Scale-dependence of non-Gaussianity

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Tomo Takahashi
Department of Physics, Saga University

Current constraints on fNL

- $f_{\rm NL}^{\rm local} = 32 \pm 21 \quad (68\% \ {\rm CL})$ $\qquad f_{\rm NL}^{\rm equil} = 26 \pm 140 \quad (68\% \ {\rm CL})$

  [WMAP7, Komatsu et al, 2010]

- $f_{\rm NL}^{\rm local} = 62 \pm 27 \quad (68\% {\rm CL})$ [NRAO VLA Sky Survey, Xia et al, 2010]

  $\vdots$

- $f_{\rm NL}$ is usually assumed to be constant.

- In some models, $f_{\rm NL}$ can be (relatively strongly) scale-dependent.

- Scale dependence of $f_{\rm NL}$ can be a discriminator of models.

Scale-dependence of fNL

- Spectral index for fNL : $n_{f_{\rm NL}} \equiv \frac{d \ln |f_{\rm NL}|}{d \ln k}$

$$\longrightarrow \quad f_{\rm NL}(k) = f_{\rm NL}(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_{f_{\rm NL}}}$$

where

$$k \equiv (k_1 k_2 k_3)^{1/3}$$

In the following, we consider "local type": $\zeta = \zeta_G + \frac{3}{5} f_{\rm NL} \zeta_G^2$

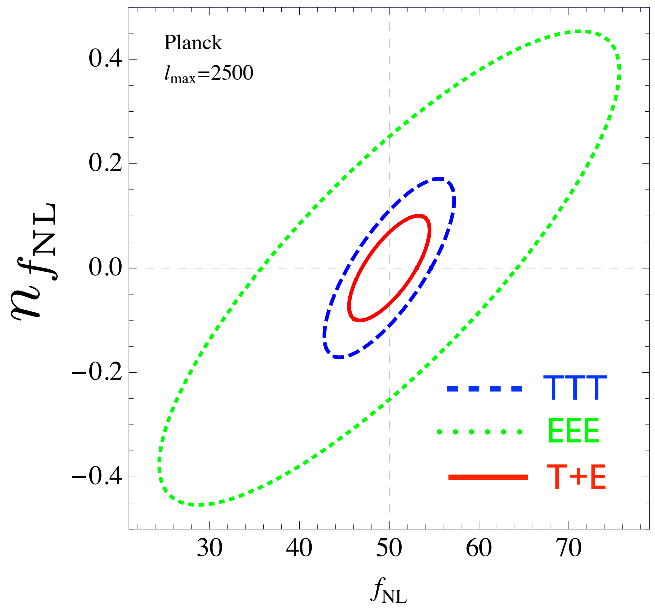
$$B_\zeta(k_1, k_2, k_3) = 2 f_{\rm NL} \Delta_\zeta^2 \left(\frac{1}{k_1^{3-(n_s-1)}} \frac{1}{k_2^{3-(n_s-1)}} + \frac{1}{k_1^{3-(n_s-1)}} \frac{1}{k_3^{3-(n_s-1)}} + \frac{1}{k_2^{3-(n_s-1)}} \frac{1}{k_3^{3-(n_s-1)}} \right)$$

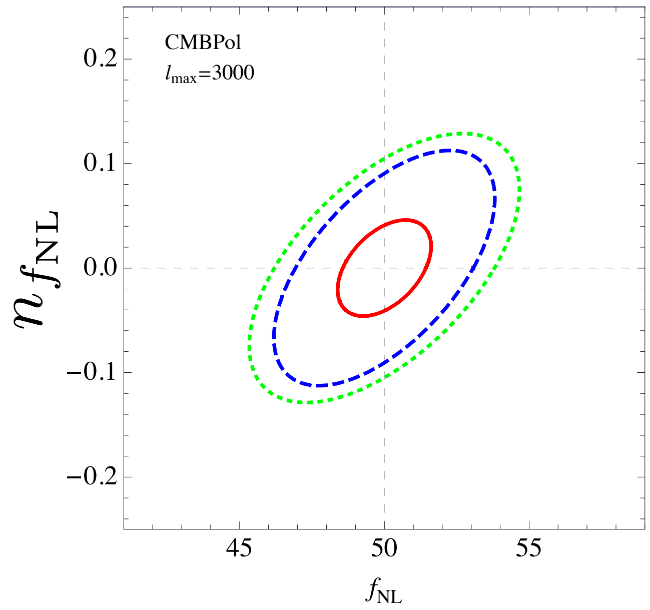
$$\left\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \right\rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Scale-dependence of fNL

