Flow equation, conformal symmetry and AdS geometries

YITP Workshop

Holography, Quantum Entanglement and Higher Spin Gravity II

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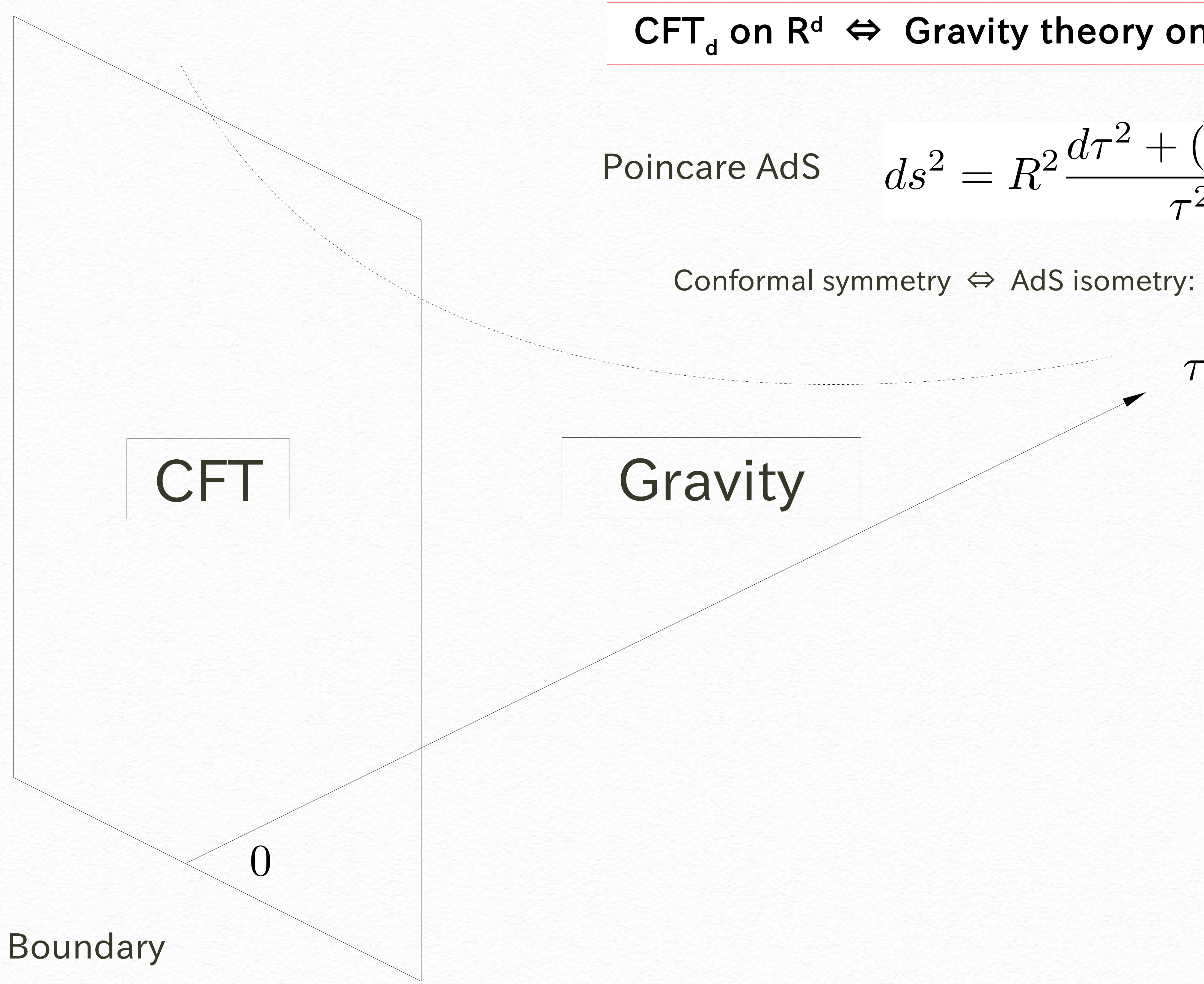
Holography and AdS/CFT

[Maldacena '97]

CFT_d on R^d ⇔ Gravity theory on AdS_{d+1}

Poincare AdS
$$ds^2 = R^2 \frac{d\tau^2 + (dx^\mu)^2}{\tau^2}$$

Conformal symmetry ⇔ AdS isometry: SO(1,d+1)



