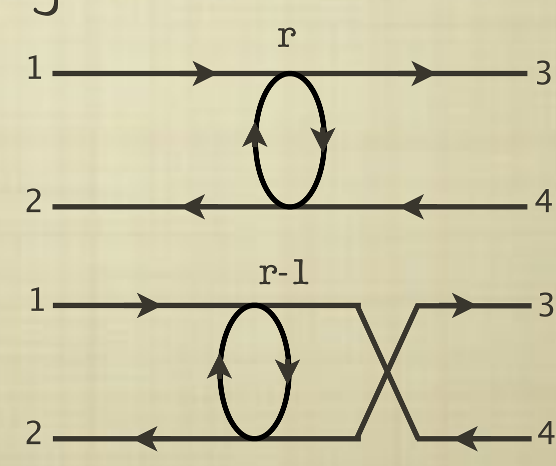

Jeff Murugan (UCT)

Chaos

[Stanford '16, Maldacena-Stanford '15, JM-Stanford-Witten '17]

Out-of-time-order thermal correlators provide a diagnostic for quantum chaos in large N systems

$$F(t) = \langle V(0)W(\beta/4+it)V(\beta/2)W(3\beta/4+it)\rangle$$

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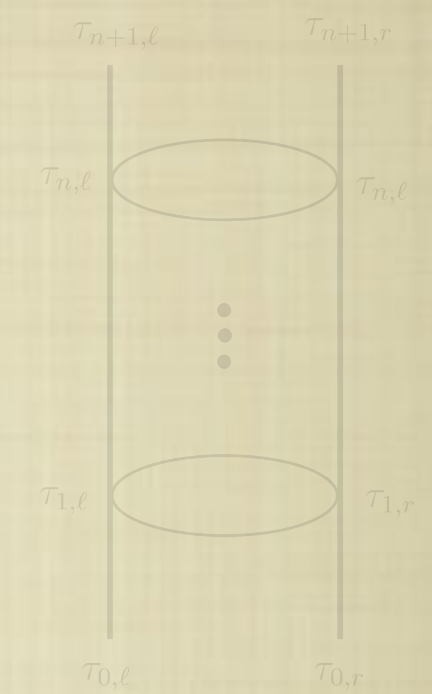

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There are two ways to compute the correlator:

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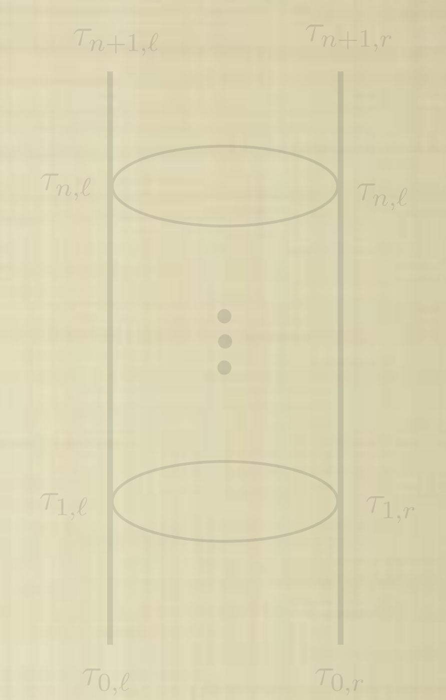