- Future attainable limit:

[Sefusatti et al., 0906.0232]



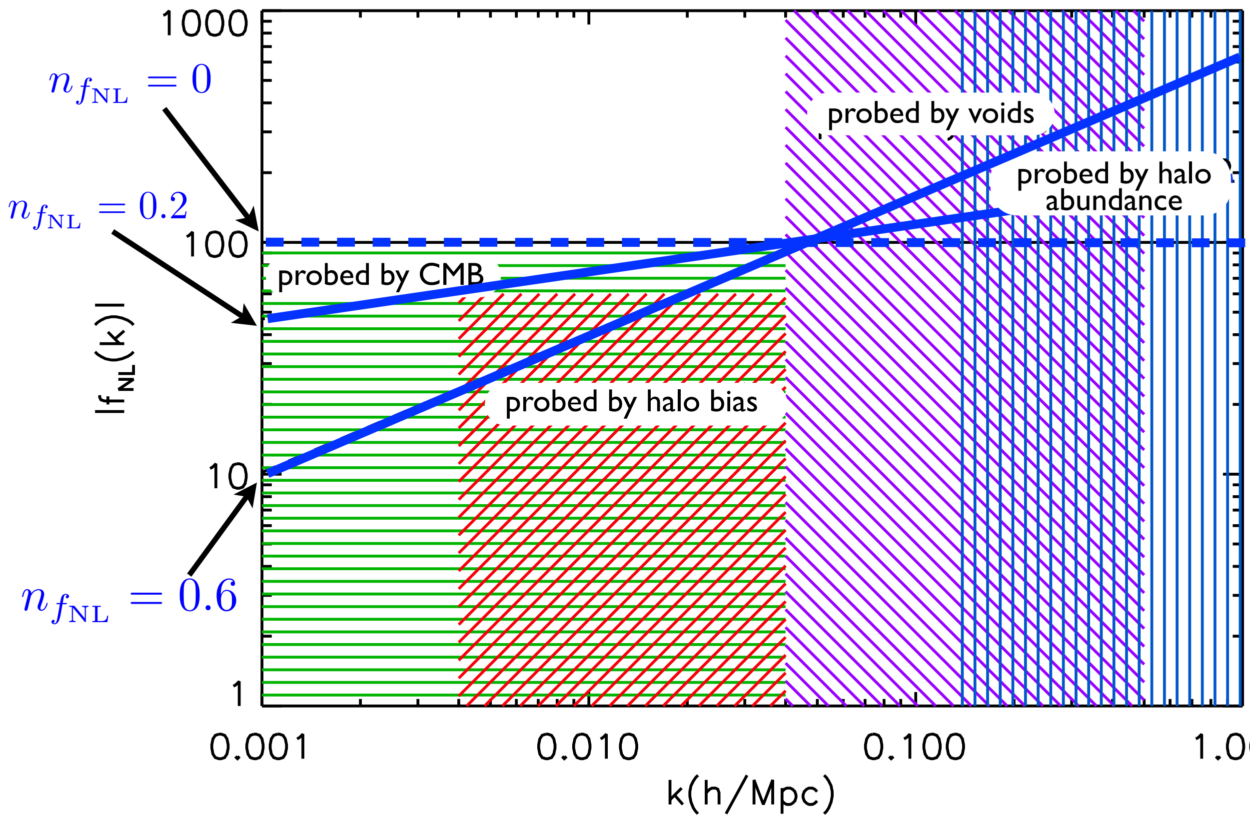


$$\Delta n_{f_{\rm NL}} = 0.1 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}}$$ (Planck)

$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}}$$ (CMBpol)

Scales probed by different approach

- Scale dependence of non-G. may give interesting implications for non-G. from LSS.



