Holography, Quantum Entanglement and Higher Spin Gravity II @ PI2018/3/14

Entanglement Spreading and Oscillation

Masahiro Nozaki (University of Chicago)

Collaborate with

Akio Tomiya (CCNU),Mitsuhiro Nishida (GIST), Yuji Sugimoto (Osaka University), Hiroyuki Fujita(Tokyo University)

KADANOFF CENTER FOR THEORETICAL PHYSICS

arXiv:1712.09899 [hep-th]

Introduction:

- EE is a candidate of **an entropy in Non-equilibrium physics**.
- In AdS/CFT correspondences, ***Entanglement*** in CFT living on the boundary is expected to be significantly related to ***Gravity*** in the bulk.

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Thermalization ⟺ ***Black Hole Physics***

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The dynamics of entanglement ⟺ The dynamics of gravity

Thermalization ***Creation of Black Hole***

Introduction:

- EE is a candidate of **an entropy in Non-equilibrium physics.**

It is important to study the ***dynamical features*** of Entanglement.

The dynamics of entanglement ⟺ The dynamics of gravity

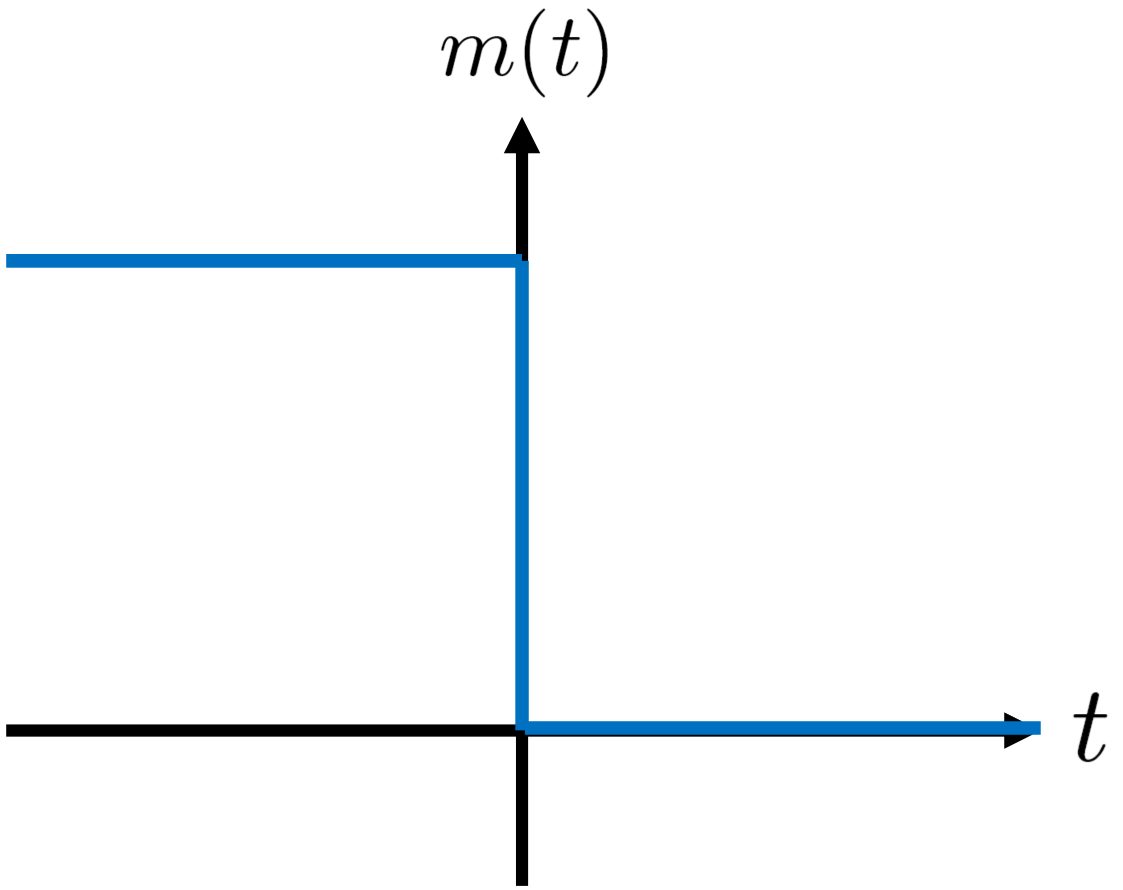
Thermalization

Black Hole Physics

Motivation

Our Motivation

In the sudden global quenches where Hamiltonian suddenly change, in the late time, the change of entanglement entropy ($\Delta S_A(t) = S_A(t) - S_A(t_{initial})$)is:



[Calabrese-Cardy, 06] [Hartman-Maldacena, 13] [Liu-Suh, 13]

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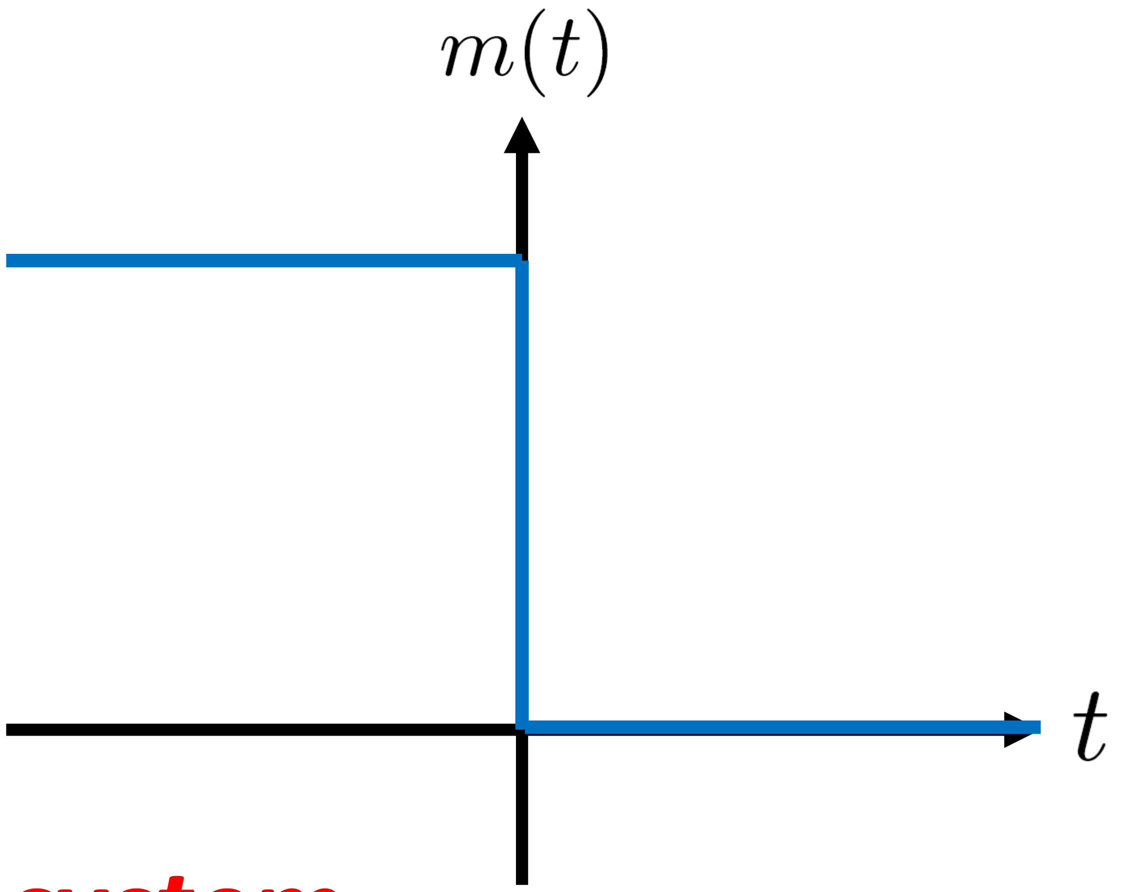

$\Delta S_A \sim$ ***Volume of Subsystem***

[Calabrese-Cardy, 06] [Hartman-Maldacena, 13] [Liu-Suh, 13]

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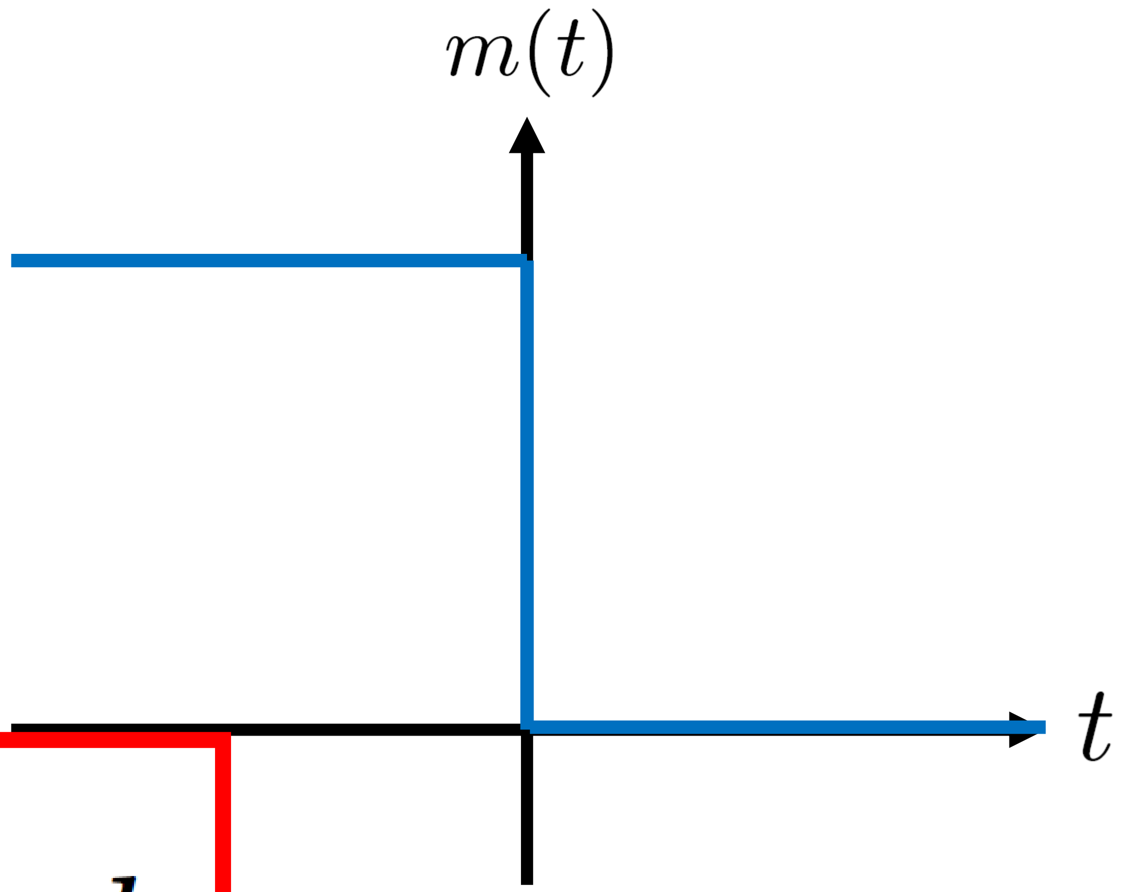
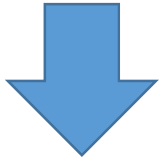


$$\Delta S_A \to S_{thermal}$$

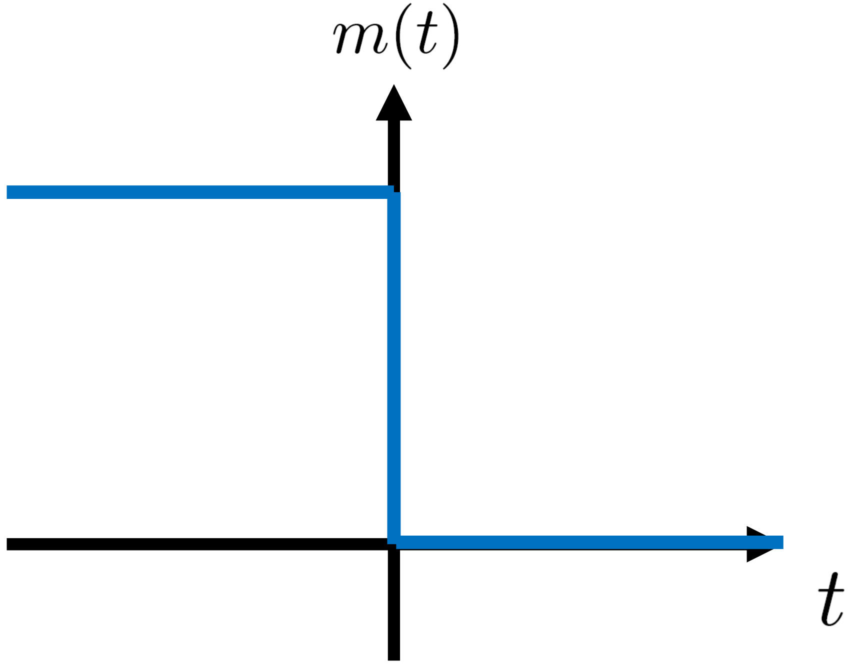
[Calabrese-Cardy, 06] [Hartman-Maldacena, 13] [Liu-Suh, 13]

Our Motivation

Sudden Quenches



Thermalizes



Our Motivation

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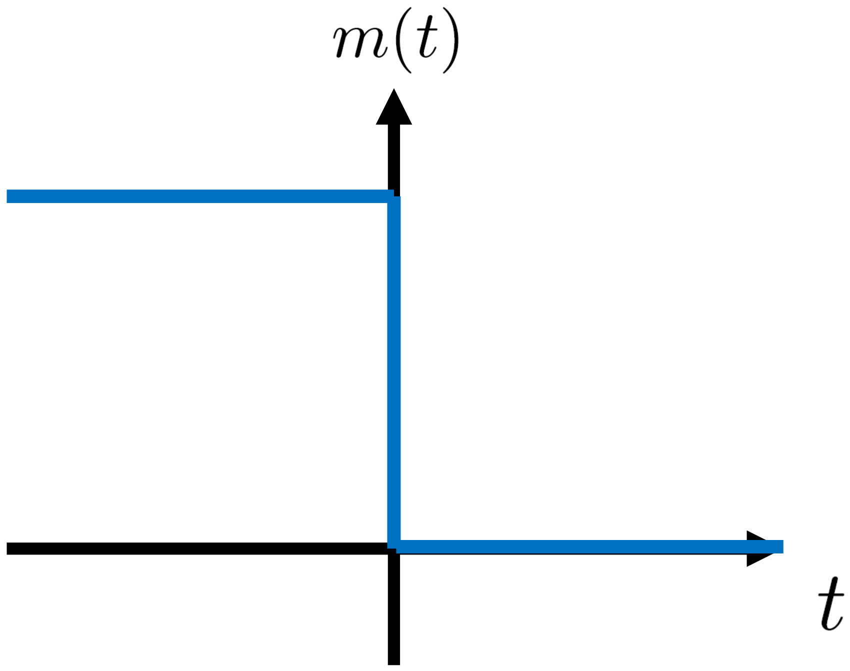


Is this unique behavior for sudden quenches?

Our Motivation

If the state is quenched gradually (smooth quenched),

is subsystem thermalized?

and

how does entanglement time-evolve?

What we have done

We have studied the time evolution of quantities (EE, LN, MI) in smooth quenches(ECP and CCP).

$$\Delta S_A(t) = S_A(t) - \underline{S_A(t_{initial})}$$

EE for initial mass

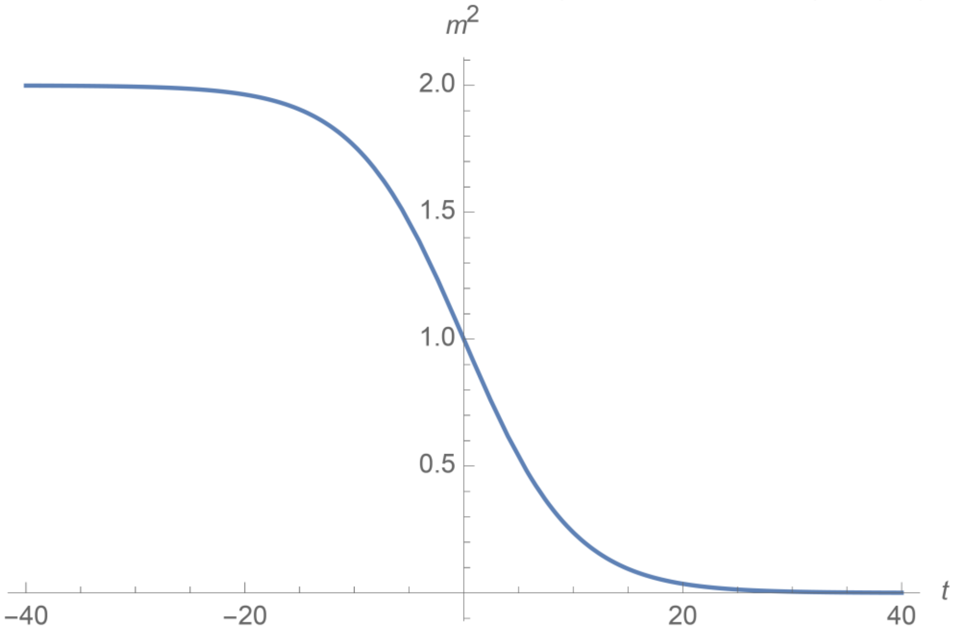
Smooth Quenches

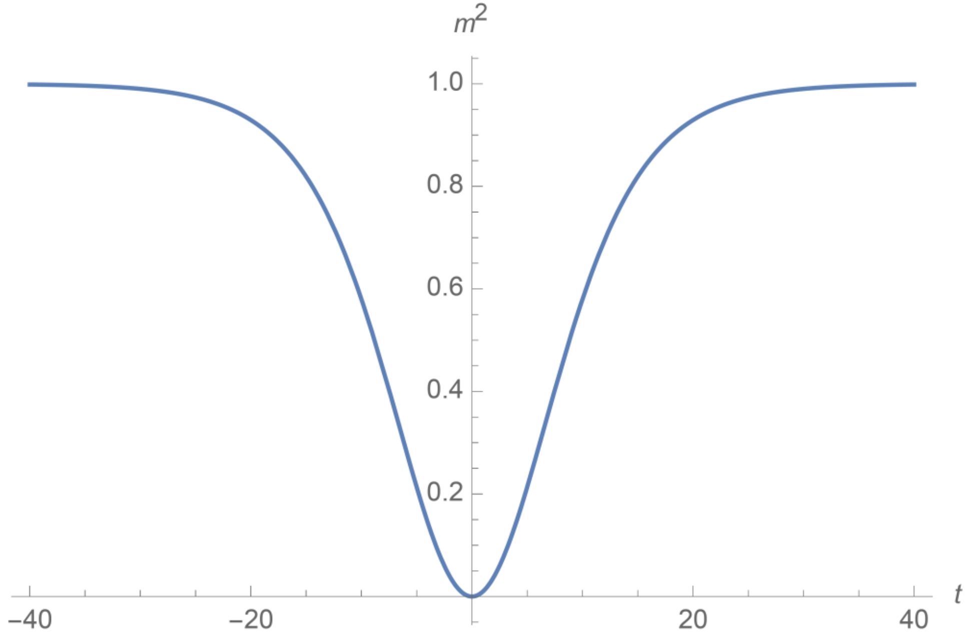
[Das-Galante-Myers, 14]

2d –Time-dependent Hamiltonian

$$H(t) = \frac{1}{2}\int dx\,\left[\Pi^2(x) + \partial_x\phi^2(x) + m^2(t)\phi^2(x)\right]$$

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
- CCP: $m^2(t) = \frac{1}{\xi^2}\tanh^2\left(\frac{t}{\delta t}\right)$





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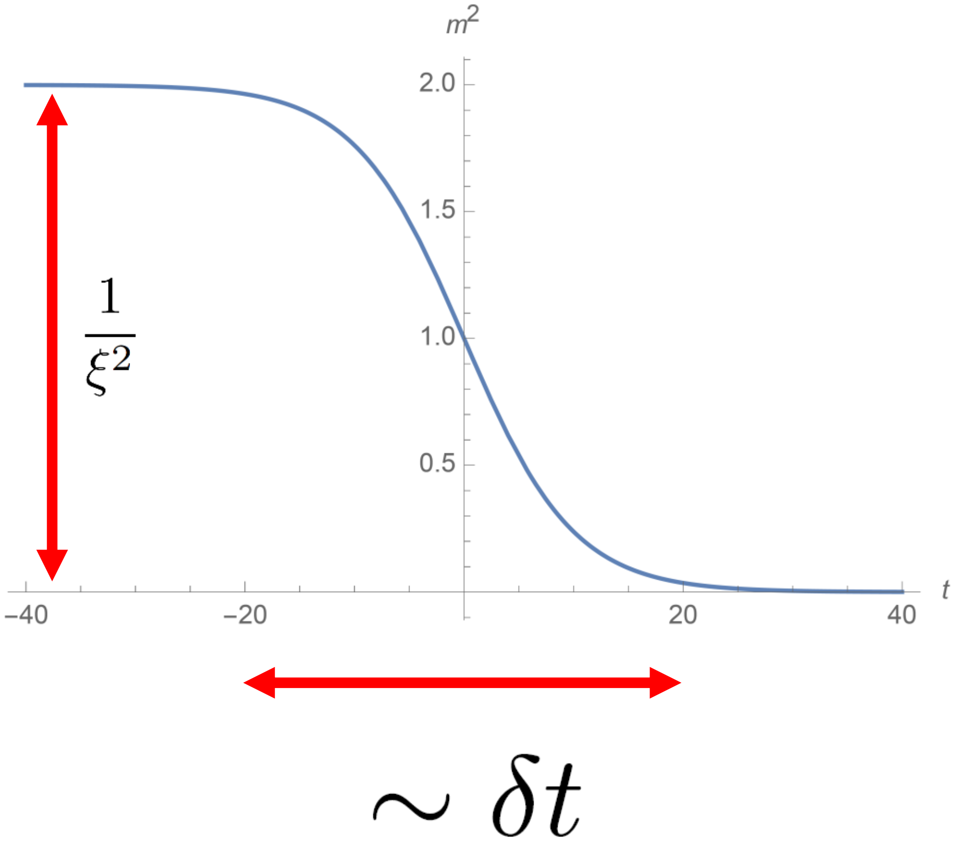

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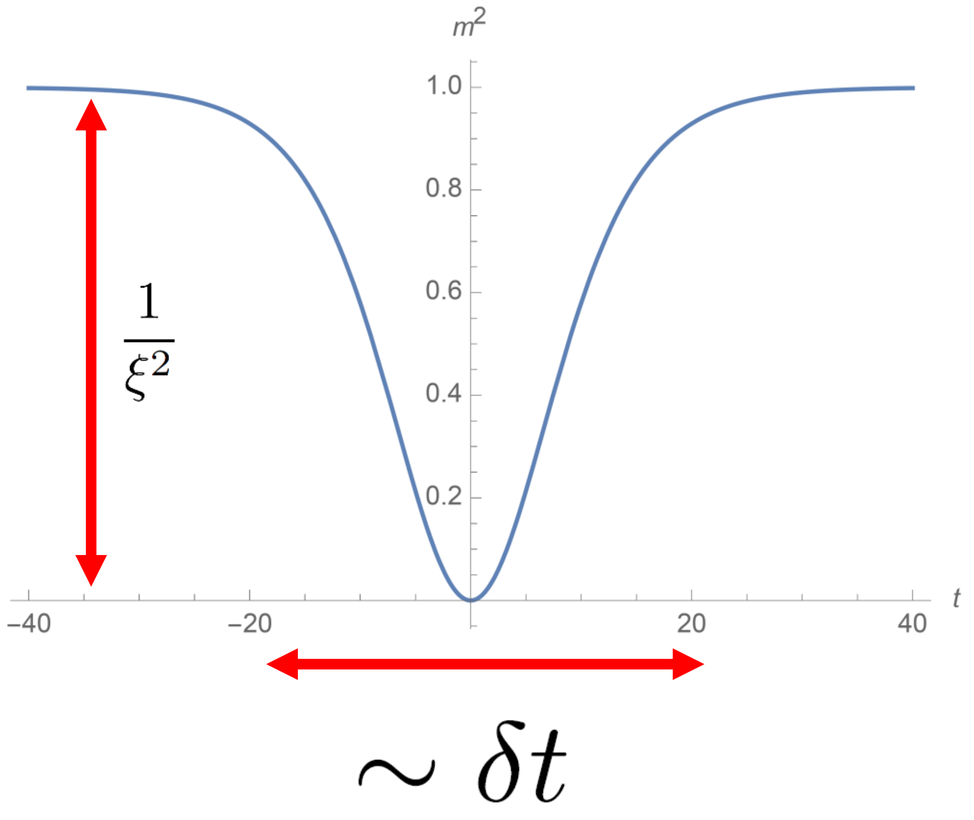

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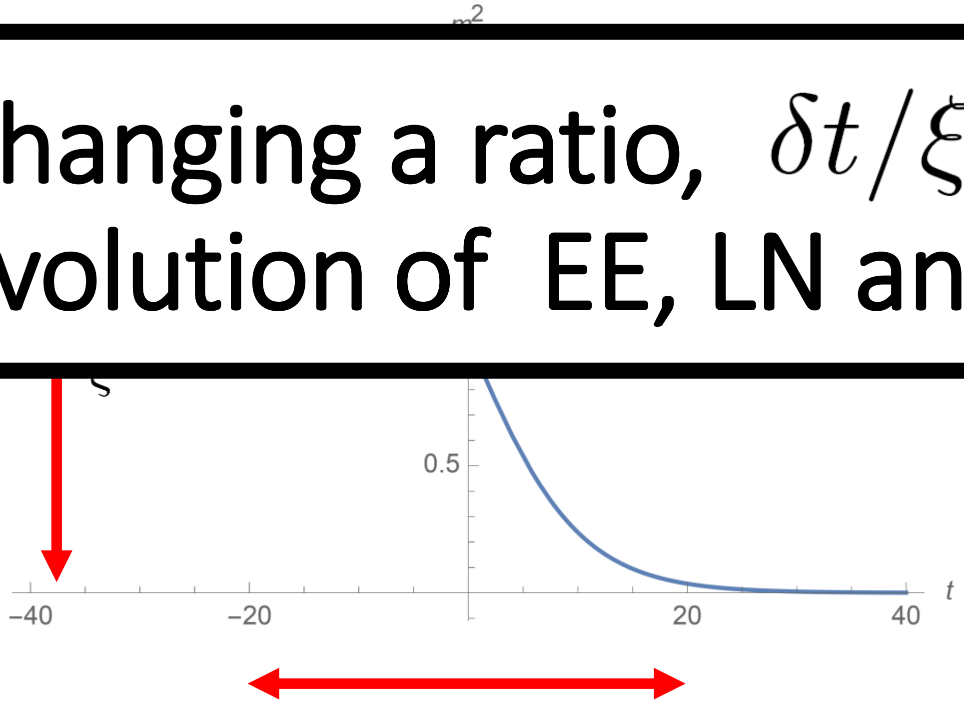
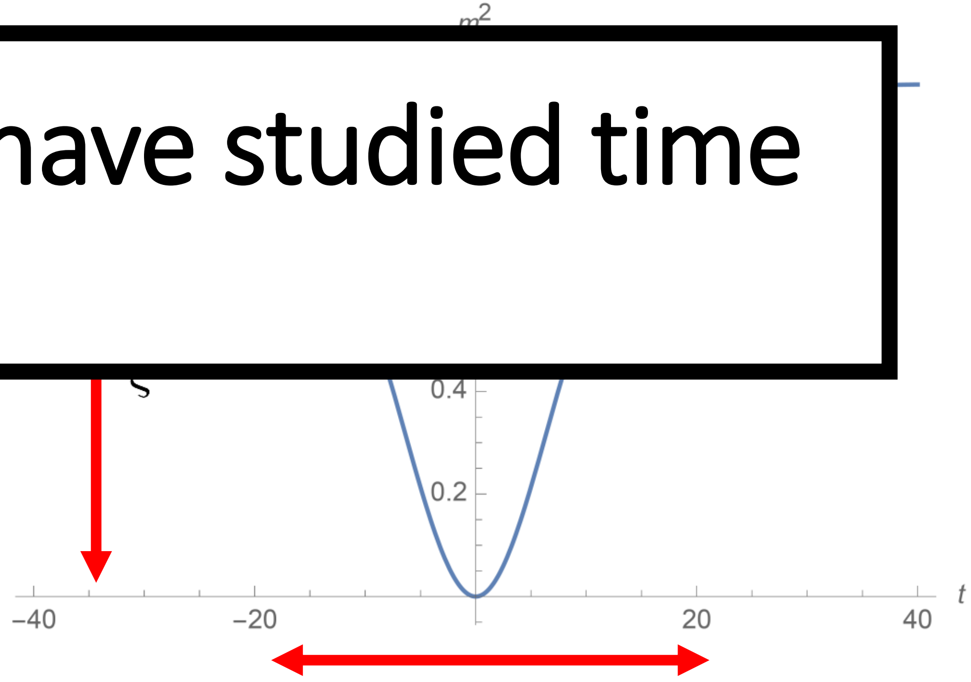
Changing a ratio, $\delta t/\xi$, we have studied time evolution of EE, LN and MI.



$\sim \delta t$



$\sim \delta t$

Smooth Quenches

[Das-Galante-Myers, 14]

- ECP:
- CCP:

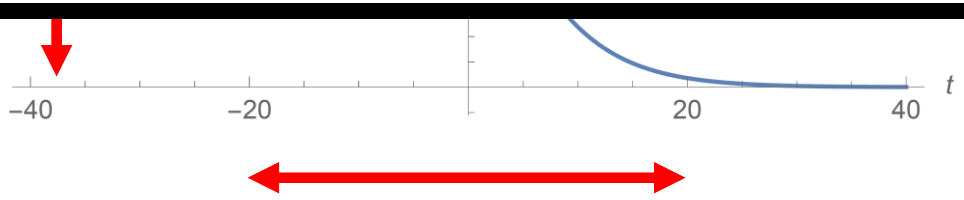
We took ***two extreme limits***:

Fast limit:

$$\omega \ll 1 \quad (\delta t \ll \xi)$$

Slow limit:

$$\omega \gg 1 \quad (\delta t \gg \xi)$$



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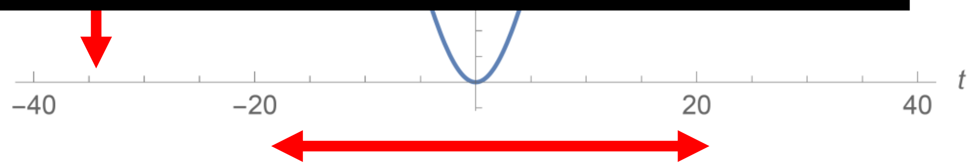

$\sim \delta t$

Results(EE)

Late-time ΔS_A in *ECP*

is proportional to subsystem size.

(***Thermalized***.)

Late-time ΔS_A in *fast-CCP* *(* $\xi \ll \delta t$ *)*

is proportional to subsystem size.

Assumptions: $\frac{1}{m \cdot a} = \frac{\xi}{a} \gg 1$, a: is a lattice spacing.

- **ECP:**

Fast limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \ll 1$

$$\Delta S_A \sim C_1 \frac{l}{\xi}$$

slow limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \gg 1$, $E_{kz} = \frac{1}{\delta t} \ll 1$

$$\Delta S_A \sim C_2 E_{kz} \cdot l$$

- **CCP:**

Fast limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \ll 1$

slow limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \gg 1$, $\xi_{kz} = \sqrt{\frac{\delta t}{m}} = \sqrt{\xi \delta t} \gg 1$

$$\Delta S_A \sim C_3(\omega) \frac{l}{\xi}$$

Assumpt **Constant.** g.

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Depends on ω

$$C_2 E_{kz} \cdot l$$

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Result1

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$$\Delta S_A \sim C_3(\omega) \frac{l}{\xi}$$

$C_3(\omega)$

In fast limit, keeping ξ constant and, ω decreases

⇒ $C_3(\omega)$ ***decreases***.

In slow limit, keeping ξ constant and, ω increases

⇒ $C_3(\omega)$ ***decreases***.

$C_3(\omega)$

In fast limit, keeping ξ constant and, ω decreases

Consistent with a number operator in late time.

In fast limit, keeping ξ constant and, ω increases

$\Rightarrow C_3(\omega)$ ***decreases.***

Results in slow ECP

- **How does the subsystem thermalize in slow ECP? (Quasi-particles are created?)**

Quasi-particles are created when the adiabaticity breaks down!
(Subsystems thermalize!)

Results in slow ECP

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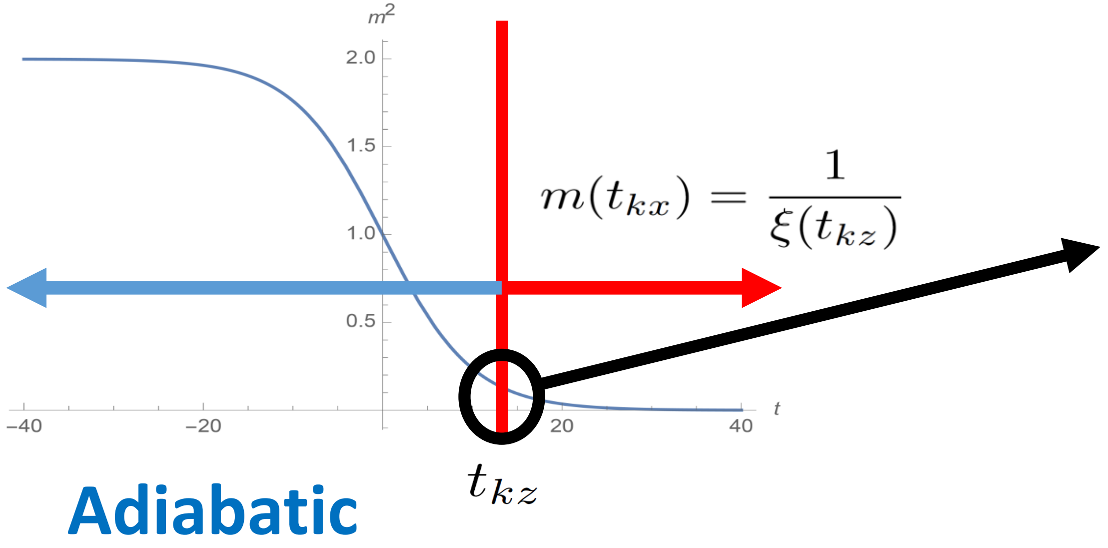


Quasi-particles are created at t= t_{kz} **and carry quantum entanglement.**

➡ **Thermalize** around $t = t_{kz} + \frac{l}{2}$

Interpretation In ECP and fast CCP

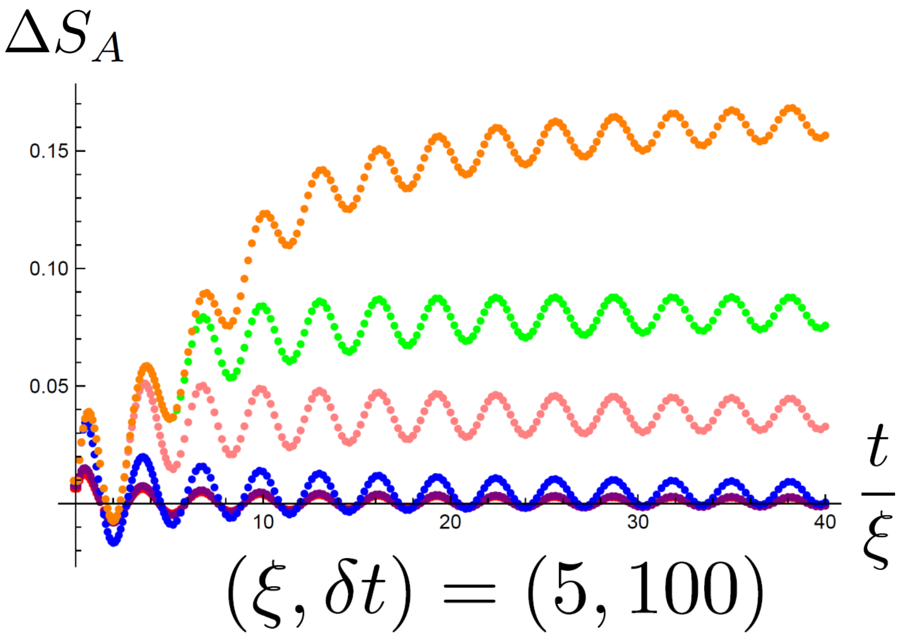
- Time Evolution of $\Delta S_A(t)$

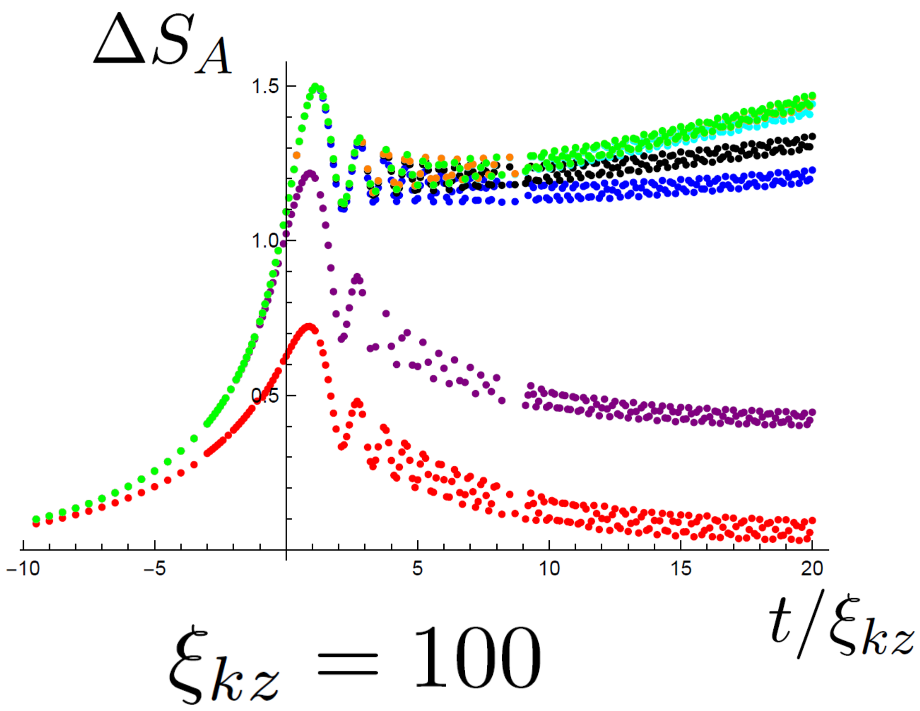
$$\Updownarrow$$

Propagation of Entangled particles

Results(EE)

Entanglement Oscillation





Results(EE)

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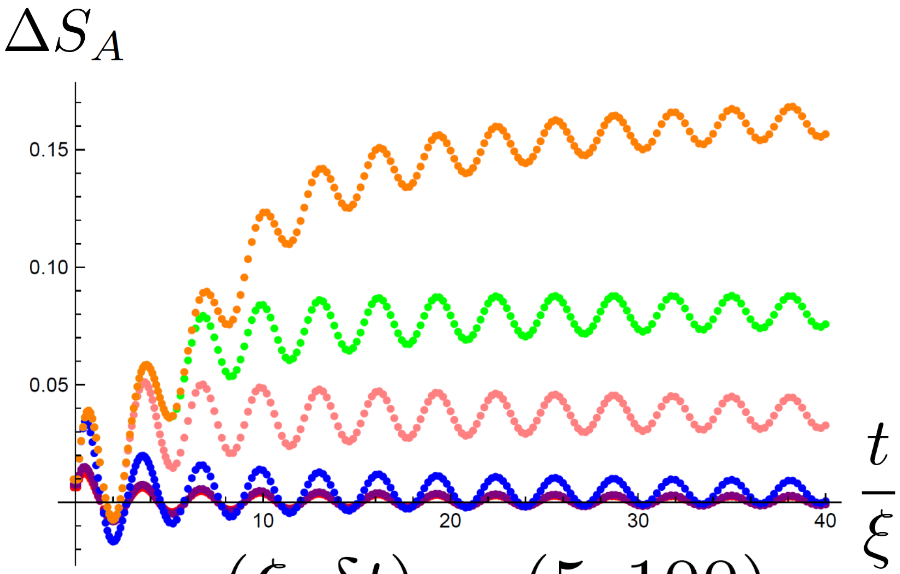
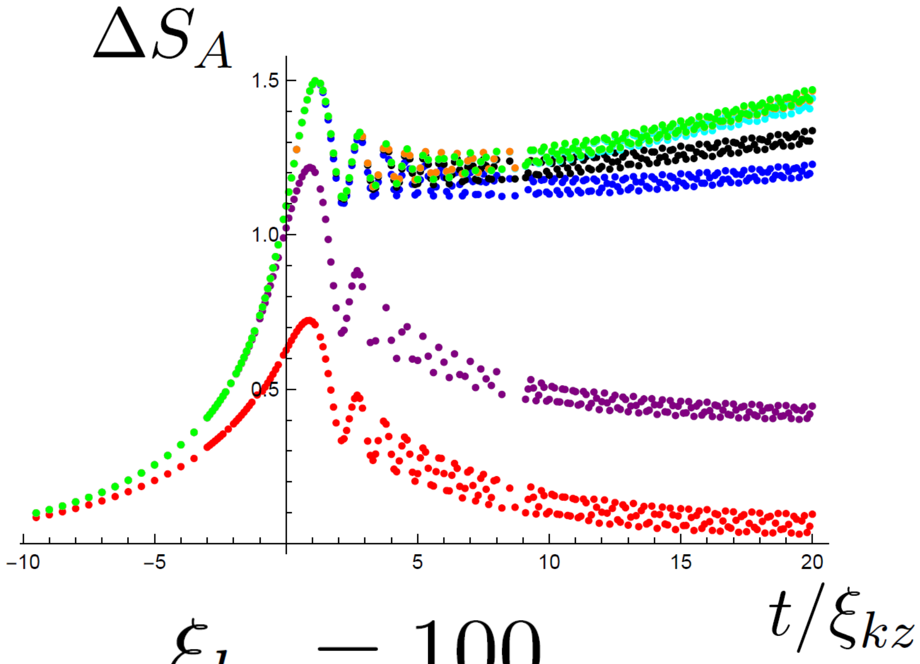


$(\xi, \delta t) = (5, 100)$



$\xi_{kz} = 100$

The period of oscillation @ late time.

