Correlators in higher spin AdS_3 holography with loop corrections

Takahiro Uetoko (Ritsumeikan University)

Based on: Universe 3 (2017) no.4, 70 [arXiv:1708.02017]
Prog. Theor. Exp. Phys. (2017) 113B03 [arXiv:1708.08657]
[arXiv:1801.08549]
with Yasuaki Hikida (YITP, Kyoto University)

Mar 16th (2018)@YITP
“Holography, Quantum Entanglement and Higher spin Gravity II”

Introduction and our results

- Higher spin gauge theory is simple holography
  - Weak/weak duality

    [Gaberdiel-Gopakumar '10]
  - Gaberdiel-Gopakumar conjecture (for 't Hooft limit)

3d Vasiliev theory ⟷ 2d large N minimal model

- Our aim is to understand quantum aspects in this conjecture

1st		⟵ expect	$1/N$ corrections
2nd	loop corrections (for semiclassical limit)	reproduce ⟶	$1/c$ corrections

Introduction and our results

1st paper analysis

- We develop a new method to compute $\langle \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} J^{(s)} \rangle$ in large *N* minimal model
  - New results at the next non-trivial order in 1/*N* expansion

2nd paper analysis

- We propose a new regularization prescription in higher spin gravity with the symmetry of dual CFT by utilizing open Wilson lines
  - We reproduce $\langle \mathcal{O}_h \bar{\mathcal{O}}_h \rangle$ from bulk gravity up to two loop order

Plan of talk

1. Introduction and our results
2. 1st paper analysis: $1/N$ corrections
3. 2nd paper analysis: loop corrections
4. Conclusion

Plan of talk

1. Introduction and our results
2. 1st paper analysis: 1/*N* corrections
3. 2nd paper analysis: loop corrections
4. Conclusion

1st paper analysis: 1/N corrections

- W_N minimal model

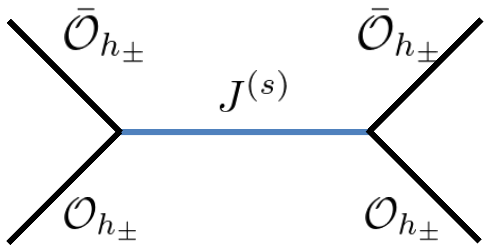
$$\frac{\mathrm{su}(N)_k \oplus \mathrm{su}(N)_1}{\mathrm{su}(N)_{k+1}}$$

  - Scalar operator: $\mathcal{O}_h$
  - Higher spin current: $J^{(s)}$ $\quad s = 2, 3, \ldots, N$
  - The spectrum of primary states are labeled as $(\Lambda_+; \Lambda_-)$

    We focus on $h_+ \equiv h(\mathrm{f}; 0)$

- Our method :

  Conformal block expansion



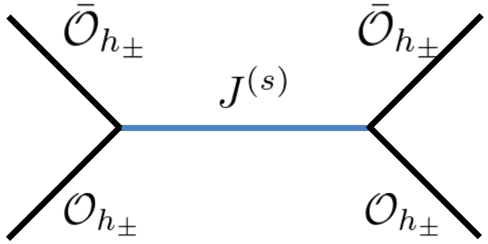
$$\langle \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} \rangle \sim \langle \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} J^{(s)} \rangle^2$$

1st paper analysis: 1/N corrections

We'd like to read off the information of quantum corrections of three point functions!!