[Verde, 1001.5217]

Scales probed by different approach

- Scale dependence of non-G. may give interesting implications for non-G. from LSS.

Table 2 Forecasts $1-\sigma$ constraints on local $f_{\rm NL}$

Data/method	$\Delta f_{\rm NL}$ $(1-\sigma)$	reference
BOSS–bias	18	Carbone et al. 2008
ADEPT/Euclid–bias	1.5	Carbone et al. 2008
PANNStarrs –bias	3.5	Carbone et al. 2008
LSST–bias	0.7	Carbone et al. 2008
LSST-ISW	7	Afshordi& Tolley 2008
BOSS–bispectrum	35	Sefusatti & Komatsu 2008
ADEPT/Euclid –bispectrum	3.6	Sefusatti & Komatsu 2008
Planck-Bispectrum	3	Yadav et al . 2007
BPOL-Bispectrum	2	Yadav et al . 2007

[Verde, 1001.5217]

Scale-dependence of non-G.

- Running of $n_{f_{\rm NL}}$:

$$\alpha_{f_{\rm NL}} \equiv \frac{dn_{f_{\rm NL}}}{d\ln k}$$

- Spectral index for $g_{\rm NL}$:

$$n_{g_{\rm NL}} \equiv \frac{d\ln|g_{\rm NL}|}{d\ln k}$$

[Byrnes et al, 1007.4277; Byrnes, Enqvist, TT, 1007.5148; Huang, 1102.4686]

Models with scale-dependent $f_{\rm NL}$

Scale-dependence of non-G can be generated from:

- Multiple sources of fluctuations
  - ▶ Models with mixed inflaton and some other field (curvaton/modulaton)

⋮

- Non-linear evolution of fluctuations after horizon exit
  - ▶ Self-interacting curvaton model

⋮

Mixed models with inflaton fluctuations

Mixed model with inflaton fluctuations

- Even in the curvaton/modulated reheating models, fluctuations of the inflaton can exist and contribute to ζ.

  - the curvature perturbation in a mixed model:

$$\zeta = \underbrace{N_\phi \delta\phi_* + \frac{1}{2} N_{\phi\phi} \delta\phi_*^2}_{\text{Inflaton contribution}} + \underbrace{N_\sigma \delta\sigma_* + \frac{1}{2} N_{\sigma\sigma} \delta\sigma_*^2}_{\text{curvaton/modulaton contribution}}$$

Mixed inflaton and curvaton model

- Power spectrum

$$P_\zeta = P_\zeta^{(\phi)} + P_\zeta^{(\sigma)} = (1+R)P_\zeta^{(\phi)}$$

$$R \equiv \frac{N_\sigma^2}{N_\phi^2} = \frac{P_\zeta^{(\sigma)}}{P_\zeta^{(\phi)}}$$

- Non-linearity parameter

$$\frac{6}{5}f_{NL} = \frac{N_\sigma^2 N_{\sigma\sigma} + N_\phi^2 N_{\phi\phi}}{(N_\sigma^2 + N_\phi^2)^2}$$

$$\simeq \left(\frac{R}{1+R}\right)^2 \frac{5}{6} f_{\rm NL}^{(\sigma)}$$

$$\frac{5}{6} f_{\rm NL}^{(\sigma)} = \frac{N_{\sigma\sigma}}{N_\sigma^2}$$

Mixed inflaton and curvaton model

- Spectral index (for power spectrum)

$$n_s = -2\epsilon + 2\eta_{\sigma\sigma} + \frac{1}{1+R}(-4\epsilon + 2\eta_{\phi\phi} - 2\eta_{\sigma\sigma})$$