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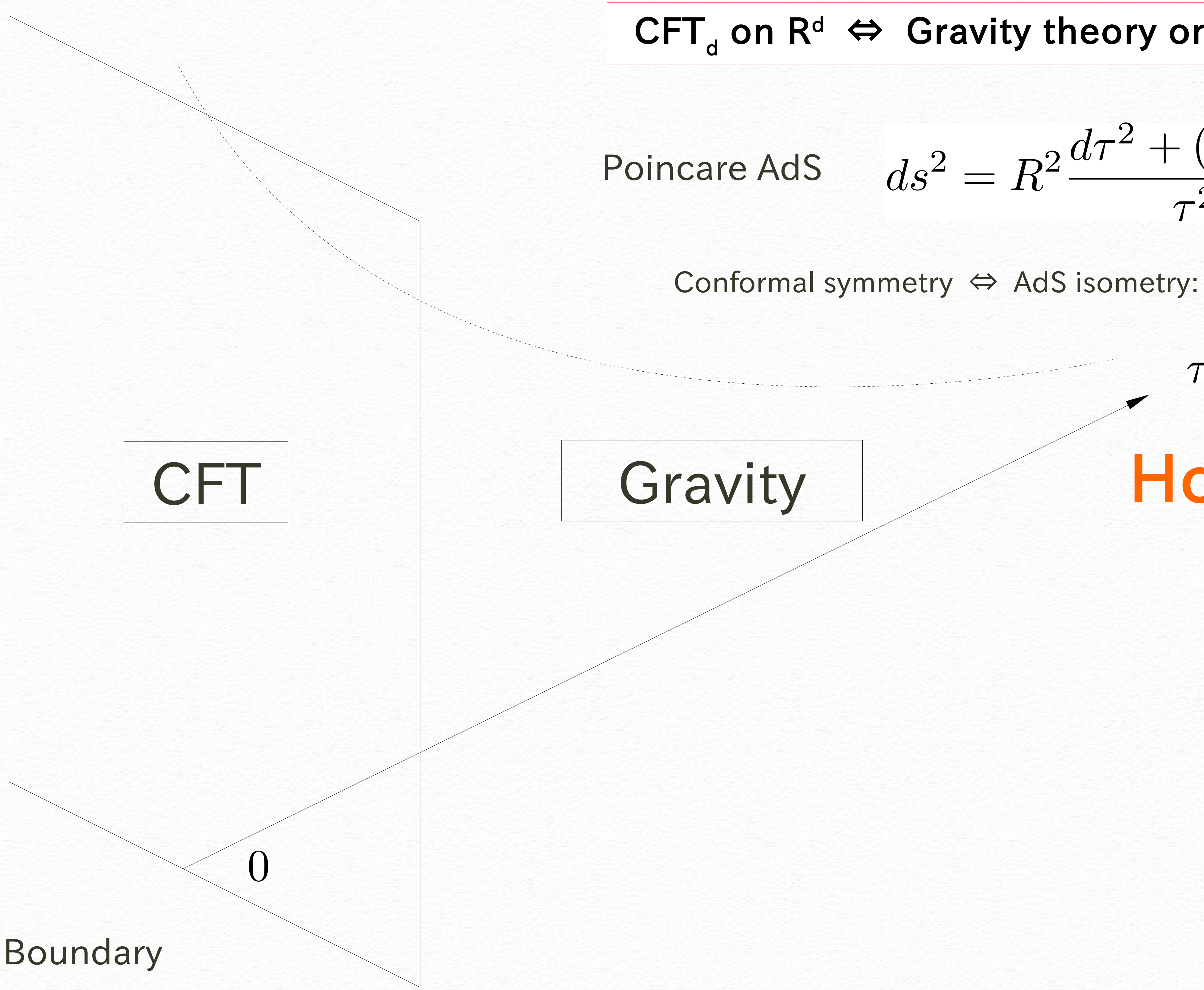

How?

AdS radial direction from CFT

1. Relevant RG flow

Construction of gravity solutions corresponding to UV and IR CFTs in the asymptotic regions.