⇔ ***The periodicity of zero mode*** $\simeq \pi\xi$

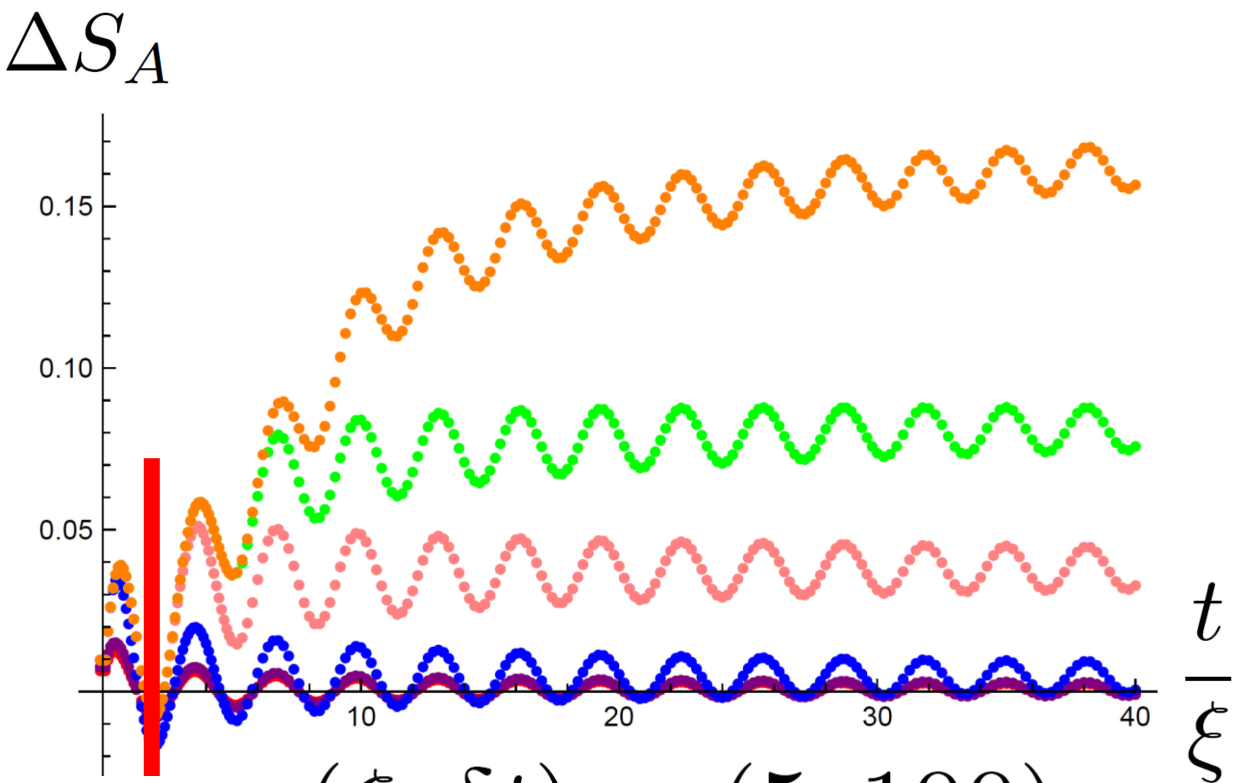
Results(EE)

Late-time ΔS_A in CCP ***oscillates*** with periodicity determined by ***a late-time mass***.

ΔS_A in fast-CCP

is ***minimized***

at $t = 2\xi$.



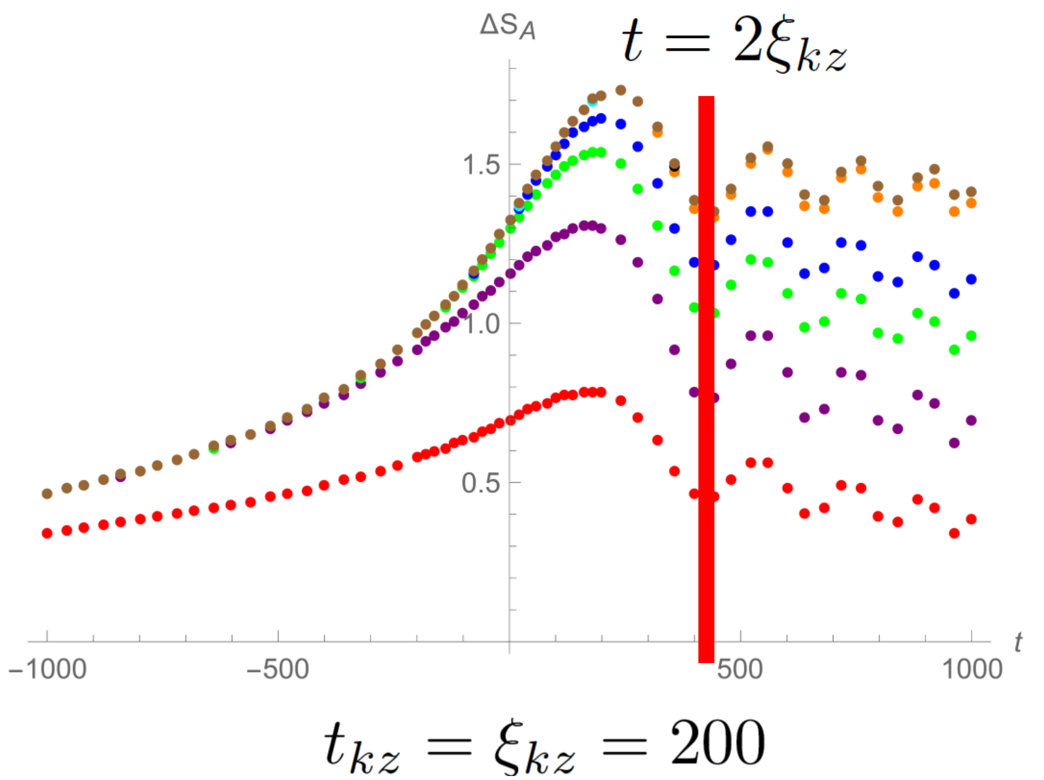
$t = 2\xi$ $(\xi, \delta t) = (5, 100)$

Results(EE)

Late-time ΔS_A in CCP ***oscillates*** with periodicity determined by ***a late-time mass***.

ΔS_A in slow-CCP starts to oscillate after $t = \underline{2\xi_{kz}}$.

Adiabaticity breaks down.



Results(EE)

Late-time ΔS_A in CCP ***oscillates*** with periodicity determined by ***a late-time mass***.

$t = 2\xi$ and $t = 2\xi_{kz}$

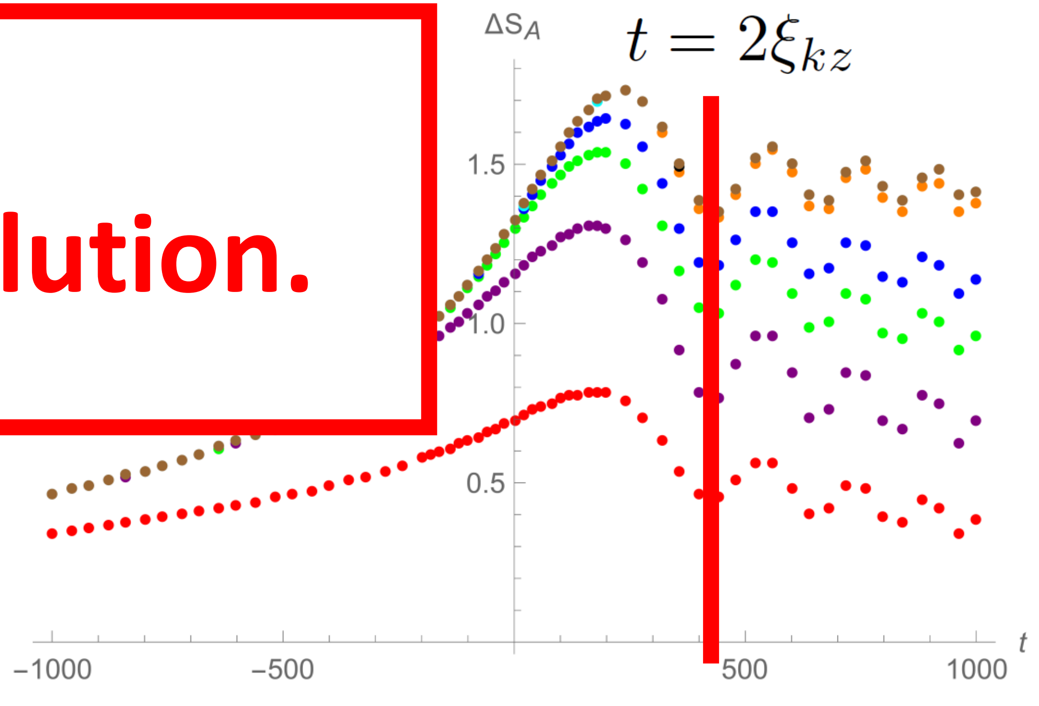
characterize time evolution.

ΔS_A

start

after $t = 2\xi_{kz}$.

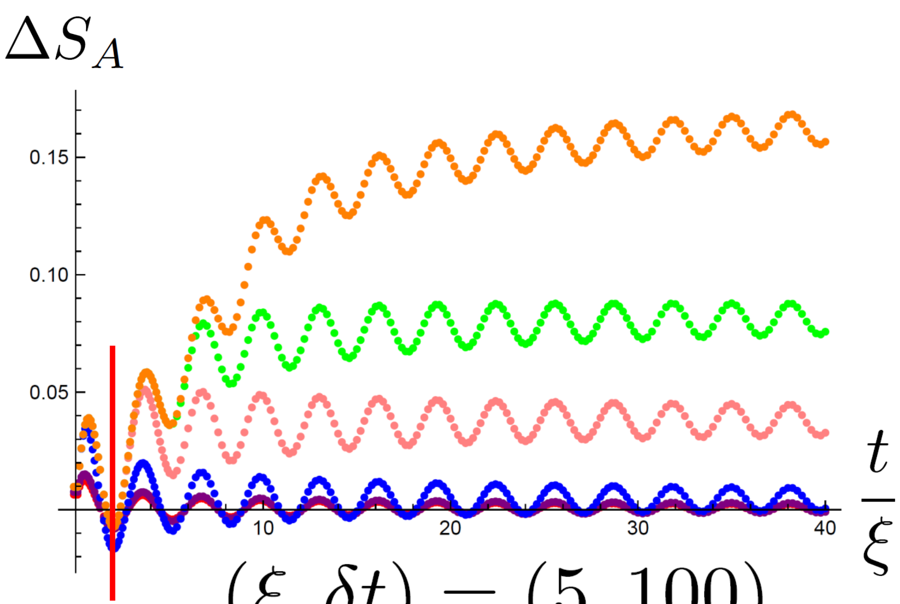
Adiabaticity breaks down.



$t_{kz} = \xi_{kz} = 200$

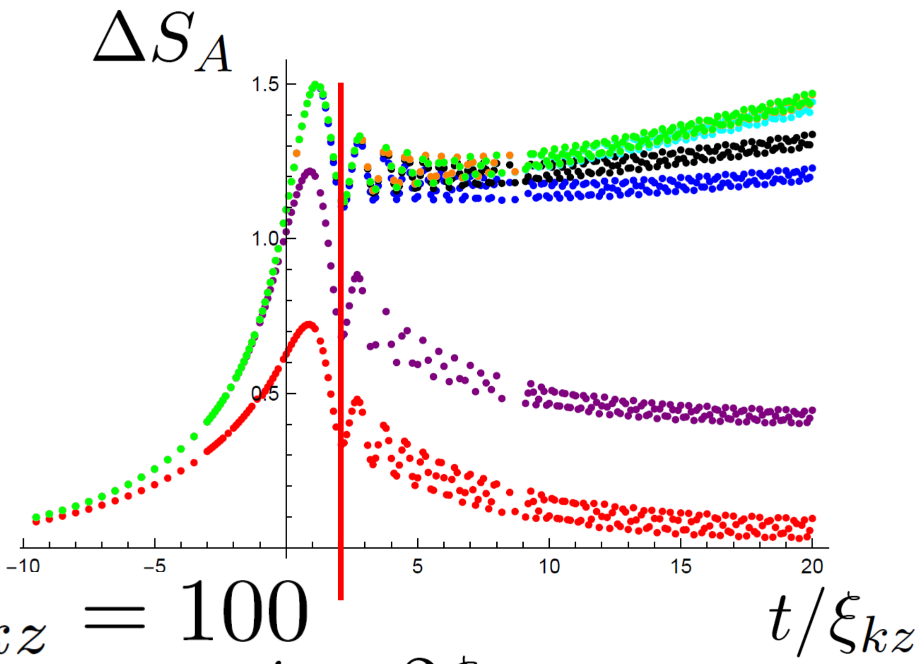
Results(EE)

Time evolution is characterized by $t \simeq 2\xi_{kz}, 2\xi$.



$(\xi, \delta t) = (5, 100)$

$t \simeq 2\xi_{kz}$



$\xi_{kz} = 100$

$t \simeq 2\xi$

After $t = 2\xi, 2\xi_{kz}$, ΔS_A oscillates.

Result(LN, MI (work in progress))

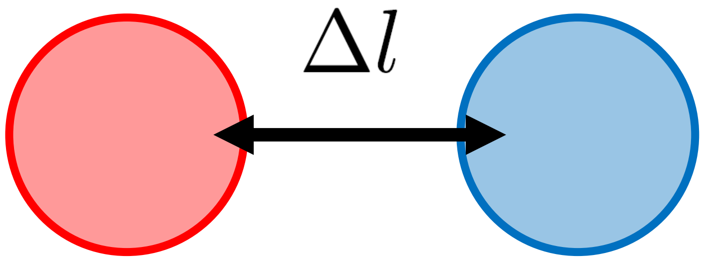
Their time-evolution in ECP can be interpreted in terms of ***relativistic propagation of quasi-particles***.

- If two subsystem are well-separated,

Late-time MI in fast-ECP ***increases*** (logarithmically?).

- If two subsystem are well-separated,

Late-time LN in fast-ECP ***decreases*** .



Result(LN, MI (work in progress))

Their time-evolution in ECP can be interpreted in terms of ***relativistic propagation of quasi-particles***.

Late-time LN weakly depends on slow modes.

Explain later.

- If two subsystem are well-separated,

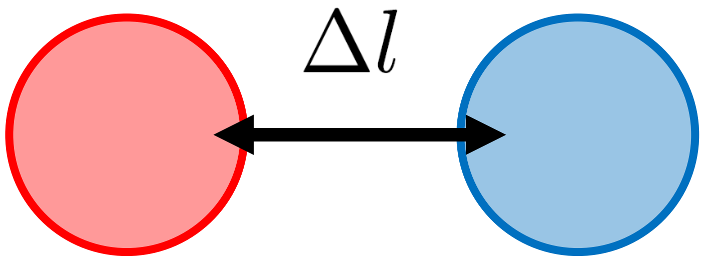
Late-time LN in fast-ECP ***decreases*** .

Result(LN, MI (work in progress))

Time evolution of LN and MI in CCP **strongly depends** on Δl.

If $\Delta l < \xi$, LN and MI ***oscillate.***

If $\Delta l \gg \xi$, oscillation is suppressed. ***Their time evolution can be interpreted in terms of relativistic propagation of quasi-particle***.



The Contents of Talk

- Introduction
- Motivation
- Results(EE)
- Results(LN, MI)
- Setup
- Method
- ECP(EE)
- CCP(EE)
- MI and LN (work in progress)
- Summary and Future directions

Setup

Smooth Quenches

- These quenches are ***more realistic***.
- Hamiltonian is not changed suddenly but is changed smoothly.
- We can excite the state ***slowly or fast***.
- This is a kind of generalization of sudden quenches.

Our protocol (Smooth Quenches)

[Das-Galante-Myers, 14]

2d –Time-dependent Hamiltonian

$$H(t) = \frac{1}{2}\int dx\,\left[\Pi^2(x) + \partial_x\phi^2(x) + m^2(t)\phi^2(x)\right]$$

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
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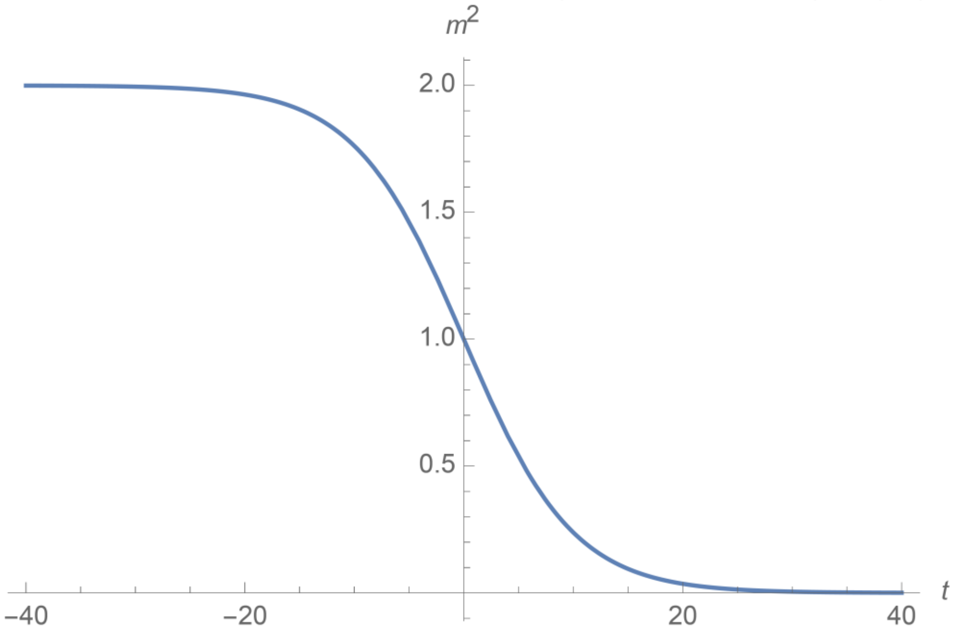

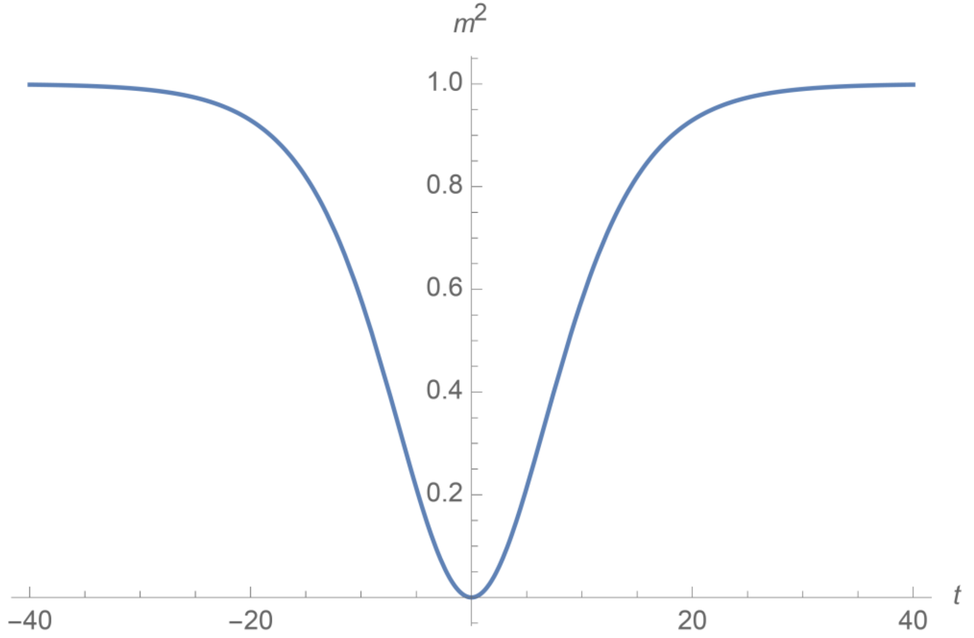

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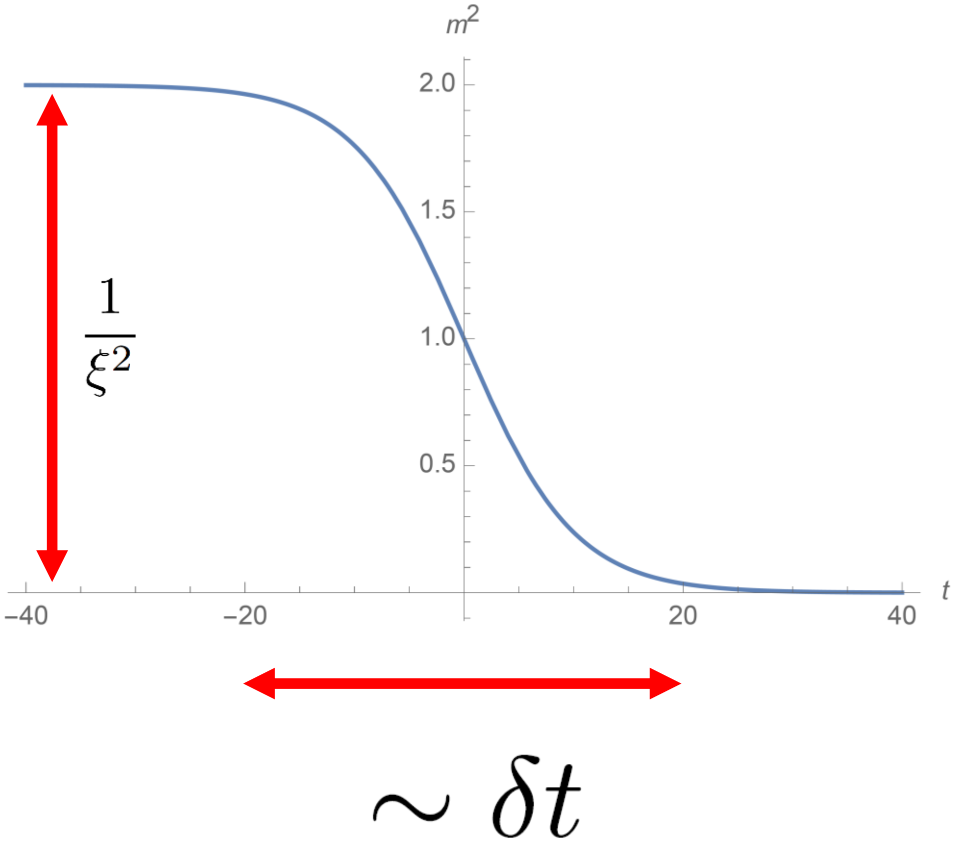

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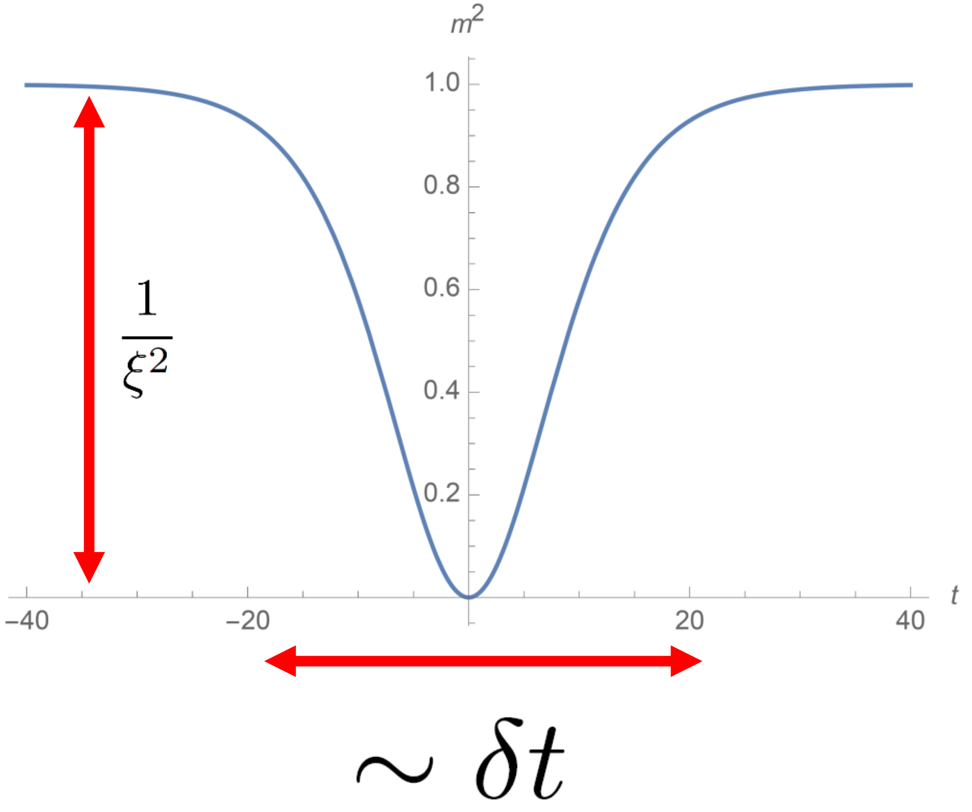
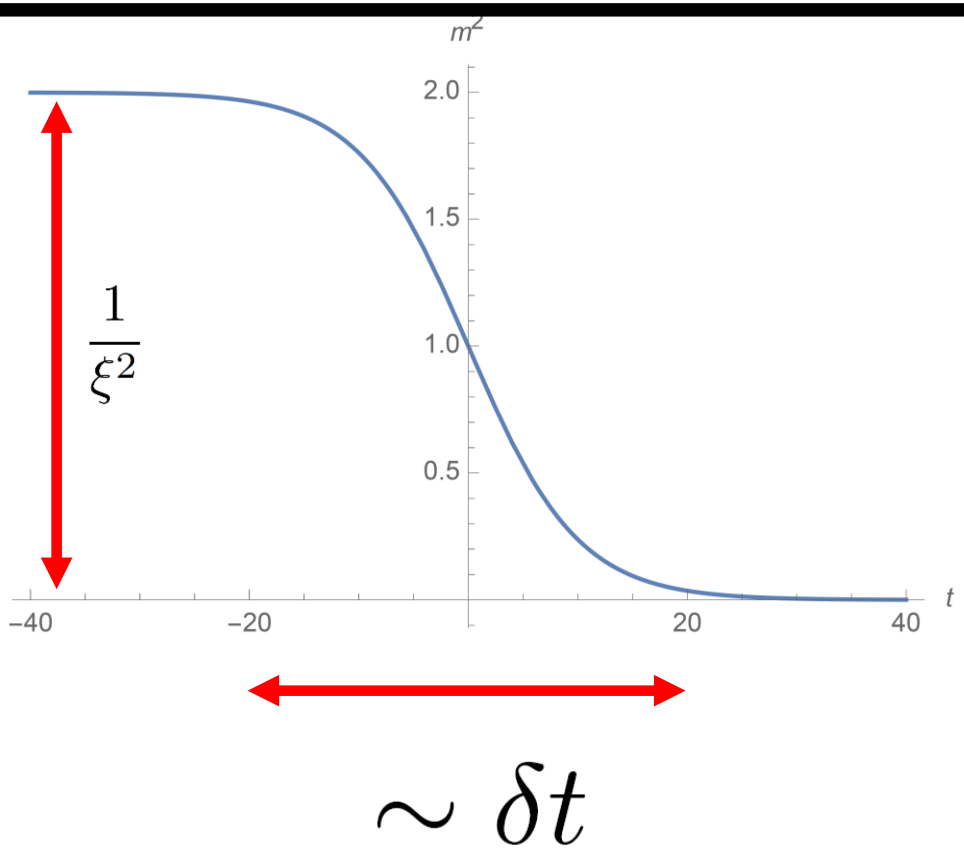
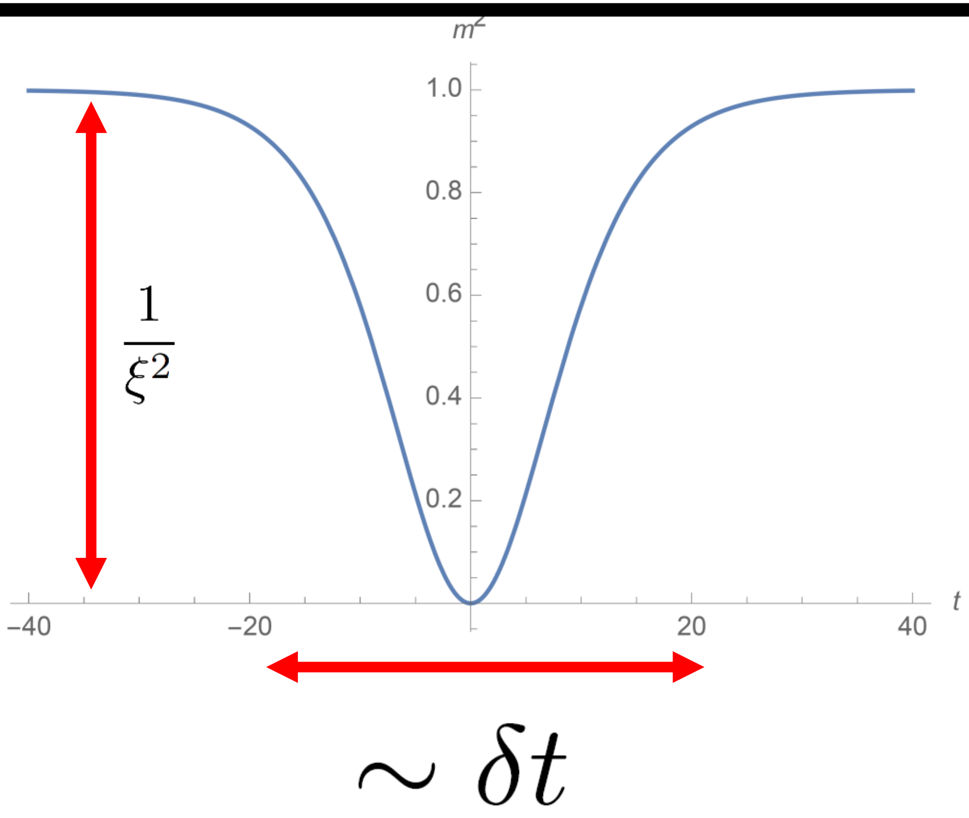


We have two tunable parameters. → **States are excited slowly and rapidly.**

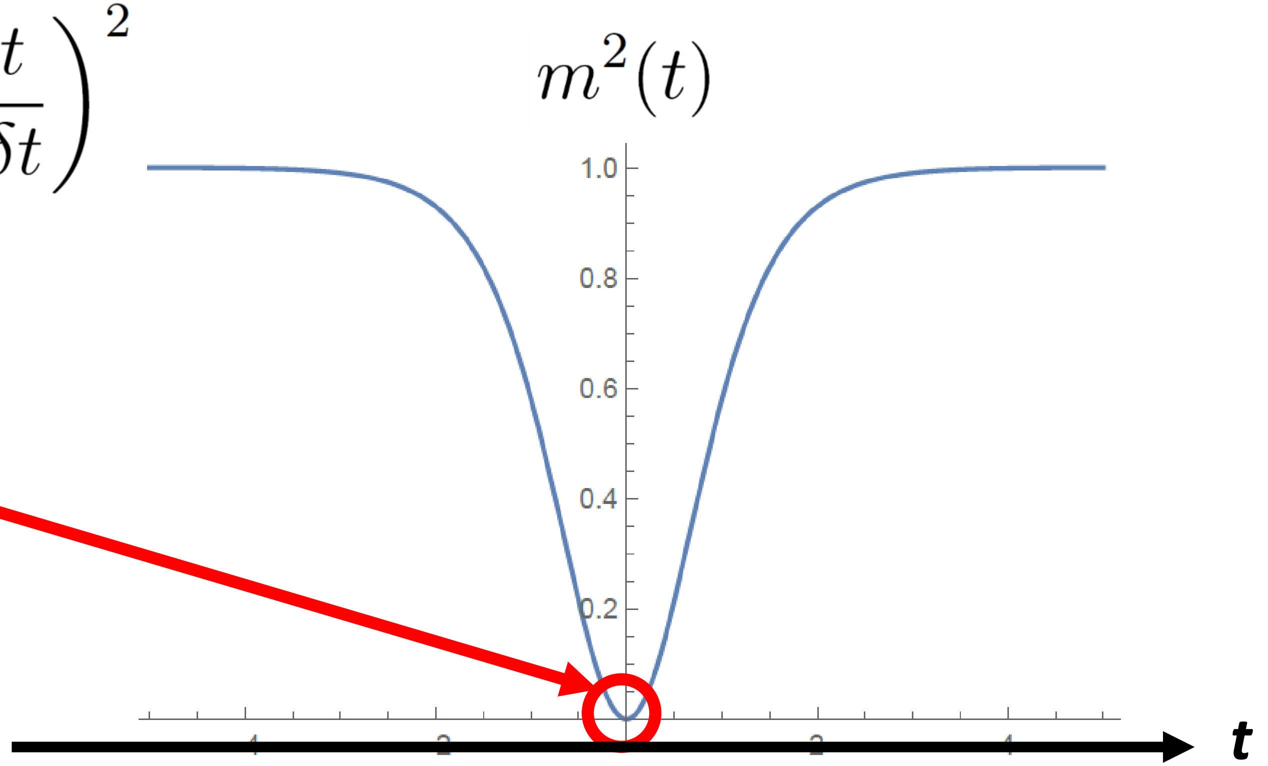




Our setup

- Theory 2d Free scalar with time dependent mass m(t).
- Put it on the lattice but take the ***thermodynamic limit***.

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$



At ***t =0***, the theory is at critical point.

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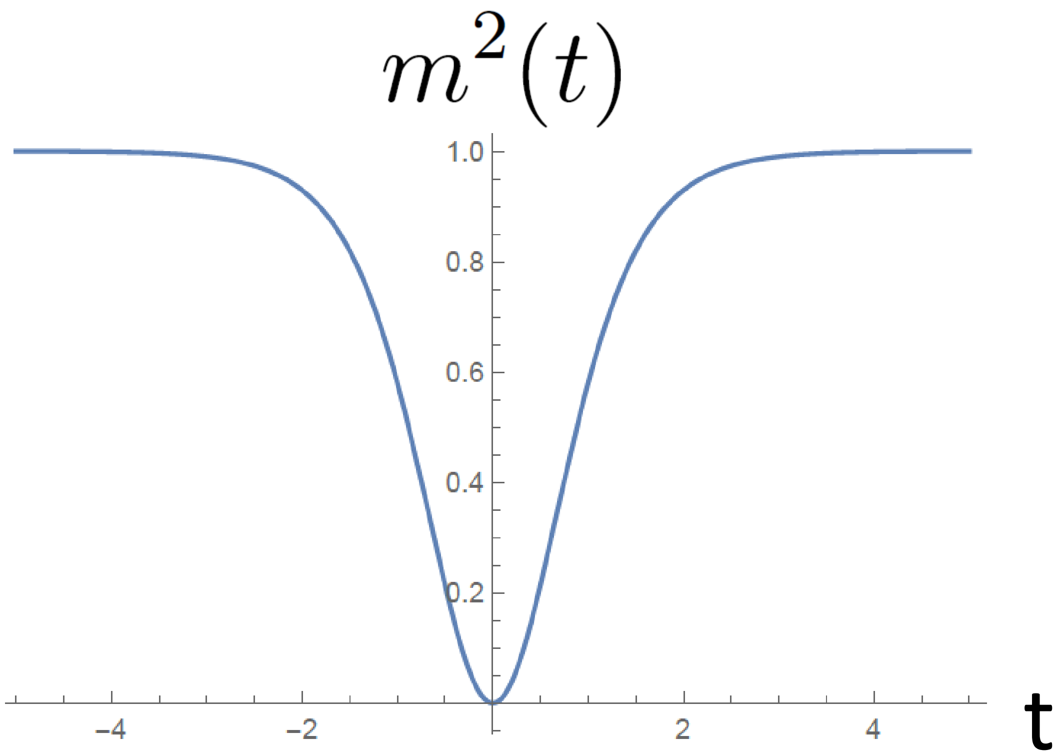
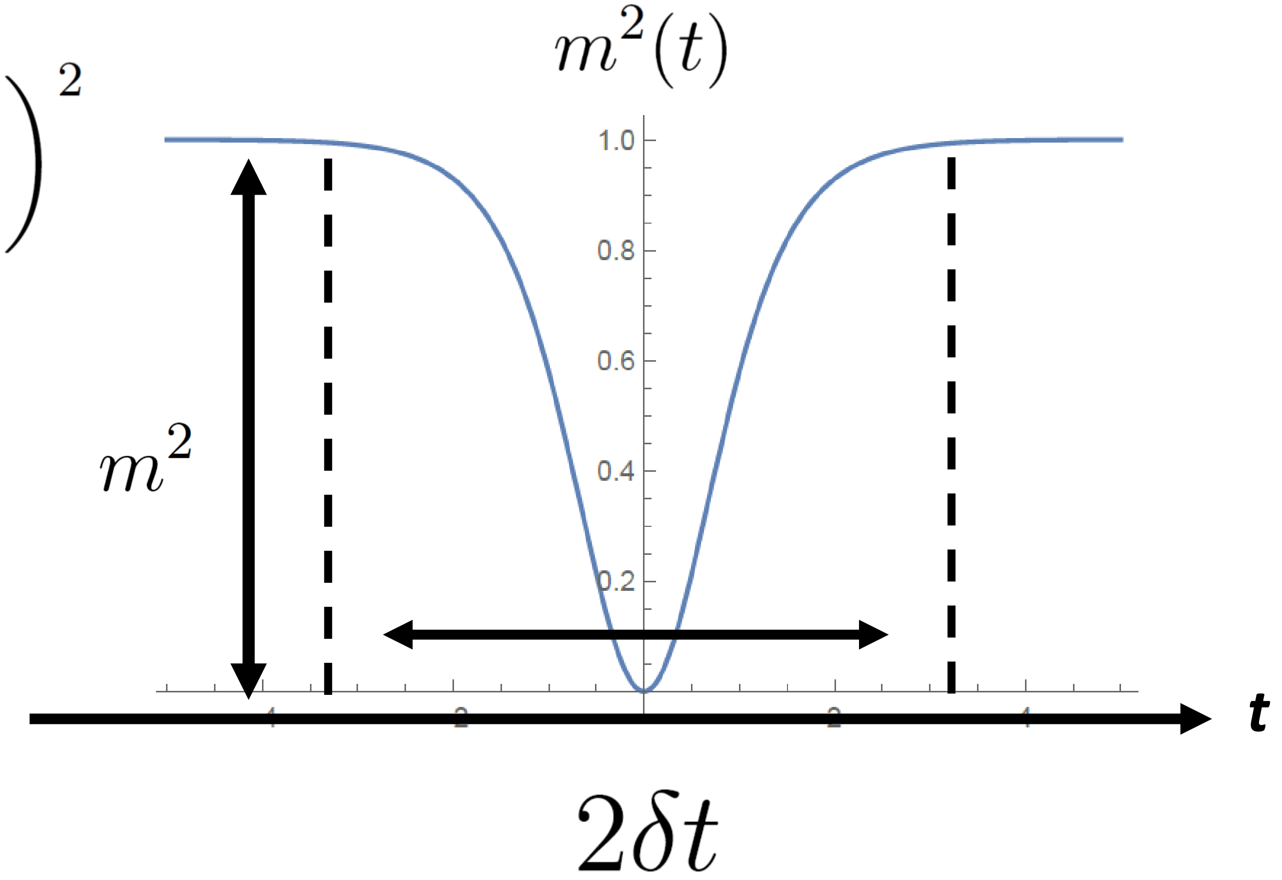


Initial state: The Ground state
for massive free scalar with mass m^2.