$$\eta_{\sigma\sigma} = M_{\rm pl}^2 \frac{1}{3H_*^2}\frac{d^2V}{d\sigma^2}$$

$$\eta_{\phi\phi} = M_{\rm pl}^2 \frac{1}{3H_*^2}\frac{d^2V}{d\phi^2}$$

- Tensor-to-scalar ratio

$$r = \frac{16\epsilon}{1+R}$$

- $R \to \infty$ limit ("pure" curvaton/modulaton limit):

$$n_s = -2\epsilon + 2\eta_{\sigma\sigma} \qquad r \to 0$$

Mixed inflaton and curvaton model

- Spectral index for $f_{\rm NL}$: $n_{f_{\rm NL}} \equiv \dfrac{d\ln|f_{\rm NL}|}{d\ln k}$

$$\frac{5}{6}f_{\rm NL} \simeq \left(\frac{R}{1+R}\right)^2 \boxed{\frac{5}{6}f_{\rm NL}^{(\sigma)}}$$

Scale-dependent

No scale-dep.
(for quadratic potential)

$$P_\zeta^{(\sigma)} \propto k^{n_s^{(\sigma)}-1}$$

$$P_\zeta^{(\phi)} \propto k^{n_s^{(\phi)}-1}$$

$$\Rightarrow \quad n_{f_{\rm NL}} = \frac{2}{1+R}\left(n_s^{(\sigma)} - n_s^{(\phi)}\right)$$

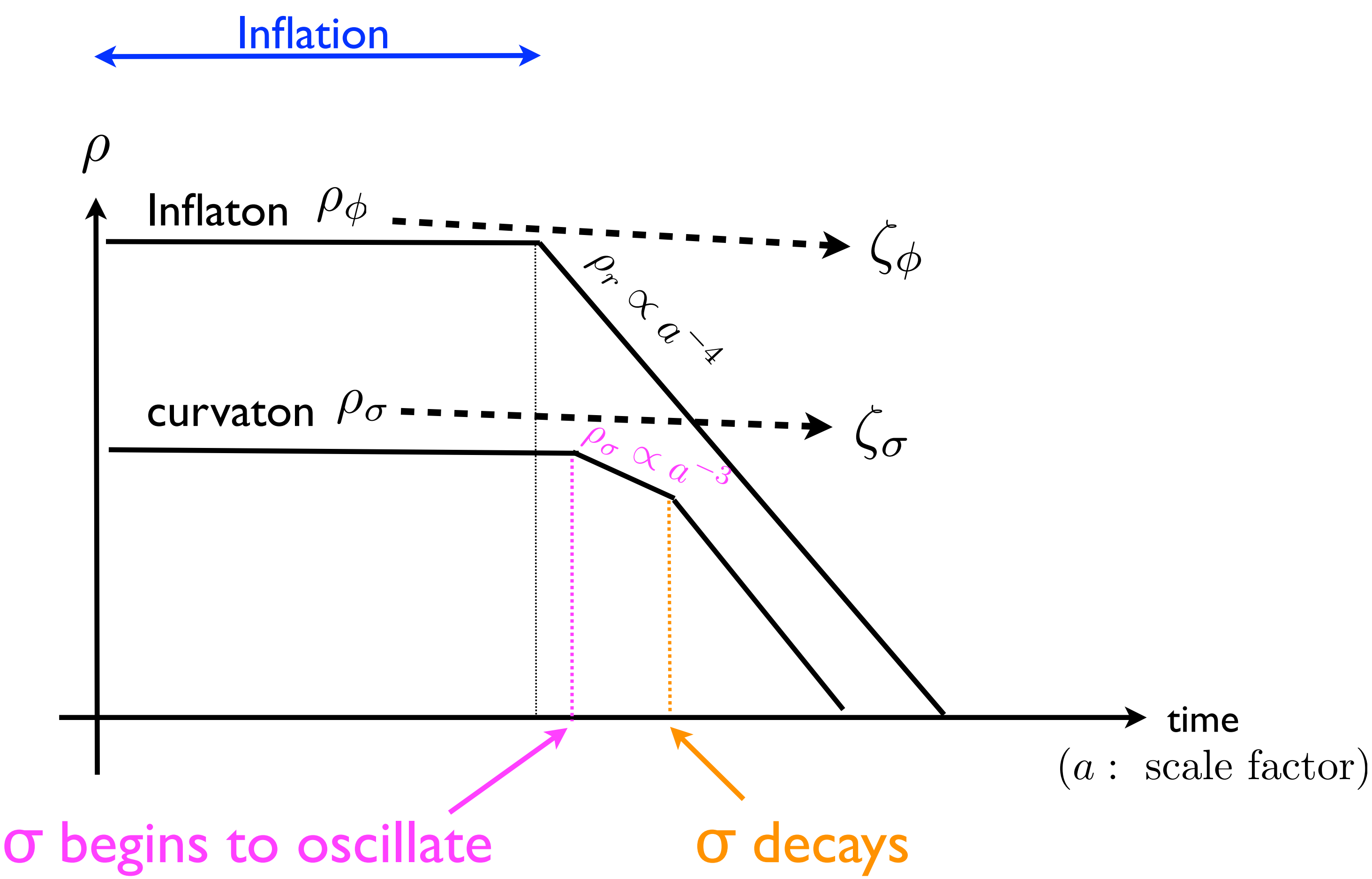
$$n_s^{(\sigma)} = -2\epsilon + 2\eta_{\sigma\sigma}$$