[Girardello-Petrini-Porrati-Zaffaroni ’98] [Distler-Zamora ’98] [de-Boer-Verlinde-Verlinde ’99] [Skenderis '00]

2. Wilsonian RG flow

The Wilsonian cut-off will correspond to sharp cut-off at the AdS radial direction

[Heemskerk-Polchinski ’10]

3. Stochastic quantization

Euclidean path integral ≡ Equilibrium limit of statistical mechanical system coupled to a heat bath.

[Lifshytz-Periwal ’00]

4. Entanglement enetropy

continuous multi-scale entanglement renormalization ansatz (cMERA) [Swingle ’09]...

5. bilocal field

Relative coordinate of bi-local field in vector models [Das-Jevicki ’03]

6. Flow equation

Smearing operators so as to resolve a UV singularity in the coincidence limit

[Aoki-Kikuchi-Onogi ’15] [Aoki-Balog-Onogi-Weisz ’16,'17]

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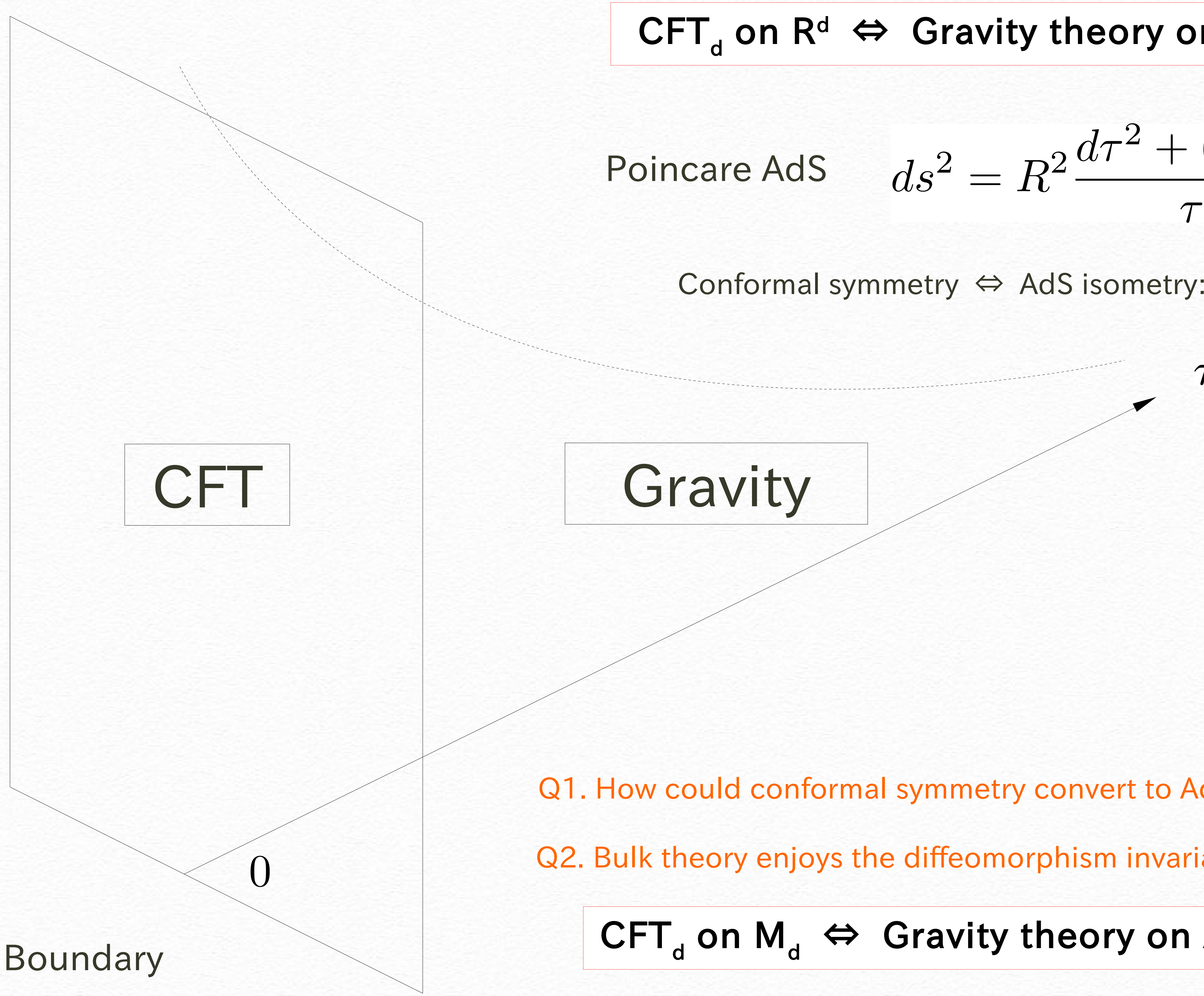

Q1. How could conformal symmetry convert to AdS isometry?