- Our method :

  Conformal block expansion



$$\langle \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} \rangle \sim \langle \mathcal{O}_{h_\pm} \bar{\mathcal{O}}_{h_\pm} J^{(s)} \rangle^2$$

1st paper analysis: 1/N corrections

- Operator product expansion

Include the information of three point function

$$\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) = \sum_p \frac{C_{12p}}{z_{12}^{h_1+h_2-h_p}\bar{z}_{12}^{h_1+h_2-\bar{h}_p}}\mathcal{A}_p(z_2) + \cdots$$

- Comparison of the both side by 1/N order

$$\langle \mathcal{O}_1(\infty)\mathcal{O}_2(1)\mathcal{O}_3(z)\mathcal{O}_4(0)\rangle = \sum_p \frac{C_{12p}C_{34p}}{|z|^{2(h_3+h_4)}}\mathcal{F}(c,h_i,h_p,z)\bar{\mathcal{F}}(c,h_i,\bar{h}_p,\bar{z})$$

Coulomb gas method

[Papadodimas-Raju '12]

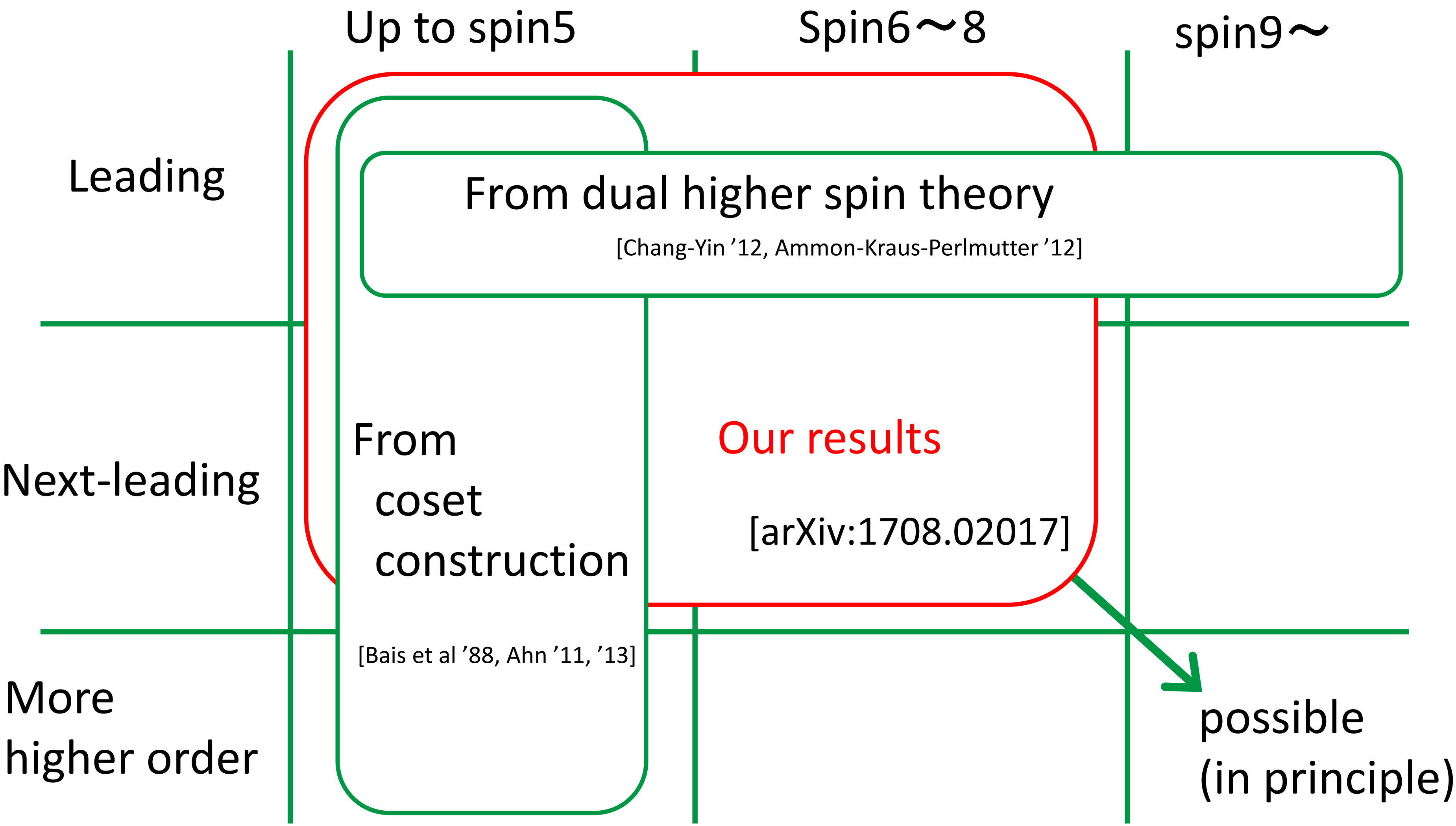
Zamolodchikov's recursion relation

[Zamolodchikov '84, Perlmutter, Beccaria-Fachechi-Macorini '15]

- We evaluate 1/N corrections of 3pt function only up to spin8

1st paper analysis: 1/N corrections

- Known results and our results



Plan of talk

1. Introduction and our results
2. 1st paper analysis: 1/*N* corrections
3. 2nd paper analysis: loop corrections
4. Conclusion

2nd paper analysis: loop corrections

- Semi classical limit of Gaberdiel-Gopakumar duality

[Castro-Gopakumar-Gutperle-Raeymaekers '11, Gaberdiel-Gopakumar, Perlmutter-Prochazka-Raeymaekers '12]

3d sl(N) Chern-Simons theory with matters	⟷	2d W_N minimal model with large c but finite N

- Non-unitary regime with $k = -1 - N + \frac{N(N^2-1)}{c} + \cdots$

- Two point function is fixed by the symmetry as

$$\langle \mathcal{O}_h(z) \bar{\mathcal{O}}_h(0) \rangle = \frac{1}{z^{2h}}$$

- Conformal weight can be obtained by coset construction

$$h = h_0 + \frac{1}{c} h_1 + \frac{1}{c^2} h_2 + \mathcal{O}(c^{-3})$$