Slow Quenches

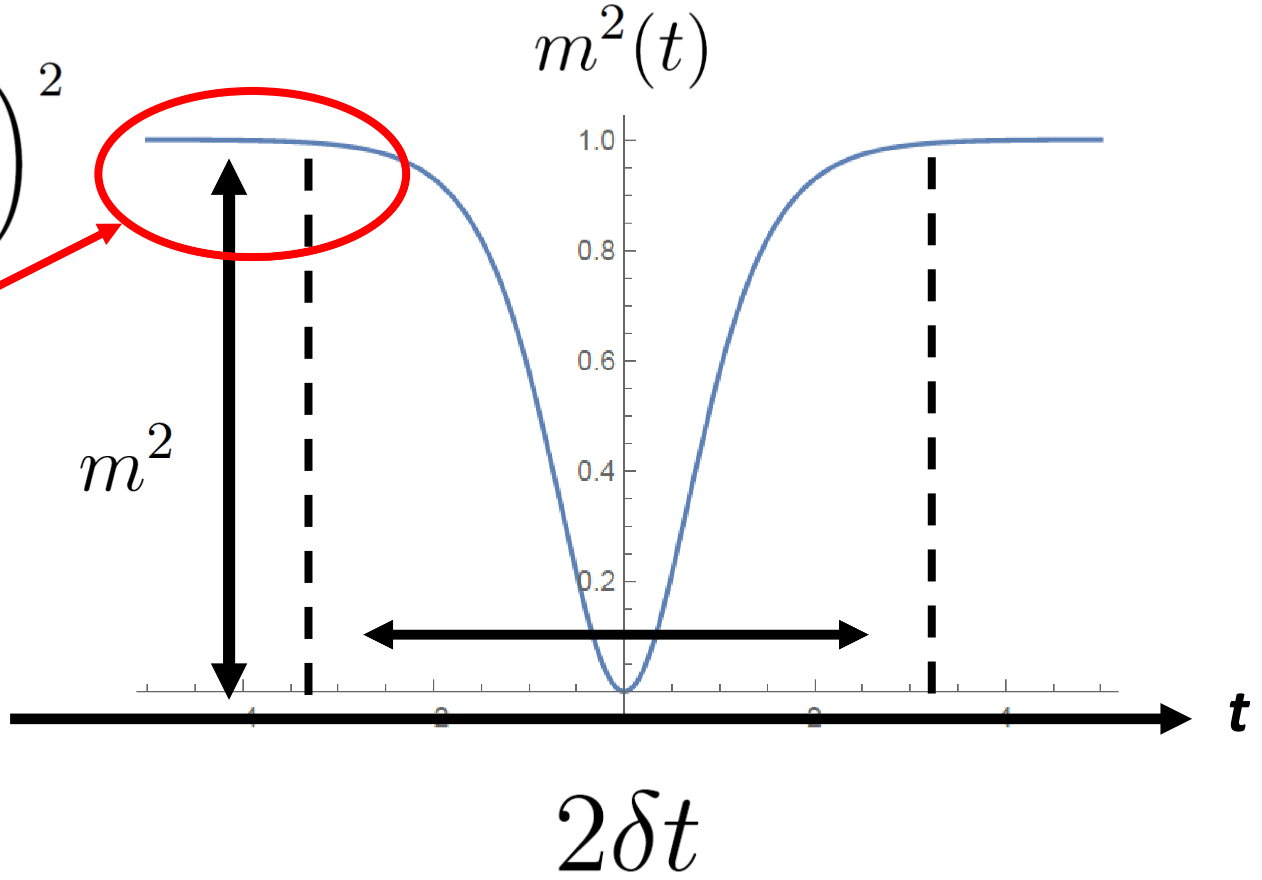
Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$



Slow Quenches

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$

Very Early time:
Observables ***can***
be computed adiabatically .



Adiabatic Expansion

$$X_{ij} = X_{ij}^{(0)} + X_{ij}^{(1)} + \cdots$$

$$P_{ij} = P_{ij}^{(0)} + P_{ij}^{(1)} + \cdots$$

$$D_{ij} = D_{ij}^{(0)} + D_{ij}^{(1)} + \cdots$$

$$\langle \phi_i \phi_j \rangle = X_{ij}$$

$$\left\langle \dot{\phi}_i \dot{\phi}_j \right\rangle = P_{ij}$$

$$\frac{1}{2} \left\langle \{\phi_i, \dot{\phi}_j\} \right\rangle = D_{ij}$$

Higher orders has higher derivative with respect to t.

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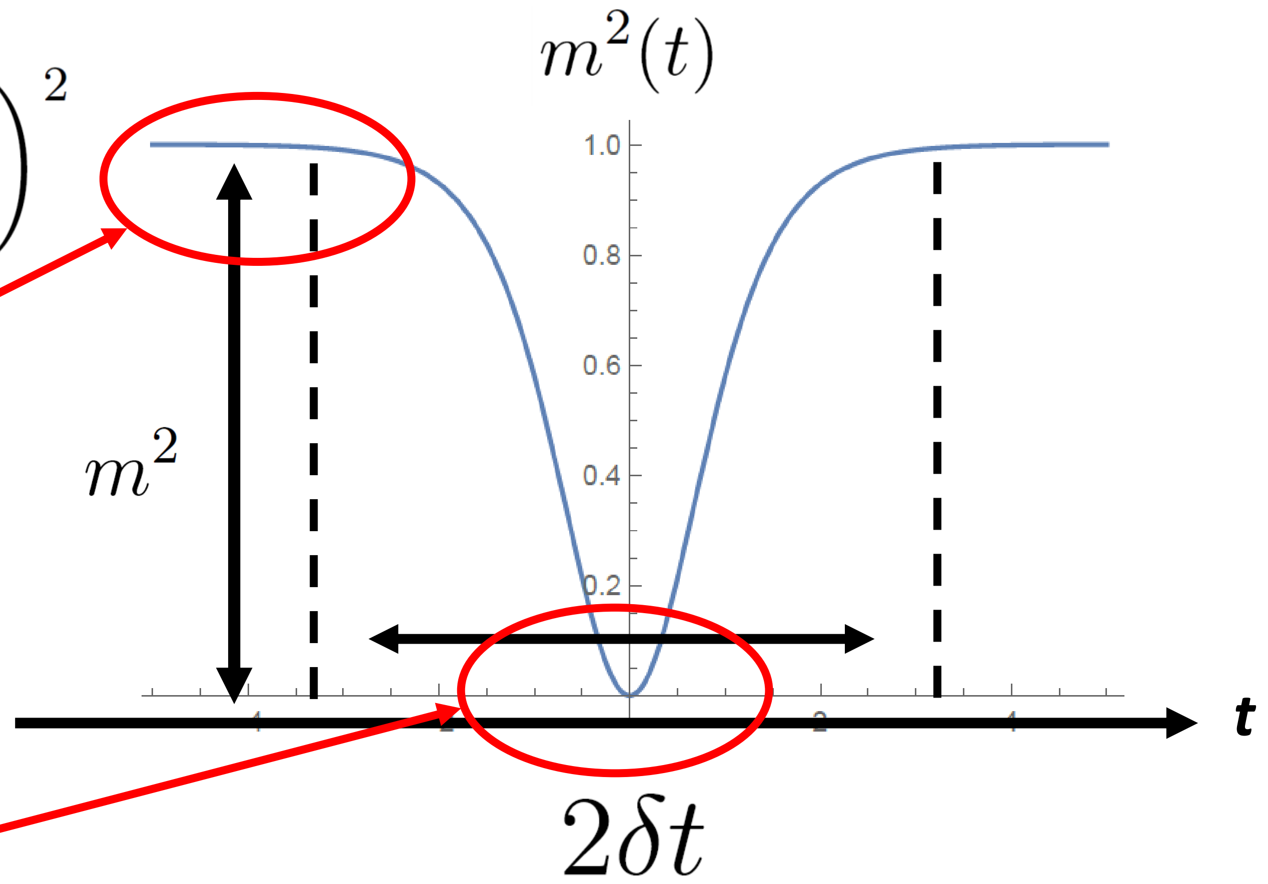
Higher orders has higher derivative with respect to t.

Landau Criteria $\frac{1}{m^2(t)} \frac{dm(t)}{dt} \ll 1$ → ***Adiabaticity holds***

Slow Quenches

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$



Very Early time:

Observables ***can*** be computed adiabatically .

Around critical point:

Observables ***can not*** be computed adiabatically .

Adiabatic Expansion

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Landau Criteria $\frac{1}{m^2(t)} \frac{dm(t)}{dt} \sim 1$ ⇒ ***Adiabaticity braeks down.***

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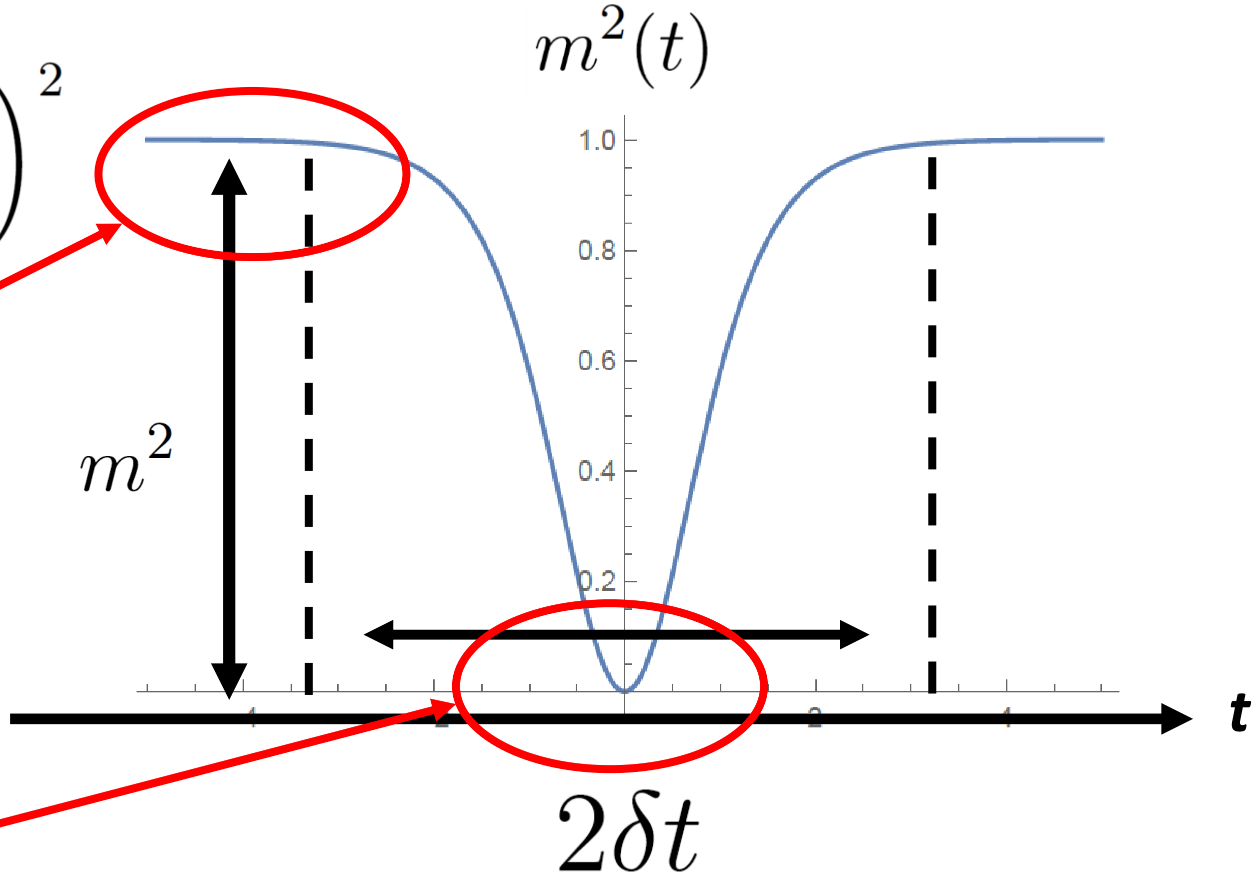
Higher orders has higher derivative with respect to t.

Landau Criteria $\frac{1}{m^2(t)} \frac{dm(t)}{dt} \sim 1$ ⇒ ***Adiabaticity braeks down.***

The time when adiabaticity breaks down is called Kibble-Zurek Time t_{kz}.

Slow Quenches

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$



Very Early time:

Observables ***can*** be computed adiabatically .

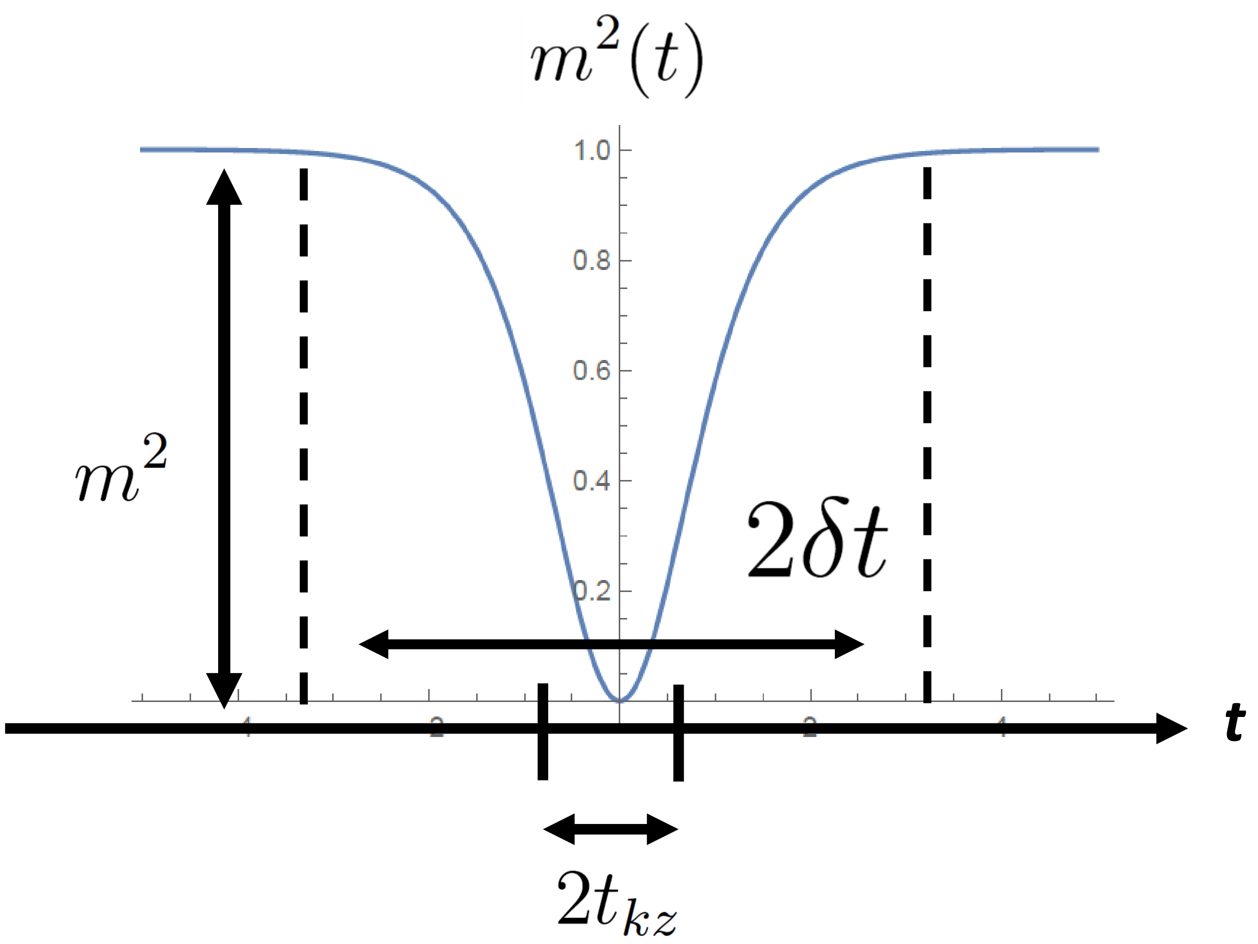
Around critical point:

We assume that at $t = -t_{kz}$, adiabaticity breaks down.

(Around $t = t_{kz}$, time process becomes adiabatic again.)

If t_{kz} *is so small,* ***the most of whole time evolution*** *is adiabatic.*

More precisely, $\frac{t_{kz}}{\delta t} \ll 1$



*If t_{kz} is so small, **the most of whole time evolution** is adiabatic.*

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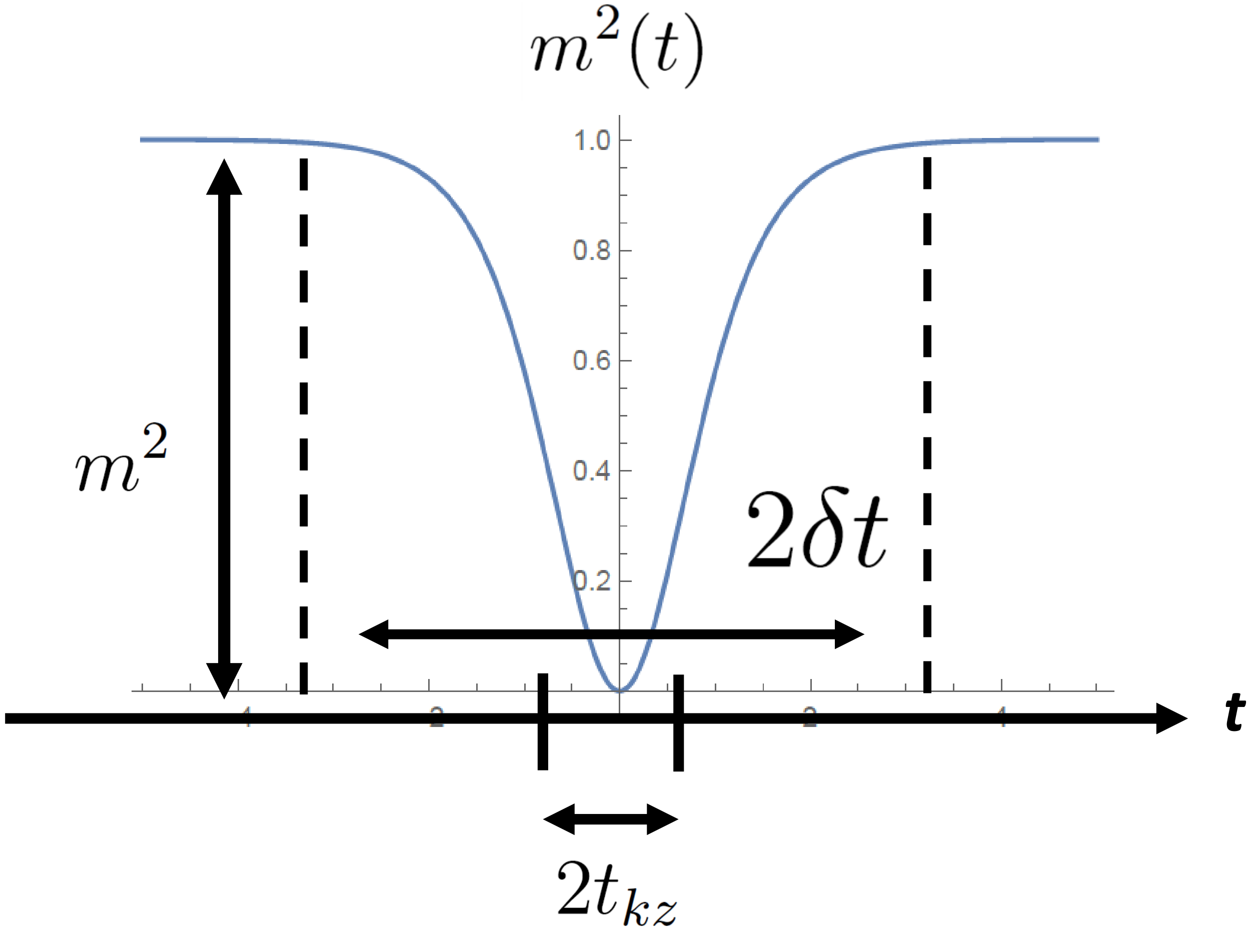
$$\omega = m\delta t \gg 1$$



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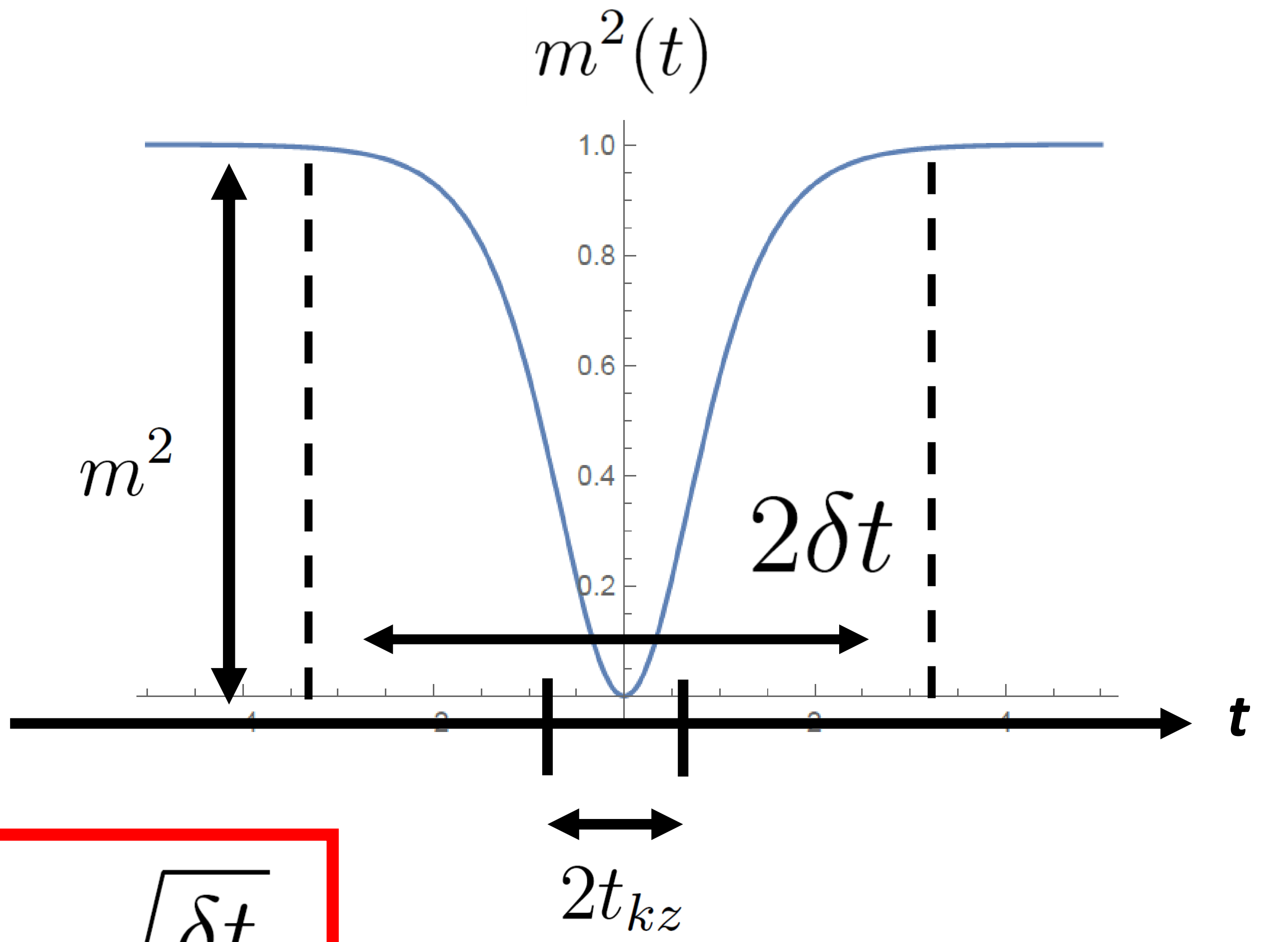
$$t_{kz} = 1/m(-t_{kz}) = \xi_{kz} = \sqrt{\frac{\delta t}{m}}$$



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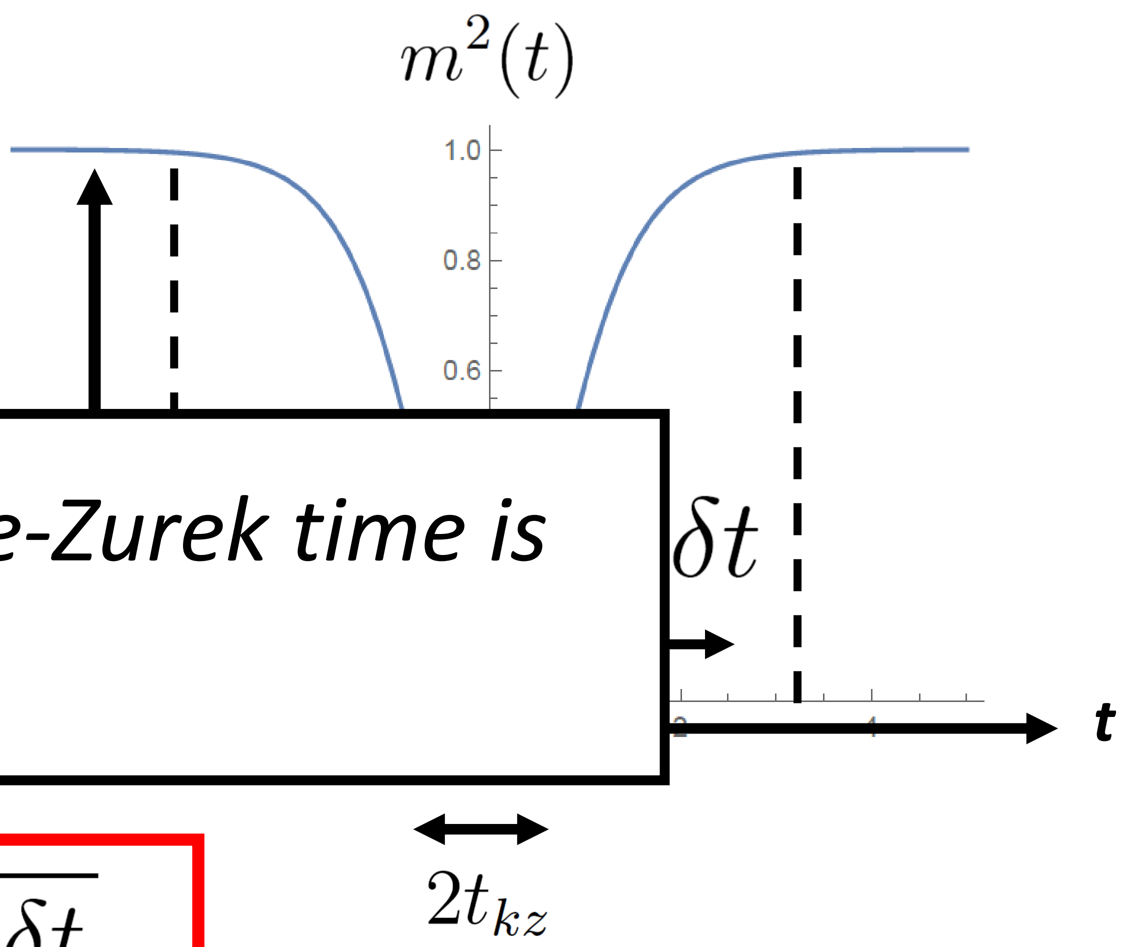
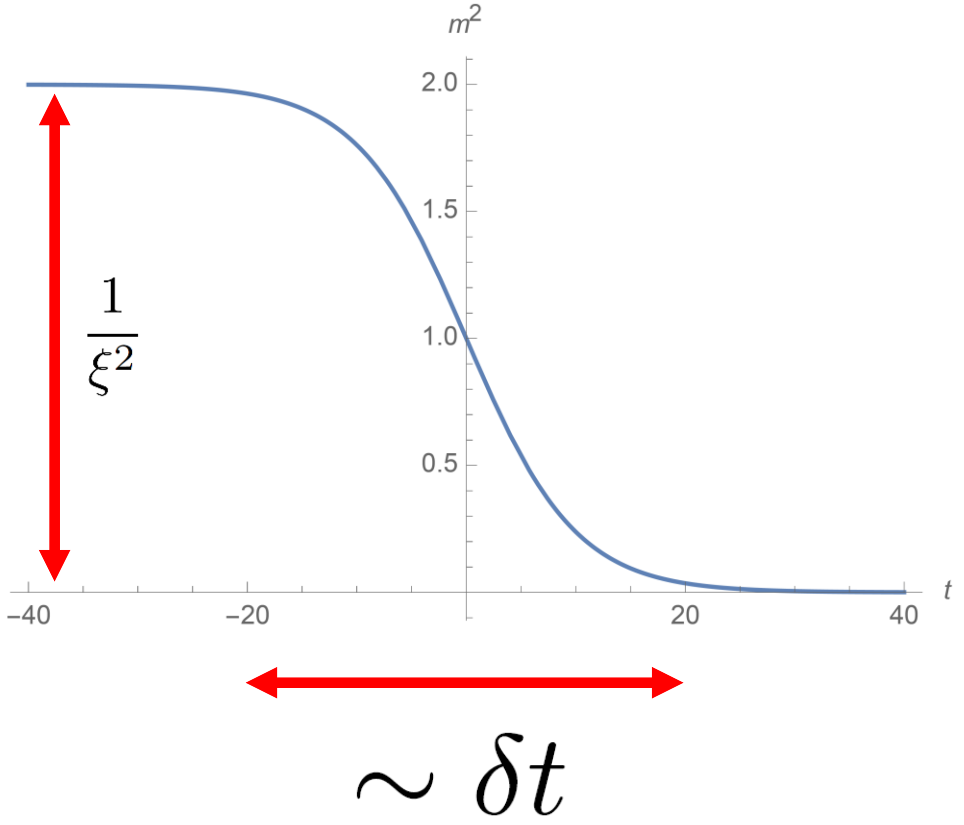


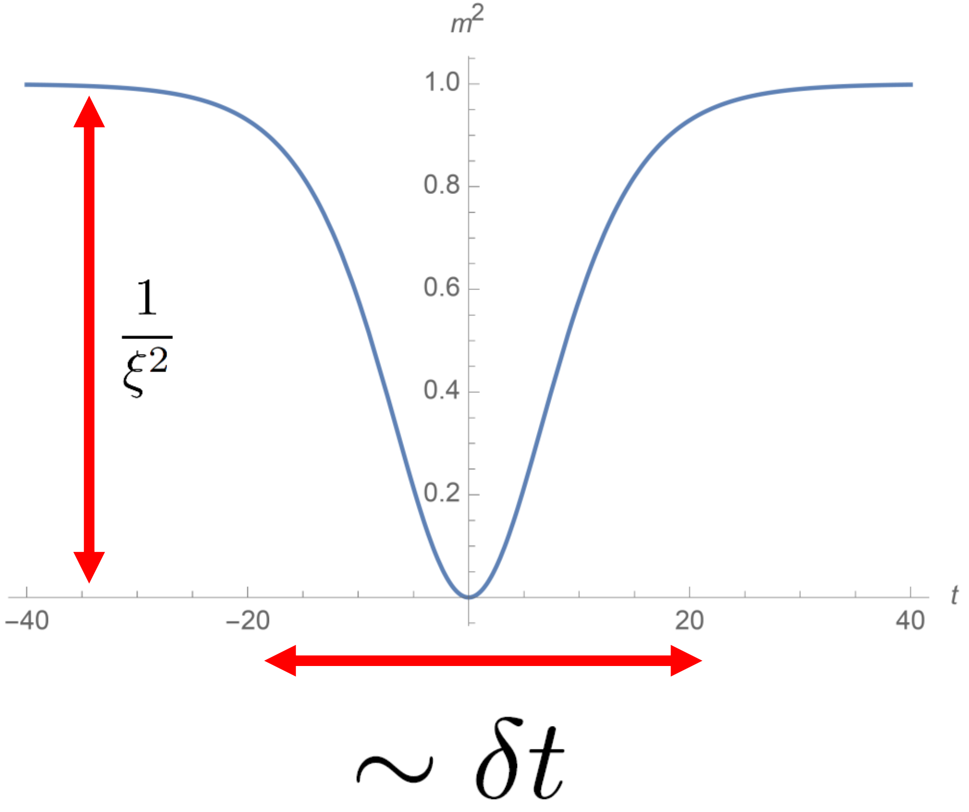
ω

*In **slow quenches**, Kibble-Zurek time is small.*

$$t_{kz} = m(-t_{kz}) = \xi_{kz} = \sqrt{\frac{\delta t}{m}}$$

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
- CCP: $m^2(t) = \frac{1}{\xi^2}\tanh^2\left(\frac{t}{\delta t}\right)$





Fast Quench limit: $\omega \ll 1 \quad (\delta t \ll \xi)$

Slow Quench limit: $\omega \gg 1 \quad (\delta t \gg \xi)$

- ECP : $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$

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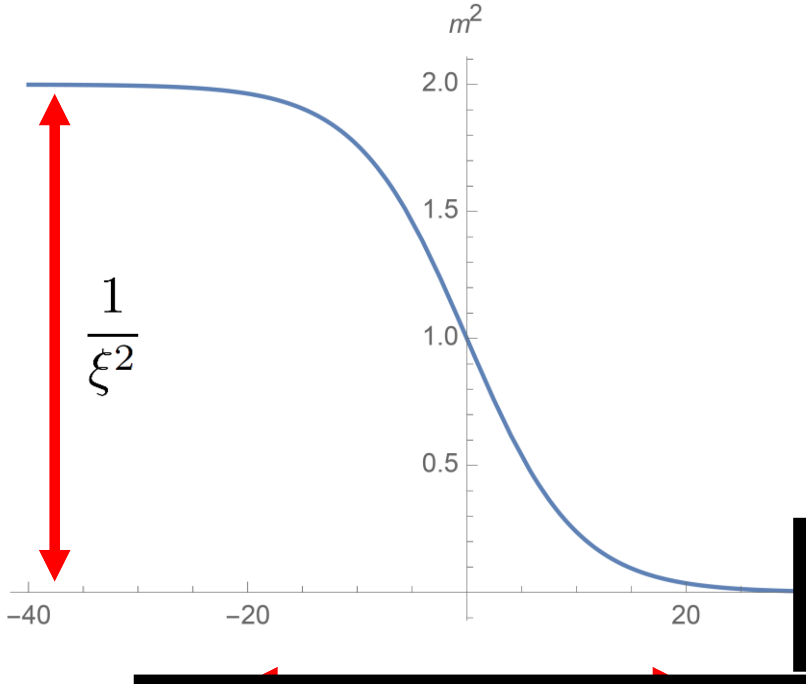

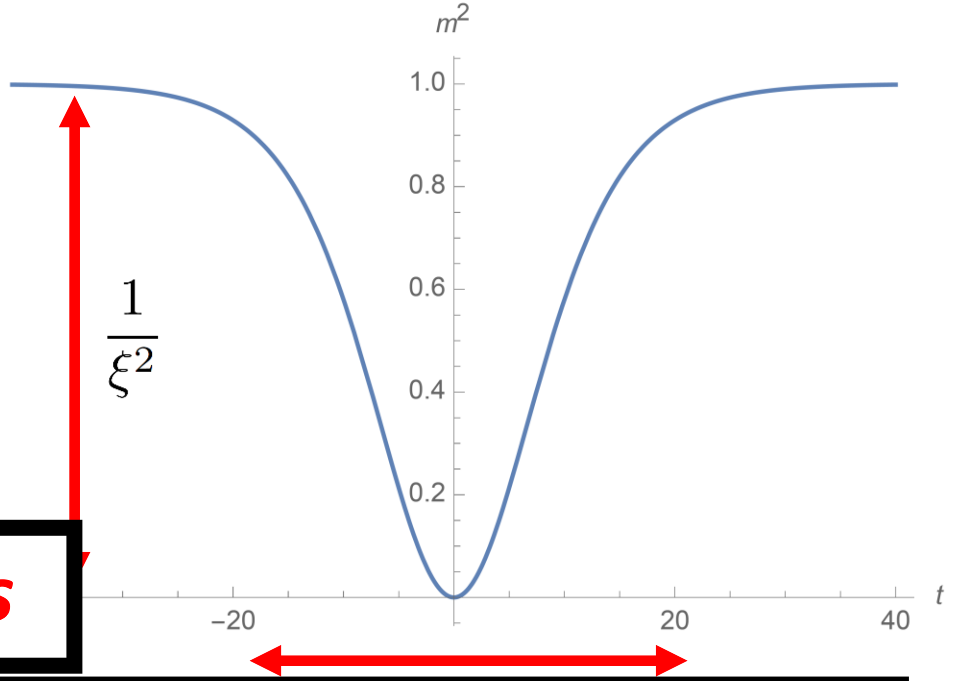

Parameters

- ECP :

$$\xi_{kz} = 1/E_{kz} = \delta t$$

$$t_{kz} = \delta t \log \omega$$

- CCP:

$$\xi_{kz} = t_{kz} = \sqrt{\delta t \cdot \xi}$$

Method

Discretize

- We put our theory on ***the lattice*** so that we compute ΔS_A by the correlator method.

Correlator method

- This is a method to compute ΔS_A by using the correlation functions.

Conditions: 1. State is ***a Gaussian state***.

2. Local observables can be computed ***by Wick theorem***.

Correlator Method

- If an initial state $|\Psi\rangle$ is given by a gaussian state:

For example, $|\Psi\rangle$ ($a_k |\Psi\rangle = 0$)

Correlator Method

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We assume that a reduced density matrix is given by

$$\rho_A = tr_B \rho \sim e^{-\sum \gamma_k b_k^\dagger b_k}$$

If ϕ_i, ϕ_j are included in A,

$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

Correlator Method

- If an initial state $|\Psi\rangle$ is given by a gaussian state:

For example, $|\Psi\rangle$ ($a_k |\Psi\rangle = 0$)

We assume that a reduced density matrix is given by

$$\rho_A = tr_B \rho \sim e^{-\sum \gamma_k b_k^\dagger b_k}$$

$$a_k = \alpha_k b_k + \beta_{-k} b^\dagger_{-k}$$

If ϕ_i, ϕ_j are included in A,

$$\langle \phi_i \phi_j \rangle = tr(\rho \phi_i \phi_j) = tr_A(\rho_A \phi_i \phi_j) = \langle \phi_i \phi_j \rangle_A$$

Correlator Method

$$\underline{\langle \phi_i \phi_j \rangle} = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \underline{\langle \phi_i \phi_j \rangle_A}$$

Determined by E.O.M and so on.

$$f\left(\gamma_k\right)$$

Correlator Method

$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

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$f(\gamma_k)$

Two point functions ⟷ γ_k

Correlator Method

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$$f(\gamma_k)$$

Two point functions ⟷ γ_k ➡ $\rho_A \sim e^{-\sum \gamma_k b_k^\dagger b_k}$

Correlator Method

$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

Two point functions ⟷ γ_k ⇒ $\rho_A \sim e^{-\sum \gamma_k b_k^\dagger b_k}$

⇓

S_A ***is determined by two point functions.***

Correlator method

Entanglement Entropy:

$$S_A = \sum_{k=1}^{l} s_A(\gamma_k)$$

$$s_A(\gamma_k) = \left(\frac{1}{2} + \gamma_k\right) \log\left(\frac{1}{2} + \gamma_k\right) - \left(-\frac{1}{2} + \gamma_k\right) \log\left(-\frac{1}{2} + \gamma_k\right)$$

$$\Gamma = \begin{pmatrix} X_{ij} & \frac{1}{2}D_{ij} \\ \frac{1}{2}D_{ji} & P_{ij} \end{pmatrix} \quad J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

$$X_{ij} = \langle \phi_i \phi_j \rangle \qquad P_{ij} = \langle \pi_i \pi_j \rangle \qquad D_{ij} = \langle \{\phi, \pi_j\} \rangle$$

- $M = iJ\Gamma$ has eigenvalues $\pm\gamma_k$.

Correlator method

Entanglement Entropy:

The subsystem size = *l*

$$S_A = \sum_{k=1}^{l} s_A(\gamma_k)$$

2*l* ×2*l* matrix

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2l ×2l matrix

$\Gamma =$

By evaluating M, we can compute S_A.

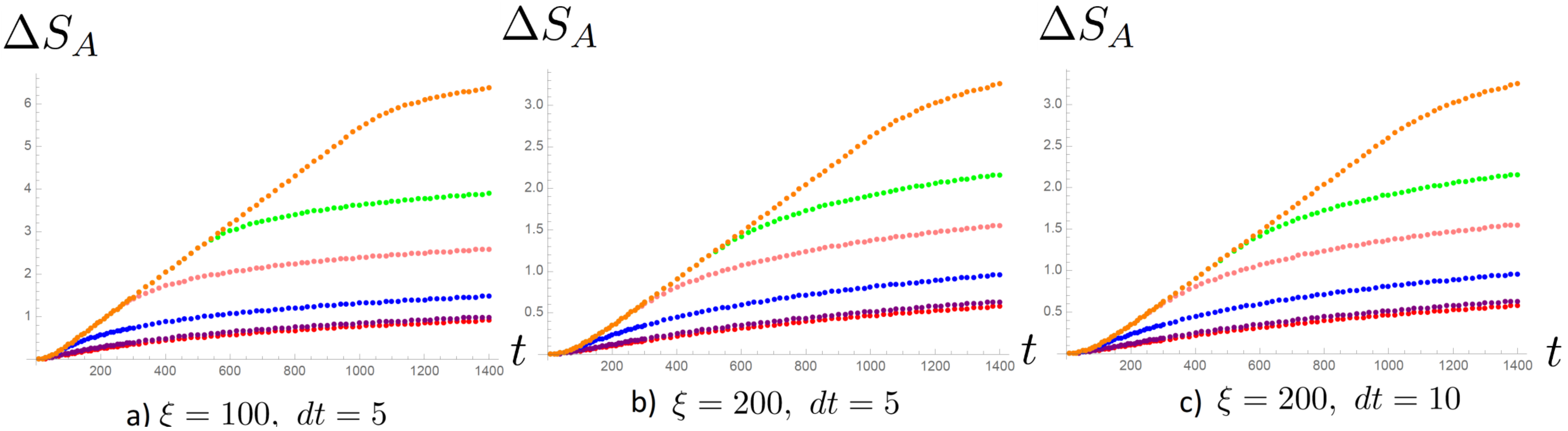
$X_{ij} = \langle \phi_i \phi_j \rangle$ $\quad P_{ij} = \langle \pi_i \pi_j \rangle$ $\quad D_{ij} = \langle \{\phi_i, \pi_j\} \rangle$

- $M = iJ\Gamma$ has eigenvalues $\pm\gamma_k$.