Q2. Bulk theory enjoys the diffeomorphism invariance

CFT_d on M_d ⇔ Gravity theory on AdS_{d+1}?

Flow equation

(Gradient) flow equation

smear operators to resolve a UV singularity in the coincidence limit

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06] [Luscher '10,'13]

“Regularization”

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"Regularization"

Consider CFT with a real scalar primary operator O in d dimensions

Free flow equation

$$\frac{\partial O(x;t)}{\partial t} = \partial^2 O(x;t) \qquad O(x;0) = O(x)$$

t: flow parameter,
O(x;t): flowed operator

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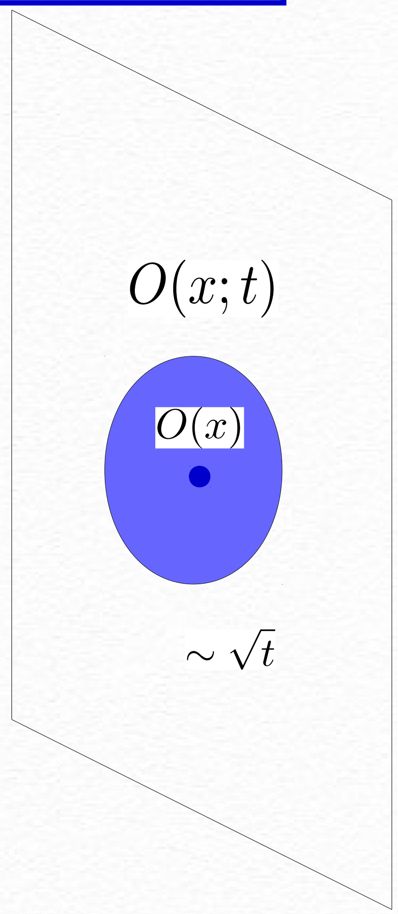
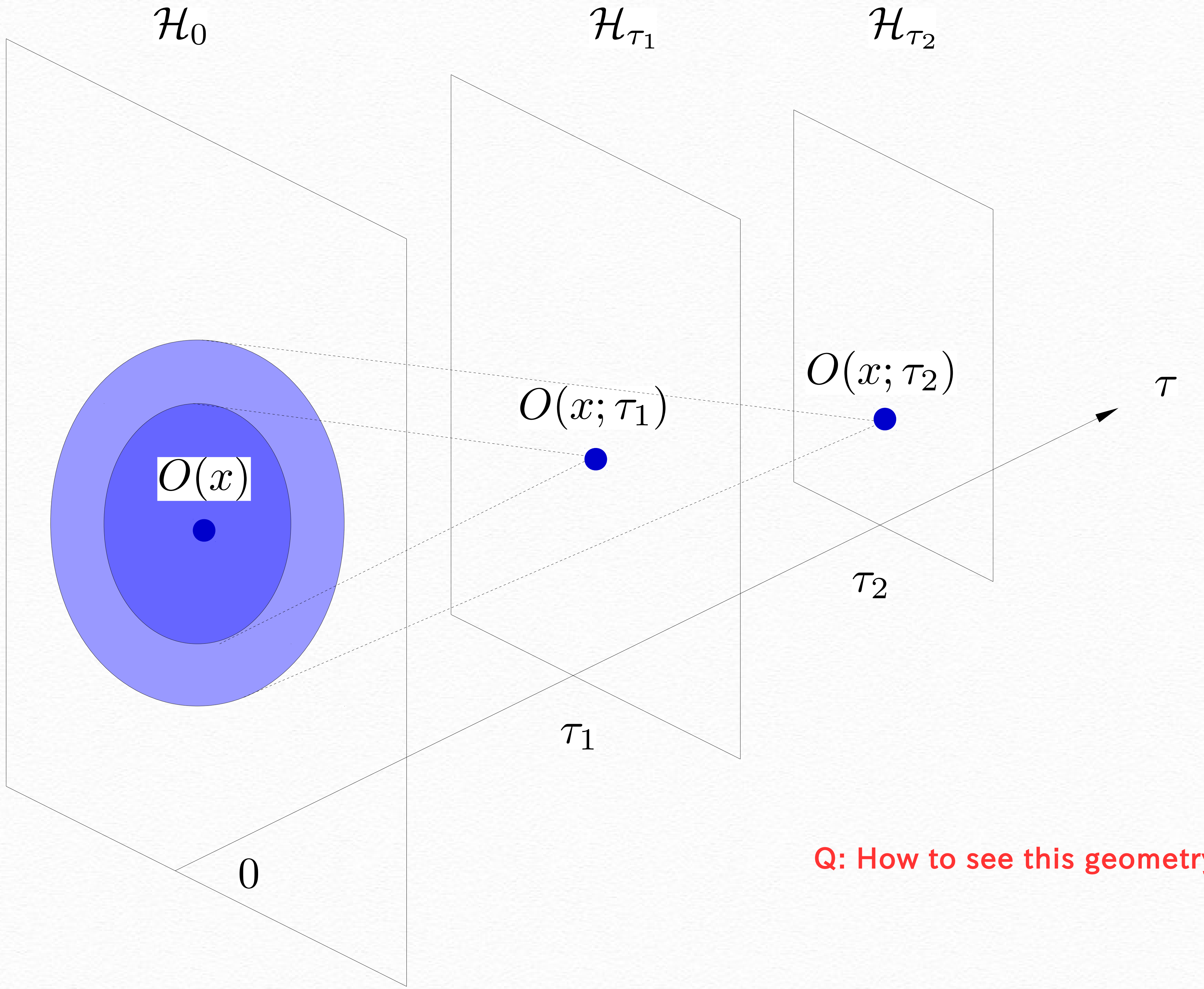
Solution