$$\langle \mathcal{O}_h(z) \bar{\mathcal{O}}_h(0) \rangle = \frac{1}{z^{2h_0}} \left[1 - \frac{1}{c} 2h_1 \log(z) + \frac{1}{c^2} (2h_1^2 \log^2(z) - 2h_2 \log(z)) \right] + \mathcal{O}(c^{-3})$$

2nd paper analysis: loop corrections

We'd like to obtain the information of quantum corrections of two point functions from the bulk gravity!!

- Two point function is fixed by the symmetry as

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \frac{1}{z^{2h}}$$

  - Conformal weight can be obtained by coset construction

$$h = h_0 + \frac{1}{c}h_1 + \frac{1}{c^2}h_2 + \mathcal{O}(c^{-3})$$

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \frac{1}{z^{2h_0}}\left[1 - \frac{1}{c}2h_1\log(z) + \frac{1}{c^2}(2h_1^2\log^2(z) - 2h_2\log(z))\right] + \mathcal{O}(c^{-3})$$

2nd paper analysis: loop corrections

- Open Wilson line method
  - Wilson line operator

$$W(z) = P\exp\left(\int_{z_i}^{z_f} dz a(z)\right) \qquad (A = e^{-\rho L_0} a(z) e^{\rho L_0} dz + L_0 d\rho)$$

  - Asymptotic AdS condition restricts the form of gauge field

$$a(z) = V_1^2 - \frac{6}{c}\sum_{s=2}^{N}\frac{1}{N_s}J^{(s)}(z)V_{s-1}^{s}$$

  - The expectation value of Wilson line computes the two and three point functions

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \langle W_{h_0}(z)\rangle, \;\; \langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)J^{(s)}(y)\rangle = \langle W_{h_0}(z)J^{(s)}(y)\rangle$$

  - Open Wilson line corresponds to particle running in the bulk

2nd paper analysis: loop corrections

- Prescription for regularization

Scale invariance is not broken

  - Regulator

$$\langle J^{(s)}(z_2) J^{(s)}(z_1) \rangle = -\frac{(2s-1)cN_s}{6} \frac{1}{z_{21}^{2s}} \to -\frac{(2s-1)cN_s}{6} \frac{1}{z_{21}^{2s-2\epsilon}}$$

