Scrambling and Relative entropy

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Holography quantum entanglement and higher spin gravity ||

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Scrambling

Scrambling refers to phenomena of quick delocalization of quantum information in thermal states. [Sekino Susskind] [Lashkari Stanford et al]….

The typical time scale of these phenomena is the scrambling time.

They have been diagnosed by the growth of a square of commutator.

[Stanford Shenker] [Kitaev]….

$$C(t) = \langle [W(t), V(0)]^2 \rangle_\beta \sim e^{\lambda t}$$

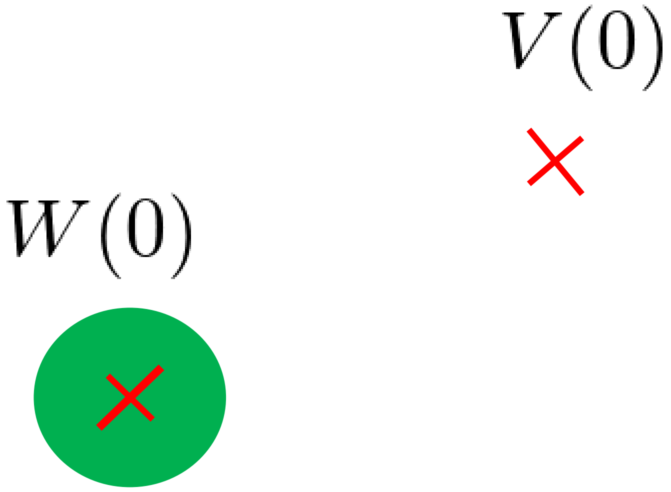
The commutator measures the size of the operator W(t) which was initially localized.

Scrambling(2)

The growth of the size of local operators means that, after the scrambling time, the thermal RDM ρ_β and its perturbation $W^\dagger(t)\rho_\beta W(t)$ become indistinguishable in any local subregion A due to the delocalization of W(t).

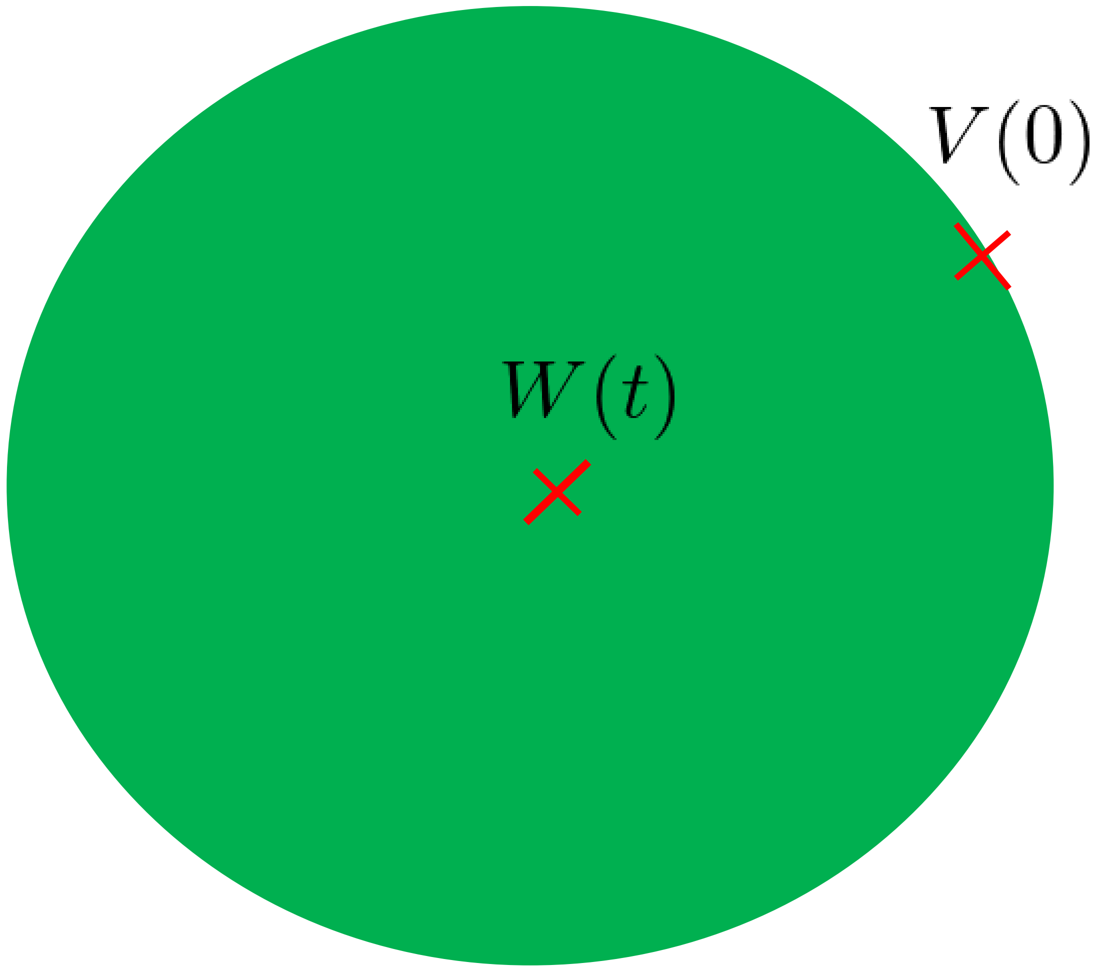
In other words, quantum information of W(t) is scrambled, and spread over the total system.