$$O(x;t) = \int d^d y \rho(x,y;t) O(y)$$

$$\rho(x,y;t) = \frac{1}{(4\pi t)^{d/2}} e^{-\frac{(x-y)^2}{4t}}$$



Sketch of smearing and extra direction



Q: How to see this geometry?

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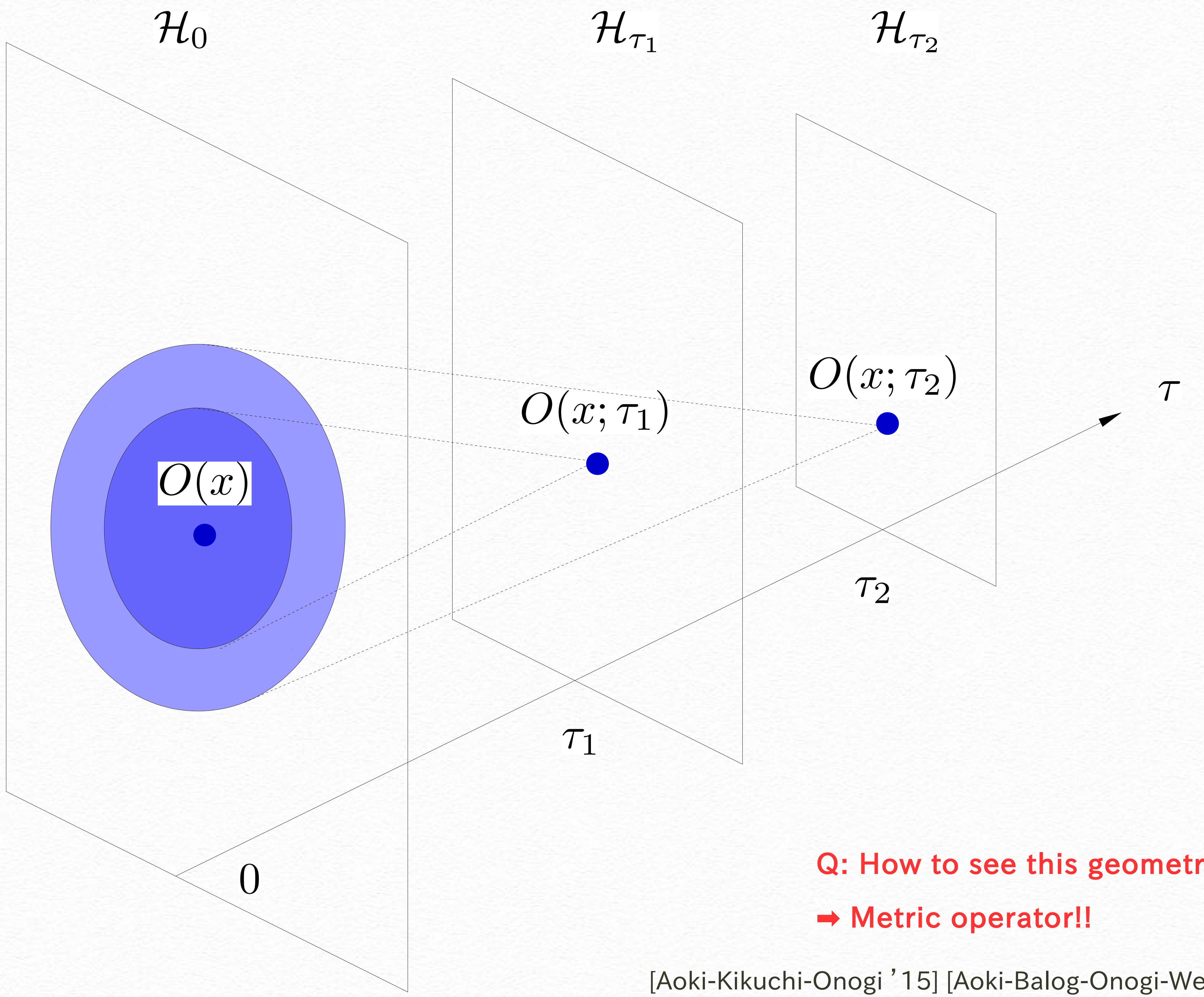

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➡ Metric operator!!

[Aoki-Kikuchi-Onogi '15] [Aoki-Balog-Onogi-Weisz '16,'17]

Metric operator & induced metric

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Def. (Dimensionless normalized operator)

$$\sigma(x;t) := \frac{O(x;t)}{\sqrt{\langle O(x;t)^2 \rangle_{CFT}}}$$

“Operator renormalization”

NOTE: $\langle \sigma(x;t)\sigma(x;t) \rangle_{CFT} = 1$

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$$\hat{g}_{MN}(x;t) := R^2 \frac{\partial \sigma(x;t)}{\partial z^M} \frac{\partial \sigma(x;t)}{\partial z^N}$$

$$g_{MN}(z) := \langle \hat{g}_{MN}(x;t) \rangle_{CFT}$$

R: constant of length dimension $z^M = (x^\mu, \tau)$ with $\tau = \sqrt{2dt}$

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Result 1 Induced metrics with the free flow equation become the AdS metric for a couple of vector models in the free or critical limit.