$$n_s^{(\phi)} = -6\epsilon + 2\eta_{\phi\phi}$$

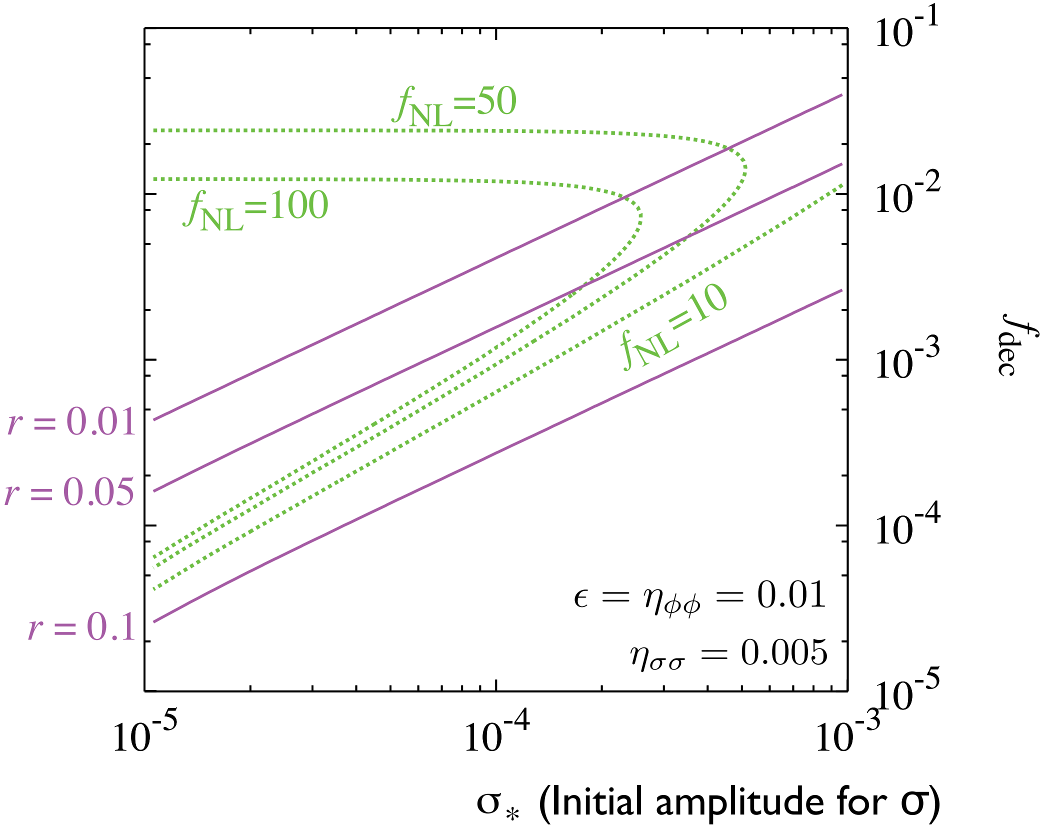
$$\Rightarrow \quad n_{f_{\rm NL}} = \frac{4}{1+R}\left(2\epsilon - \eta_{\phi\phi} + \eta_{\sigma\sigma}\right)$$