EE in ECP

EE in fast ECP

Plot of EE in Fast ECP



Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

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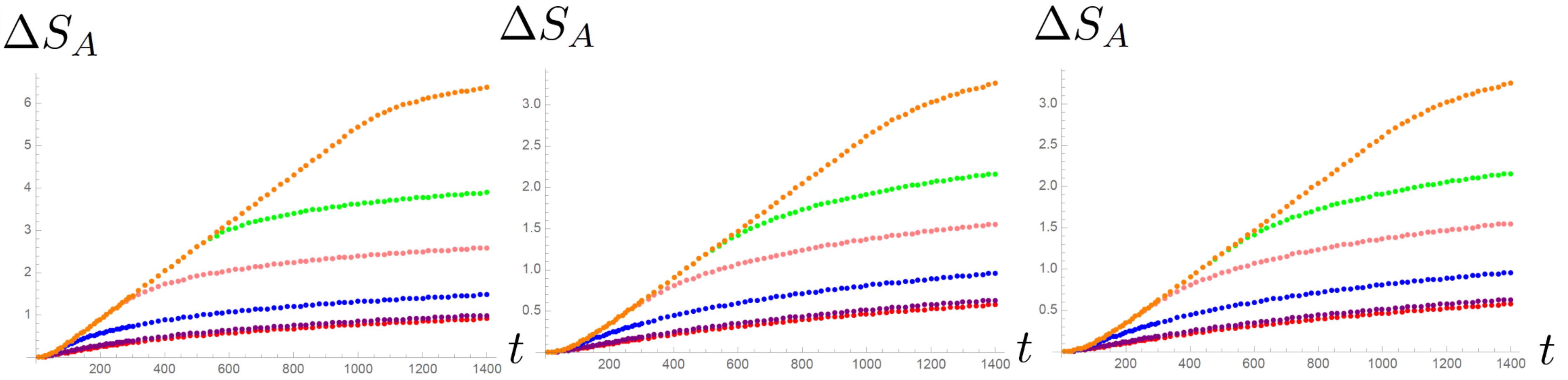


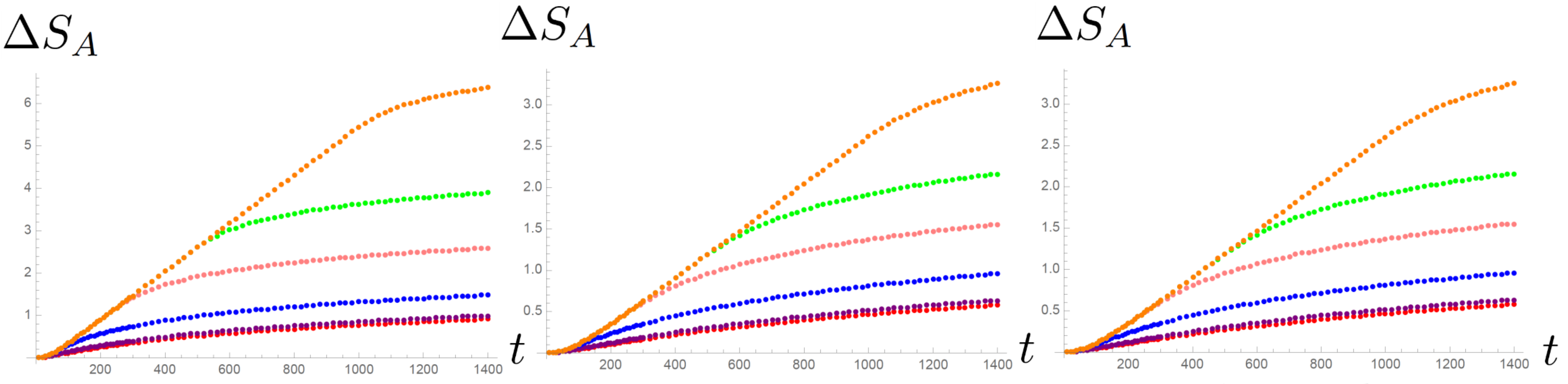
a) $\xi = 100, \ dt = 5$ b) $\xi = 200, \ dt = 5$ c) $\xi = 200, \ dt = 10$

If l is sufficiently larger than ξ and $\xi \ll t \leq l/2$,

ΔS_A does not depend on l and linearly increases with time.

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5



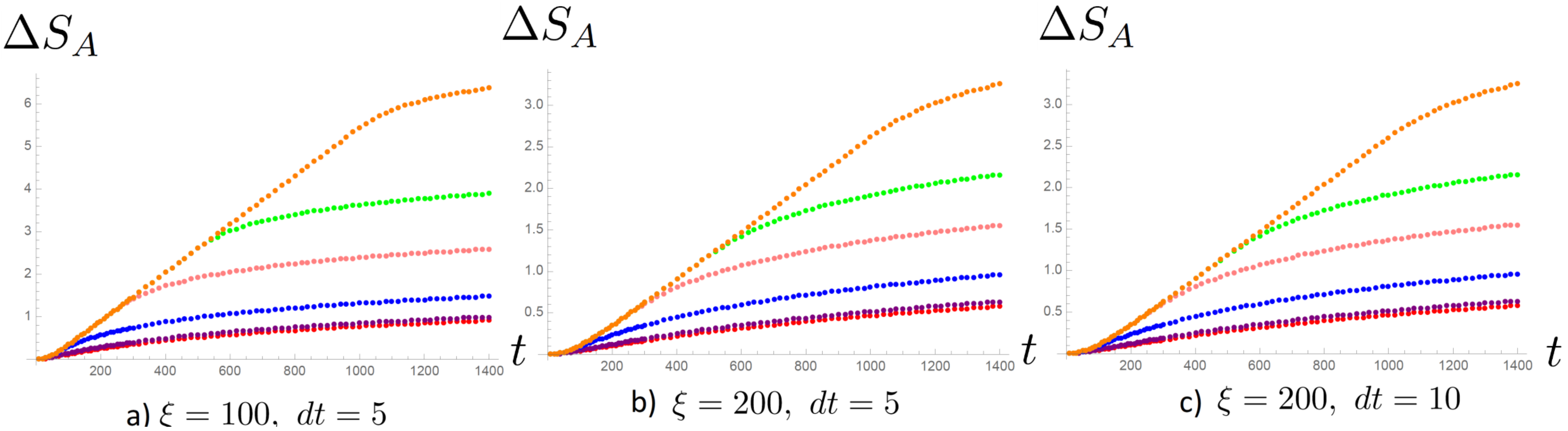
a) $\xi = 100, \ dt = 5$ b) $\xi = 200, \ dt = 5$ c) $\xi = 200, \ dt = 10$

If l is sufficiently larger than ξ and $\xi \ll t \leq l/2$,

$$\Delta S_A(t) \sim 0.57 \times \frac{t}{\xi}$$

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5



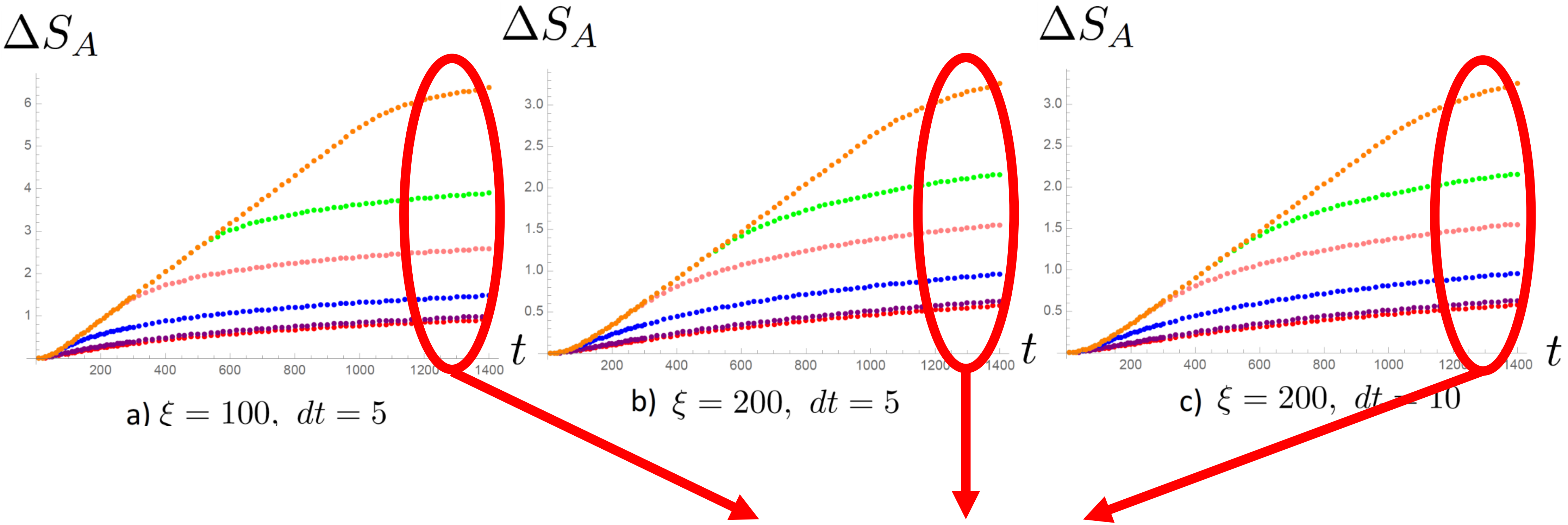
If t is sufficiently larger than l/2 ($t \gg l/2$),

$$\Delta S_A(l) \sim 0.28 \times \frac{l}{\xi}$$

Thermalize

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

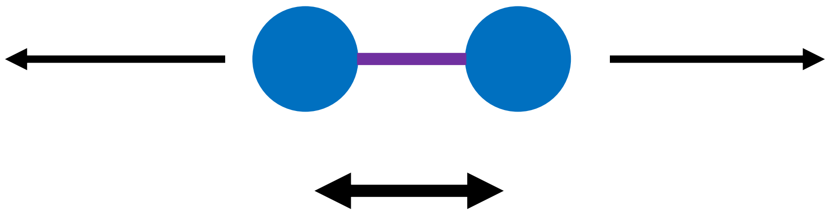
Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

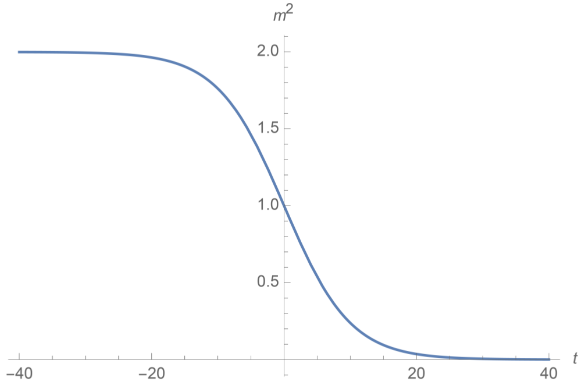


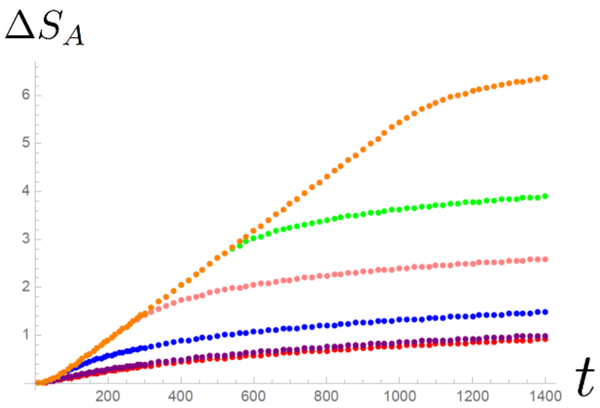
Slowly increase

Quasi-Particle Interpretation

- **As in sudden quenches, around t=0, entangled quasi-particle are created everywhere.**

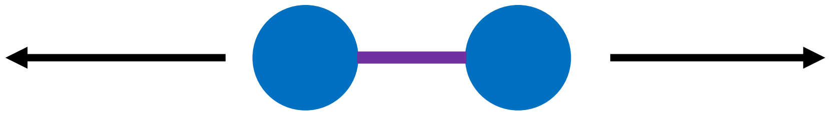






- Entangled pair is constructed two particles.

They propagate in the opposite directions with v :



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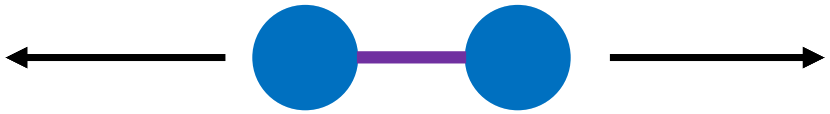
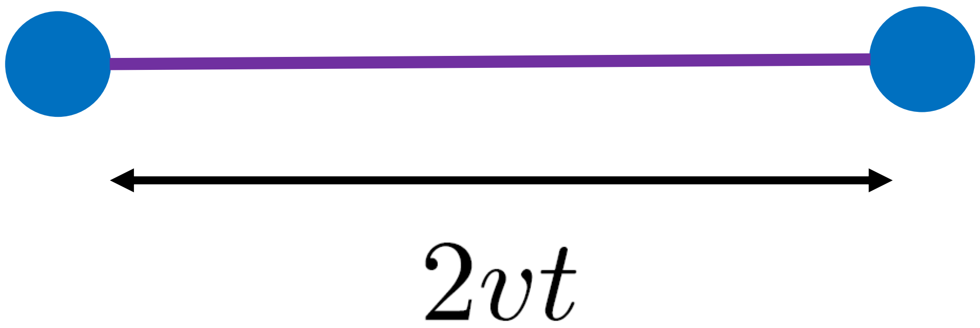


If one of them is included in A and the other is out of A,

Entangled pair can contribute to entanglement entropy.

At l/2 > t > 1/m, the distance between entangled particles is given 2vt:



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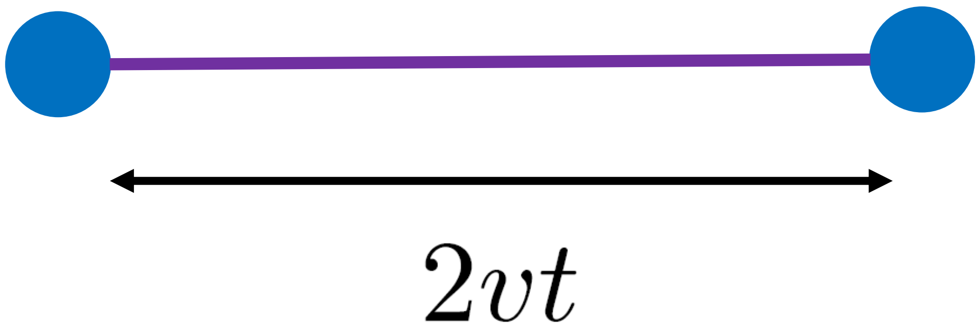
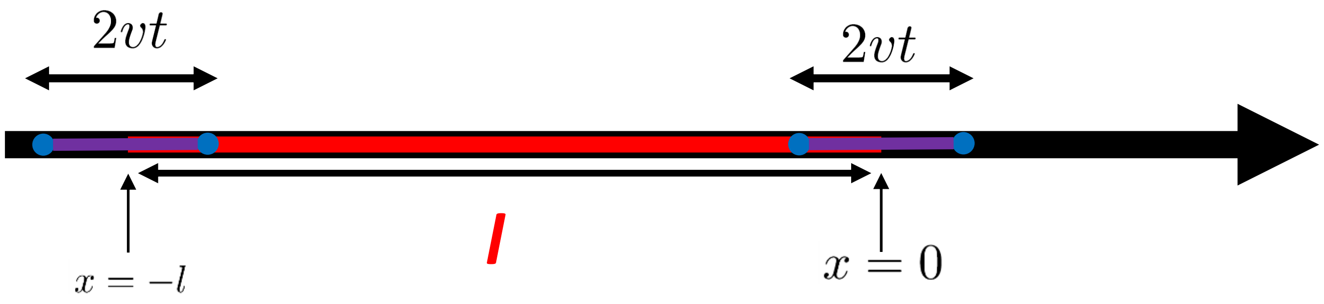


The particle created at the boundary at t=0 is at $x = vt$ or $x = -l - vt$.



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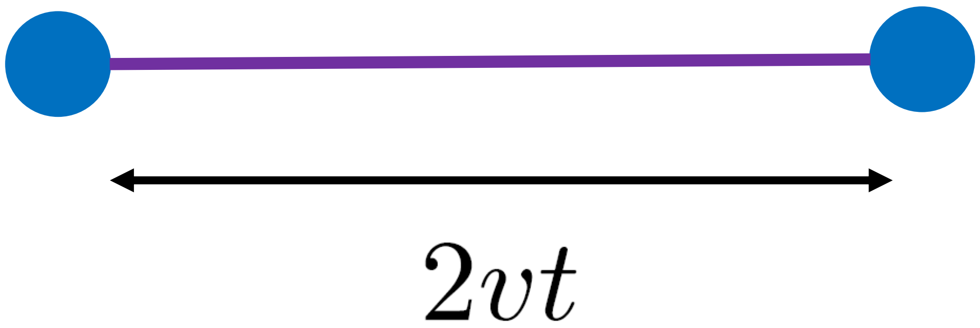
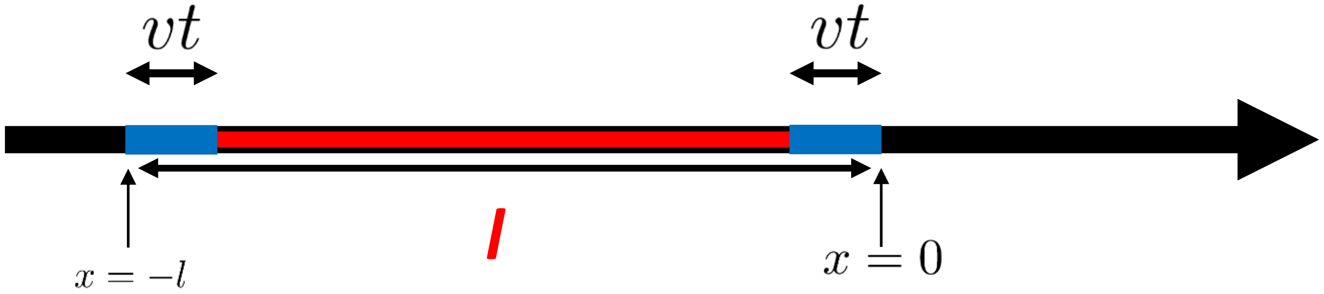


The entangled pairs in the blue region can contribute to S_A .



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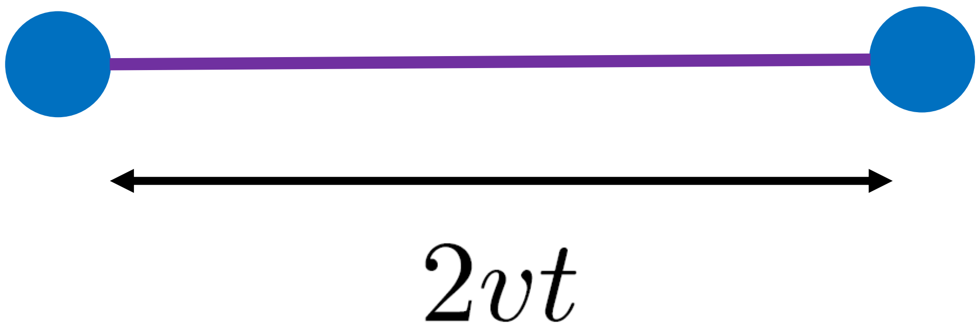

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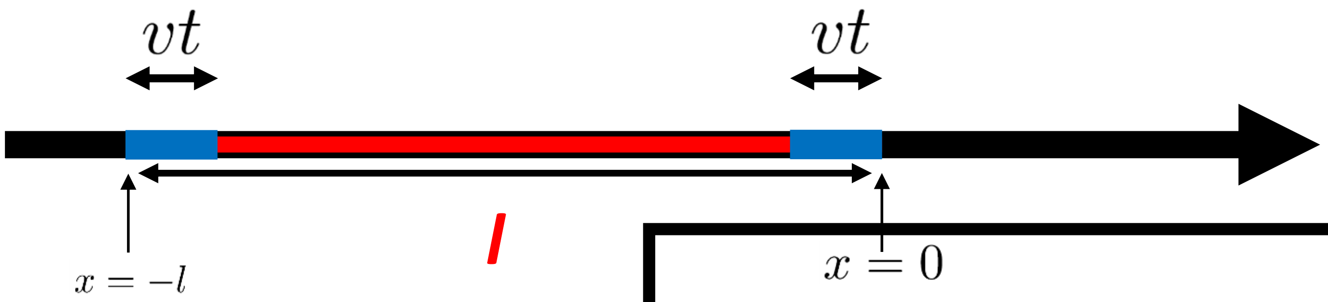
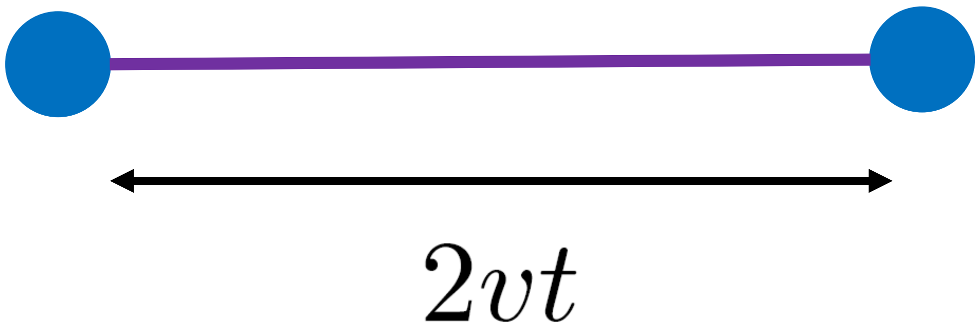


# of entangled pair $\sim$ t

At l/2 > t > 1/m, the distance between entangled particles is given 2vt:



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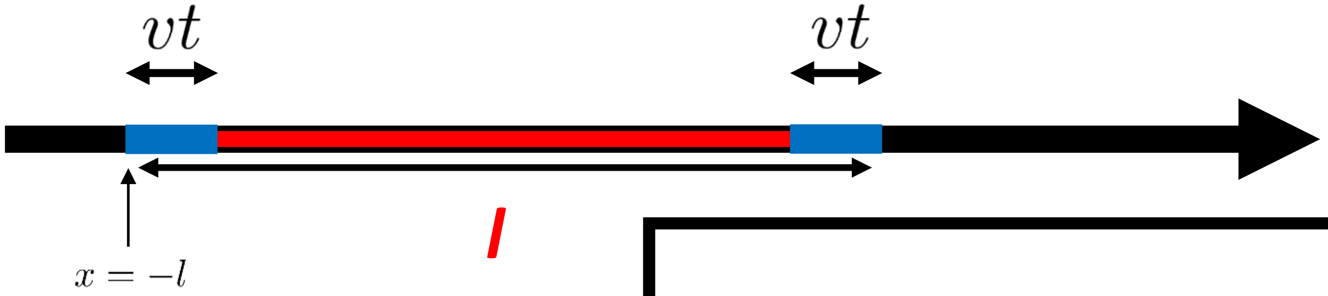
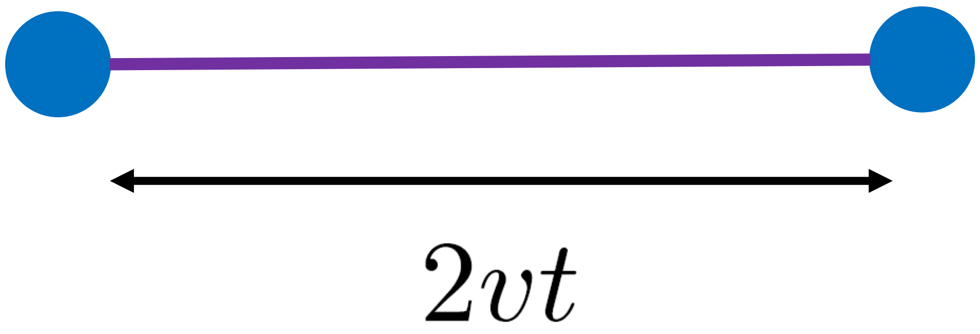


# of entangled pair $\sim$ t

At l/2 > t > 1/m, the distance between entangled particles is given 2vt:



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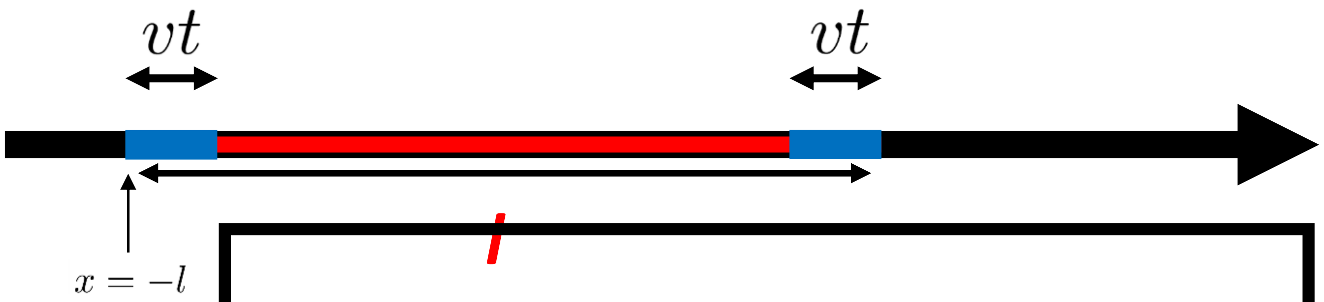
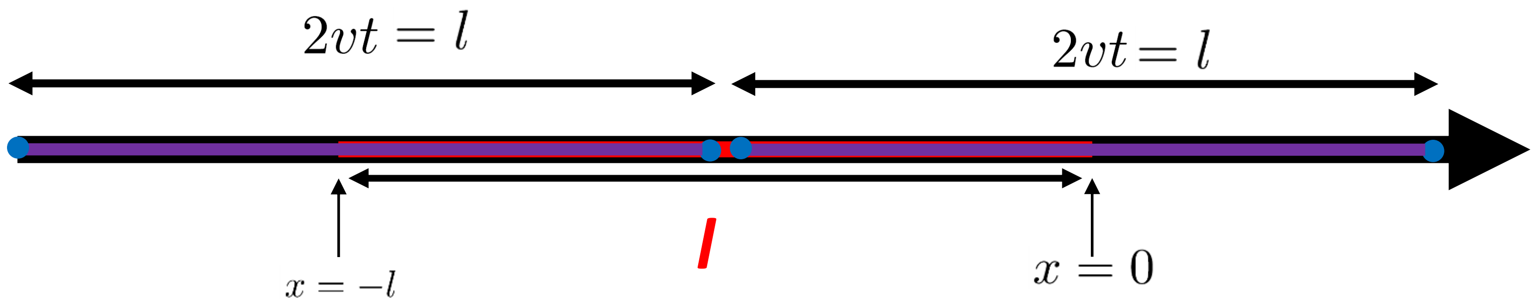


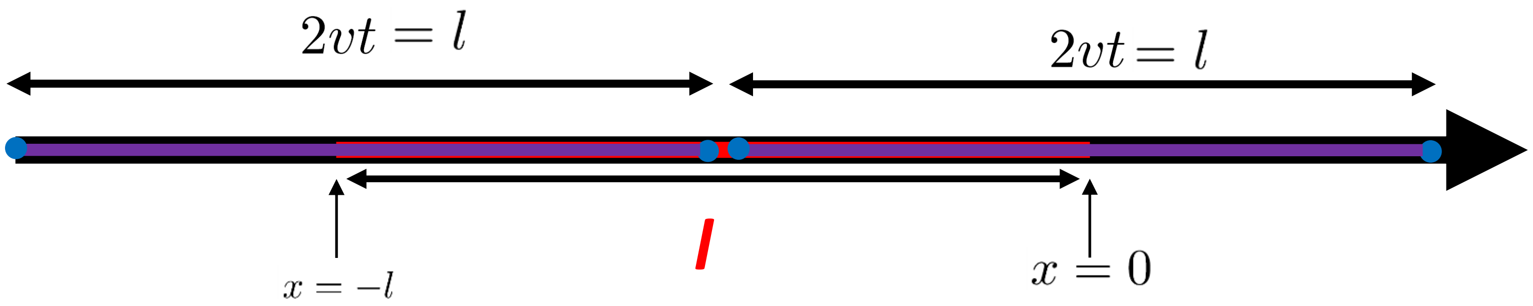
# of entangled pair $\sim$ t

S_A ***linearly grows with t***

At t=l/2v, the distance between quasi-particles is the subsystem size.



At t=l/2v, all entangled pairs in the region A can contribute to .



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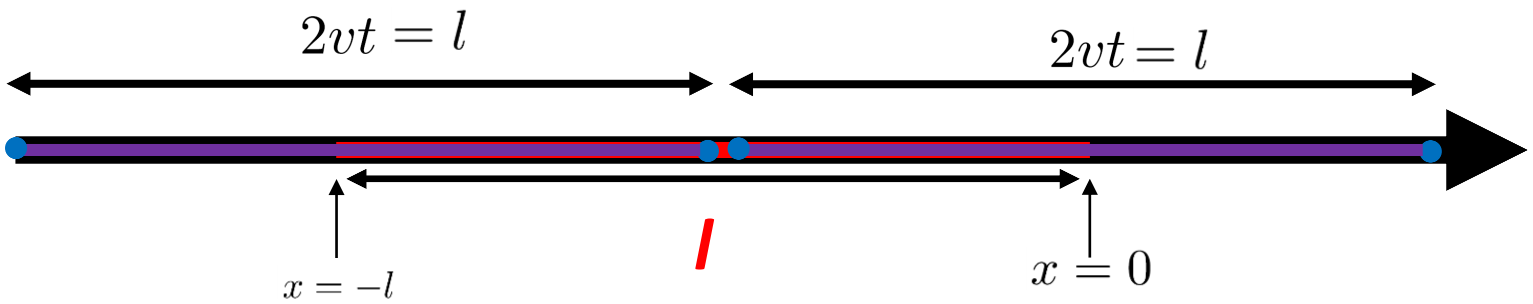
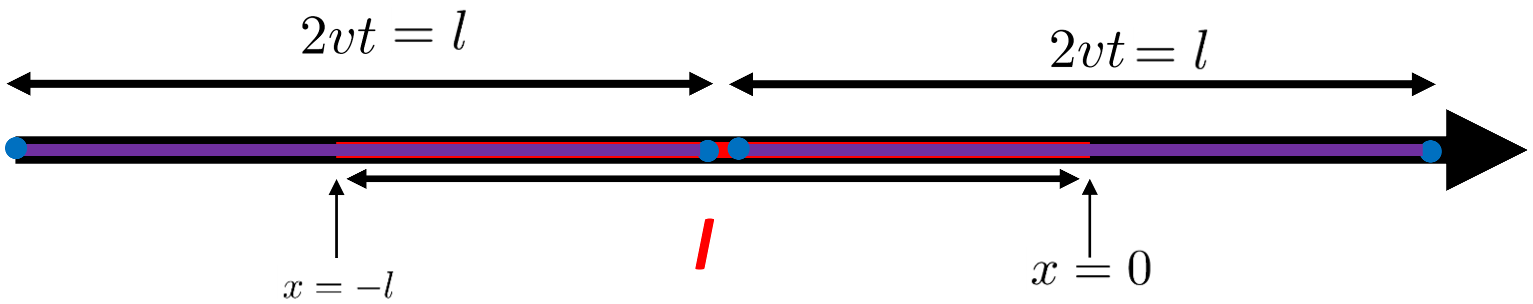


# of entangled pair $\sim$ l

At t=l/2v, all entangled pairs in the region A can contribute to .



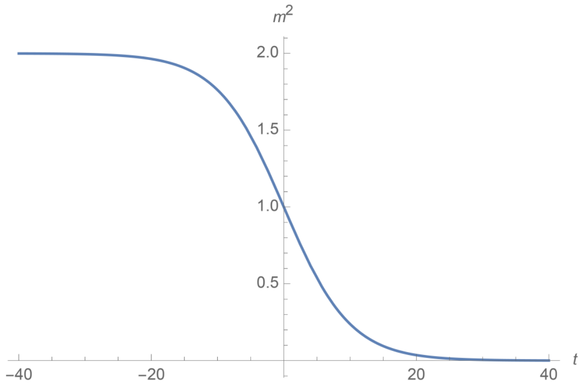
# of entangled pair $\sim$ l

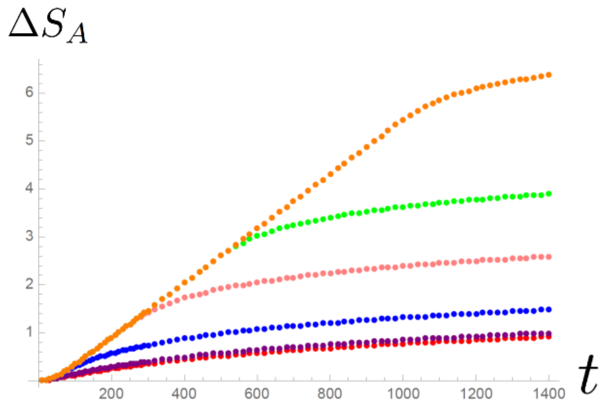
S_A **has volume law.**

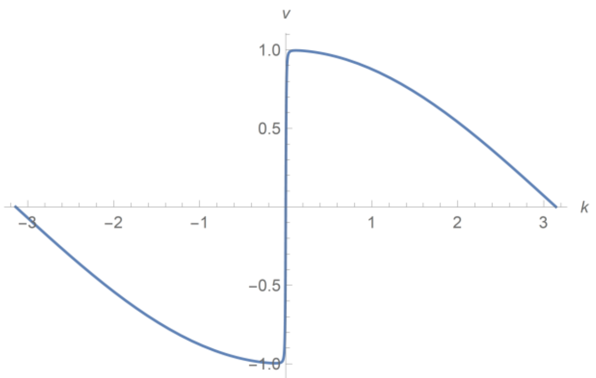
Quasi-Particle Interpretation

- **As in sudden quenches, around t=0, entangled quasi-particle are created everywhere.**
- **Their speed is given by the group velocity at t=0**

$$v_k = \frac{d\omega_k(t)}{dk}, \quad \omega_k(t) = \sqrt{4\sin^2\left(\frac{k}{2}\right) + m^2(t)}.$$







[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]

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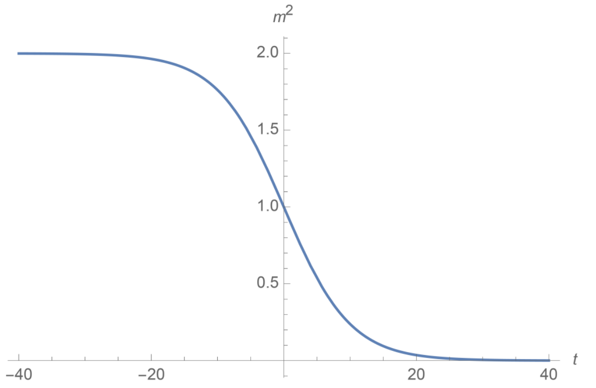
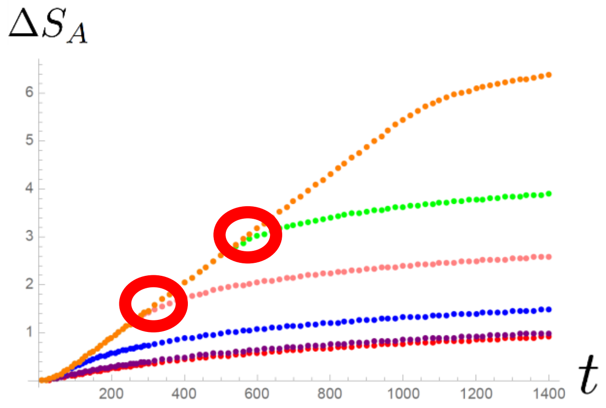
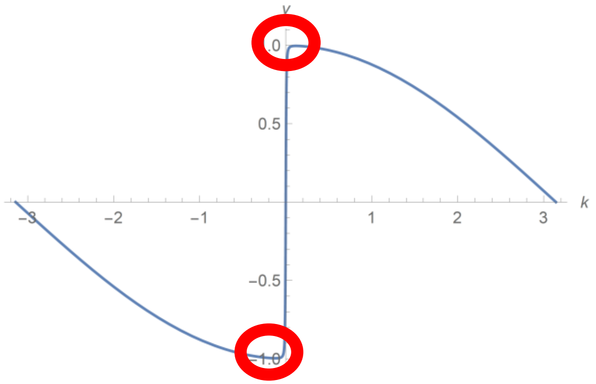
$$|v_{max}| \sim 1$$

Around t =l/2, the time evolution of ΔS_A changes.