[Aoki-Kikuchi-Onogi '15] [Aoki-Balog-Onogi-Weisz '16,'17]

Result 2 An induced metric becomes quantum information metric for a general QFT.

[Aoki-SY '17]

Result 3 An induced metric defined in this way becomes AdS for a general CFT.

[Aoki-SY '17]

Induced metric = information metric

[S.Aoki-SY ‘17]　PTEP (2018) 031B01

Induced metric = information metric

Def. (Bure metric for a density matrix)

$$D(\rho, \rho + d\rho)^2 = \frac{1}{2}\mathrm{tr}(d\rho\, G)$$

ρ : density matrix G: hermitian 1 form operator satisfying $\rho\, G + G\, \rho = d\rho$

For a pure state, G is given by $G = d\rho$

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In CFT setup, the density matrix for a state localized at z = (x, τ) is

$$\rho_z = |\sigma(x;t)\rangle\langle\sigma(x;t)|, \quad |\sigma(x;t)\rangle := \sigma(x;t)|0\rangle, \quad \langle\sigma(x;t)| := \langle 0|\sigma(x;t)$$

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cf. Fischer information metric $|\langle\Psi_\lambda|\Psi_{\lambda+\delta\lambda}\rangle|^2 = 1 - 2G_{\lambda\lambda}\delta\lambda^2 + \cdots$

$|\Psi_\lambda\rangle$: Vacuum state for $H_0 + \lambda V$

Q1:Conformal symmetry ➡ AdS isometry?