Mixed inflaton and curvaton model

A brief thermal history of the curvaton scenario



n_s and r in mixed inflaton-curvaton model



$$f_{\rm dec} = \left. \frac{3\rho_\sigma}{4\rho_{\rm rad} + 3\rho_\sigma} \right|_{\rm decay}$$

Inflation: $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$

Curvaton: $U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2$

► Sizable r *(>O(0.01))* and large f_{NL} *(>O(10))* can both possible

[Ichikawa, Suyama, TT, Yamaguchi, 0802.4138]

Mixed inflaton and curvaton model

■ Spectral index for $f_{\rm NL}$:

- Small-field inflation case $(\epsilon \ll \eta)$

--- $n_{f_{\rm NL}} \simeq -2(n_s - 1) \sim 0.06 - 0.1$

→ may be detectable with Planck!

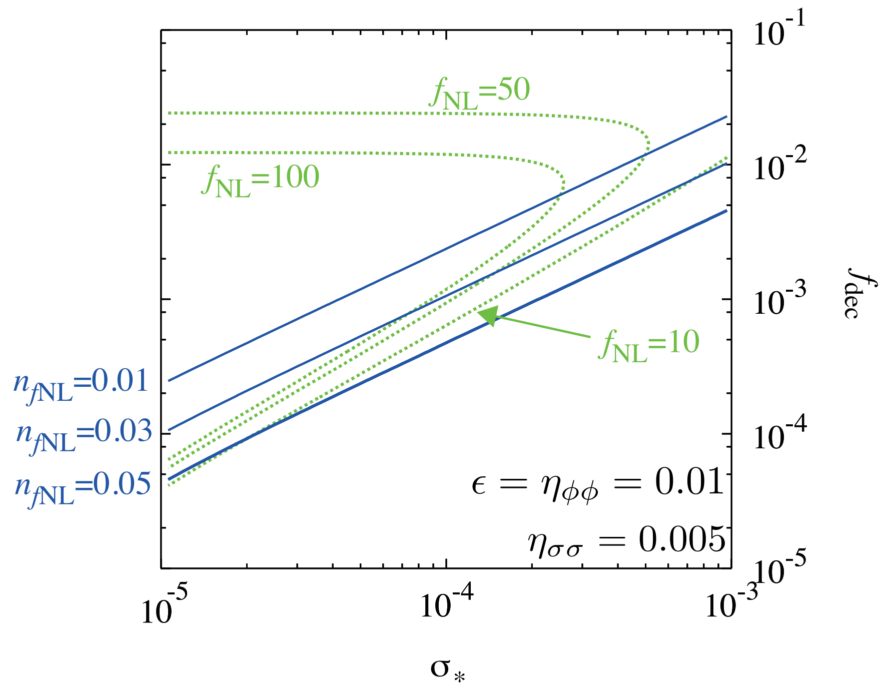
--- $r \ll \mathcal{O}(1)$

[Byrnes et al. 2009]

Mixed inflaton and curvaton model

■ Spectral index for $f_{\rm NL}$:

- Large-field inflation case (e.g. chaotic inflation) $(\epsilon \sim \eta)$



Inflation: $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$

Curvaton: $U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2$

Self-interacting curvaton model

Self-interacting curvaton model

- In some curvaton models, the curvaton potential can deviate from a (purely) quadratic form.