T=0



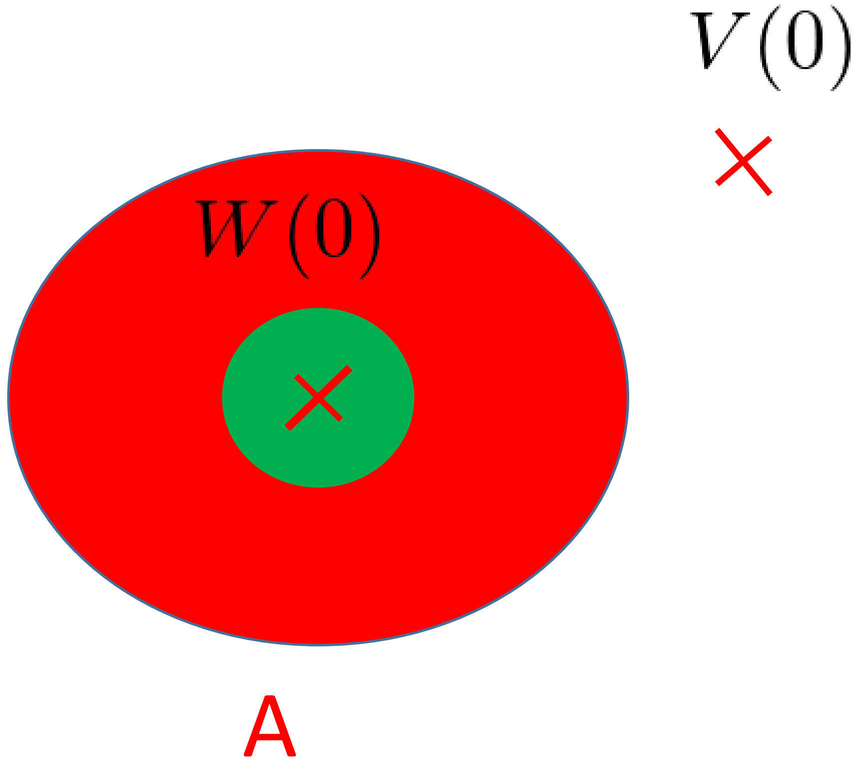
$$[W(0), V(0)] = 0$$

T=t



$$[W(t), V(0)] = e^{\lambda t}$$

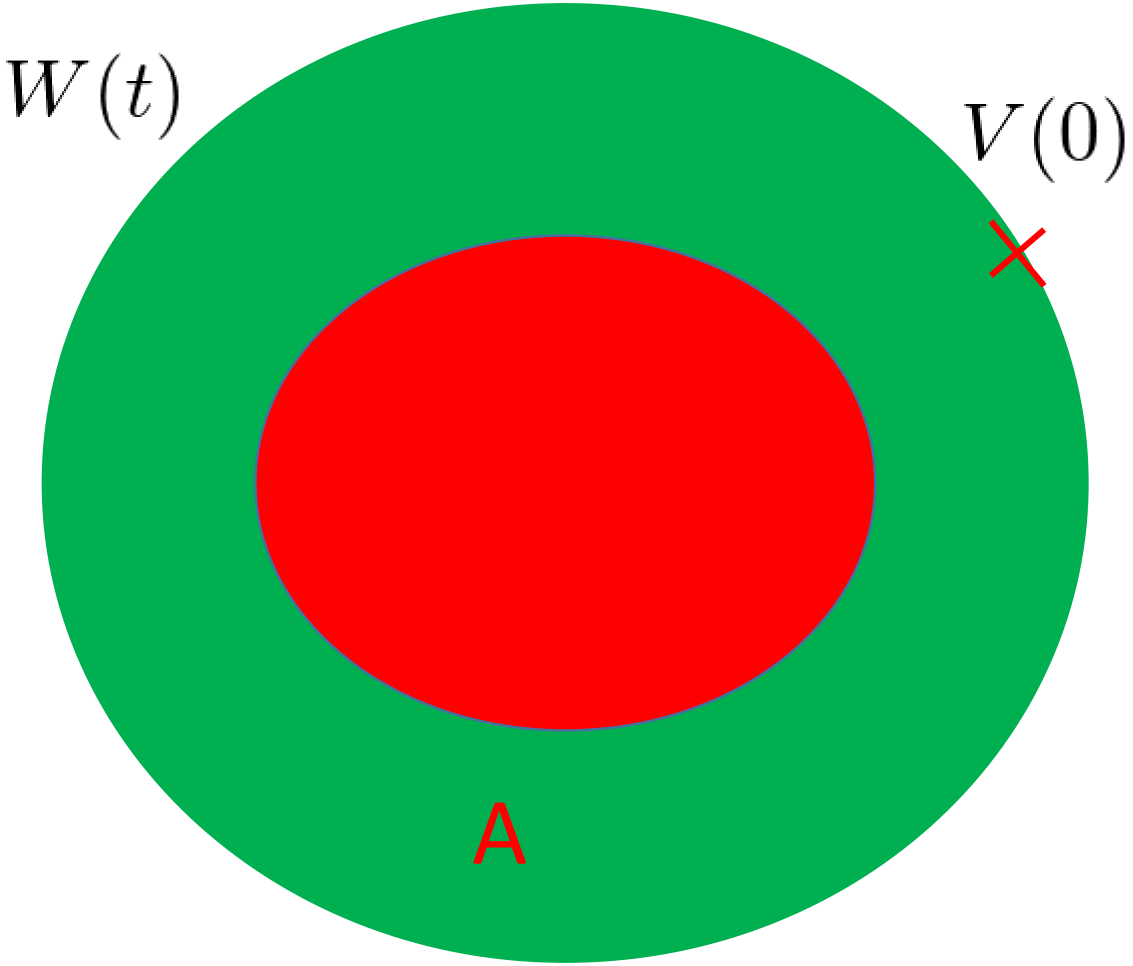
T=0



$$\rho_\beta \neq W(t)\rho_\beta W^\dagger(t)$$

On the subregion A

T=t



$$\rho_\beta \sim W(t)\rho_\beta W^\dagger(t)$$

On the subregion A

Scrambling(2)

The distance between the thermal RDM ρ_β and its perturbation $W^\dagger(t)\rho_\beta W(t)$ is suitable to characterize the scrambling.

The decay rate of the distance characterizes the chaotic nature of the system.

The distance between two density matrices is measured by relative entropy.

Motivated by this observation, we studied the dynamics of scrambling using the relative entropy in 2d CFT as well as several spin chain models.