[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]







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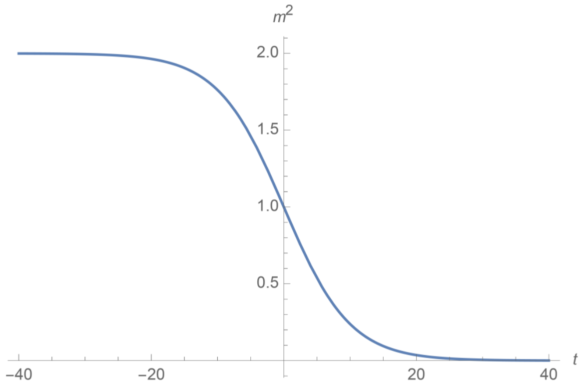
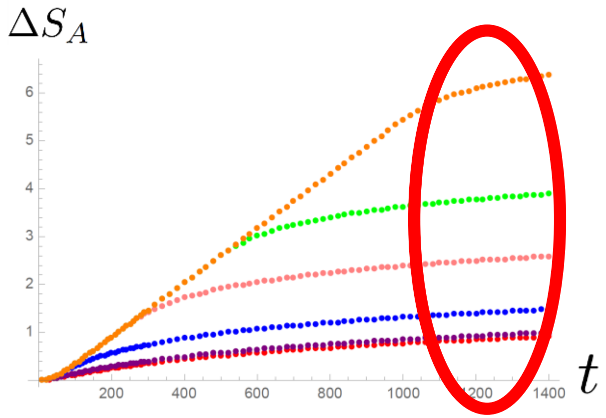
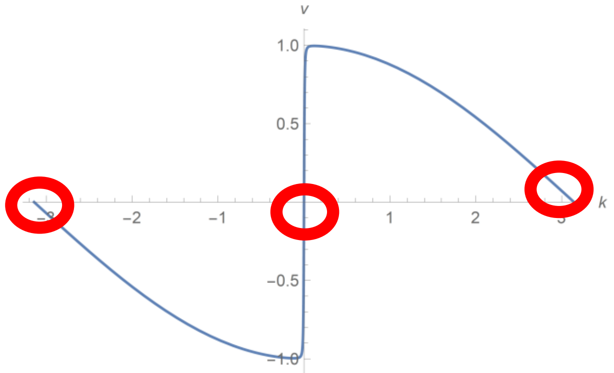
Slow mode (∼ zero mode and large k mode)

➡ Slowly increases in the late time

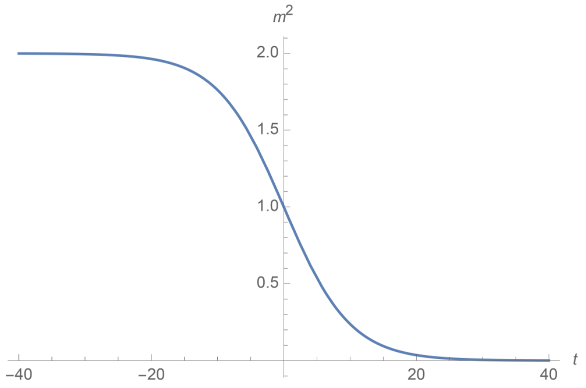
[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]







Quasi-Particle Interpretation



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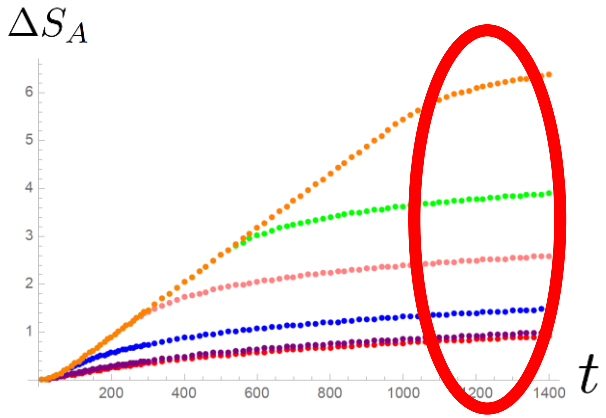
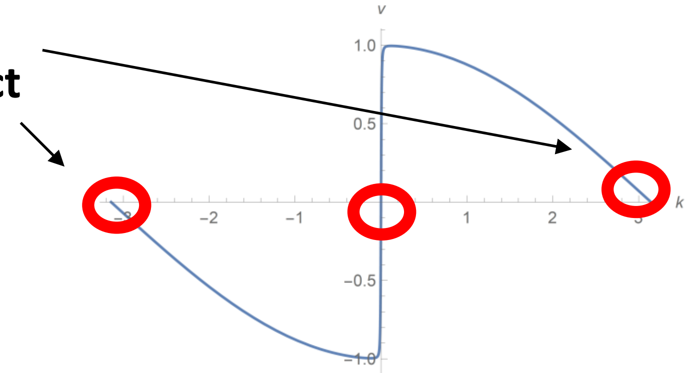


$$v_k = \frac{d\omega_k(t)}{dk}, \quad \omega_k(t) = \sqrt{4\sin^2\left(\frac{k}{2}\right) + m^2(t)}.$$

Lattice effect



Slow mode (～ zero mode and large k mode)

Slowly increases in the late time

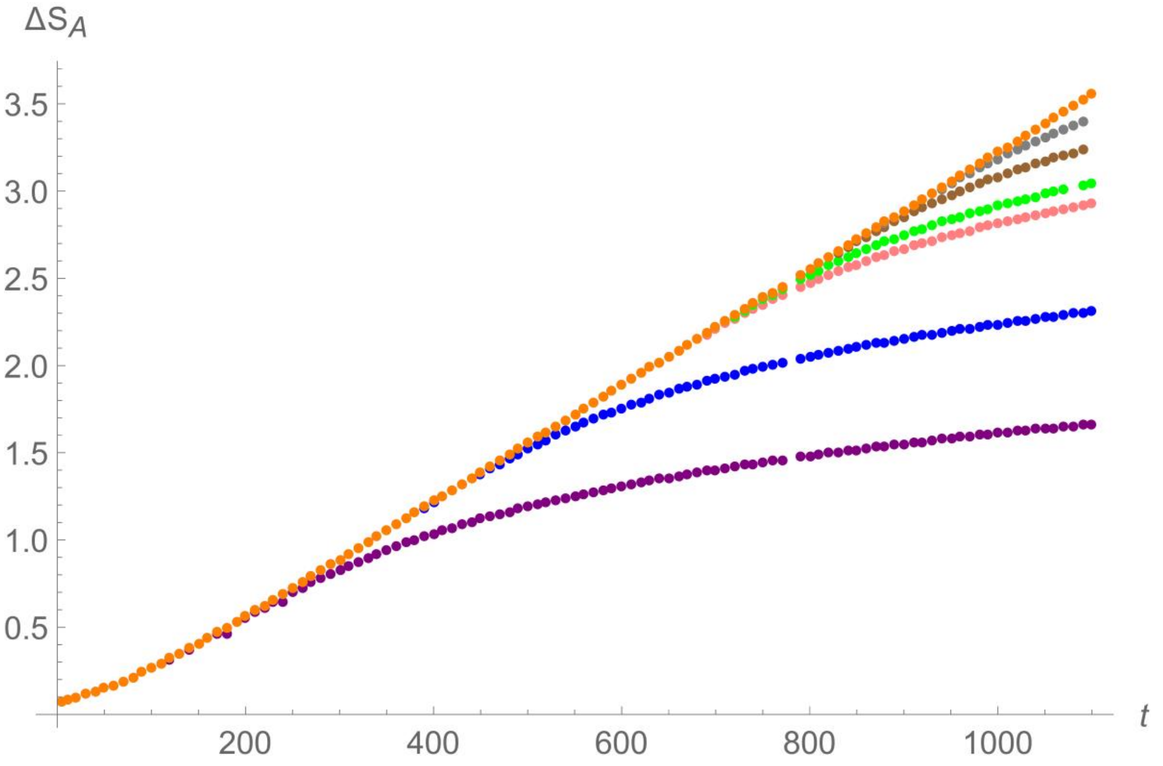
[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]

EE in slow ECP

Plot of EE in Slow ECP

Orange Curve: l=2000, Gray Curve: l=1000, Brown Curve: l= 800,

Green Curve: l=600, Pink Curve: l=500, Blue Curve: l=100, Purple Curve: l=10



$$(\xi, \delta t) = (1, 100)$$

Plot of EE in Slow ECP

Orange Curve: l=2000, Gray Curve: l=1000, Brown Curve: l= 800,

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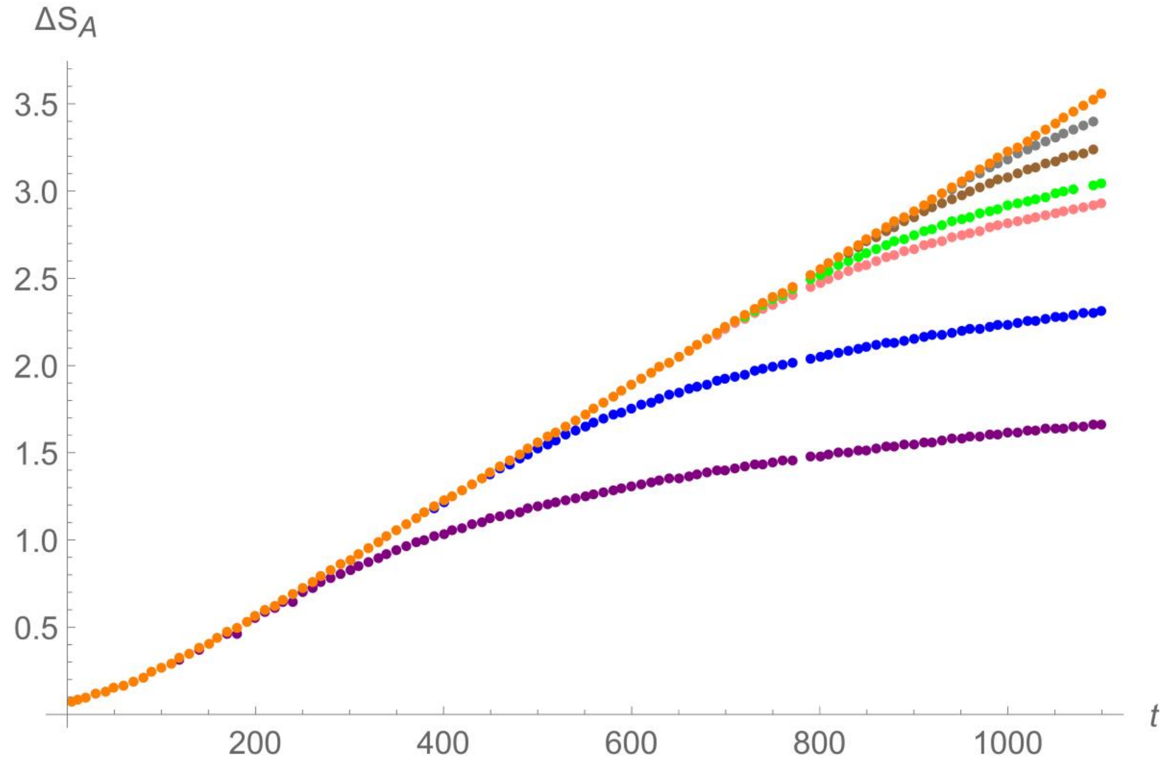
$$t_{kz} + \frac{l}{2} = \delta t \log \omega + \frac{l}{2} > t \gg \frac{1}{E_{kz}},$$

ΔS_A **does not depend on *l*.**



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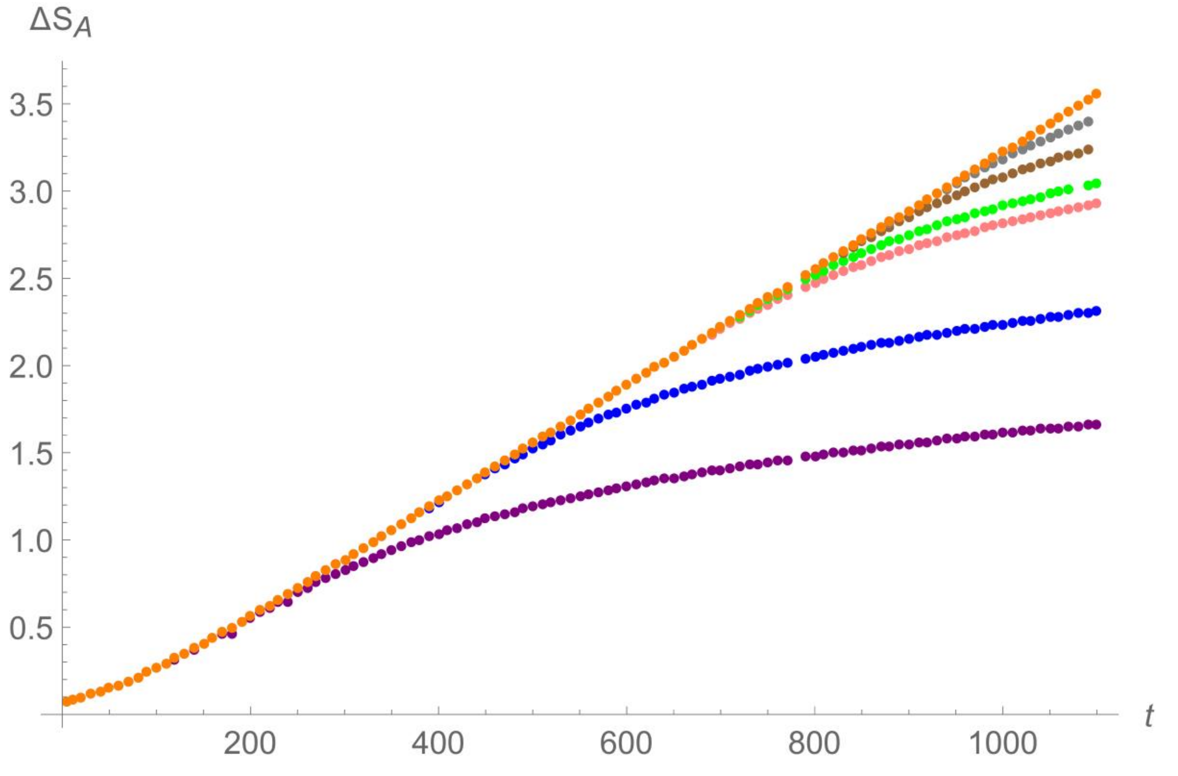
$$\Delta S_A \sim \frac{1}{3} E_{kz} \cdot t$$



$$(\xi, \delta t) = (1, 100)$$

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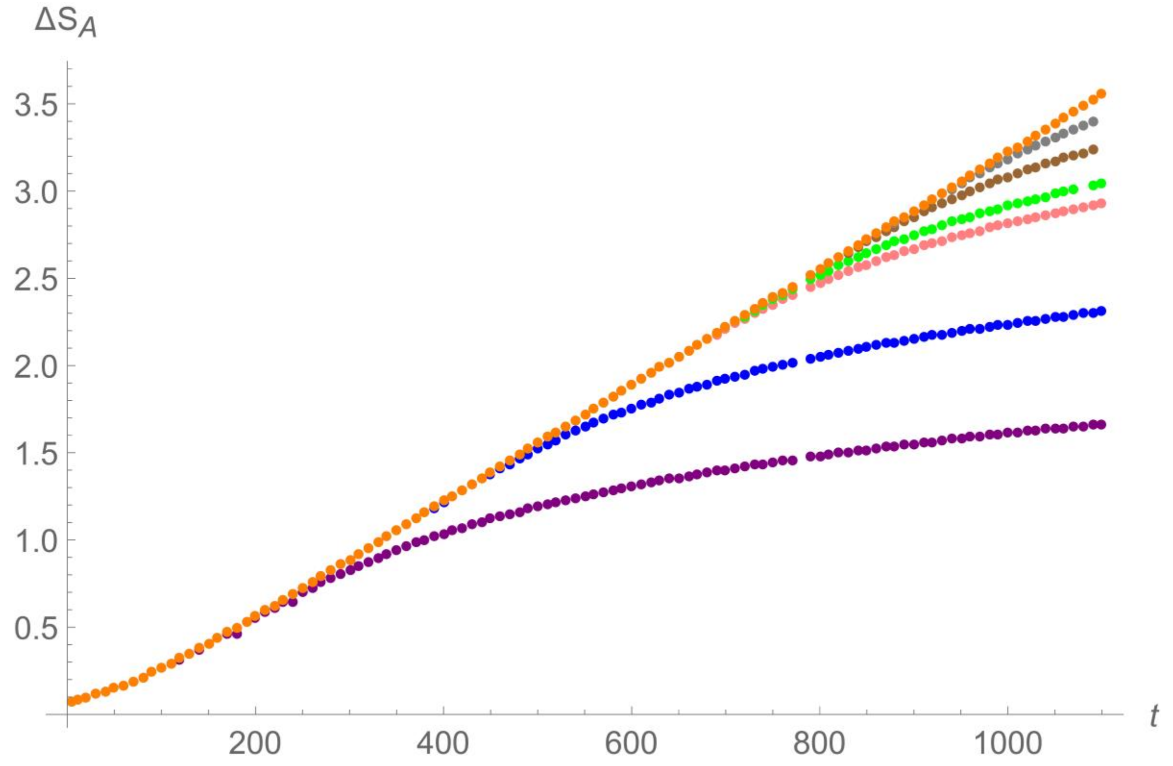
Adiabaticity breaks down.

$$t_{kz} + \frac{l}{2} = \delta t \log \omega + \frac{l}{2} > t \gg \frac{1}{E_{kz}},$$

$$\Delta S_A \sim \frac{1}{3} E_{kz} \cdot t$$

$$\frac{1}{\xi} \to E_{kz}$$



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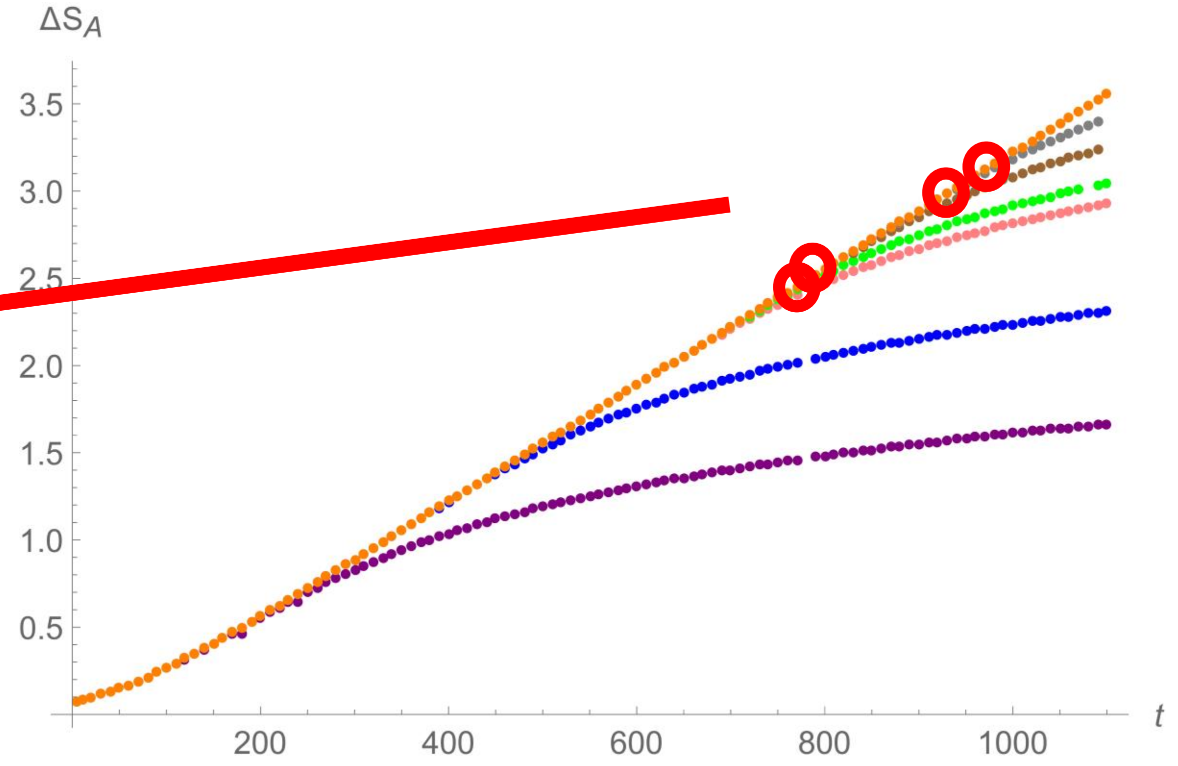
After $t = t_{kz} + \frac{l}{2}$,

ΔS_A depends on l.



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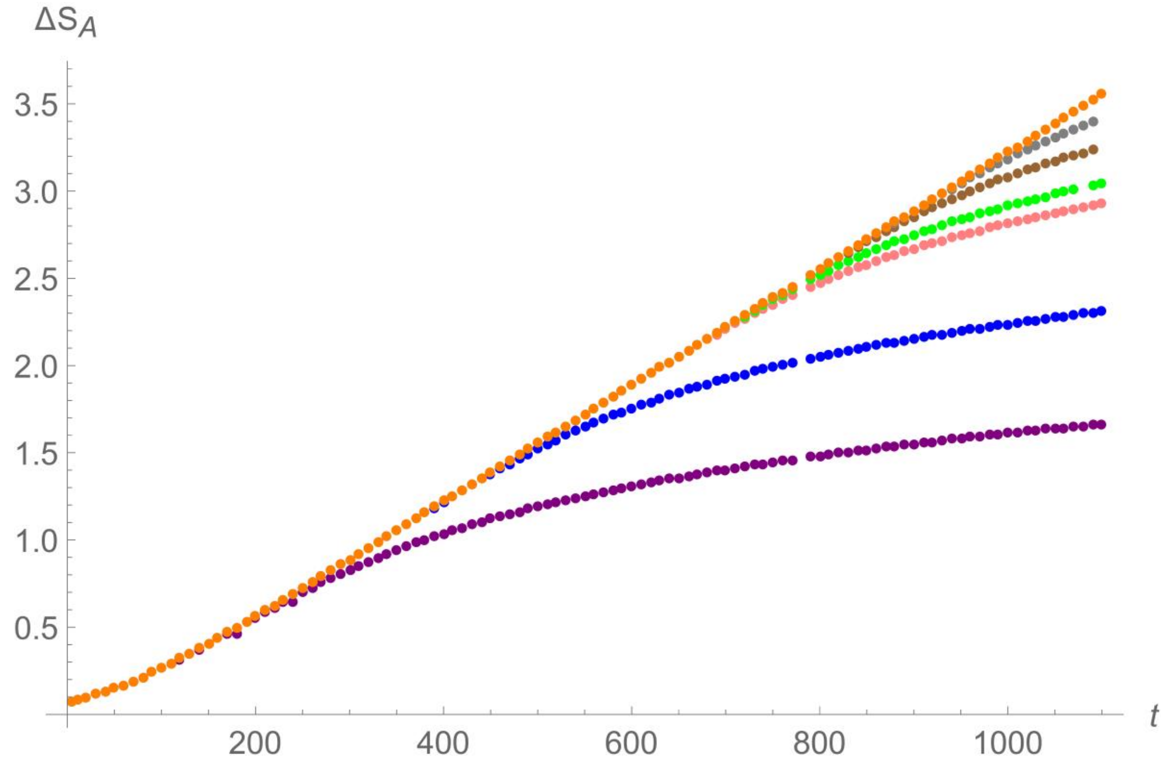
$$t \gg t_{kz} + \frac{l}{2},$$

$$\Delta S_A \sim \frac{1}{6} E_{kz} \cdot l$$

$$\frac{1}{\xi} \to E_{kz}$$



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Quasi-Particle Interpretation

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➡ **Quasi-particles are created at** $t = t_{kz}$**.**

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➡ **Quasi-particles are created at** $t = t_{kz}$.

➡ ***When adiabaticity breaks down, quasi-particles are created.***

Proportionality Coefficient

The proportionality coefficient of ***l*** or ***t*** is set by

an initial correlation length ξ in the fast limit,

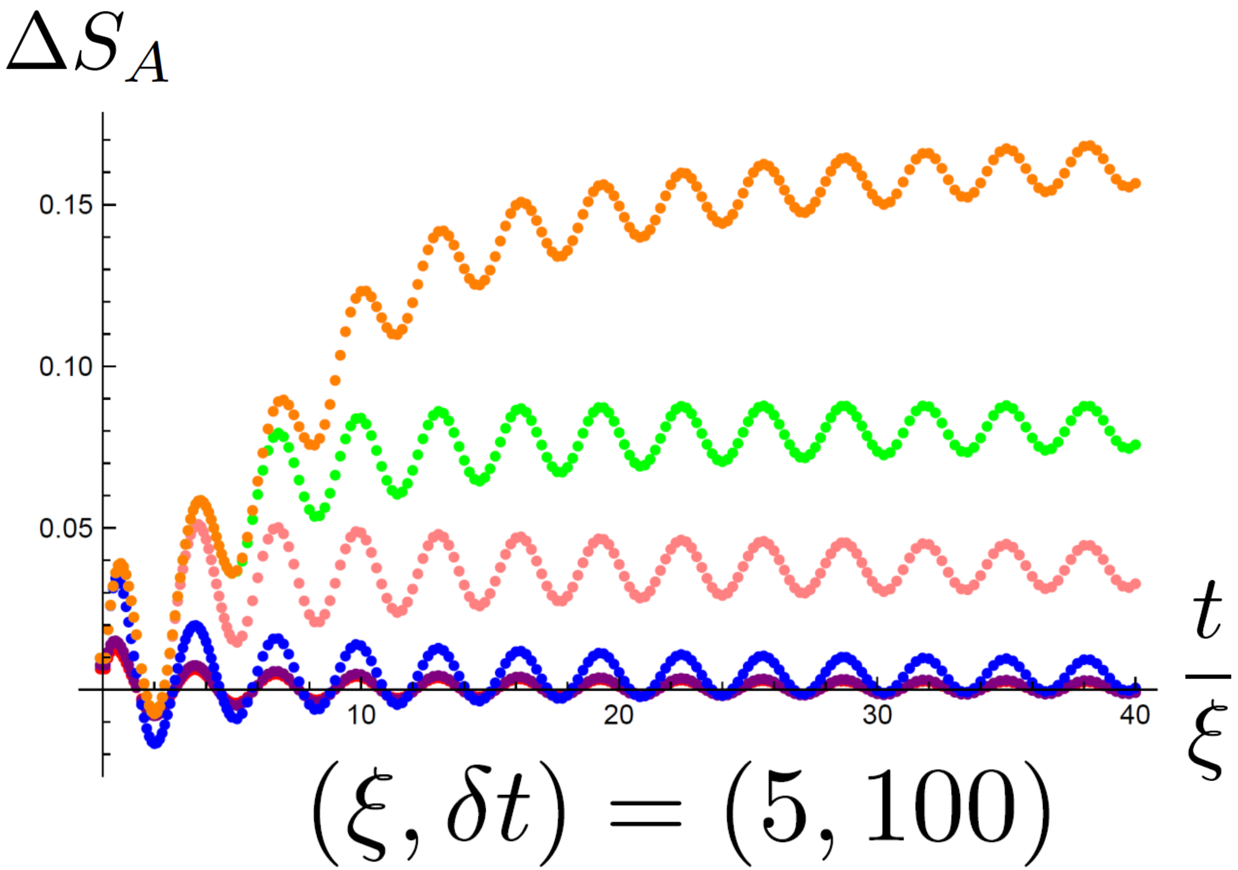
a scale generated when adiabaticity breaks down, E_{kz} **,** in the slow limit.

EE in fast CCP

Plot of EE in fast CCP

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5



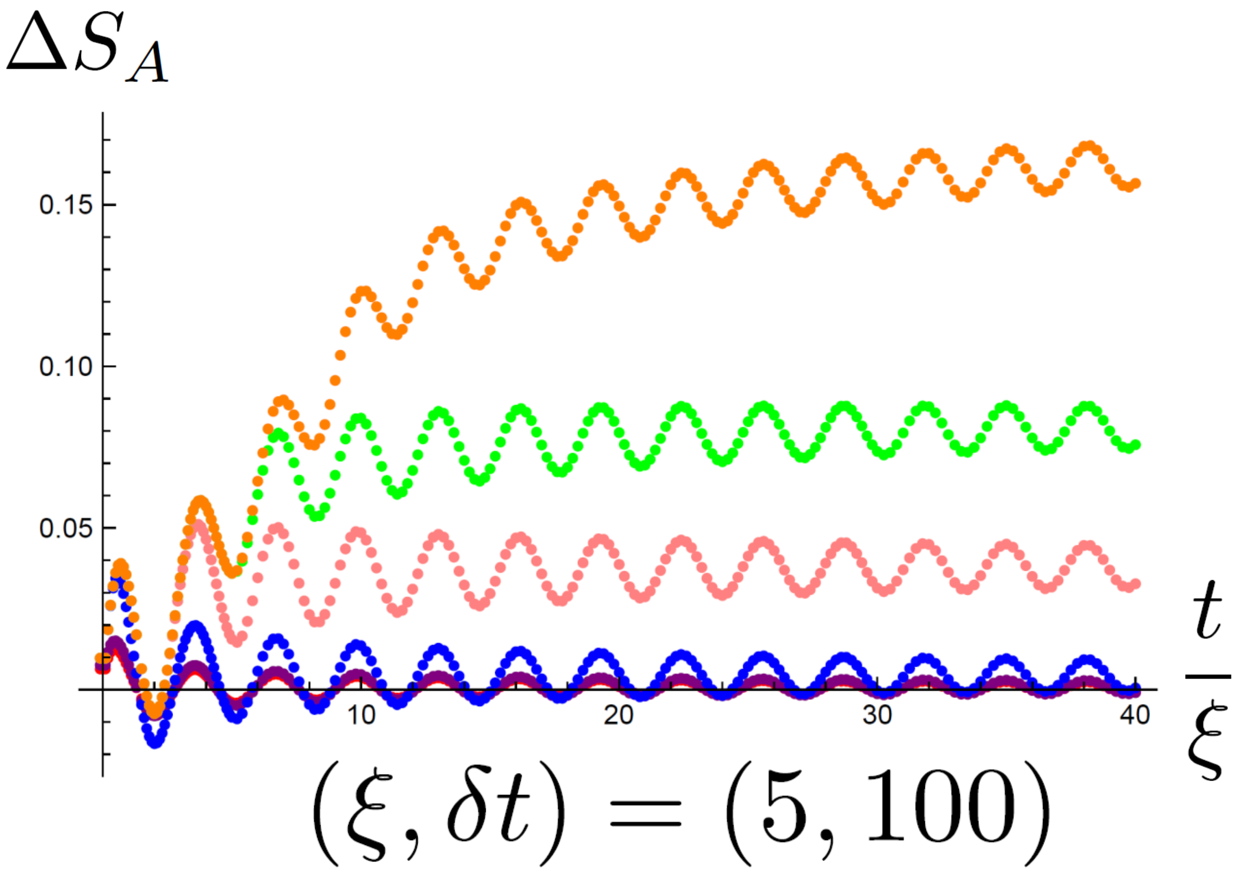
Plot of EE in fast CCP

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

- **If** $\xi \ll t \leq l/2$ **,**

ΔS_A **doesn't depend on *l*** .



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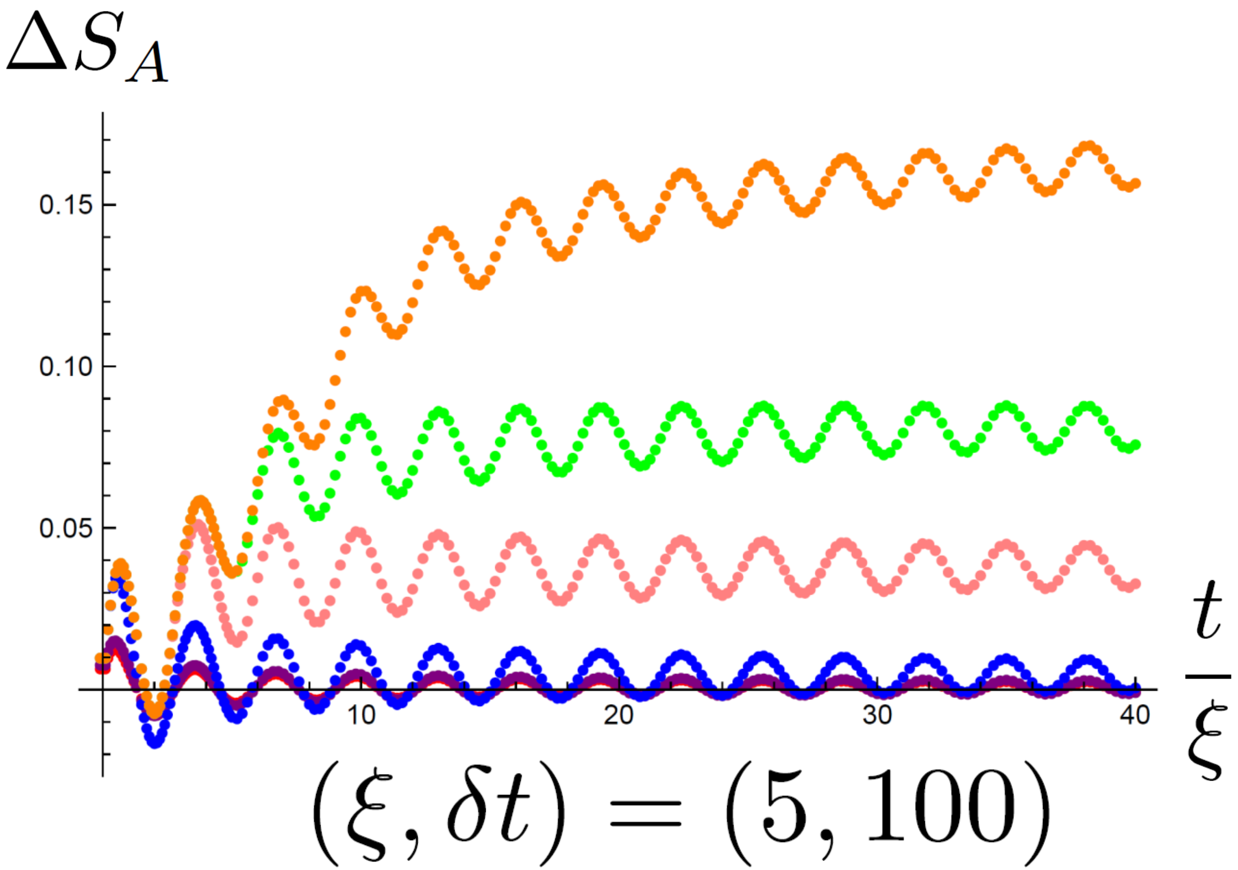
Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

- **If** $t \gg l/2,$

$$\Delta S_A \sim -\omega^2 \log(\omega) \times \frac{l}{\xi}$$



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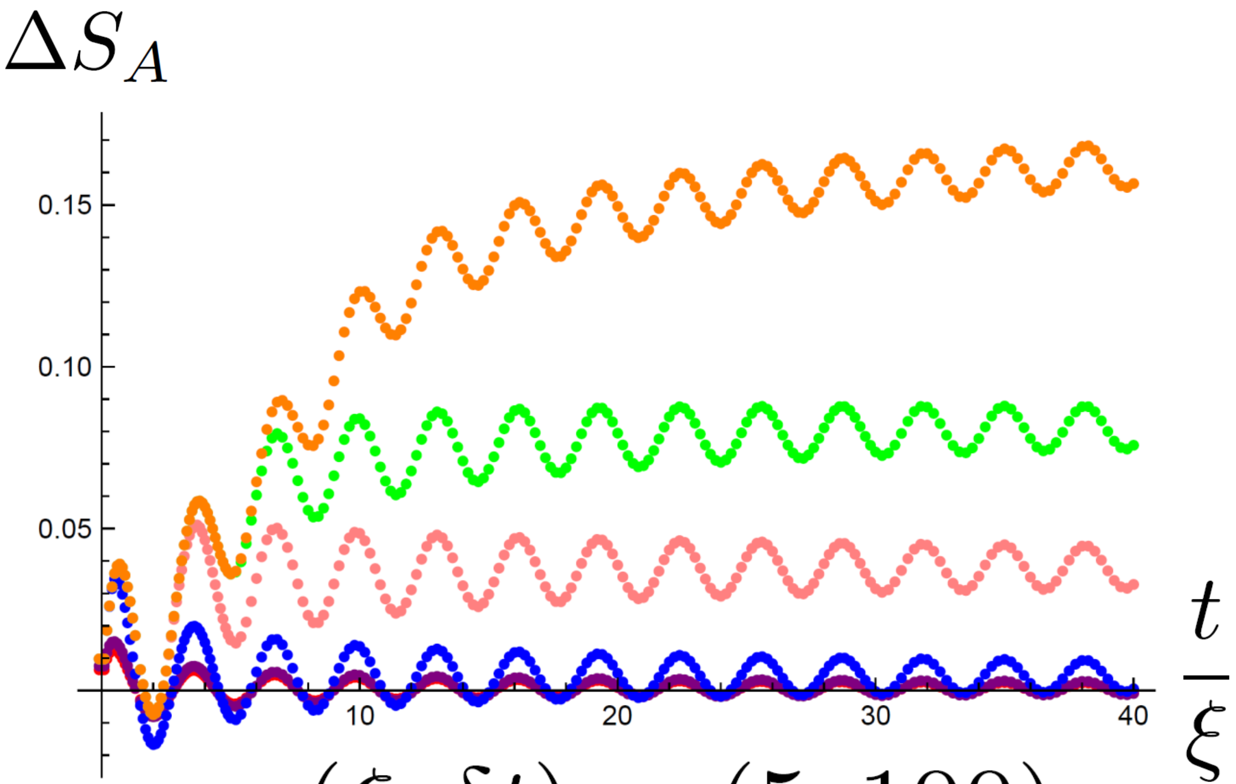
Thermalized



Entangled particles are created around t=0.

$$(\xi, \delta t) = (5, 100)$$

Plot of EE in fast CCP

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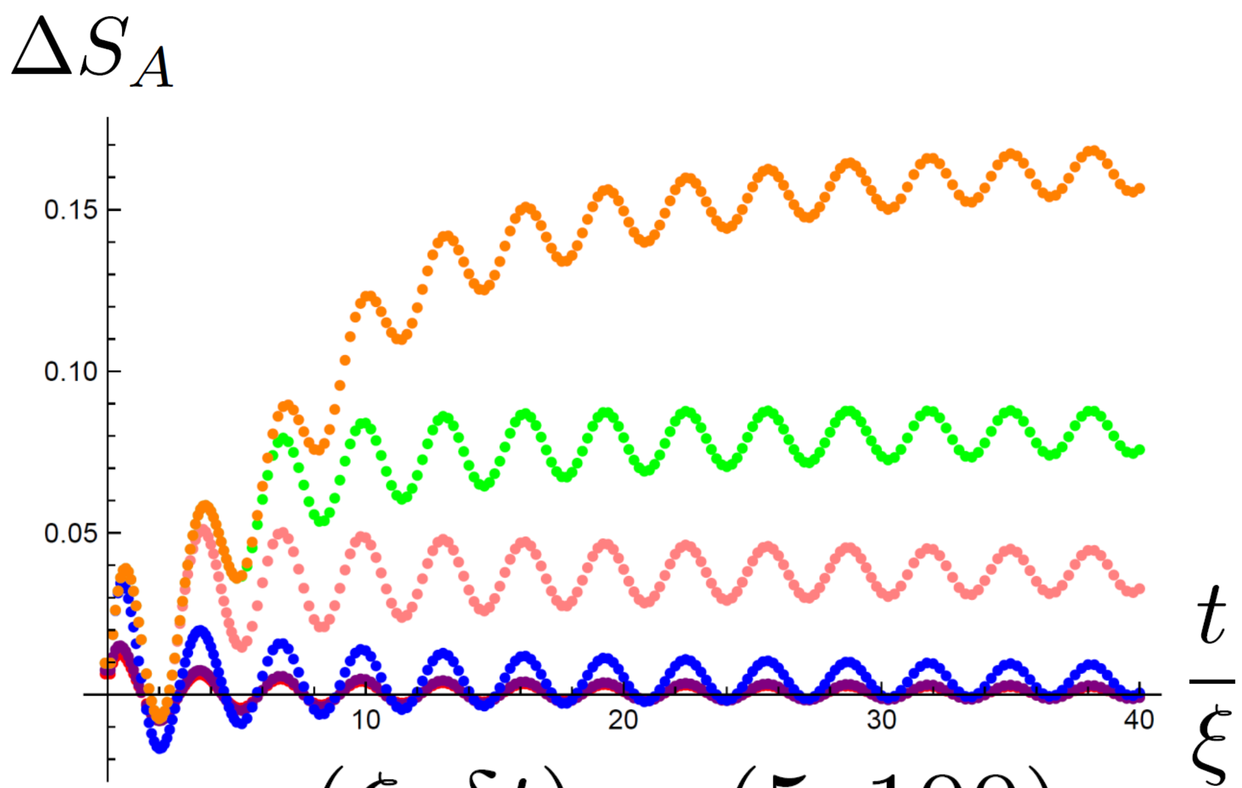


$(\xi, \delta t) = (5, 100)$

If ω ***decreases (*** ξ ***is fixed)***,

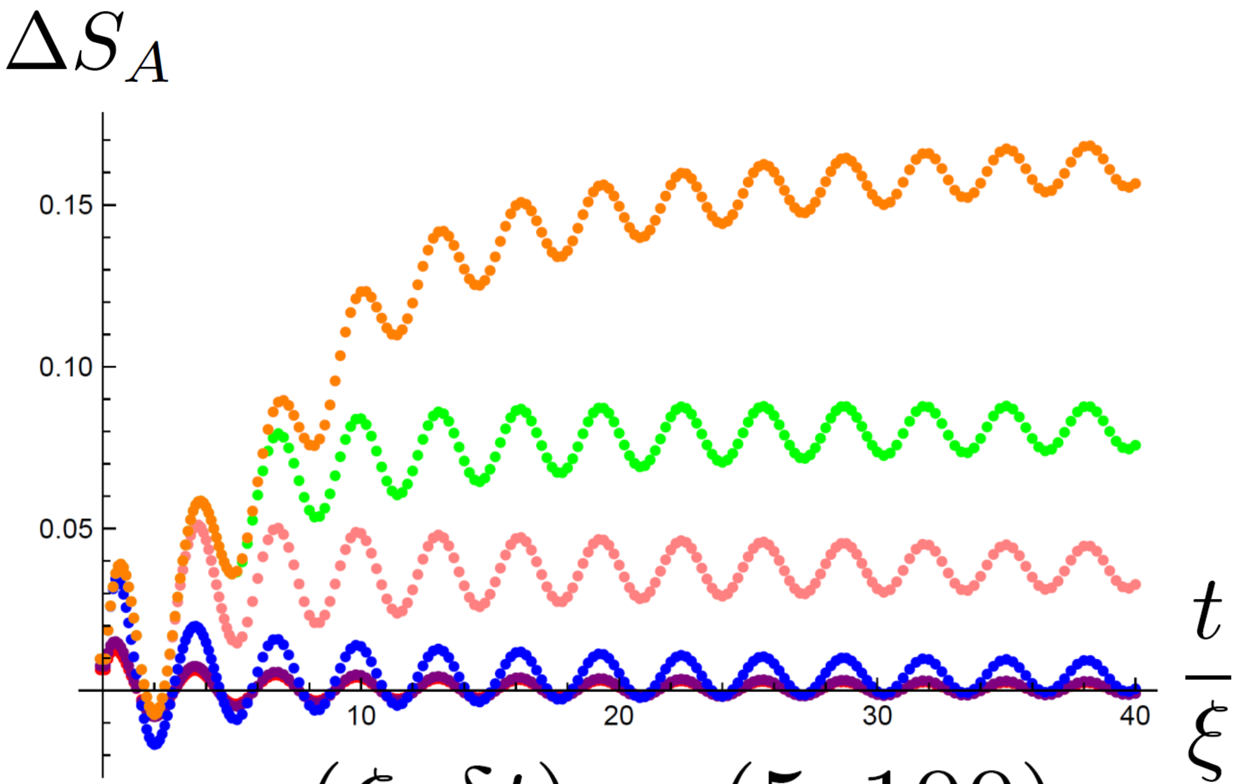
$-\omega^2 \log(\omega)$ ***decreases***.