  - Shift parameters in open Wilson line

$$W_{h_0}(z) = P \exp \left[\int_0^z dz' (V_1^2 - \frac{6}{c} \sum_{s=2}^{N} \frac{c_s}{N_s} J^{(s)}(z') V_{-s+1}^s) \right]$$

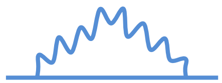
  - Divergences are absorbed order by order

$$c_s = 1 + \frac{1}{c} c_s^{(1)} + \frac{1}{c^2} c_s^{(2)} + \mathcal{O}(c^{-3})$$

→ We fixed the finite parts of c_s

2nd paper analysis: loop corrections

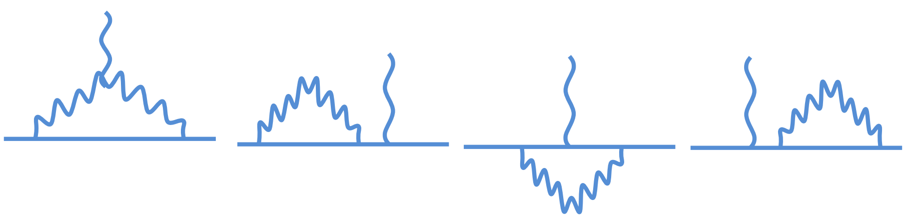
- Ambiguity to choose c_s (N=2,3)
  - 1-loop corrections of 2pt functions