Relative entropy

- Relative entropy between two density matrices is

$$S(\rho||\sigma) = \mathrm{tr}\left(\rho \log \rho\right) - \mathrm{tr}\left(\rho \log \sigma\right)$$

- This quantity measures the distance between the two, therefore one can use this to see whether they are distinguishable or not.

A quantum generalization of the KL divergence.

Modular Hamiltonian

- In some sense relative entropy is a generalization of free energy.

$$\begin{aligned} S(\rho||\sigma) &= \mathrm{tr}\rho \log \rho - \mathrm{tr}\rho \log \sigma \\ &= [\langle \rho K_\sigma \rangle - \langle \sigma K_\sigma \rangle] - [S(\rho) - S(\sigma)] \\ &\equiv \langle \Delta K_\sigma \rangle - \Delta S \end{aligned}$$

$$K_\sigma = -\log \sigma \qquad S(\rho) = -\mathrm{tr}\rho \log \rho$$

K_σ is called modular Hamiltonian of σ .

When $\sigma = e^{-\beta H}$ the relative entropy indeed reduced to free energy.

Set up.

We start from a TFD state, and perturb it
By a local operator W(t).

$$|\Psi_1\rangle = \quad |TFD\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\frac{\beta}{2}E_n}|E_n\rangle_L|E_n\rangle_R$$

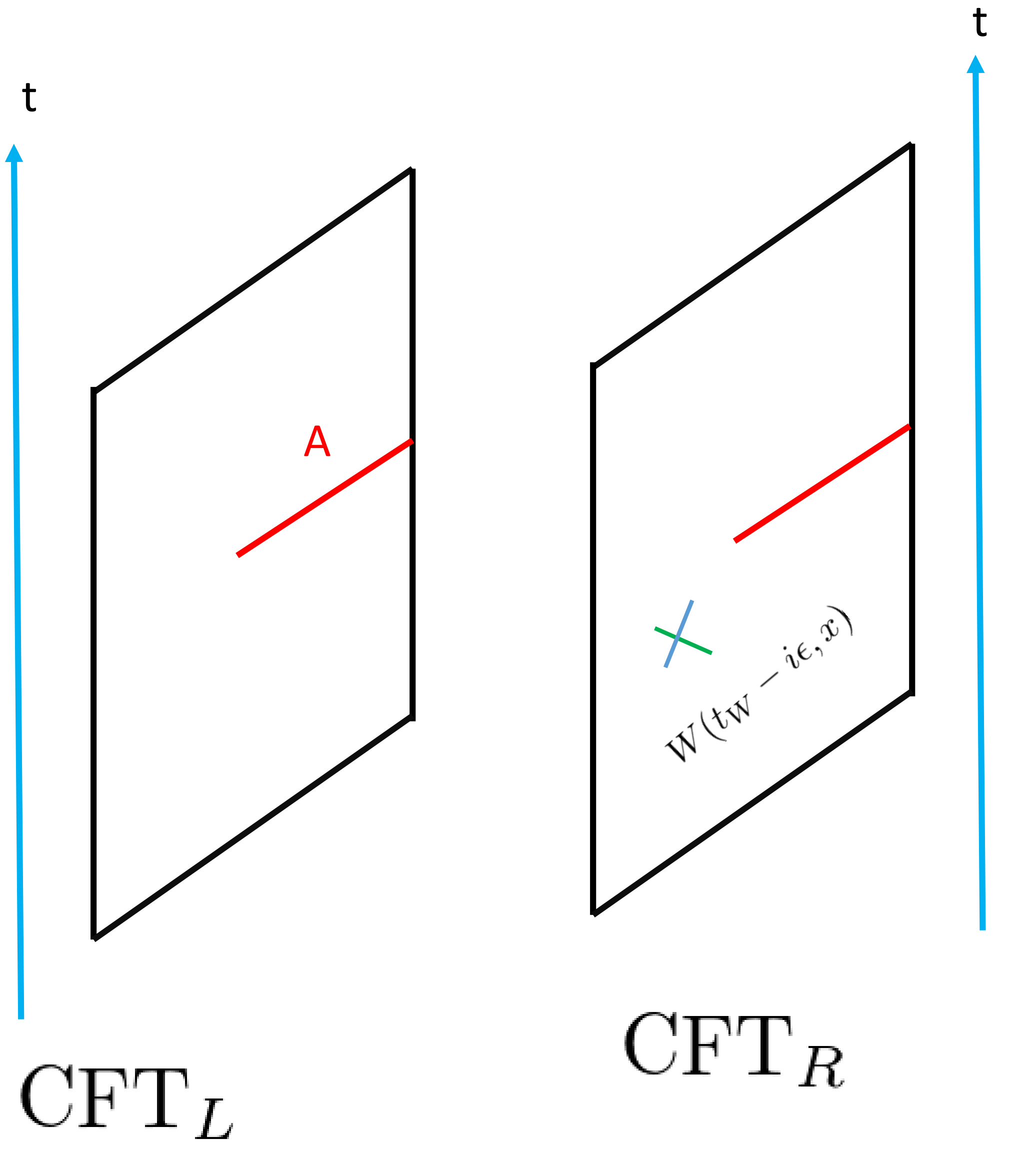
$$|\Psi_2\rangle = W(t_W - i\epsilon, x)|TFD\rangle$$