[S.Aoki-SY '17] PTEP (2018) 031B01

cf. [Jevicki-Kazama-Yoneya '98]

[Das-Jevicki '03]

Conformal symmetry ➡ AdS isometry?

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Conformal transformation:

$$\delta x^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu (b_\nu x^\nu),$$

$$\delta^{\rm conf} O(x) = -\,\delta x^\mu \partial_\mu O(x) - \frac{\Delta}{d}(\partial_\mu \delta x^\mu) O(x)$$

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cf. [Das-Jevicki '03] or Das's talk in 'Strings and Fields 2017'

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Vanish by taking VEV!!

⬇

$$\langle \delta^{\rm conf}\hat g_{MN}(x;t)\rangle = \langle \delta^{\rm diff}\hat g_{MN}(x;t)\rangle .$$

Q2: Generalization to non-trivial curved boundary?

[S.Aoki-SY ‘17]

Generalization to curved background

A curved manifold needs to admit CFT to live. ➡ Restrict ourselves to a conformally flat manifold.

We need to construct a flow equation associated with the curved manifold.

$$\frac{\partial O(x;t)}{\partial t} = \partial^2 O(x;t) \quad \rightarrow \quad ???$$

Generalization to curved background

A curved manifold needs to admit CFT to live. ➡ Restrict ourselves to a conforamlly flat manifold.

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"Canonical" free flow equation **(Primary flow equation)**

$$\frac{\partial}{\partial \tilde{t}} O(\Omega_x; \tilde{t}) = g^{-\frac{1}{d}}(x) g^{-\frac{\Delta}{2d}}(x) \partial^2 \cdot g^{\frac{\Delta}{2d}}(x) O(\Omega_x; \tilde{t}), \qquad O(\Omega_x; 0) = O(\Omega_x)$$

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The induced metric:

$$\begin{aligned}
\tilde{g}_{\tilde{\tau}\tilde{\tau}}(z) =& R^2 \frac{\Delta}{\tilde{\tau}^2}, \\
\tilde{g}_{\tilde{\tau}\mu}(z) =& g_{\mu\tilde{\tau}}(z) = -R^2 \frac{\Delta}{\tilde{\tau}} \frac{\partial}{\partial x^\mu} \log\{g^{\frac{1}{2d}}(x)\}, \\
\tilde{g}_{\mu\nu}(z) =& R^2 \Delta \left[\frac{\partial}{\partial x^\mu} \log\{g^{\frac{1}{2d}}(x)\} \frac{\partial}{\partial x^\nu} \log\{g^{\frac{1}{2d}}(x)\} + \frac{\delta_{\mu\nu} g^{\frac{1}{d}}(x)}{\tilde{\tau}^2} \right],
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This is the (local) AdS metric whose radius is $R\sqrt{\Delta}$ with the boundary M_d!!

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This is the (local) AdS metric whose radius is $R\sqrt{\Delta}$ with the boundary M_d!!

This AdS metric can be obtained from the usual Poincare AdS by a bulk diffeomorphism

$$ds^2(\tilde{t},x) = ds^2_{\mathrm{PAdS}}(t,x)|_{t=g^{-1/d}(x)\tilde{t}}$$

cf. PBH transformation [Penrose '88] [Brown-Henneaux '86]

Summary

- An induced metric corresponds to the **quantum information metric**.

- For a general CFT, the induced metric for the free flow always becomes **AdS.**

- Conformal transf. converts to AdS isometry **after quantum averaging**.

- We have constructed a canonical flow equation for a primary scalar operator on a conformally flat manifold, called the **primary flow equation**.

- The induced metric associated with the primary flow equation becomes AdS with **the conformally flat boundary**.

- AdS with the conformally flat boundary is obtained from the usual Poincare AdS **by a bulk finite diffeomorphism**.

Future works

- How to encode dynamics beyond geometry? For excited states?

  working in progress [Aoki-Balog-SY]

- How to reconstruct bulk operator?

- Canonical choice of a normalized field?

- Spin 1,2 field? Fermion?

- de Sitter construction? Application to the real world?

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