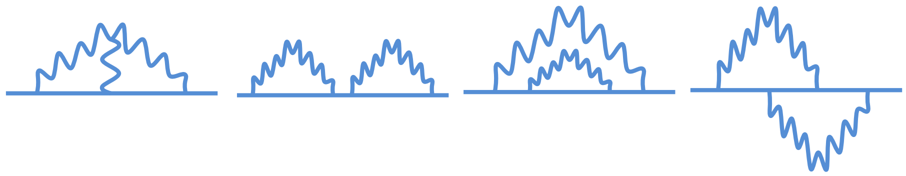
$$c_s = 1 + \mathcal{O}(c^{-1})$$

renormalize overall factor

  - 1-loop corrections of 3pt functions



  - 2-loop corrections of 2pt functions



$$c_s = 1 + \frac{1}{c} c_s^{(1)} + \mathcal{O}(c^{-2})$$

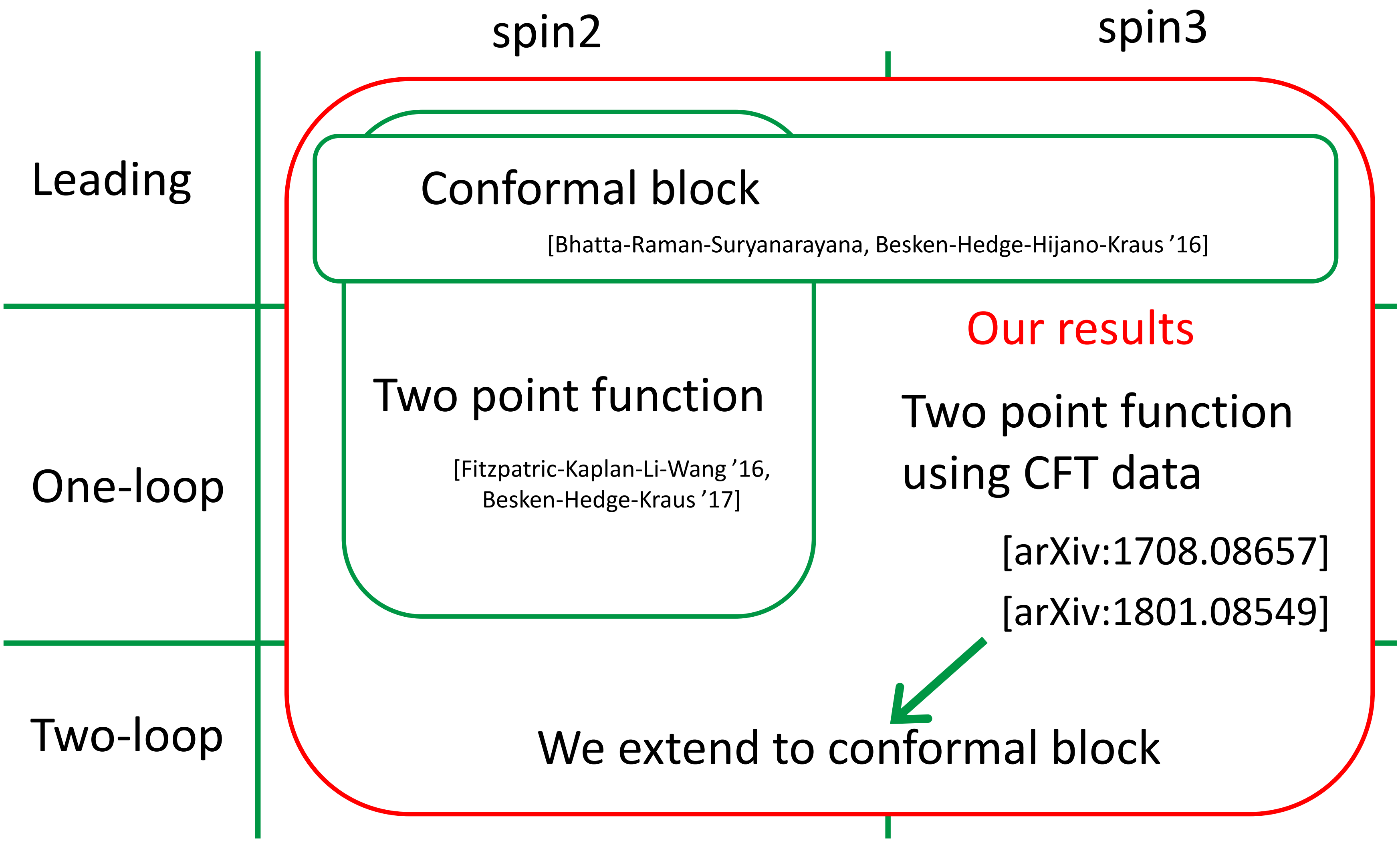
3pt function is fixed
by conformal Ward identity

fix $c_s^{(1)}$
+ renormalize overall factor

- $1/c^2$ corrections of 2pt functions are reproduced using CFT data

2nd paper analysis: loop corrections

- Known results and our results from open Wilson line



Plan of talk

1. Introduction and our results
2. 1st paper analysis: $1/N$ corrections
3. 2nd paper analysis: loop corrections
4. Conclusion

Conclusion

- We develop a simple and systematic method to calculate 3pt functions in W_N minimal model

- We propose a new regularization prescription in sl(N) Chern-Simons theory with dual CFT data

- Futurework
  - A way to determine the interaction parameter without referring to explicit boundary data
  - Supersymmetric case (in progress)