Plot of EE in fast CCP

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

- ΔS_A **is oscillating**



$$(\xi, \delta t) = (5, 100)$$

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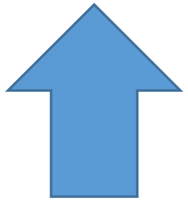
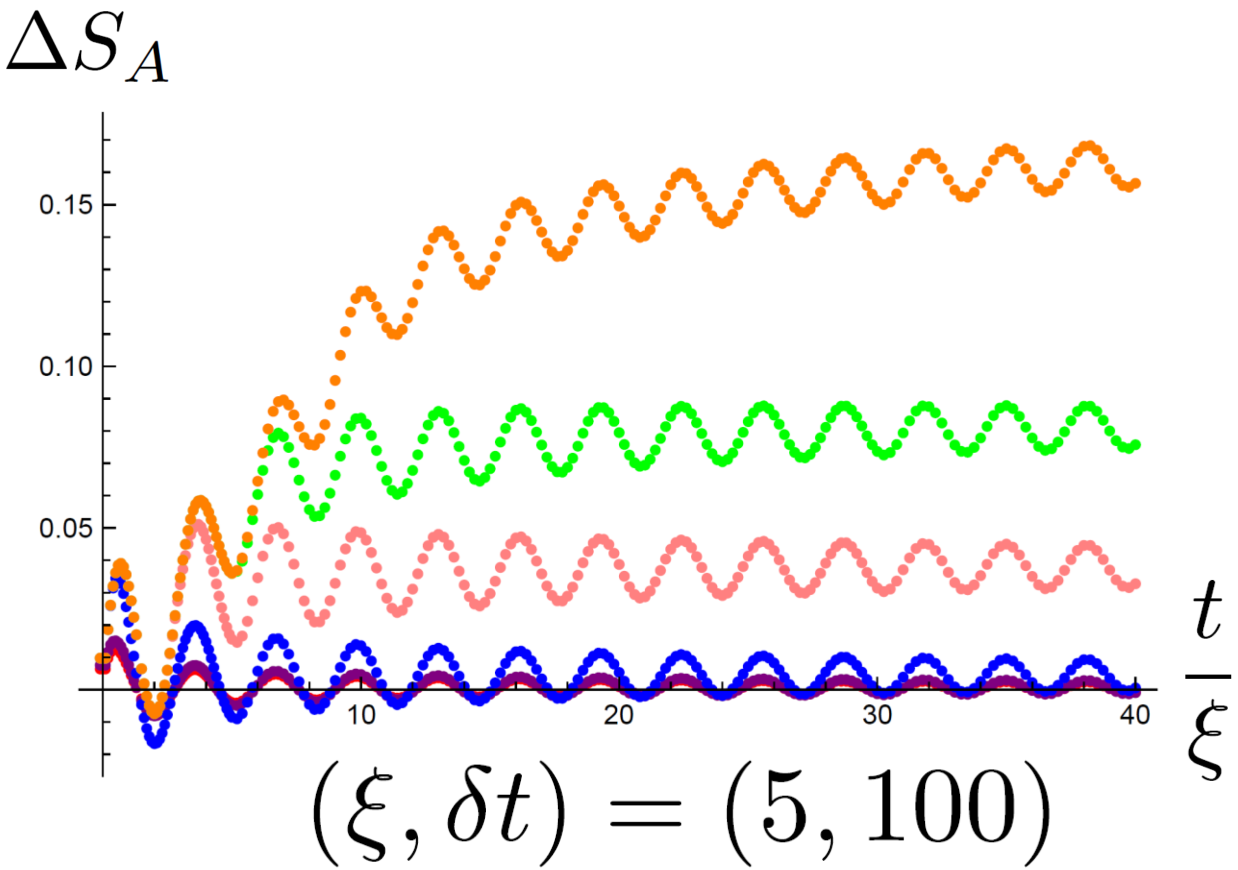


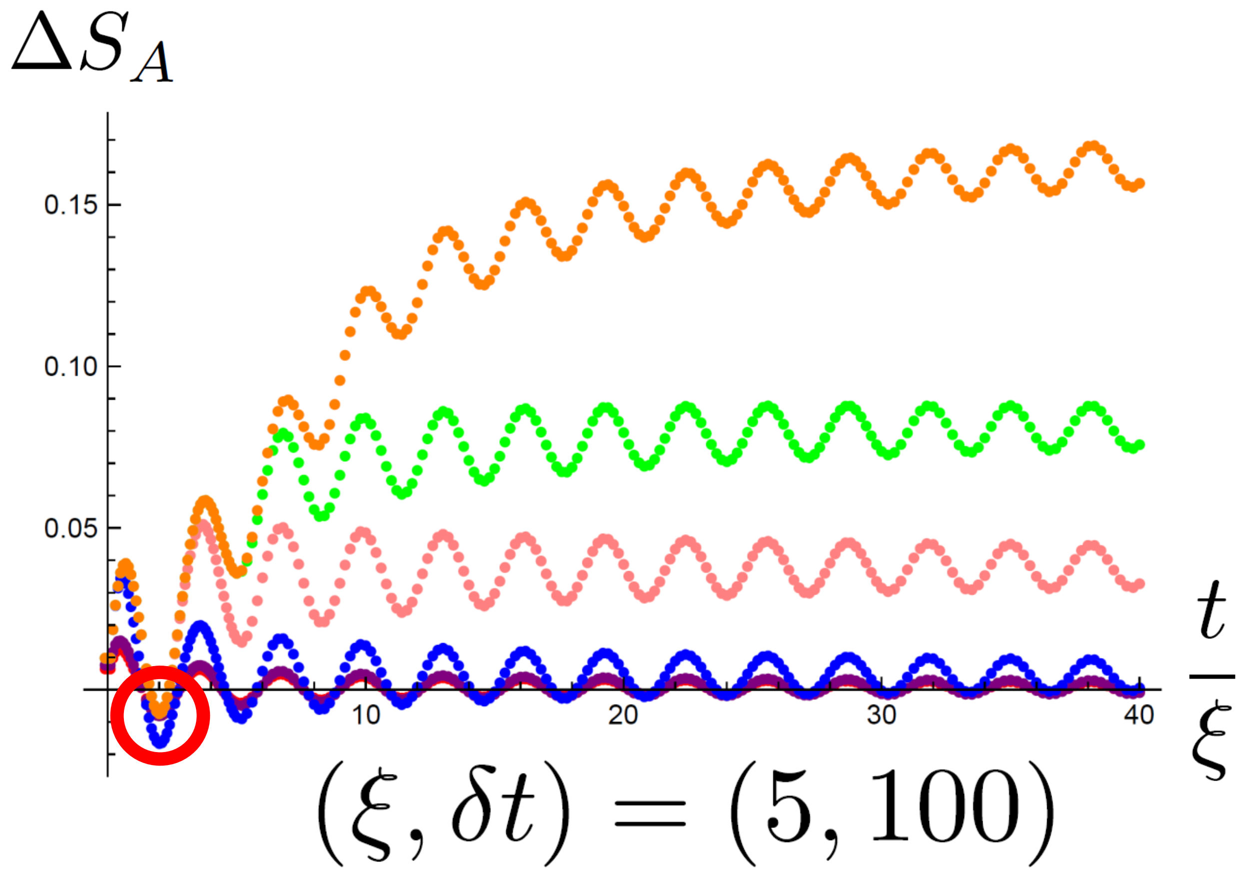
- **Frequency is determined by final mass.**

$$periodicity \sim \pi\xi$$



Minimum Value

- Minimum value of ΔS_A is at $t = 2\xi$.



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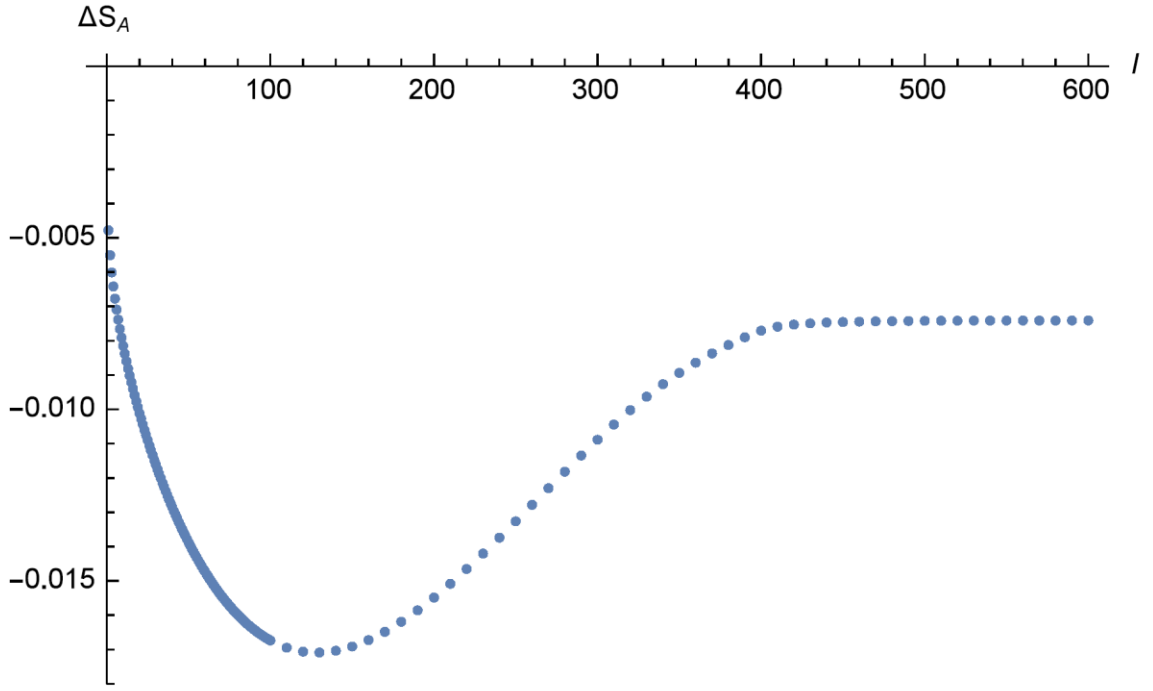
The plot for *l-dependence of* ΔS_A at $t = 2\xi$



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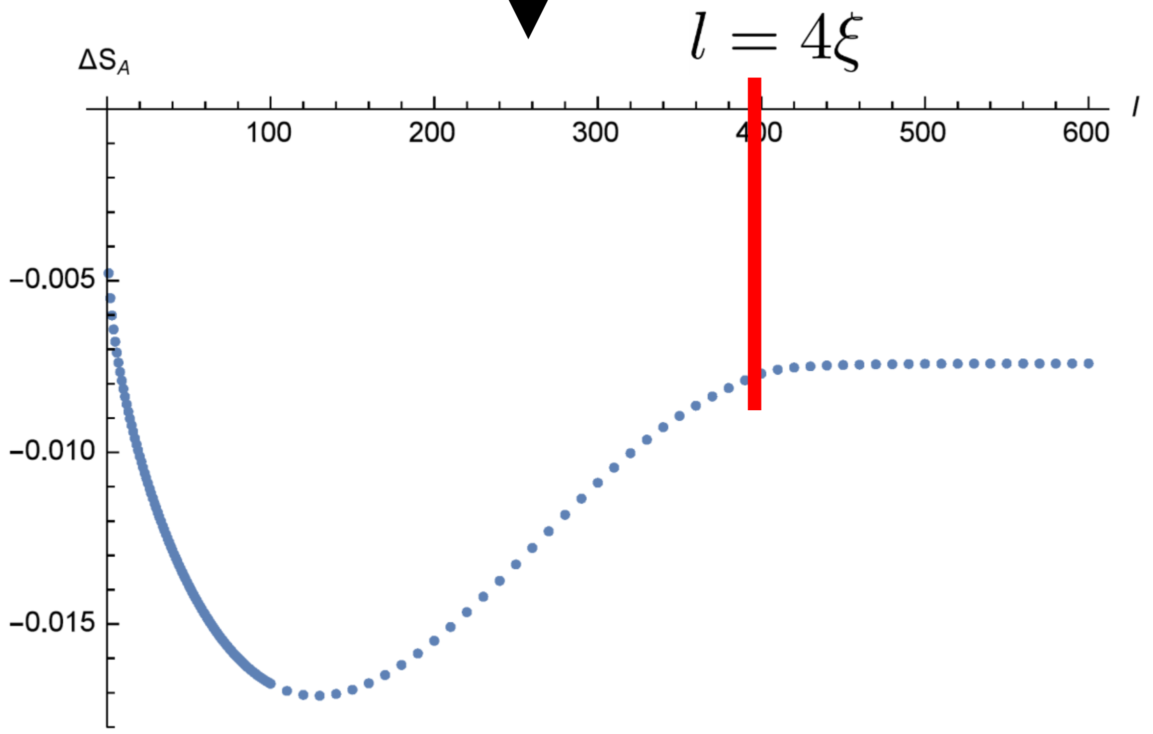
The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$

- Around $l = \xi$,
  ΔS_A is ***minimized***.

- Around $l = 4\xi$,
  ΔS_A is constant.



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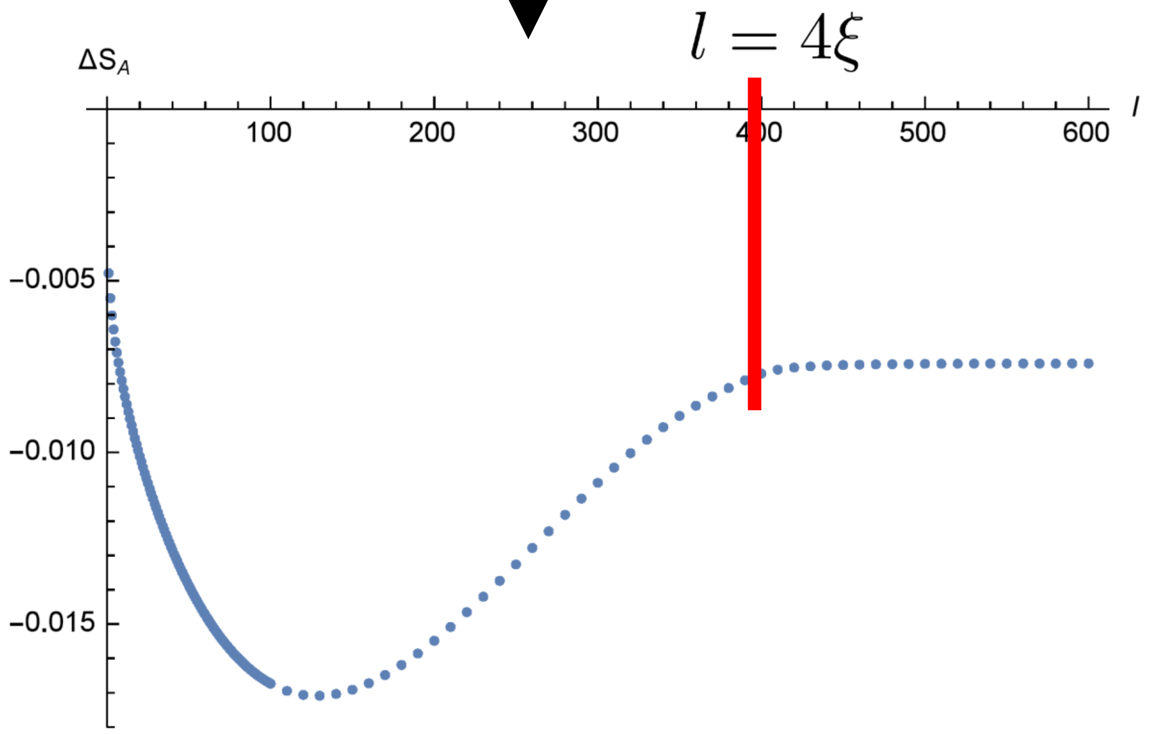
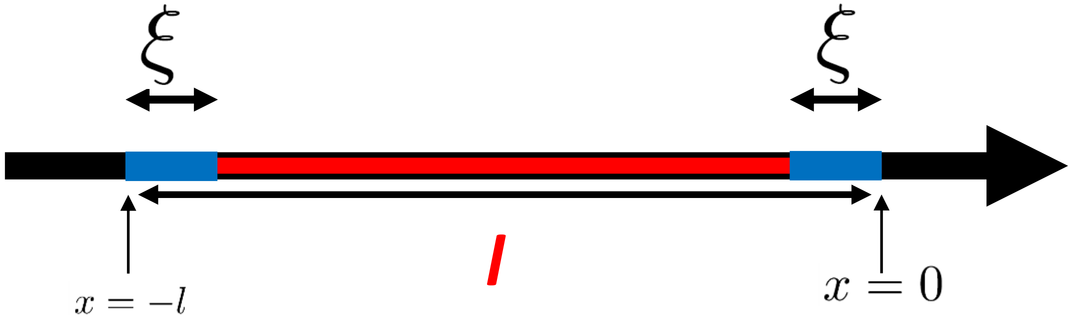


Minimum Value

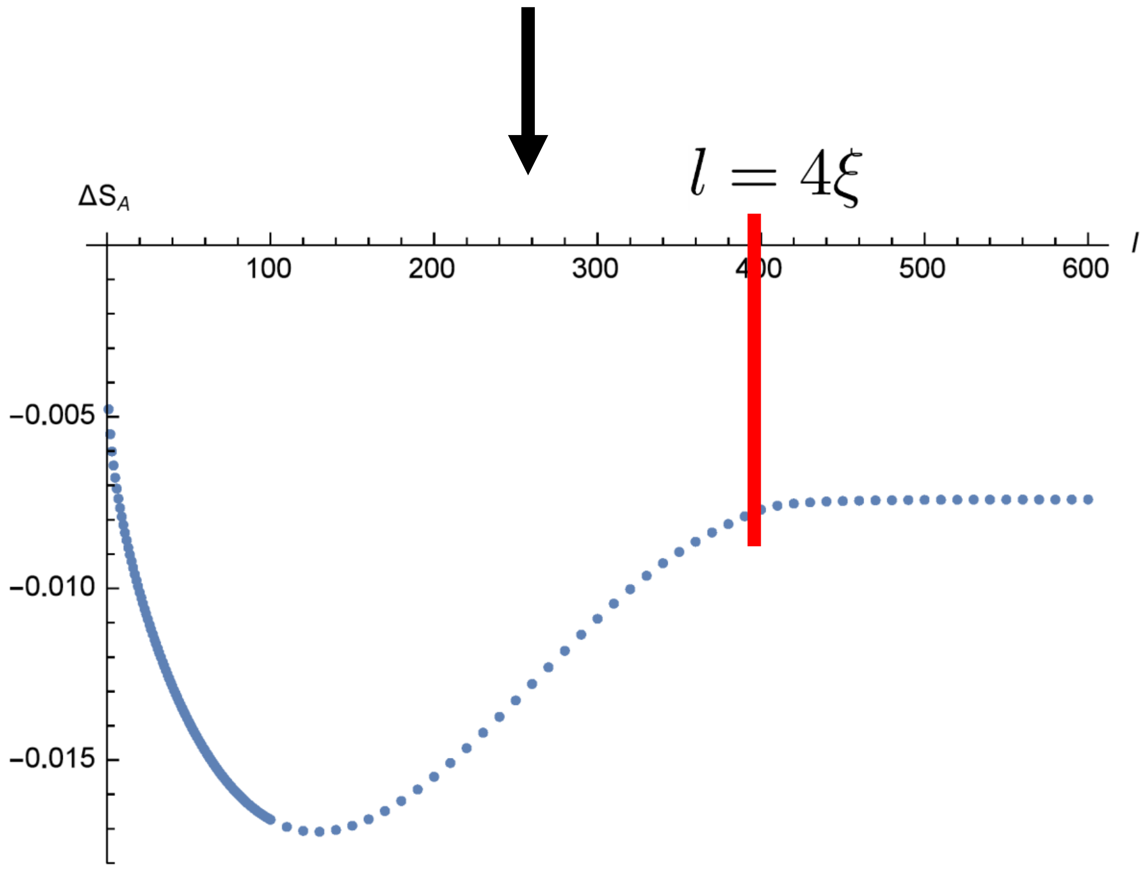
- Around $l = 4\xi$, ΔS_A is constant.

Initially, the blue region of the subsystem ***A*** is entangled with the complemental region.

$l > \xi$, S_A is constant.



The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



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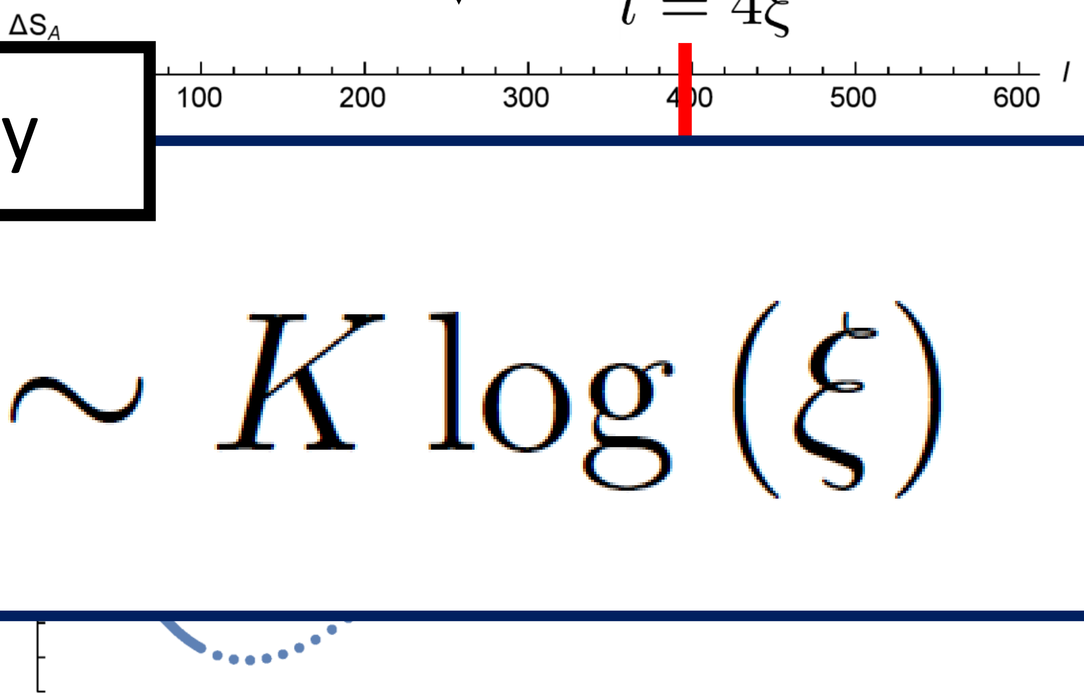
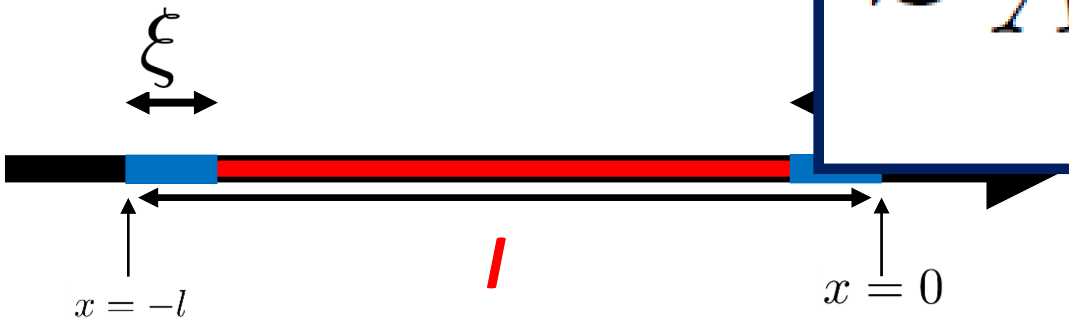


Initially, the blue region of the subsystem A … the complementary region.

2d Massive Theory

$$S_A \sim K \log(\xi)$$

$l > \xi$, S_A is constant.



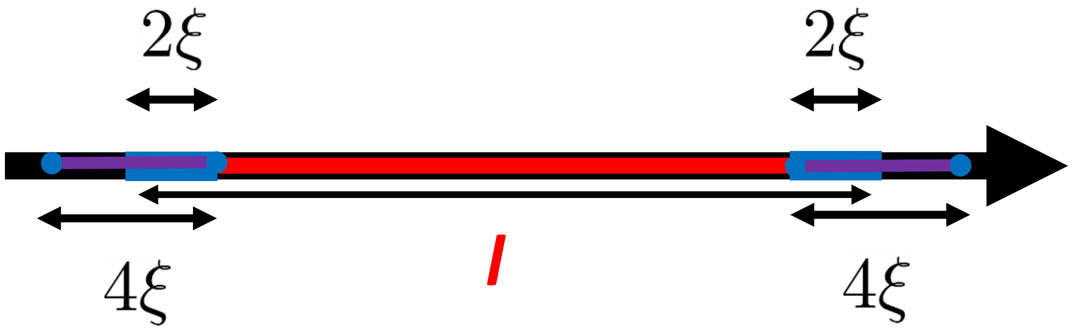
Minimum Value

- Around $l = 4\xi$, ΔS_A is constant.

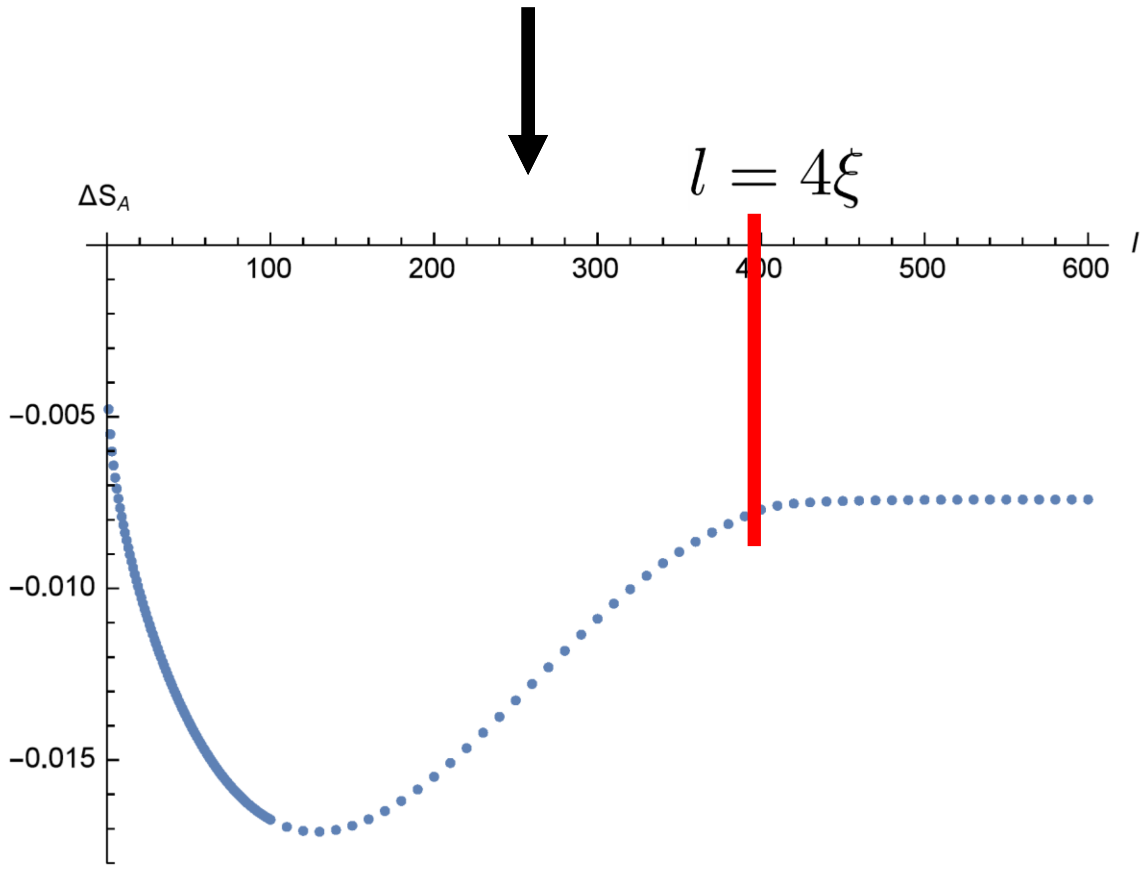
Entangled particle picture

At $t = 2\xi$, the blue region of the subsystem ***A*** is entangled with the complemental region.

$l > 4\xi$, ΔS_A is constant (<0).



The plot for *l-dependence of* ΔS_A ***at*** $t = 2\xi$



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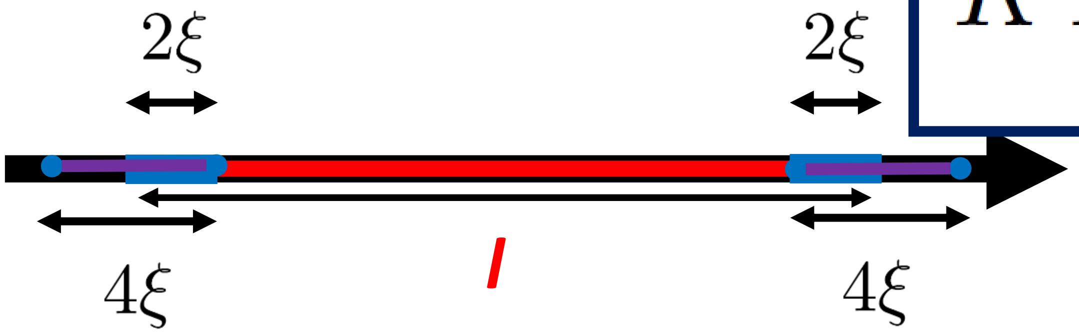
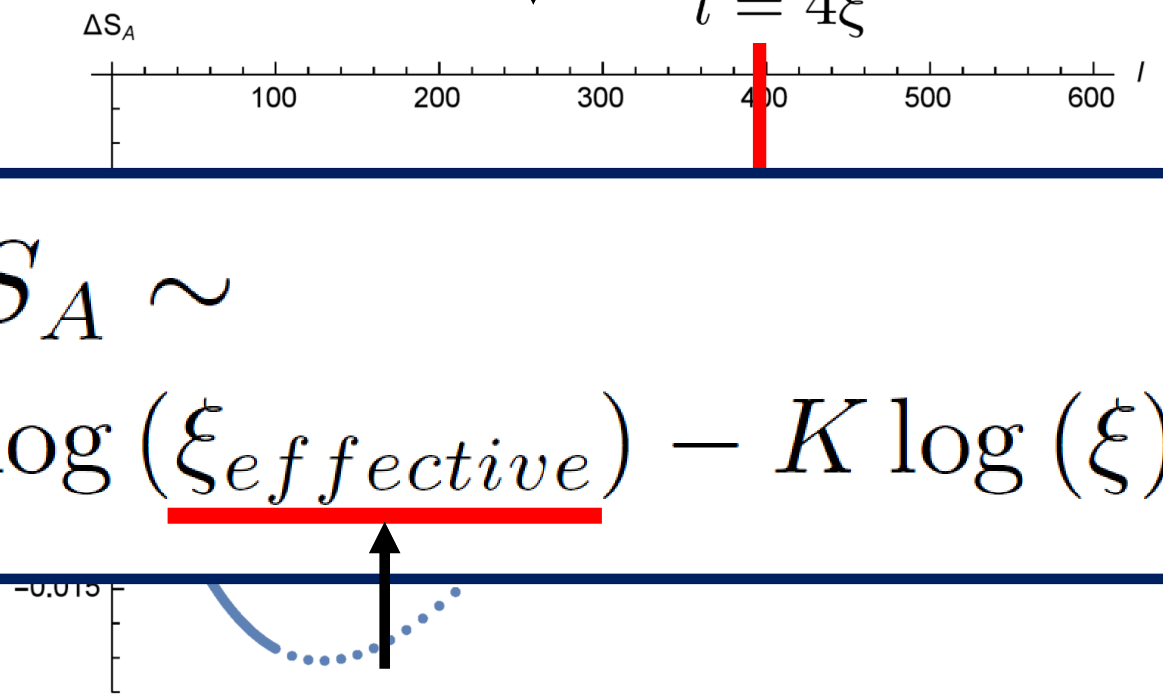


The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



$$\Delta S_A \sim K \log\left(\xi_{effective}\right) - K \log\left(\xi\right)$$

related with a distance between entangled pair.

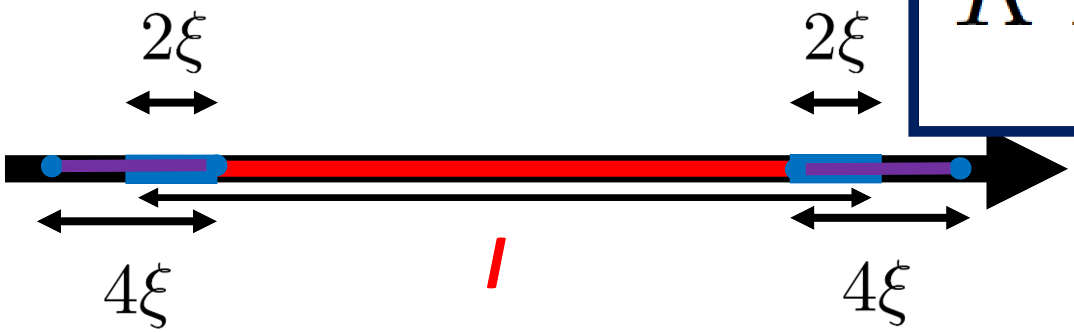
Minimum Value

- Around $l = 4\xi$, ΔS_A is constant.

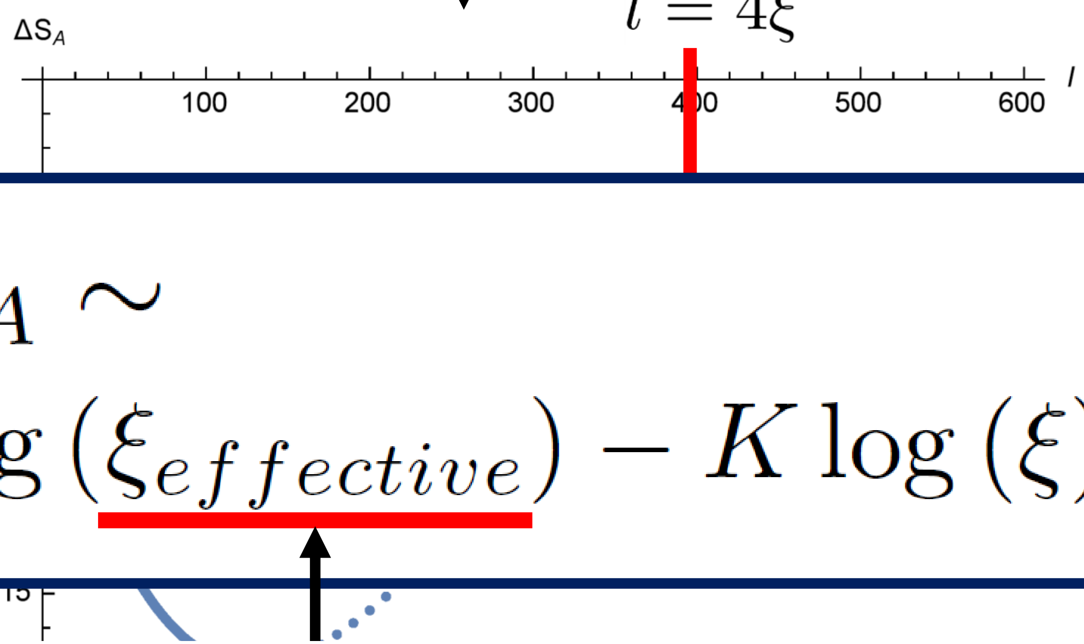
Entangled particle picture

At $t = 2\xi$, the blue region of the subsystem ***A*** is entangled with the complemental region.

$l > 4\xi$, ΔS_A is constant (<0).



The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



$$\Delta S_A \sim K \log\left(\xi_{effective}\right) - K \log\left(\xi\right)$$

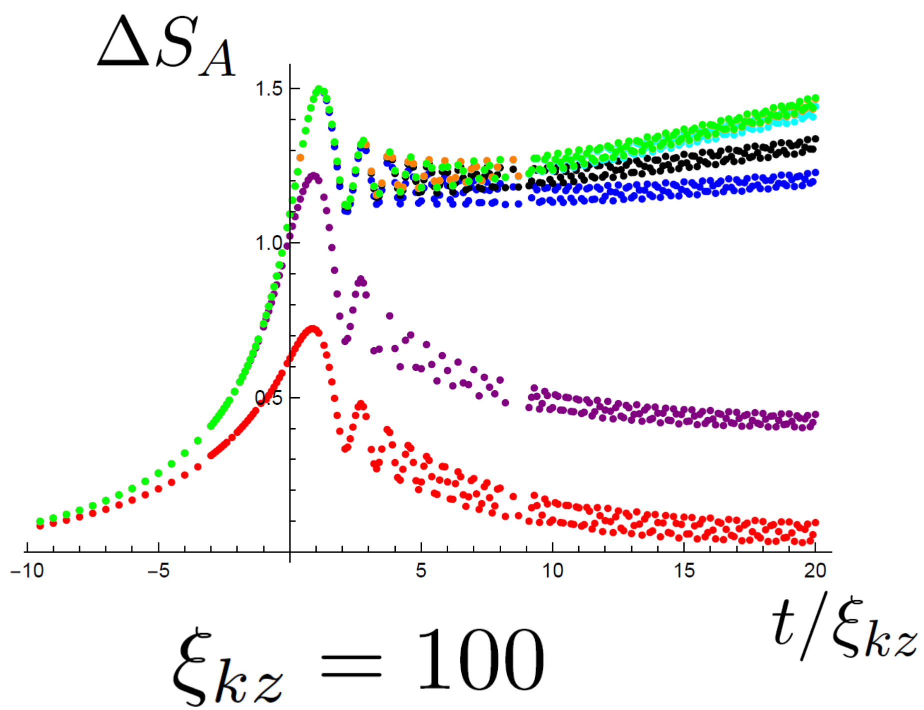
$$\xi_{effective} < \xi$$

EE in slow CCP

Plot of EE in Slow CCP

Green Curve: l=3000, Orange Curve: l=2500, Black Curve: l= 1000,

Blue Curve: l=500,Purple Curve: l=100, Red Curve: l=10



Plot of EE in Slow CCP

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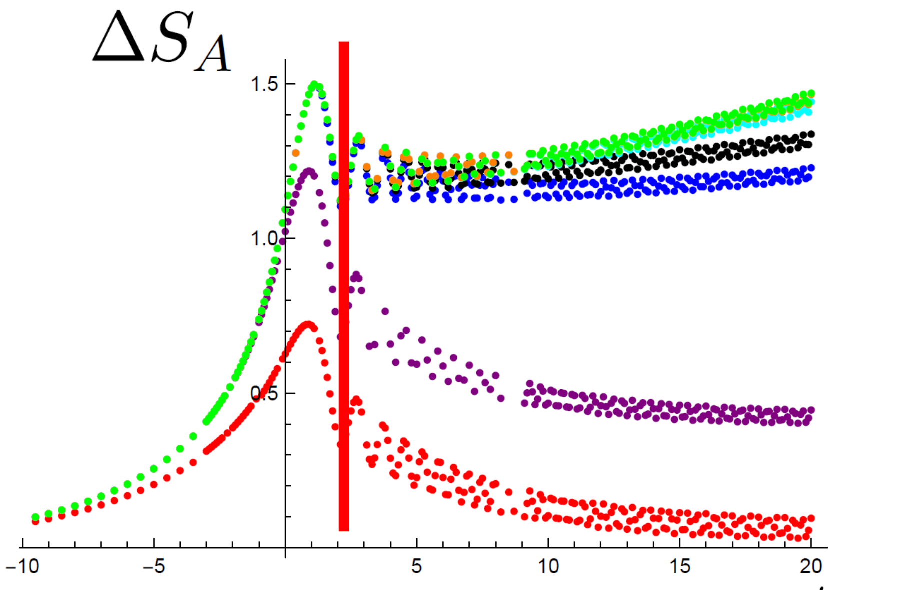
After $t = 2\xi_{kz}$,

ΔS_A starts to oscillate.

$$t \simeq 2\xi_{kz}$$



$$\xi_{kz} = 100$$

Plot of EE in Slow CCP

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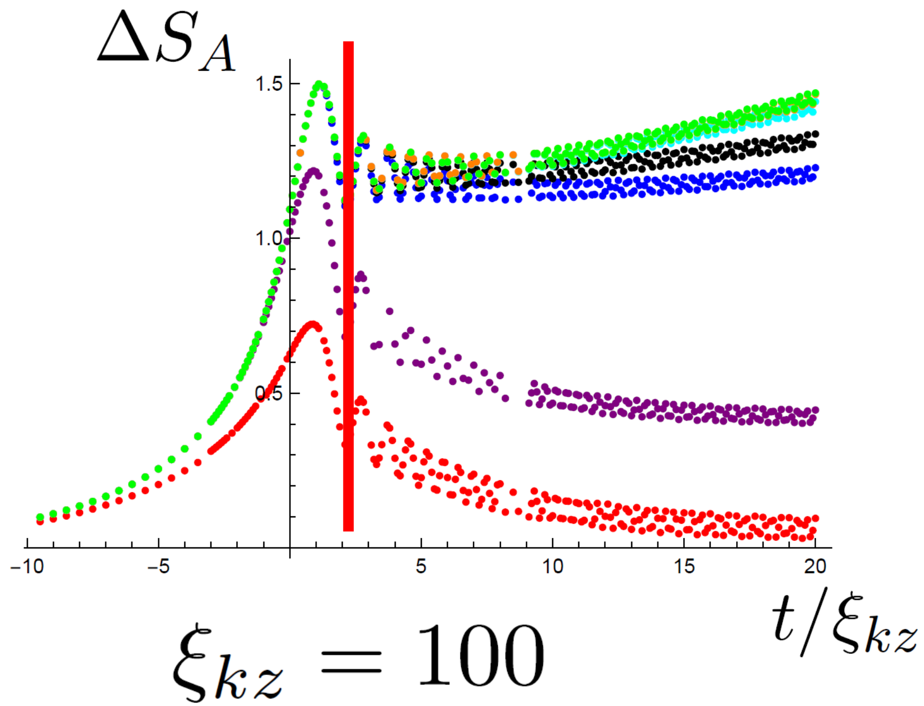
After $t = 2\xi_{kz}$,

ΔS_A starts to oscillate.