Evolve both CFTs forward in time.

Subsystem A: half space in both CFTs

Relative entropy between two RDMS.

$$S(\rho_1||\rho_2)$$



The relative entropy in the large C limit

In the large central charge limit, we can use the gravity description to calculate the relative entropy.

A TFD state with a local operator insertion $W(t_W - i\epsilon, x)|TFD\rangle$ has a simple gravity dual : a BTZ black hole with a shock wave.

$$ds^2 = -\frac{4}{(1+uv)^2}dudv + \left(\frac{1-uv}{1+uv}\right)^2 dy^2 + 4\delta(u)h(y)du^2$$

The entanglement entropy part can be easily computed by the RT formula.

This entropy has been computed in the context of local quench on thermal backgrounds.

Modular Hamiltonian

- The modular Hamiltonian in the set up is local, since it is related to the vacuum one via conformal map.

$$K_L = -\frac{\beta}{\pi}\int_C dy \frac{\cosh \pi\frac{y-t}{\beta}\sinh \pi\frac{t+y-L}{\beta}}{\cosh \pi\frac{L-2t}{\beta}} T(y)$$

In the expression we need to choose the correct contour C, which manifests the causality in the result.

This part is universal, in dependent of the CFT in question.

The relative entropy in the large C limit(2)

- When $t \gg \log \frac{\beta^2}{\epsilon}$ the relative entropy decays exponentially,

$$S(\rho_1||\rho_2) = 3c\left(\frac{h_W \pi}{c}\frac{e^{x+t_W}}{(\sin\epsilon)\cosh t}\right)^2$$

- The relative entropy become O(1) at the scrambling time. t=log c. After this, quantum corrections to the RT formula become important, in order to further follow the time evolution.

Some Numerics in spin chain models.