For example,

- When an MSSM flat direction is the curvaton, its potential can be given as:

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda^2 \sigma^{2(n-1)}}{2^{n-1} M^{2(n-3)}}$$

➡ The form of the potential can deviate from the quadratic one.

- Interesting prediction for the scale-dependence of non-Gaussianity.

$n_{f_{\rm NL}}$ in the self-interacting curvaton

In the following, we assume: $U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda\sigma^n$

■ Scale-dependence of $f_{\rm NL}$

$$n_{f_{\rm NL}} = \frac{U'''(\sigma_*)}{3H_*^2}\left(\frac{\sigma_{\rm osc}\sigma'_{\rm osc}}{(\sigma'_{\rm osc})^2 + \sigma_{\rm osc}\sigma''_{\rm osc}}\right)$$

▶ Non-linear evolution outside the horizon may give large $n_{f_{\rm NL}}$.

- When the potential is purely quadratic,
  (almost) no scale-dependence

- Non-zero $n_{f_{\rm NL}}$ may indicate a self-interacting curvaton.

$n_{f_{\rm NL}}$ in the self-interacting curvaton

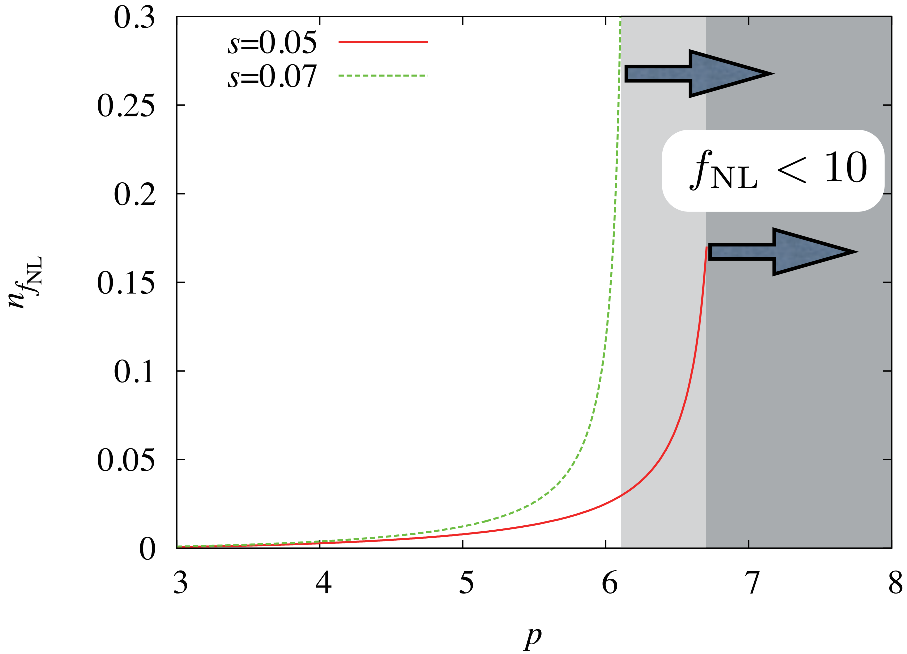
- To characterize the size of NQ term, we define s:

$$s \equiv \frac{U(\mathrm{non-quadratic})}{U(\mathrm{quadratic})} = 2\lambda \left(\frac{\sigma_*}{m_\sigma}\right)^{p-2}$$

($s \to 0$: quadratic limit)