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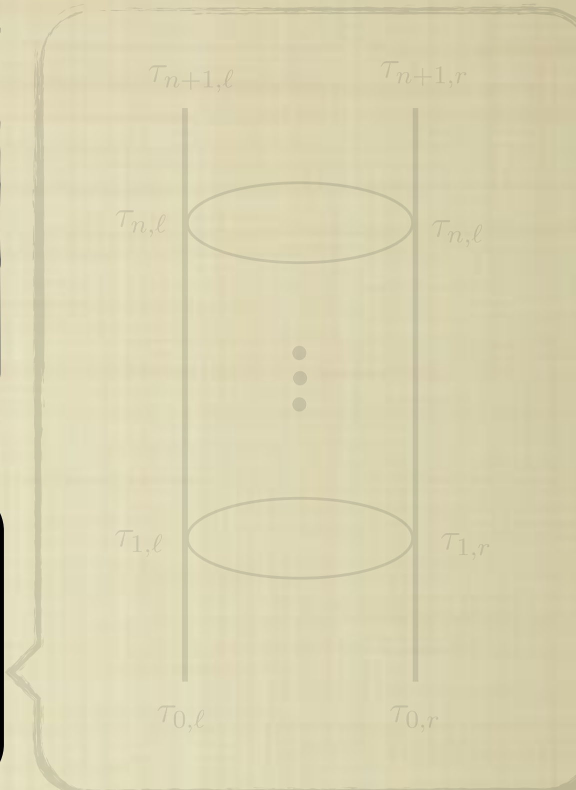

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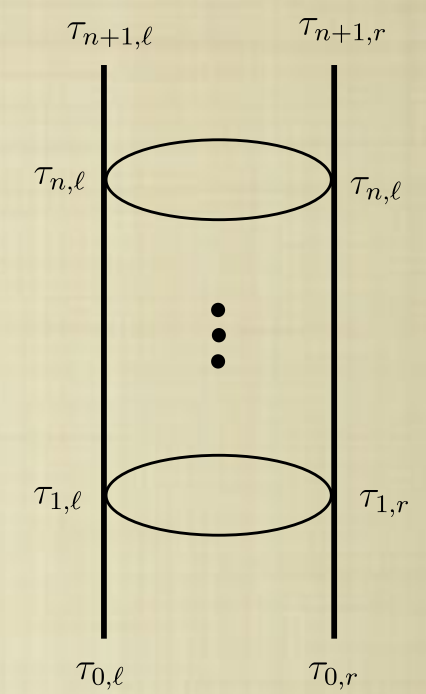

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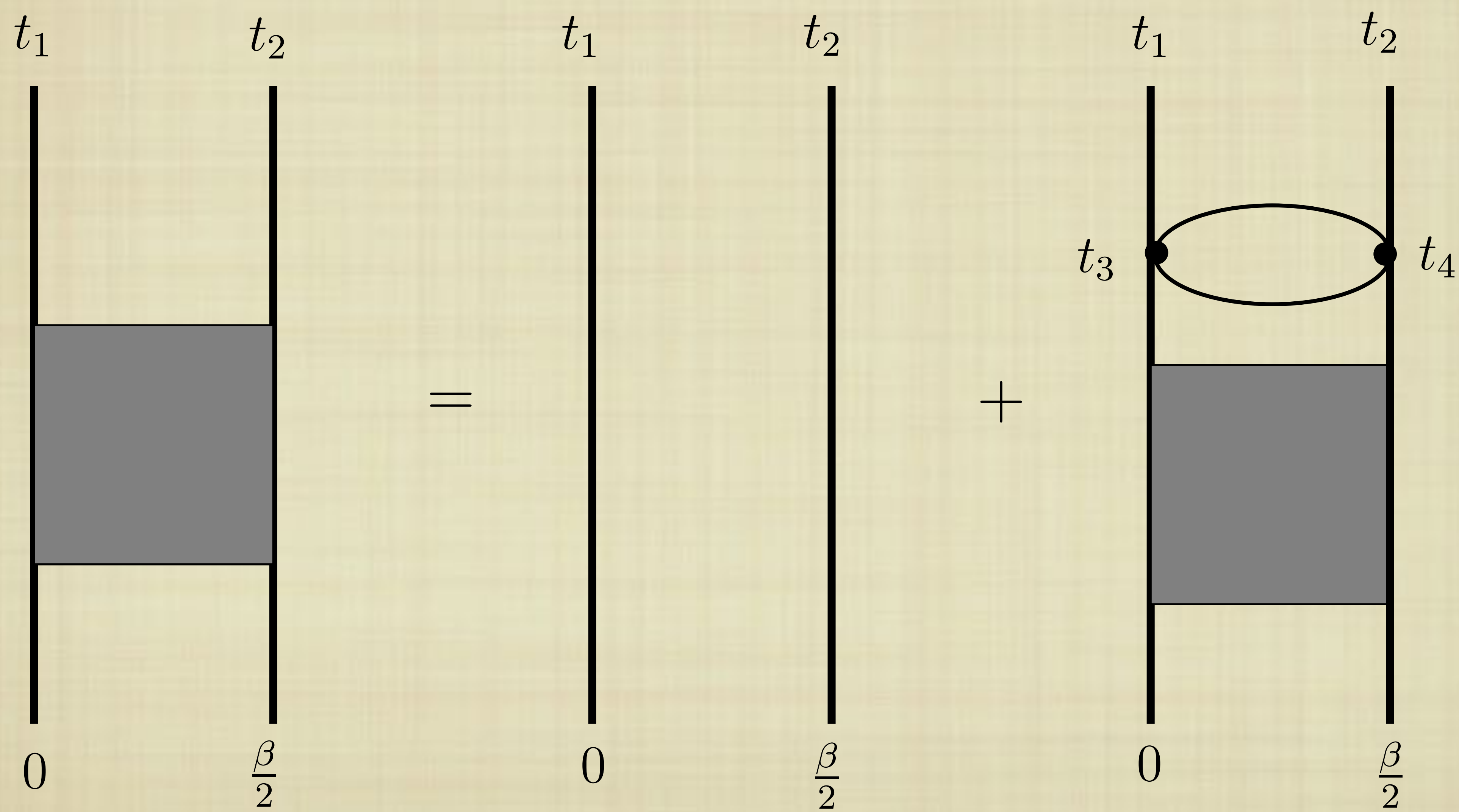