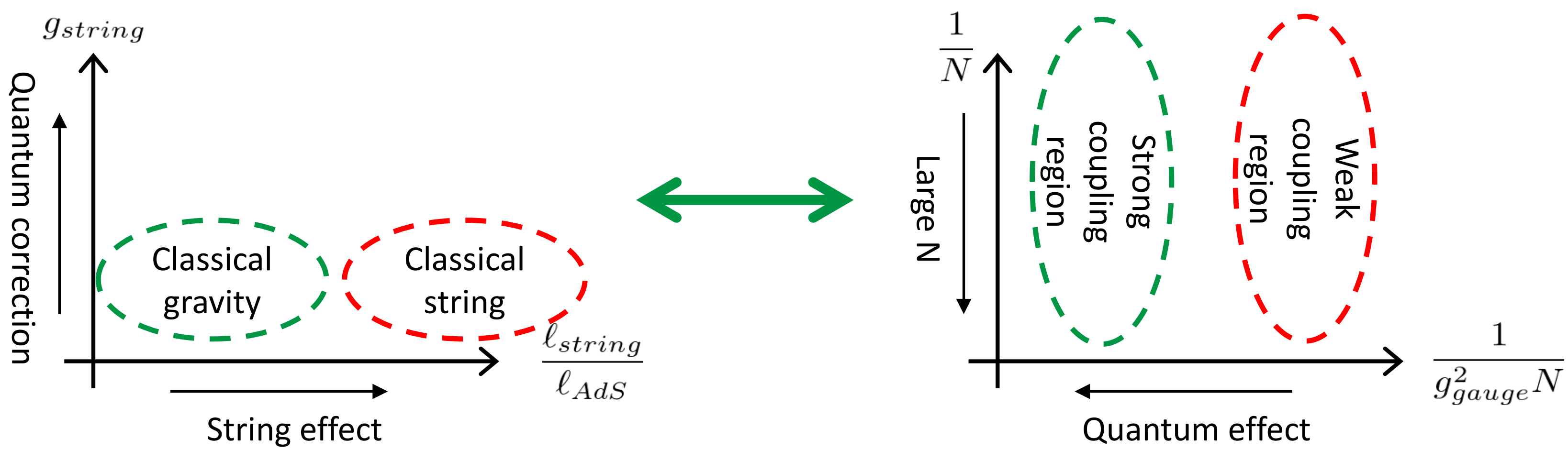
Back up slides

Higher spin and string

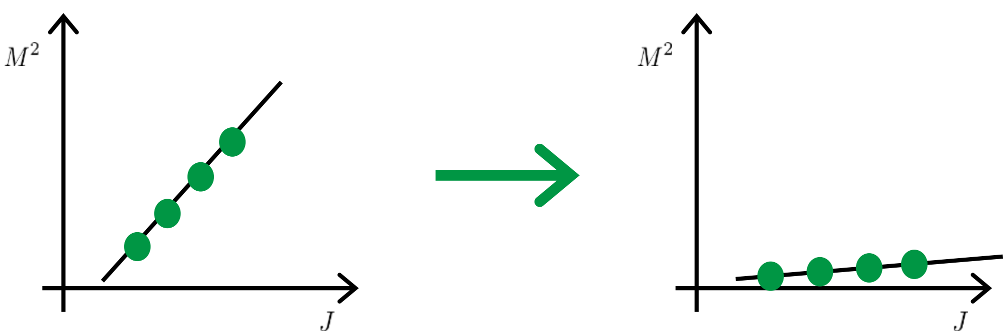
- AdS/CFT correspondence

Superstring theory on AdS

Gauge theory



- Tension less limit → Massless higher spin



Weak/weak duality!

Higher spin AdS_3 holography

- The relation of parameters

$$c = (N-1)\left(1 - \frac{N(N+1)}{(N+k)(N+k+1)}\right) \qquad \lambda = \frac{N}{N+k}$$

- 't Hooft limit, $1/N$ expansion is almost same ac $1/c$ expansion

$$c \sim N(1-\lambda^2) + \mathcal{O}(N^0)$$

- Semi classical limit, N is finite

$$k = -1 - N + \frac{N(N^2-1)}{c} + \cdots$$

Higher spin AdS_3 holography

- Triality relation [Gaberdiel-Gopakumar '12]

Sl(N) CS theory

Vasiliev theory

Drinferd-Sokolov reduction

$$W_\infty[N] \simeq W_\infty\left[\tfrac{N}{N+k}\right] \simeq W_\infty\left[-\tfrac{N}{N+k+1}\right]$$

Triality relation