Similar to the results in fast quenches.

$$t \simeq 2\xi_{kz}$$



Plot of EE in Slow CCP

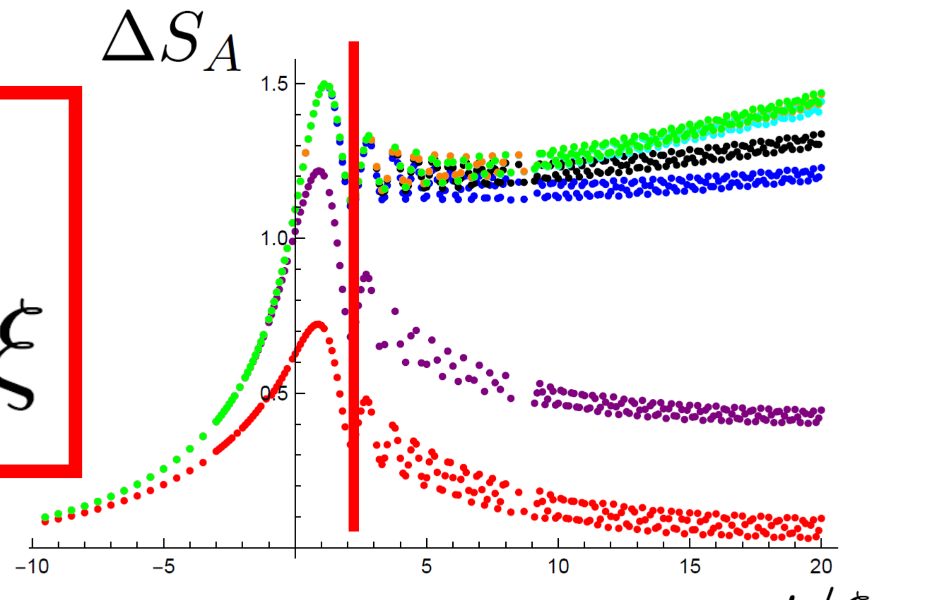
Green Curve: l=3000, Orange Curve: l=2500, Black Curve: l= 1000,

Blue Curve: l=500,Purple Curve: l=100, Red Curve: l=10

Periodicity of ΔS_A at late time

~ Periodicity of zero mode $\simeq \pi\xi$

$t \simeq 2\xi_{kz}$



$\xi_{kz} = 100$

$$\Delta S_A(t = 2\xi_{kz})$$

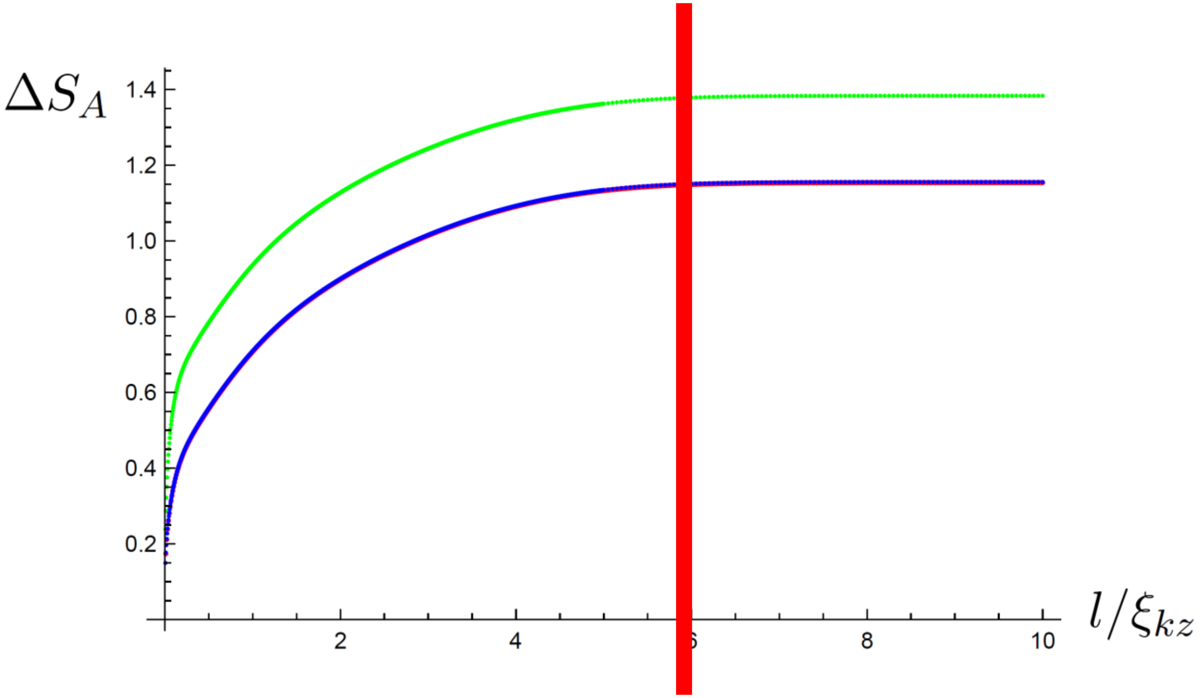
- $l > 6\xi_{kz}$

ΔS_A is a constant (>0).

Red: $(\omega, \xi_{kz}) = (100, 100)$

Blue: $(\omega, \xi_{kz}) = (100, 200)$

Green: $(\omega, \xi_{kz}) = (400, 200)$



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- Entangled particle interpretation

Adiabaticity breaks down.

@ $t \sim -t_{kz}$

Entangled particles are created.

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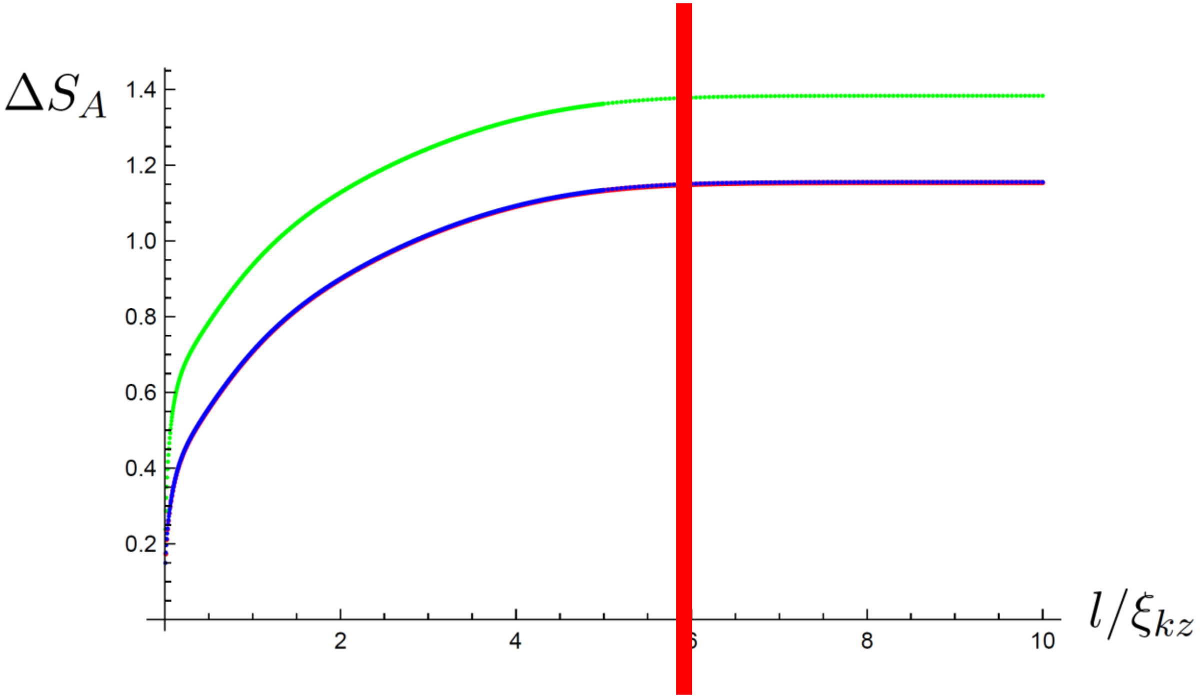

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Entangled particles are created.

$$\Delta S_A \sim \boxed{\xi_{effective} > \xi}$$

$$K \log\left(\xi_{effective}\right) - K \log\left(\xi\right)$$

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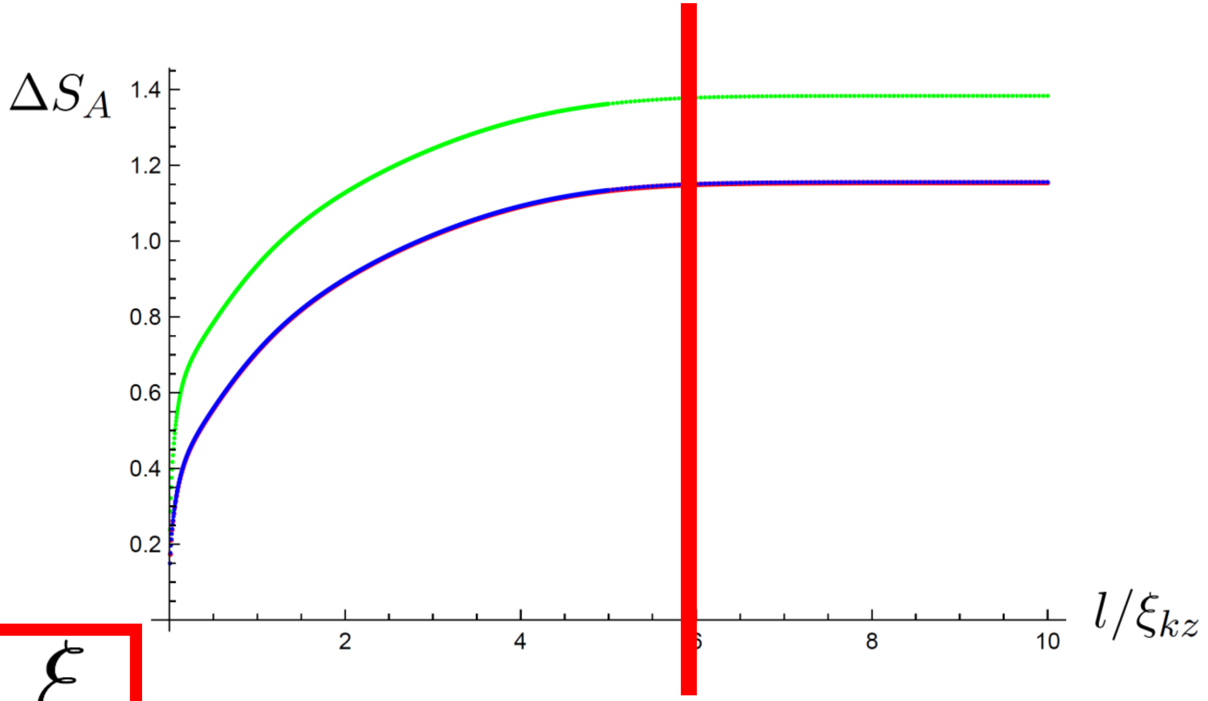
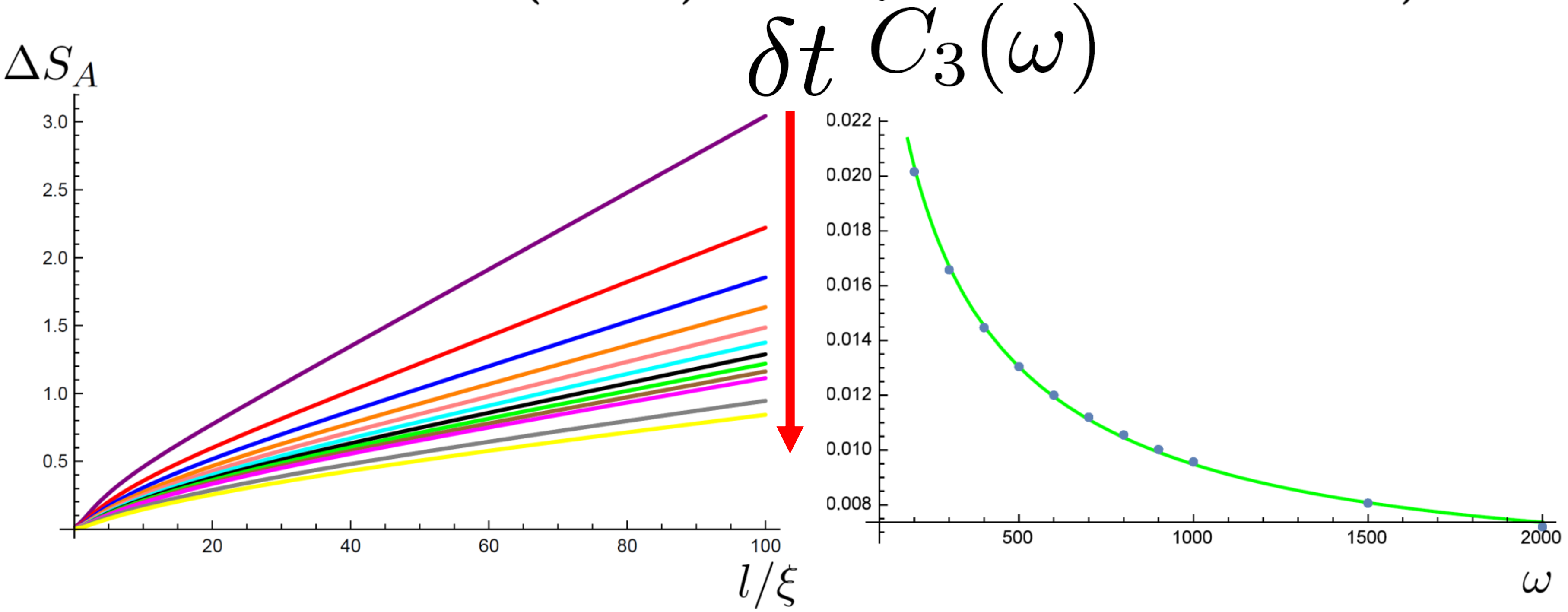


$l = 6\xi_{kz}$

Late time in CCP

$$(\xi, t) = (10, 1000000)$$



ΔS_A Is fitted by $\Delta S_A \sim \dfrac{C_3(\omega)}{\xi} l + B$

Late time in CCP

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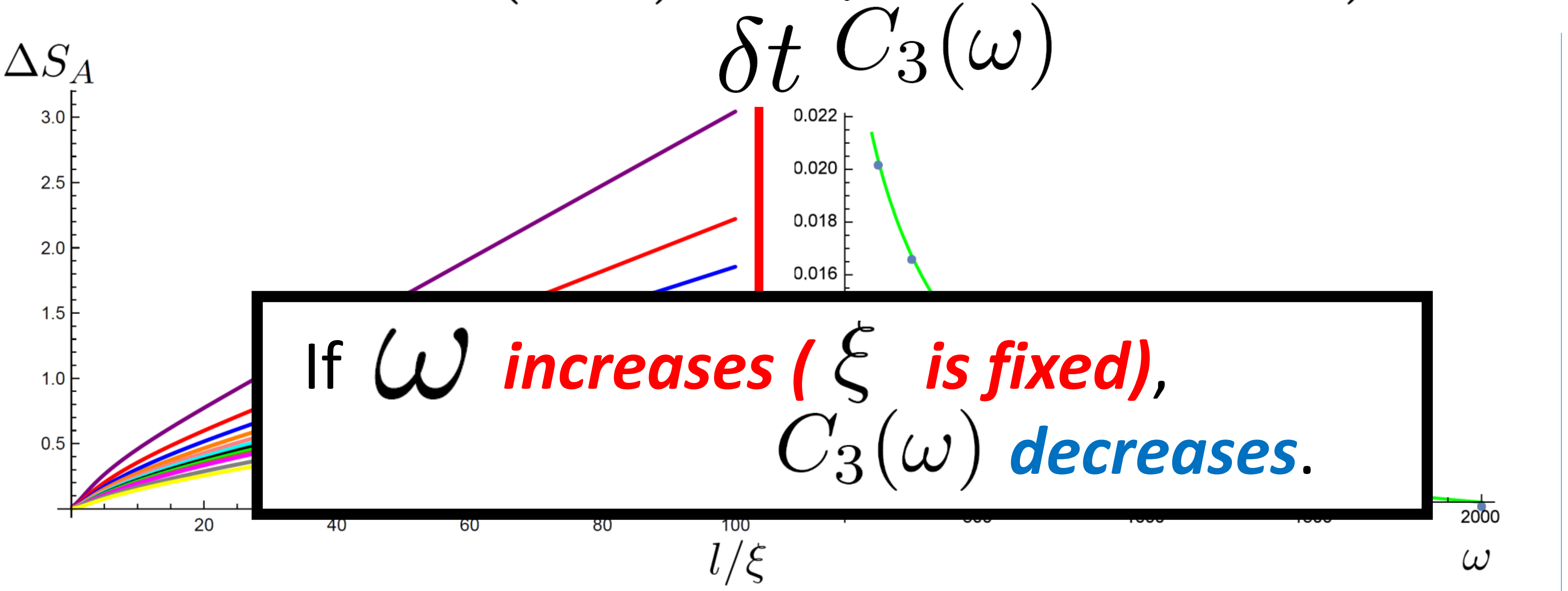

If ω **increases (ξ is fixed)**, $C_3(\omega)$ **decreases**.

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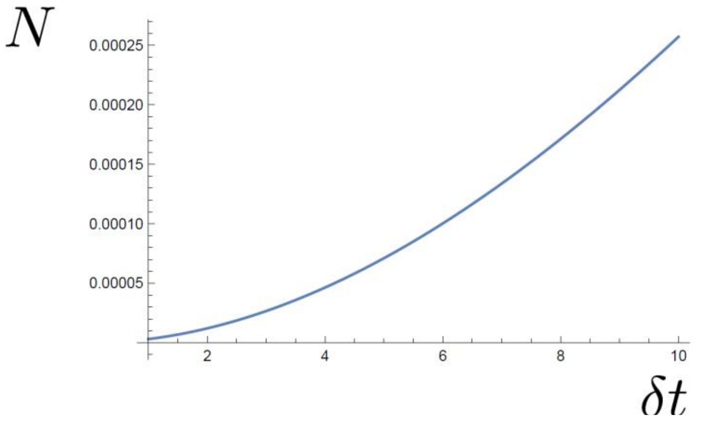
Volume law in Fast and slow limits

- In the fast limit , Fitting function: $\Delta S_A \sim -\omega^2 \log(\omega) \times \frac{l}{\xi}$
- In the slow limit, Fitting function: $\Delta S_A \sim \frac{C_3(\omega)}{\xi} l + B$

N is the number operator/ Volume at late time. $\xi = 100$



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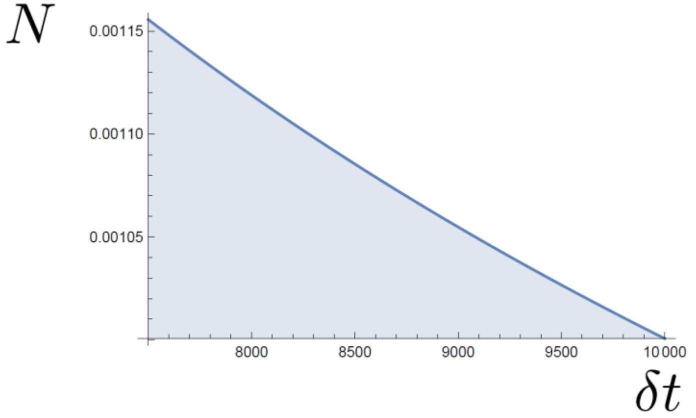

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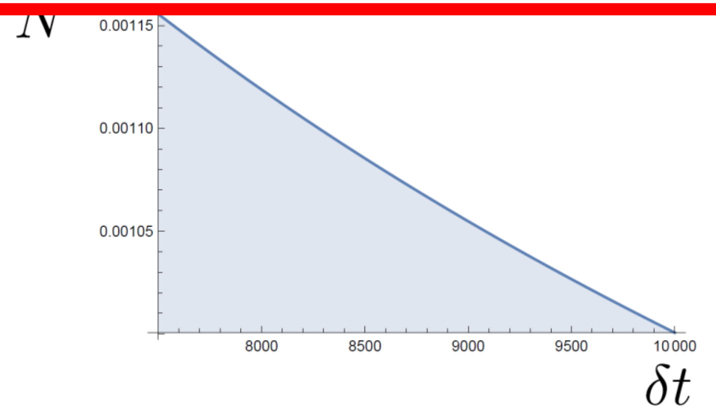
N is the number operator/ Volume at late time. $\xi = 100$

If ω ***decreases (*** ξ ***is fixed),***

The behavior of entanglement entropy at late time
is consistent with the behavior of number operator at late time.



If ω ***increases (*** ξ ***is fixed),***
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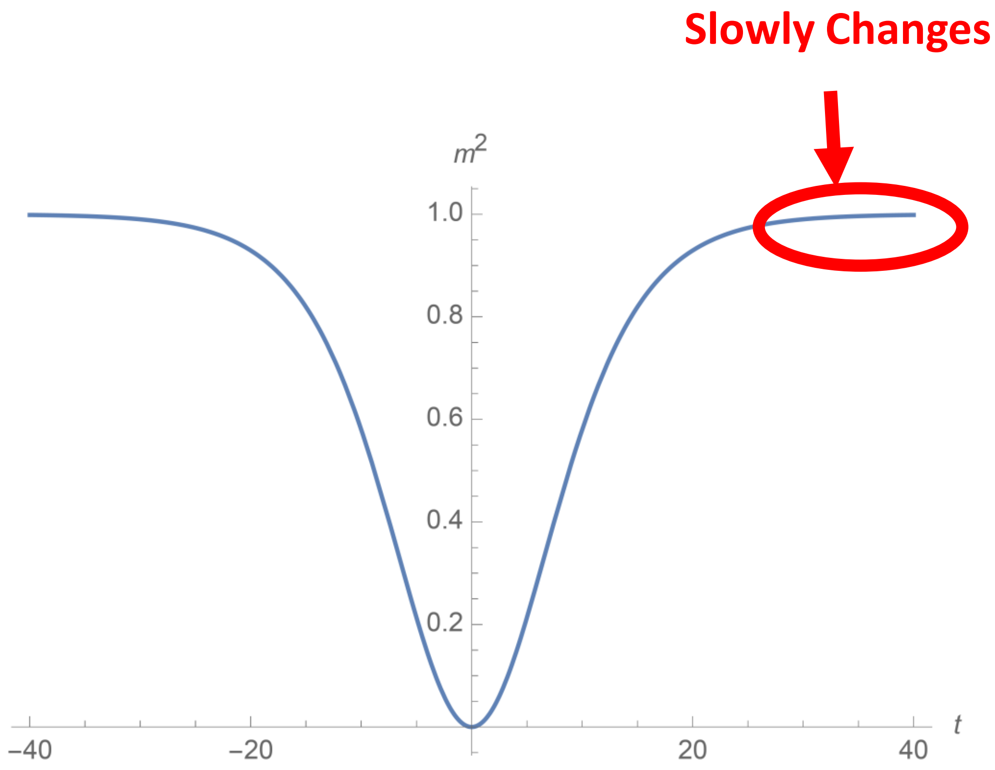
Oscillation (naive)

- **In the late time, the mass profile slowly changes.**

- **Physical quantities can be computed adiabatically.**



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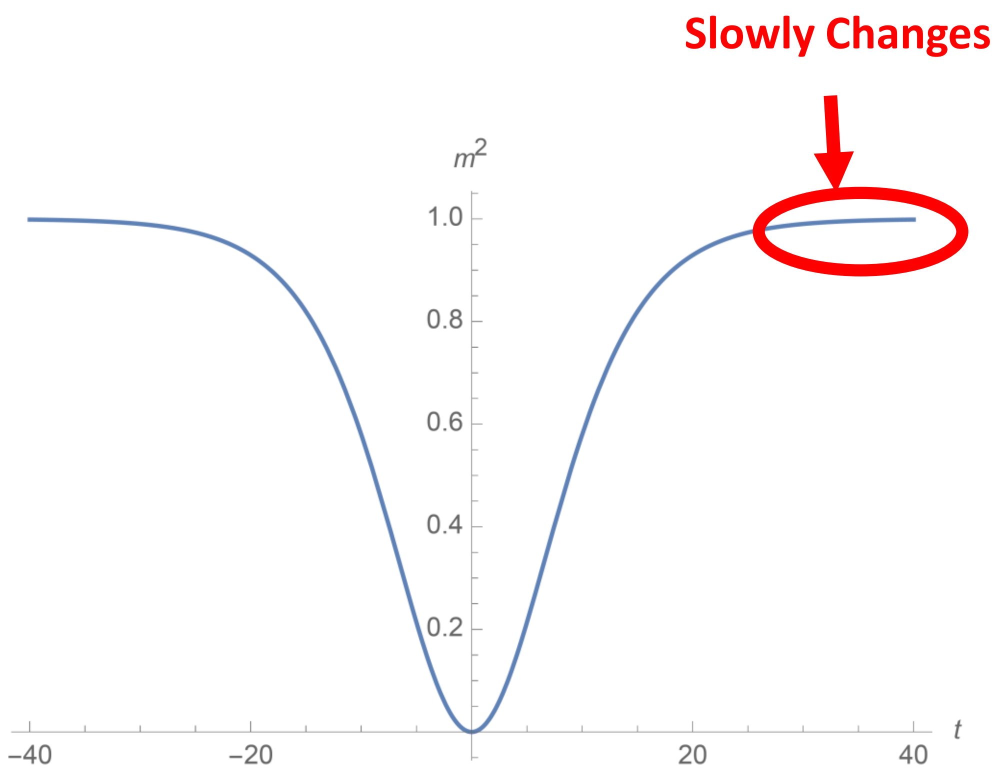
$$v_k = \partial_k \omega_k \quad \omega_k = \sqrt{4\sin^2\left(\frac{k}{2}\right) + m_f^2}$$



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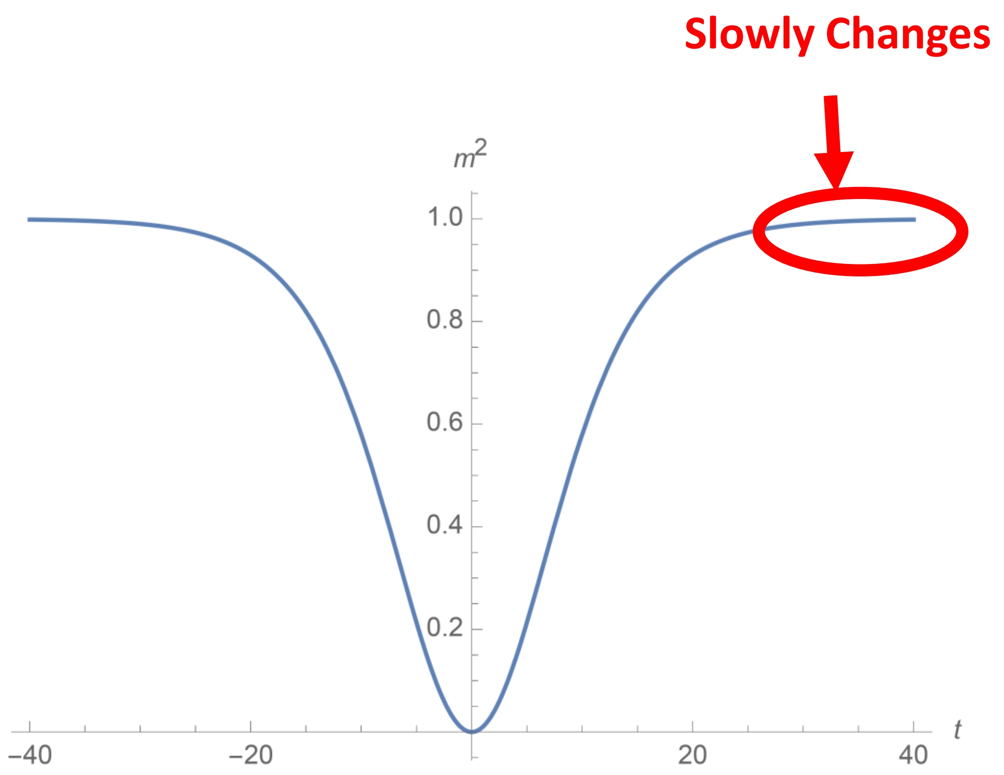

・As in ECP, in the late time, slow mode (zero mode) contribute to ΔS_A.

zero mode: $e^{-i\omega_k t} \sim e^{-im_f t}$

Oscillation

Entanglement Entropy:

$$S_A = \sum_{k=1}^{l} s_A(\gamma_k)$$

$$s_A(\gamma_k) = \left(\frac{1}{2} + \gamma_k\right) \log\left(\frac{1}{2} + \gamma_k\right) - \left(-\frac{1}{2} + \gamma_k\right) \log\left(-\frac{1}{2} + \gamma_k\right)$$

$$\Gamma = \begin{pmatrix} X_{ij} & \frac{1}{2}D_{ij} \\ \frac{1}{2}D_{ji} & P_{ij} \end{pmatrix} \quad J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

$$X_{ij} = \langle \phi_i \phi_j \rangle \qquad P_{ij} = \langle \pi_i \pi_j \rangle \qquad D_{ij} = \langle \{\phi, \pi_j\} \rangle$$

- $M = iJ\Gamma$ has eigenvalues $\pm\gamma_k$.

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Oscillation

$$X_{ab}(t) = \langle X_a(t) X_b(t) \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} X_k \cos\left(k\left|a-b\right|\right) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left|f_k(t)\right|^2 \cos\left(k\left|a-b\right|\right),$$

$$P_{ab}(t) = \langle P_a(t) P_b(t) \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} P_k \cos\left(k\left|a-b\right|\right) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left|\dot{f}_k(t)\right|^2 \cos\left(k\left|a-b\right|\right),$$

$$D_{ab}(t) = \frac{1}{2} \langle \{X_a(t), P_b(t)\} \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} D_k \cos\left(k\left|a-b\right|\right) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} Re\left[\dot{f}_k^*(t) f_k(t)\right] \cos\left(k\left|a-b\right|\right)$$

In the late limit,

$$\boxed{f_k(t) \simeq \mathcal{A}_k e^{i\omega_k t} + \mathcal{B}_k e^{-i\omega_k t}}$$

Left moving mode + Right moving mode

Oscillation

$$X_{ab}(t) = \langle X_a(t) X_b(t) \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} X_k \cos\left(k\left|a-b\right|\right) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left|f_k(t)\right|^2 \cos\left(k\left|a-b\right|\right),$$

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In the late limit,

$$X_k, P_k \simeq \mathcal{C}_k^{x,p} + \mathcal{D}_k^{x,p} \cos\left(2\omega_0 t + \Theta_k^{x,p}\right),$$

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slow mode (physical) = zero mode (k=0)

Oscillation

$$X_{ab}(t) = \langle X_a(t) X_b(t) \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} X_k \cos(k|a-b|) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} |f_k(t)|^2 \cos(k|a-b|),$$

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Oscillation

$$X_{ab}(t) = \langle X_a(t) X_b(t) \rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} X_k \cos\left(k\,|a-b|\right) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left|f_k(t)\right|^2 \cos\left(k\,|a-b|\right),$$

$P_{ab}(t) = \langle P$ …

$D_{ab}(t) = \frac{1}{2}$ … $\cos\left(k\,|a-b|\right)$

X_0, P_0, D_0 oscillate with $\pi\xi$.

➡ ΔS_A oscillate with $\pi\xi$.

X_k, P_k … $\cos\left(2\omega_0 t + \Theta_k\right)$,

$$D_k \simeq \mathcal{D}_k^d \cos\left(2\omega_0 t + \Theta_k^d\right),$$

$$\omega_0 = m$$

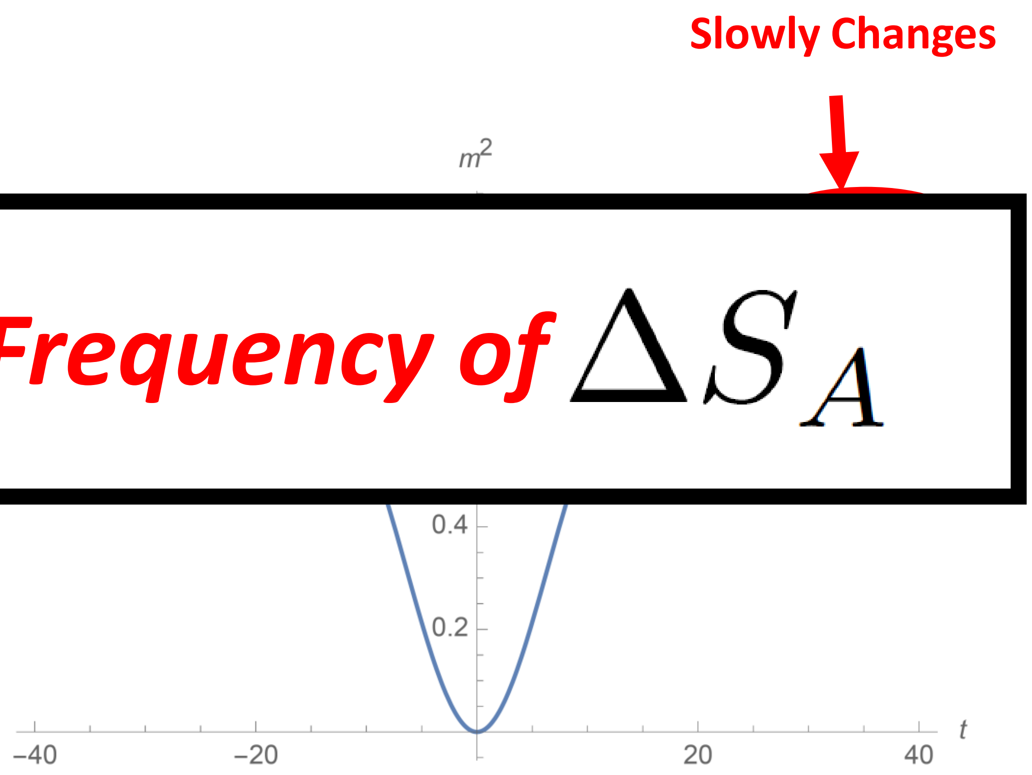
slow mode (physical) = zero mode (k=0)

Oscillation

- **In the late time, the mass profile slowly changes.**

- **Physical quantities can be computed**

Frequency of zero mode ⟷ ***Frequency of*** ΔS_A



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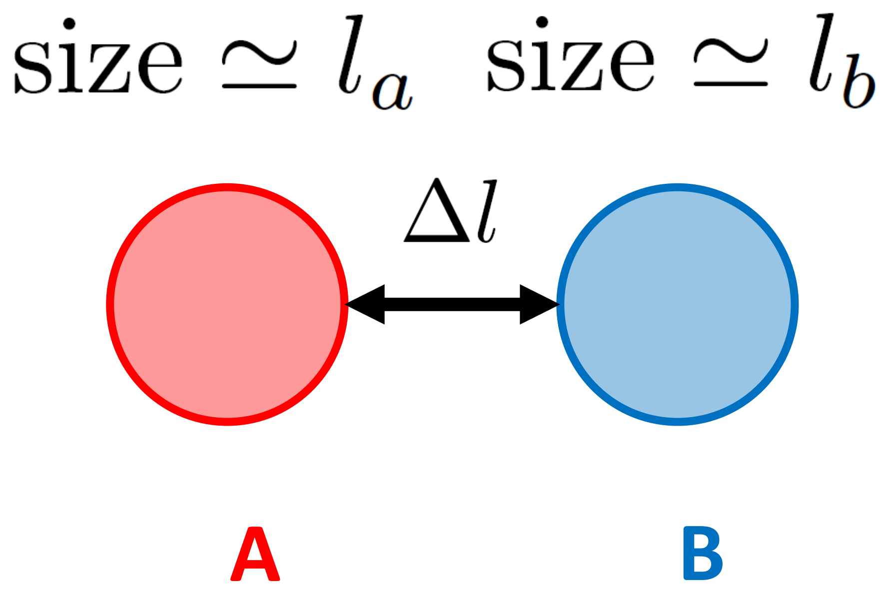
zero mode: $e^{-i\omega_k t} \sim e^{-im_f t}$

MI and LN

Time evolution of MI and LN



- **Mutual Information:**

$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

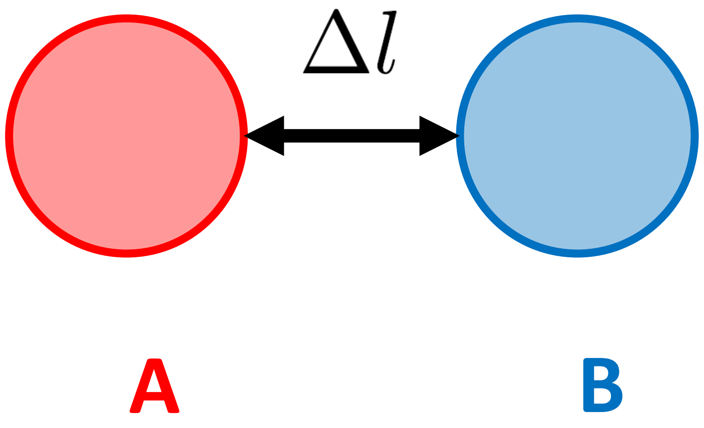
- **Logarithmic Negativity:**

$$\mathcal{E} = \log ||\rho_{A\cup B}^{T_B}|| = \log \sum_i |\lambda_i|$$

$$\rho_{A\cup B} = \rho_{ij,kl} \left|i\right\rangle \left\langle j\right|_A \otimes \left|k\right\rangle \left\langle l\right|_B \text{,} \quad \rho_{A\cup B}^{T_B} = \rho_{ij,lk} \left|i\right\rangle \left\langle j\right|_A \otimes \left|k\right\rangle \left\langle l\right|_B$$

Time evolution of MI and LN

$\text{size} \simeq l_a \quad \text{size} \simeq l_b$



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Eigenvalues of $\rho_{A\cup B}^{T_B}$

$$\rho_{A\cup B} = \rho_{ij,kl} \left|i\right\rangle \left\langle j\right|_A \otimes \left|k\right\rangle \left\langle l\right|_B \text{,} \quad \rho_{A\cup B}^{T_B} = \rho_{ij,lk} \left|i\right\rangle \left\langle j\right|_A \otimes \left|k\right\rangle \left\langle l\right|_B$$

What we are studying

Change of MI and LN:

$$\Delta I_{A,B}(t) = I_{A,B}(t) - I_{A,B}(t_{in})$$

$$\Delta \mathcal{E}(t) = \mathcal{E} - \mathcal{E}(t_{in})$$

What we are studying

Change of MI and LN:

$$\Delta I_{A,B}(t) = I_{A,B}(t) - \underline{I_{A,B}(t_{in})}$$

$$\Delta \mathcal{E}(t) = \mathcal{E} - \underline{\mathcal{E}(t_{in})}$$

← ***MI and LN for initial mass***

Time evolution of MI and LN in Fast-ECP

Parameter : $(\xi, \delta t, l_a, l_b) = (100, 5, 1000, 1000)$

$\Delta l = 0 :$ **Red**

$\Delta l = 10 :$ **Purple**

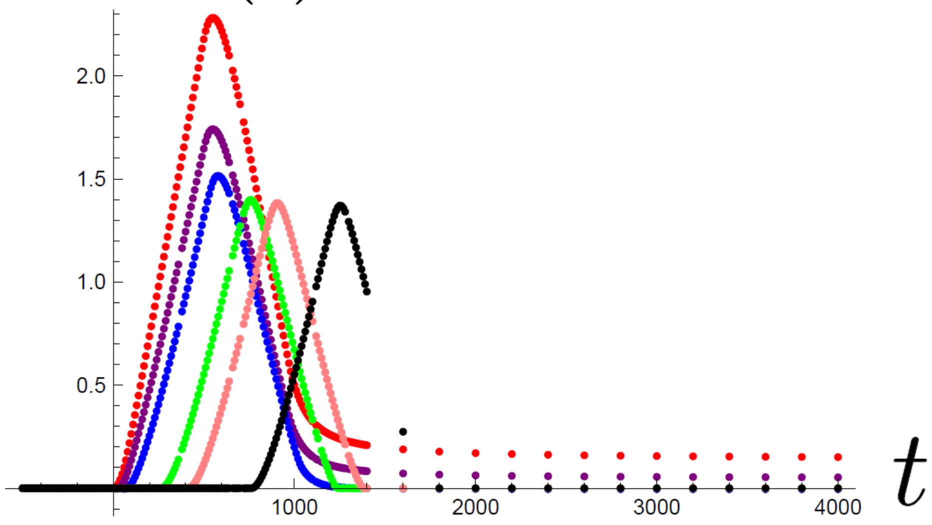
$\Delta l = 100 :$ **Blue**

$\Delta l = 500 :$ **Green**

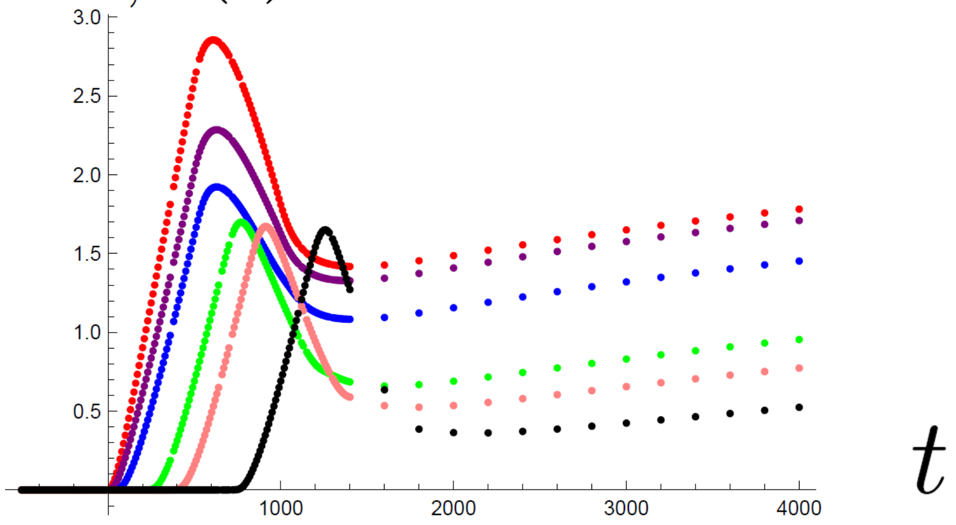
$\Delta l = 800 :$ **Pink**

$\Delta l = 1500 :$ **Black**

$\Delta \mathcal{E}(t)$



$\Delta I_{A,B}(t)$



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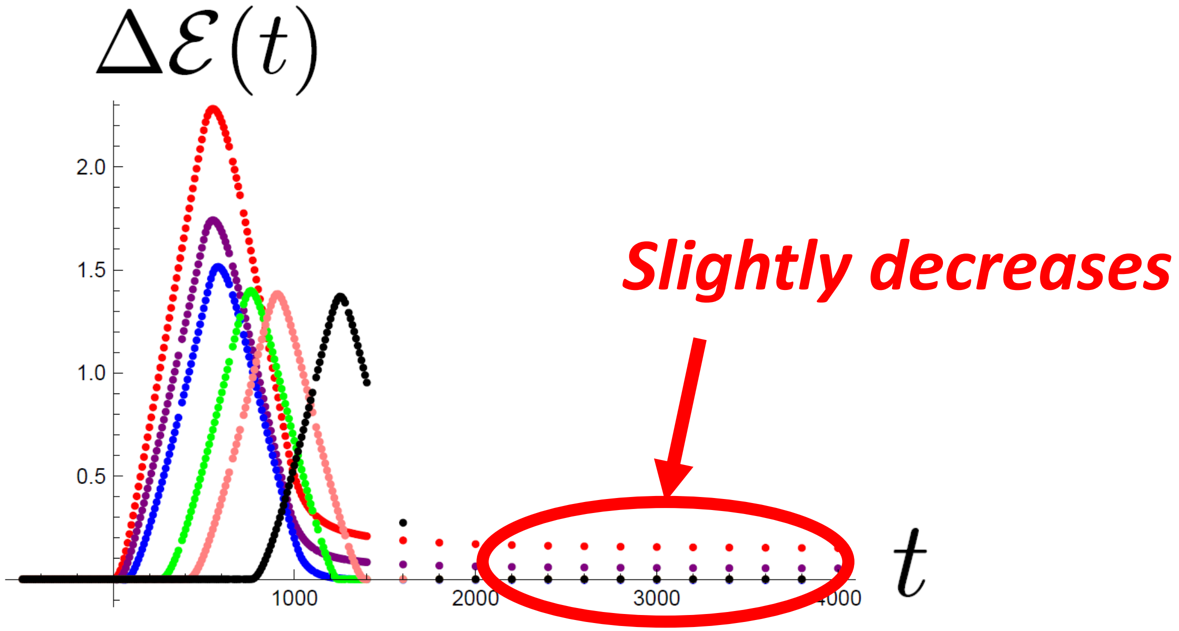