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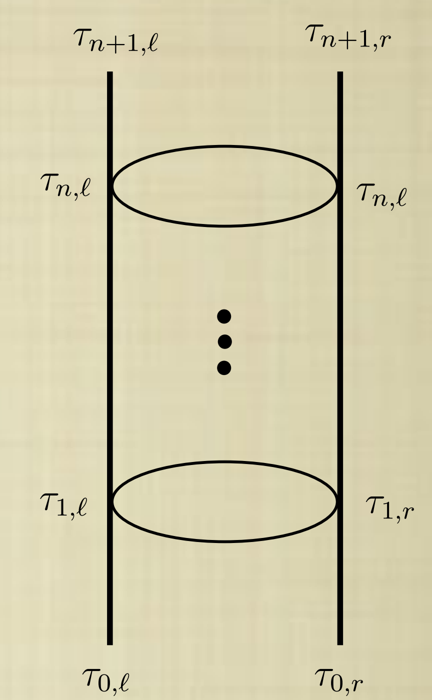
$$W_+(t_1,t_2) = \frac{\exp\left(-\frac{h}{2}(t_1+t_2)\right)}{\left(2\cosh\frac{1}{2}(t_1-t_2)\right)^{2\Delta-h}}$$

$$W(t_1,t_2) = W_0(t_1,t_2) + \int_0^{t_1} dt_3 \int_0^{t_2} dt_4 \; K_R(t_1,t_2,t_3,t_4)W(t_3,t_4)$$