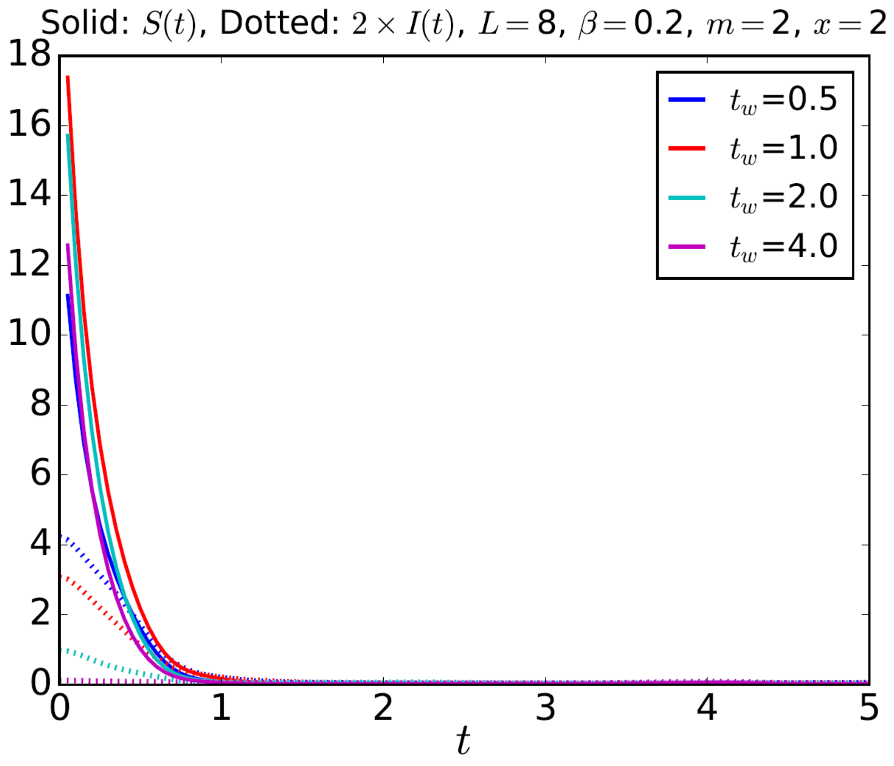
We also performed numerical calculation of the relative entropy in several spin chain models.

$$H = -\sum_{i=1}^{N} (Z_i Z_{i+1} + g X_i + h Z_i)$$

The Hamiltonian is integrable when $h = 0$.

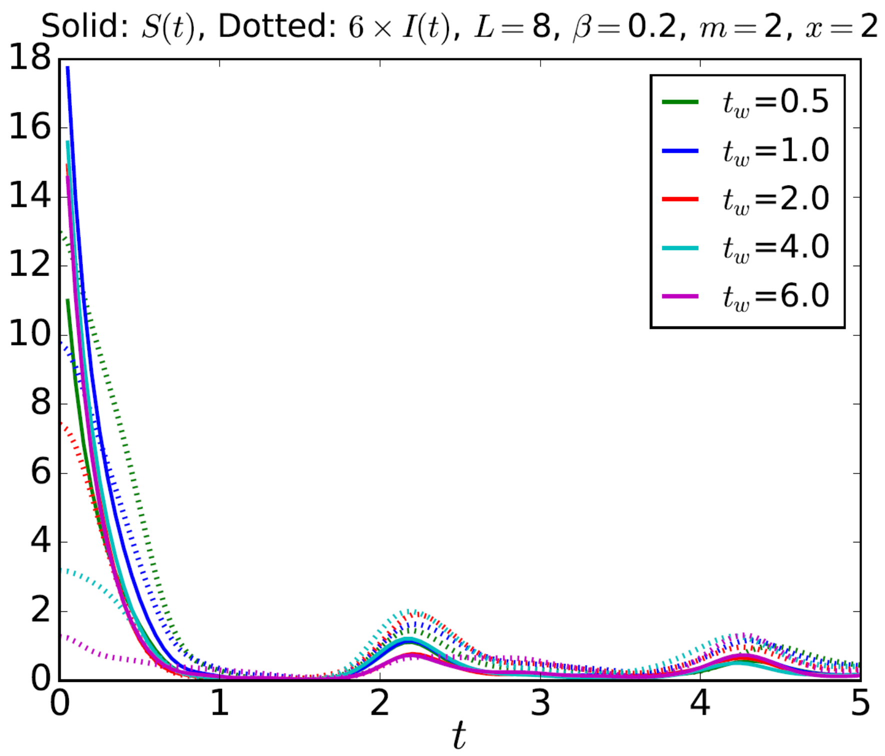
Mostly chaotic point (g,h)= (-1.05, 0.5).

Non integrable case



No oscillations in the late time regime

Integrable case



Oscillations in the late time regime

In non chaotic systems, we observe recursions of the relative entropy , however in chaotic systems, there are no recursions.

Conclusions

- I explained the application of relative entropy   to the physics of scrambling .

- Can we generalize the computation to higher dimensions?

- Probably the holographic relative entropy decays fastest. Can we show

This ?

 Can we show this is a general feature of black hole microstates or

There can be a meta stable one?

Thank you!