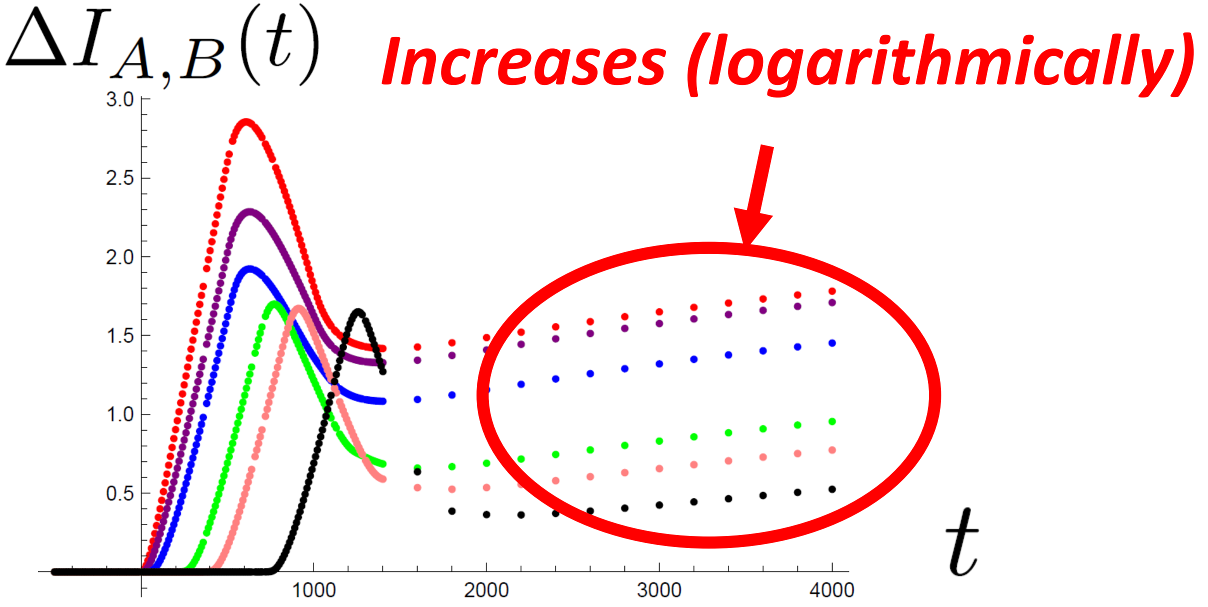

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$\Delta l = 0 :$ Red $\qquad \Delta l = 10 :$ Purple

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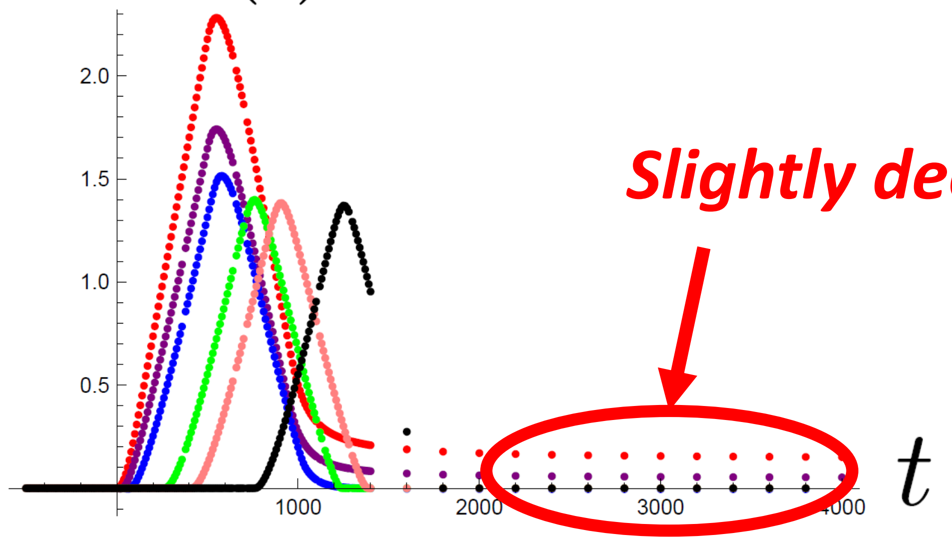
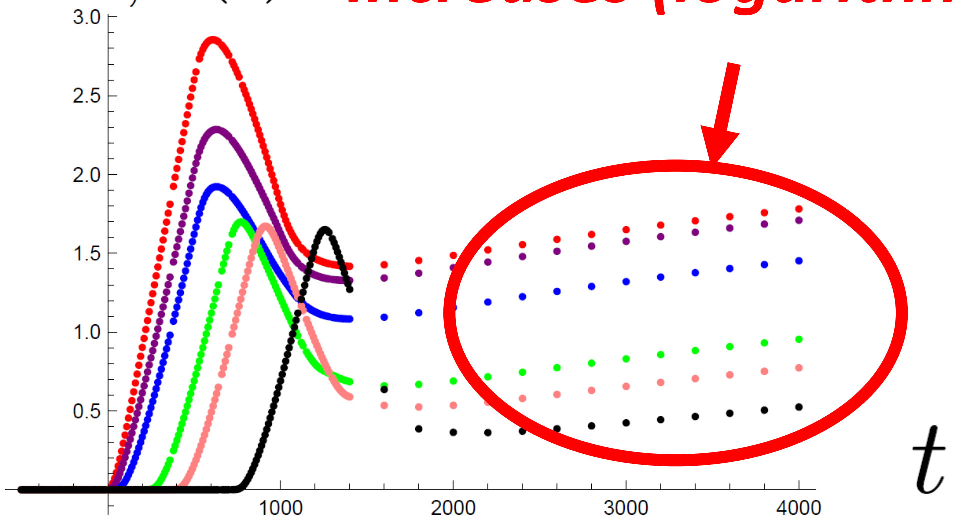
Δ

Slow mode contributes to late-time behavior.

$\Delta\mathcal{E}(t)$



$\Delta I_{A,B}(t)$



Time evolution of MI and LN in Fast-ECP

Parameter : $(\xi, \delta t, l_a, l_b) = (100, 5, 1000, 1000)$

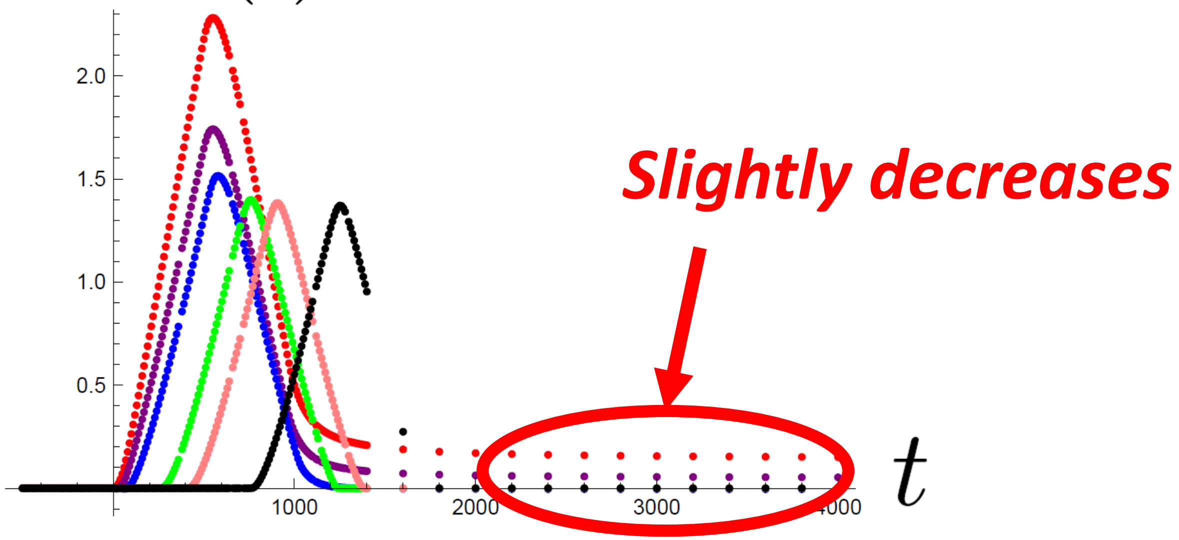
$\Delta l = 0 :$ **Red** $\Delta l = 10 :$ **Purple**

$\Delta l = 100 :$ **Blue** $\Delta l = 500 :$ **Green**

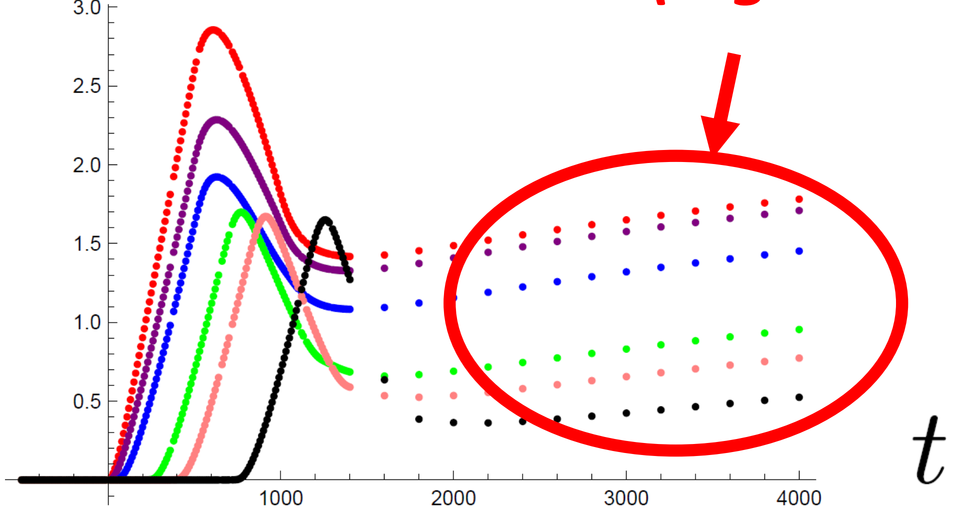
$\Delta l =$

LN is independent of slow mode.

$\Delta \mathcal{E}(t)$



$\Delta I_{A,B}(t)$ **Increases (logarithmically)**



Time evolution of MI and LN in Fast-ECP

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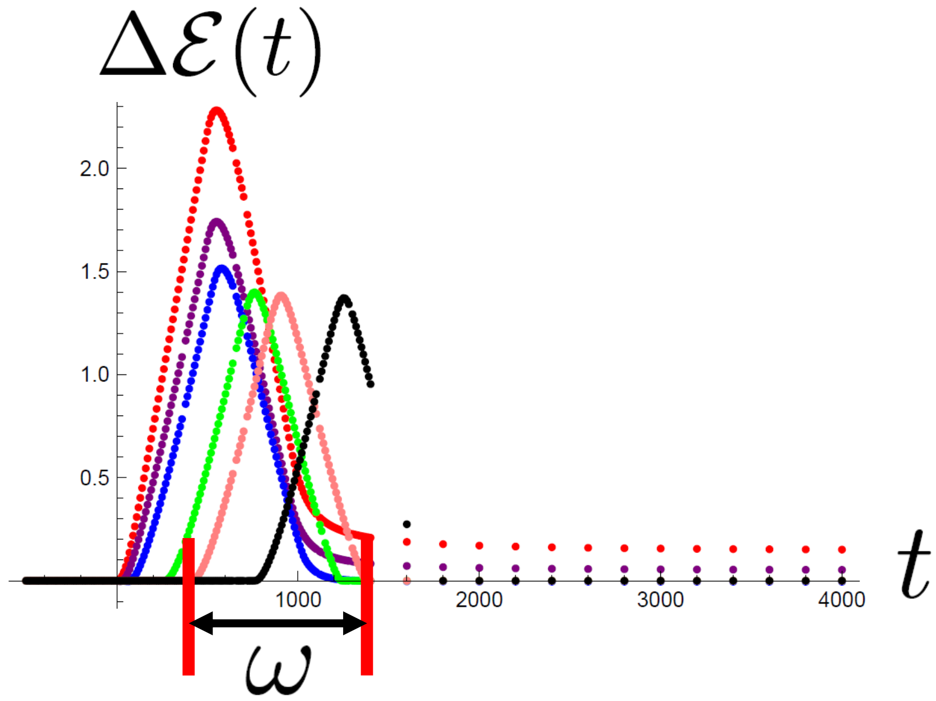
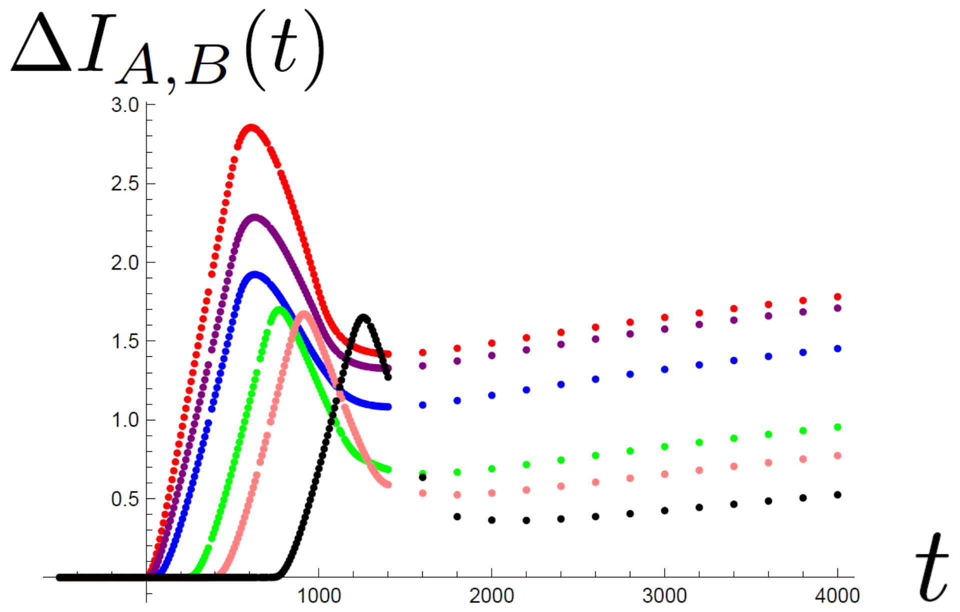


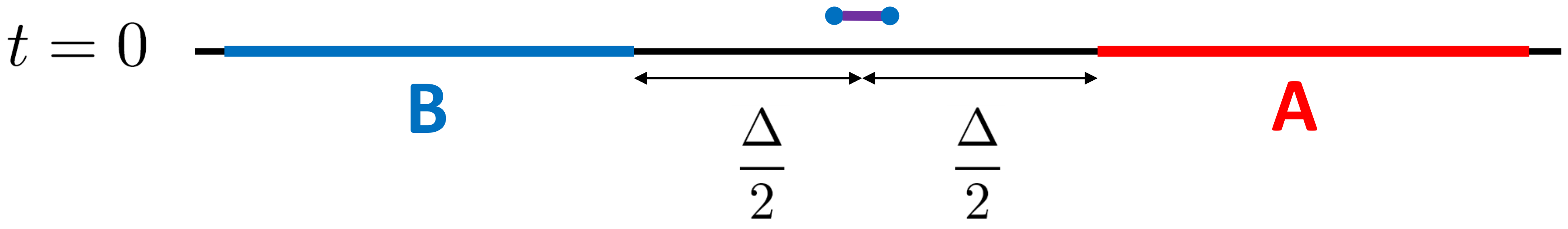


$\Delta\mathcal{E}(t)$ and $\Delta I_{A,B}(t)$ for $\Delta l \gg \xi$ ***increase*** after $t = \frac{\Delta}{2}$.

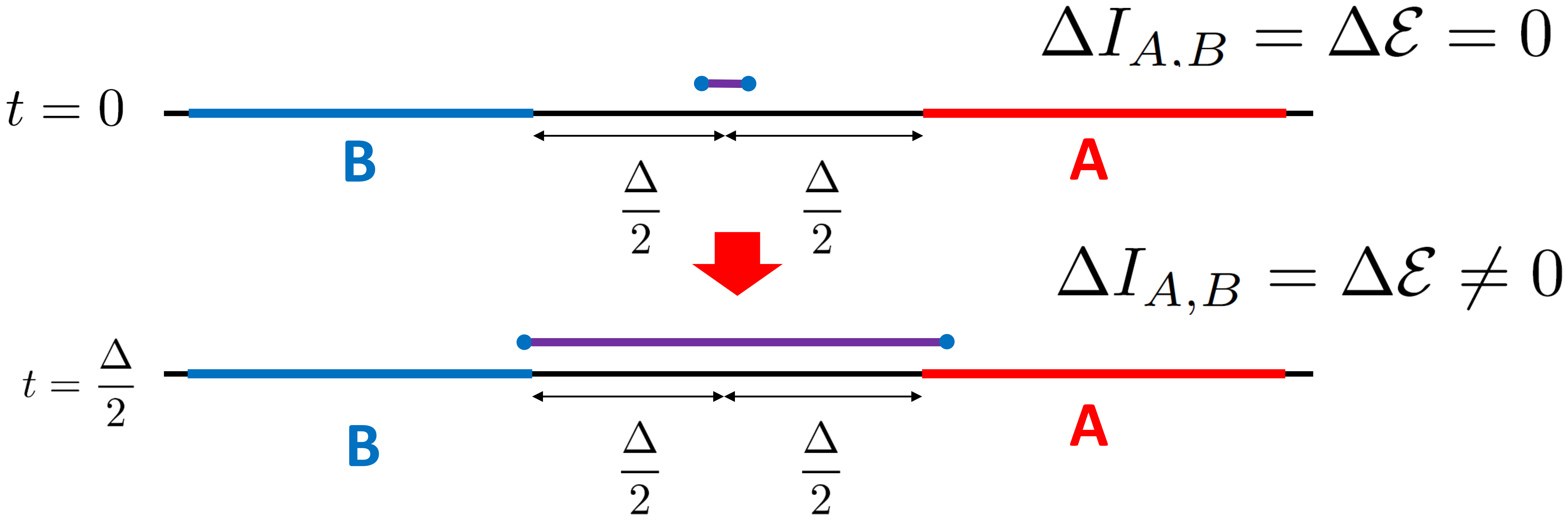
The width of wavelet is $\omega = l_a$

Quasi-particle picture

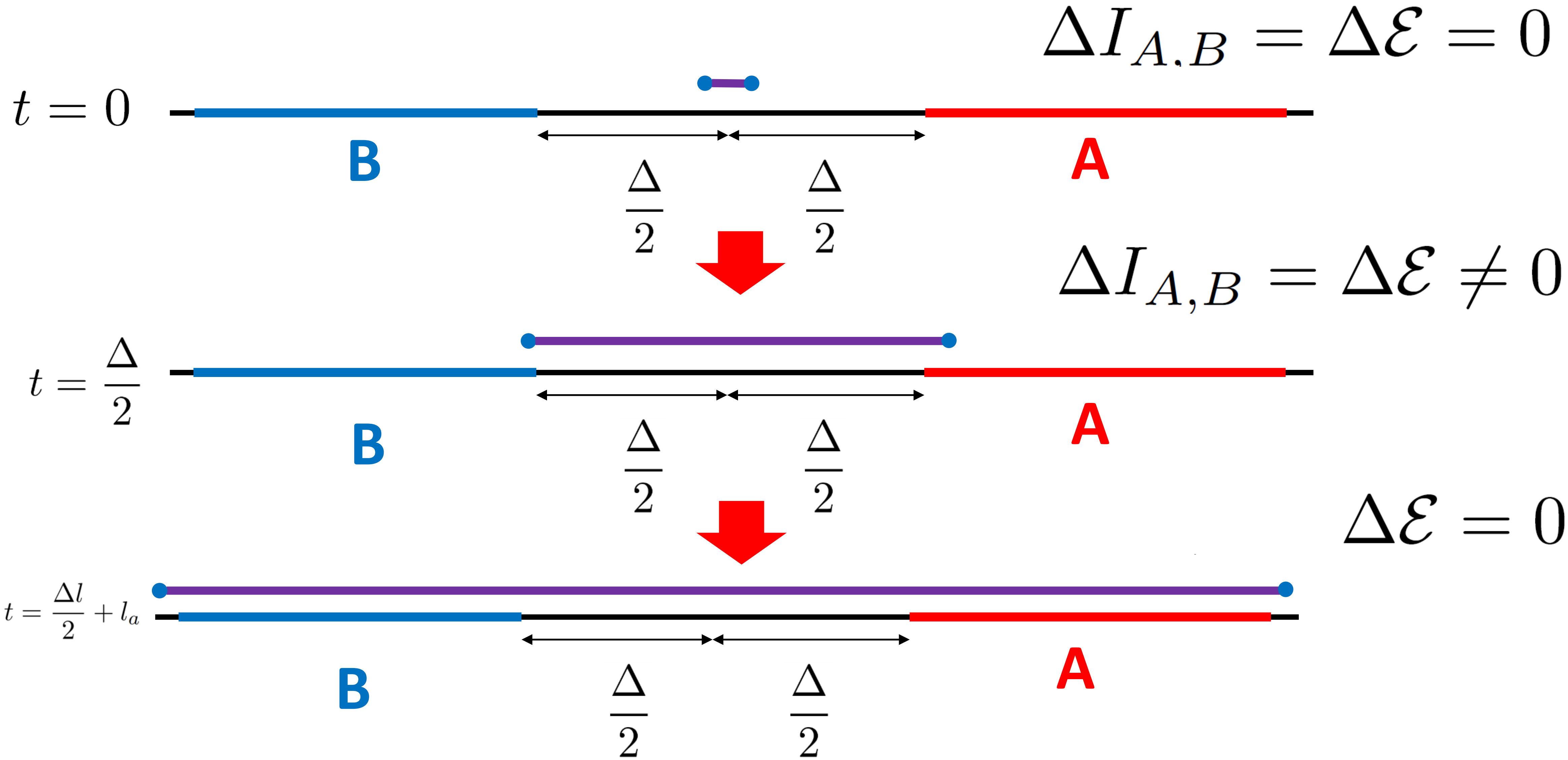
$$\Delta I_{A,B} = \Delta \mathcal{E} = 0$$



Quasi-particle picture



Quasi-particle picture



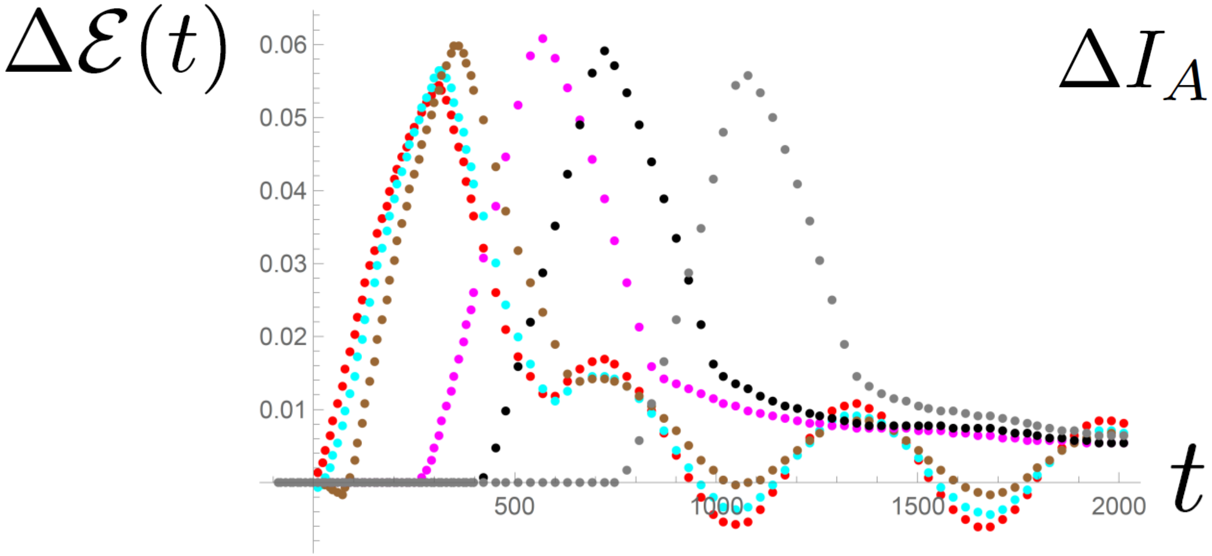
Time evolution of MI and LN in Fast-CCP

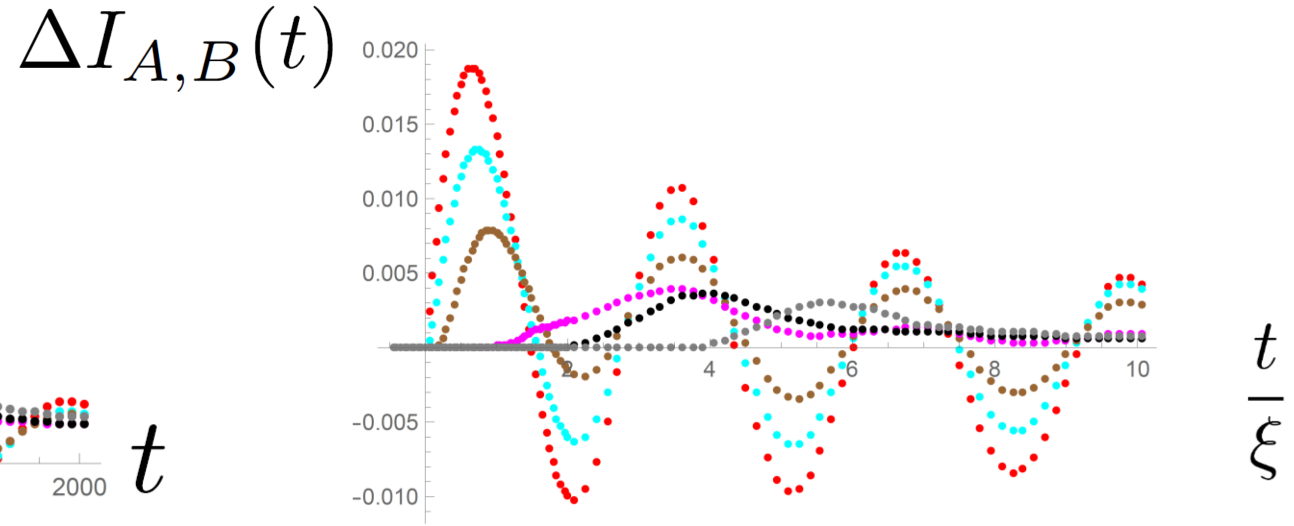
Parameter : $(\xi, \delta t, l_a, l_b) = (200, 5, 600, 600)$

$\Delta l = 0:$ **Red** $\Delta l = 10:$ **Cyan**

$\Delta l = 100:$ **Brown** $\Delta l = 500:$ **Magenta**

$\Delta l = 800:$ **Black** $\Delta l = 1500:$ **Cray**





Time evolution of MI and LN in Fast-CCP

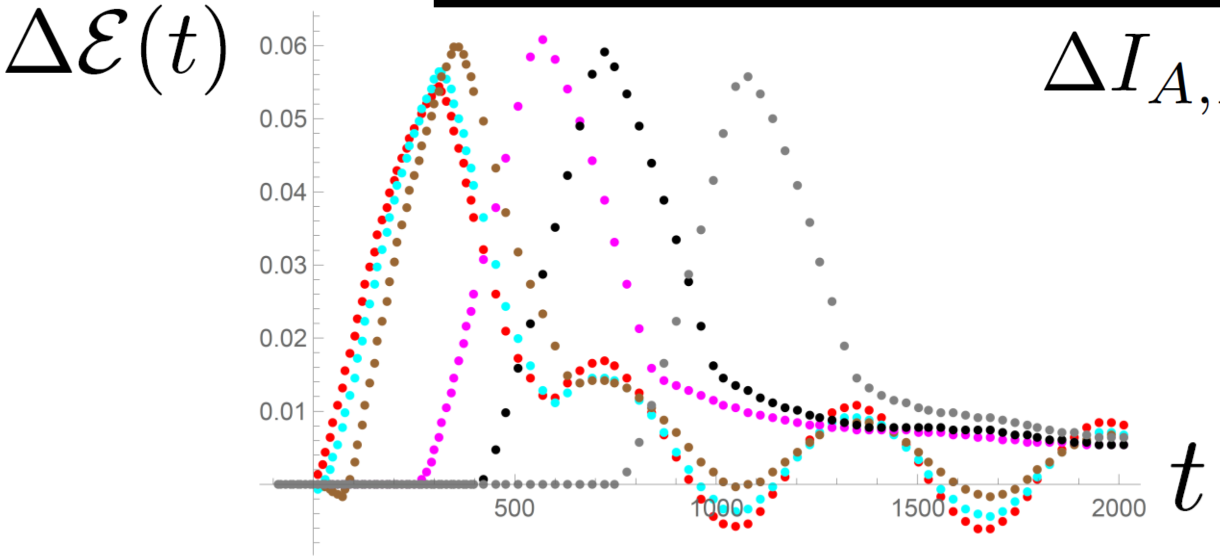
Parameter : $(\xi, \delta t, l_a, l_b) = (200, 5, 600, 600)$

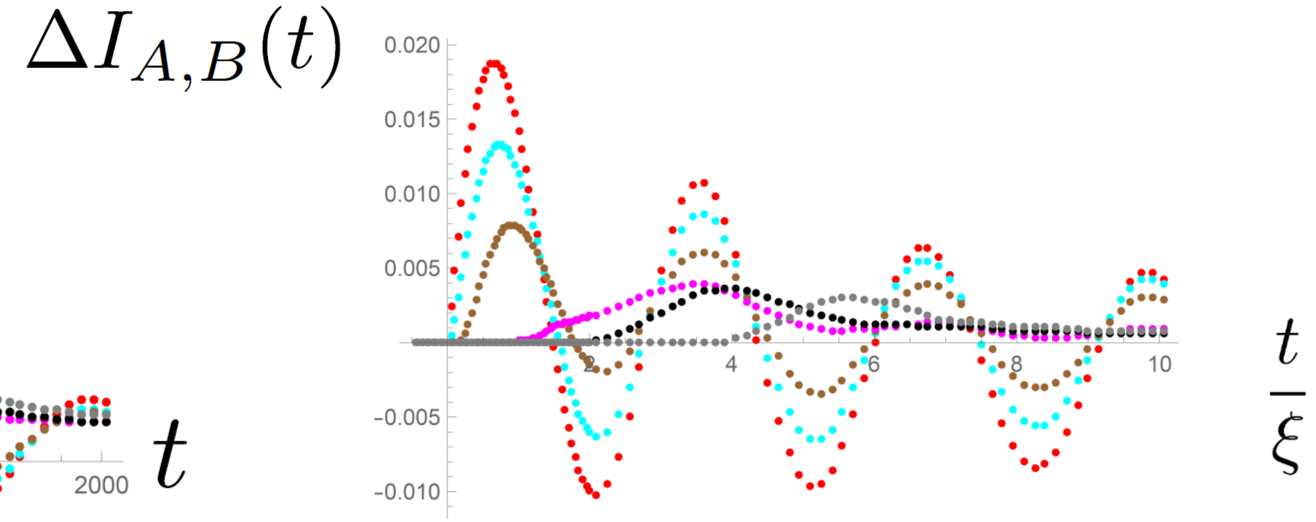
$\Delta l = 0 :$

$\Delta l = 100$ … enta

$\Delta l = 80$

Periodicity = $\pi\xi$





Time evolution of MI and LN in Fast-CCP

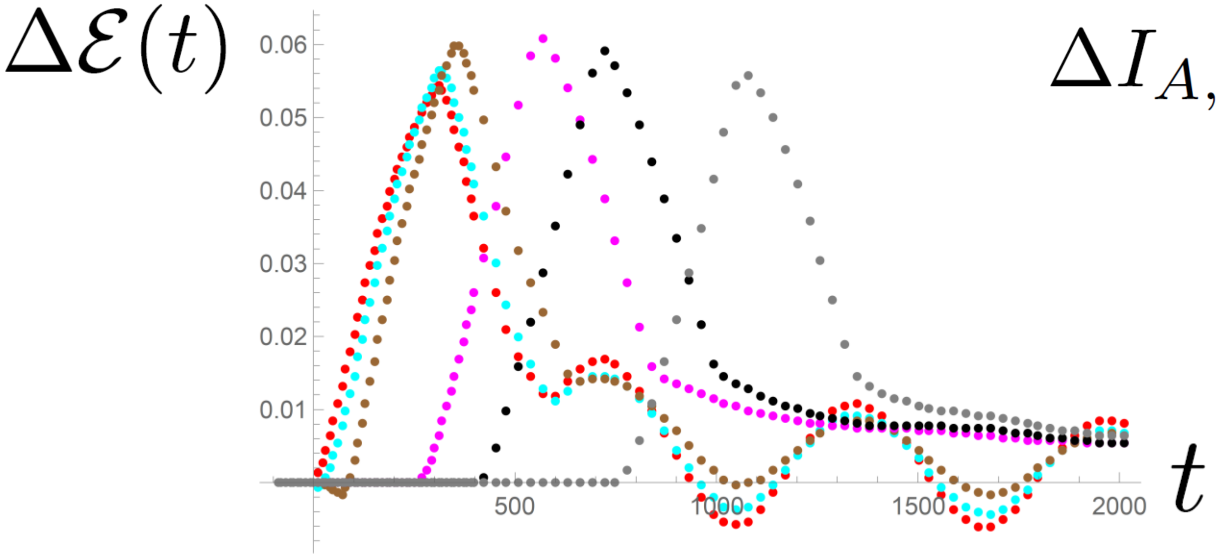
Parameter : $(\xi, \delta t, l_a, l_b) = (200, 5, 600, 600)$

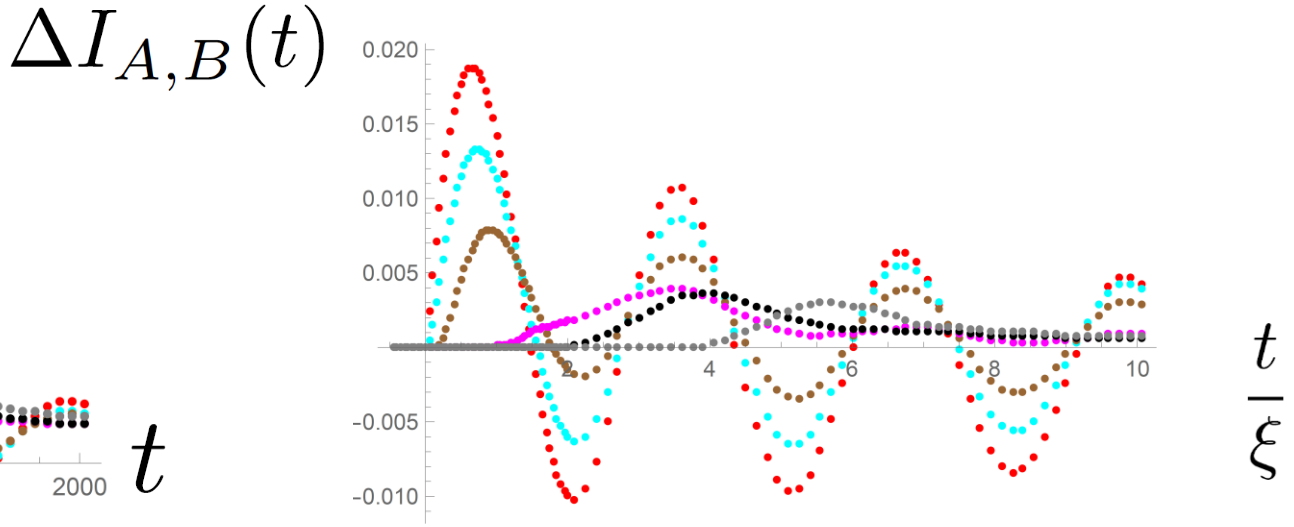
$\Delta l = 0:$ **Red** $\Delta l = 10:$ **Cyan**

$\Delta l = 10$ ... **enta**

$\Delta l = 80$ ...

$\Delta l \gg \xi$, ***oscillation vanishes.***



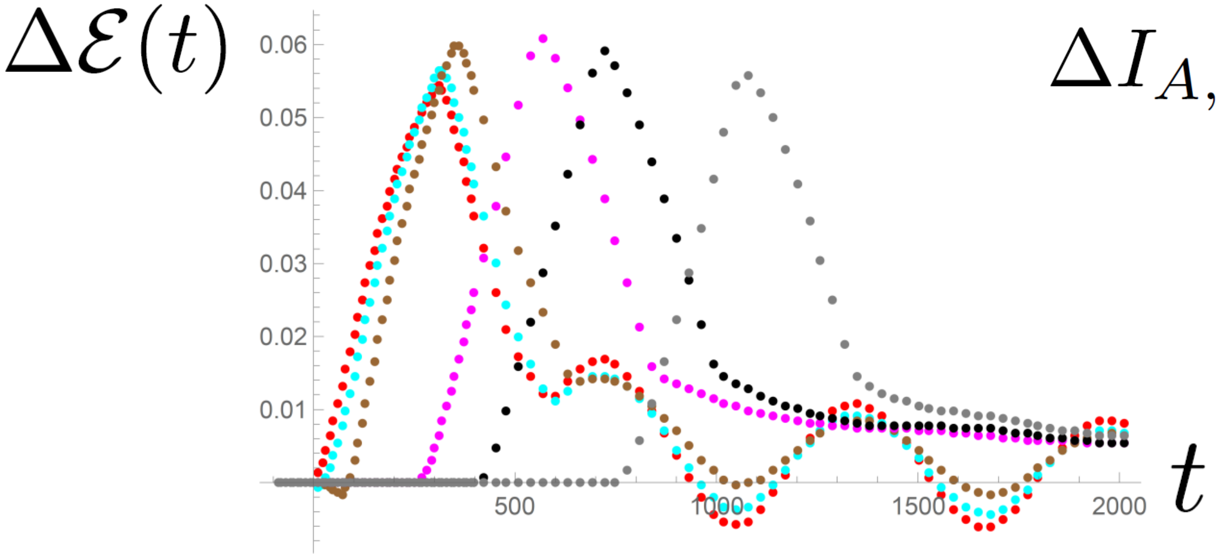


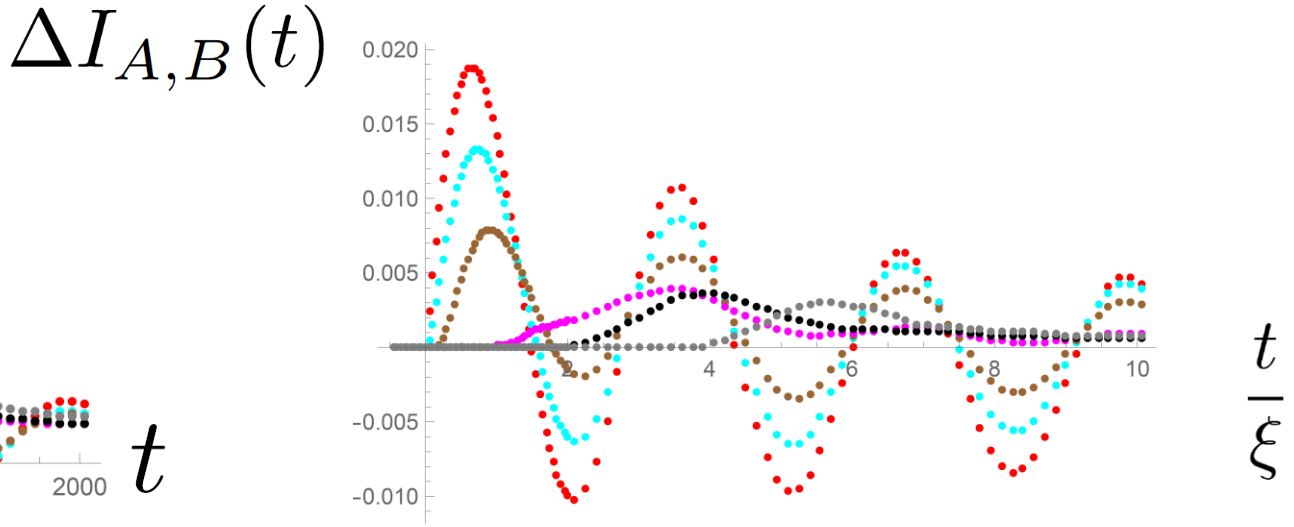
Time evolution of MI and LN in Fast-CCP

Parameter : $(\xi, \delta t, l_a, l_b) = (200, 5, 600, 600)$

$\Delta l = 0$: **Red** $\Delta l = 10$: **Cyan**

Short-range entanglement → Oscillation





Summary and Future directions

Summary

- We study what makes quasi-particles.

In slow ECP, adiabaticity-breaking plays an important role.

- Scaling of EE depend on scales
  when adiabaticity breaks down.

- Late time behavior depends on slow mode (zero mode).

Future directions

- Why does change of EE oscillate after $t = 2\xi_{kz}$, $t = 2\xi$?
- Interacting theories
- Holographic Dual
- Floquet type potential

Thank you for your attention