► Larger s, more non-linear evolution of $\delta\sigma$

$n_{f_{\rm NL}}$ in the self-interacting curvaton

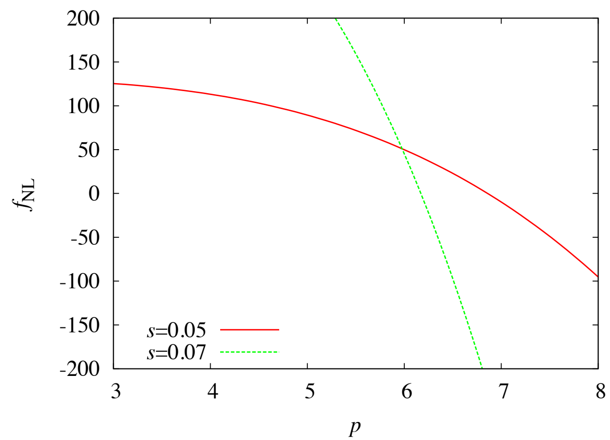


where

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^p$$

$$s \equiv 2\lambda \left(\frac{\sigma_*}{m_\sigma} \right)^{p-2}$$

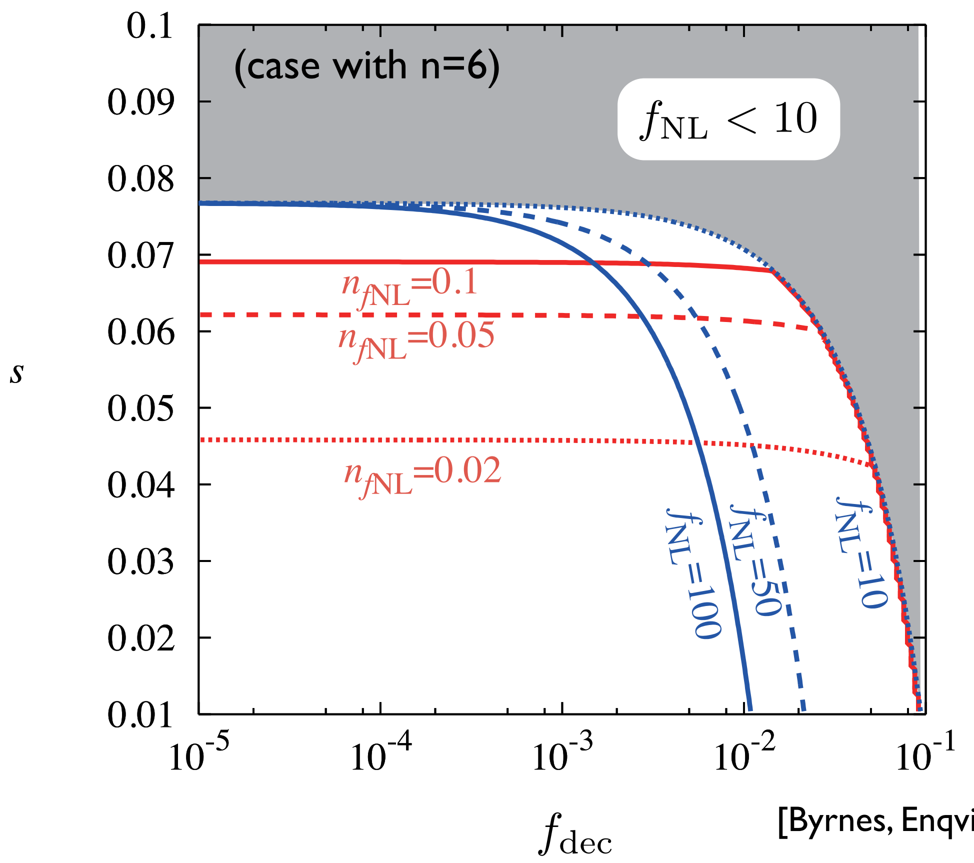
[Byrnes, Enqvist, TT, 1007.5148]



- Detectable region (CMBpol):

$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}}$$

$n_{f\mathrm{NL}}$ in the self-interacting curvaton



[Byrnes, Enqvist, TT, 1007.5148]

▸ s controls the size of nfNL. fNL is determined by r.

Summary

- Scale-dependence of non-Gaussianity ($n_{f_{\rm NL}}$) can be useful to discriminate models of large non-G.
- Some models (e.g.. mixed inflaton-curvaon, self-interacting curvaton) predict (relatively) large $n_{f_{\rm NL}}$, which can be testable with future obs.
- Scale-dependence of non-G would give interesting information for models of primordial fluctuations.