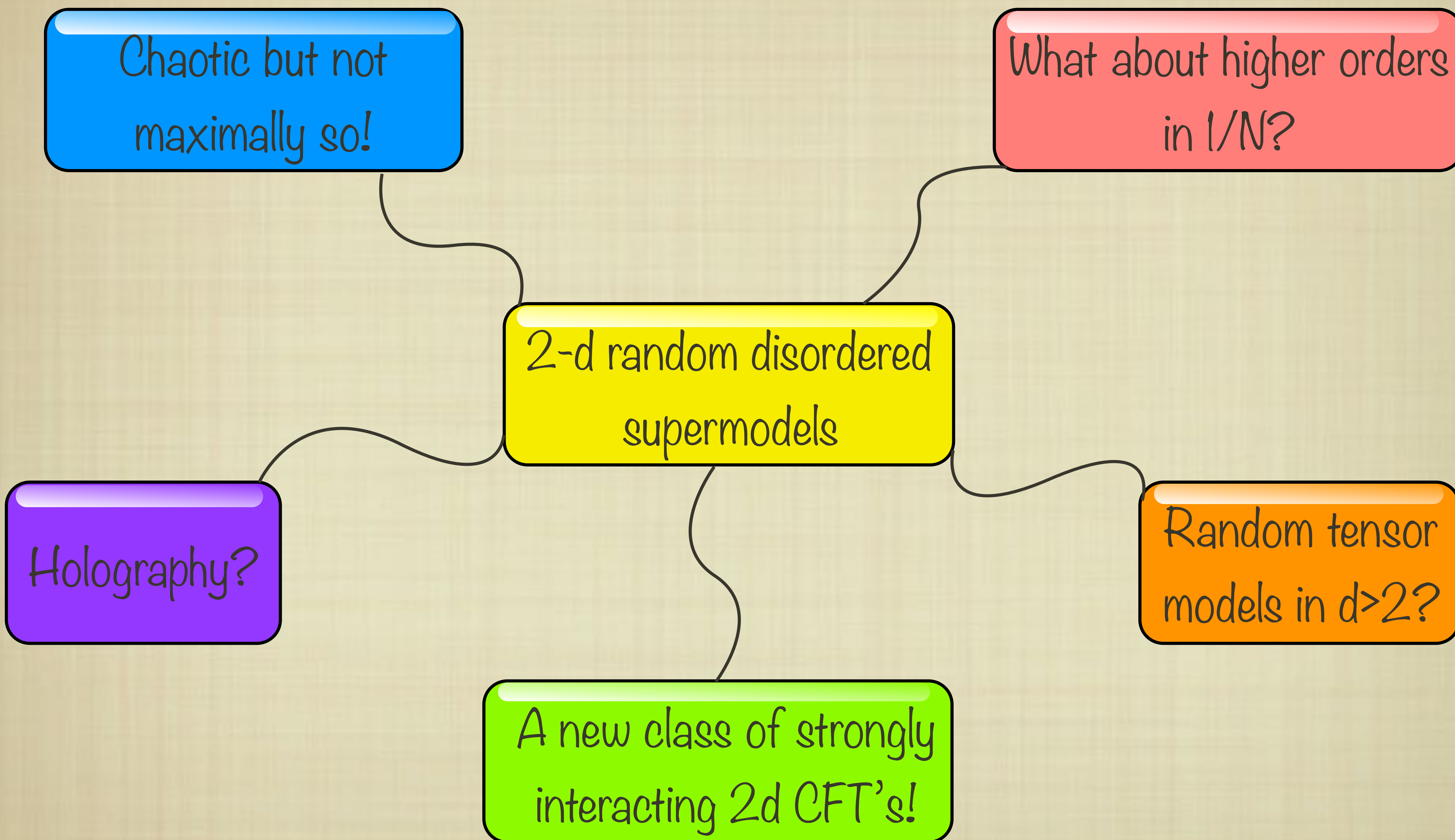
In **SYK-like models** we get this information from the correlator

$$W(t_1,t_2) = \langle (\psi_i(\varepsilon)-\psi_i(-\varepsilon))\,\psi_j(it_1)\,(\psi_i(\beta/2+\varepsilon)-\psi_i(\beta/2-\varepsilon))\psi_j(\beta/2+it_2)\rangle$$

There are two ways to compute the correlator:

- The **retarded kernel** and an integral equation
- A **direct analytic continuation** from the Euclidean 4-point function.

Conclusions and Future work



Hristo! 감사합니다 Шնորհ спасибі! תודה
நன்றி Ndiyabulela! Ke a leboha! Paldies
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धन्यवाद ¡Gracias धन्यवाद Grazie! ありがとう
Eskerrik asko!
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