Evolution of Cosmological Perturbations in Kinetic Gravity Braiding Model

Cosmological Perturbation and Cosmic Microwave Background at Kyoto 3/21/2011

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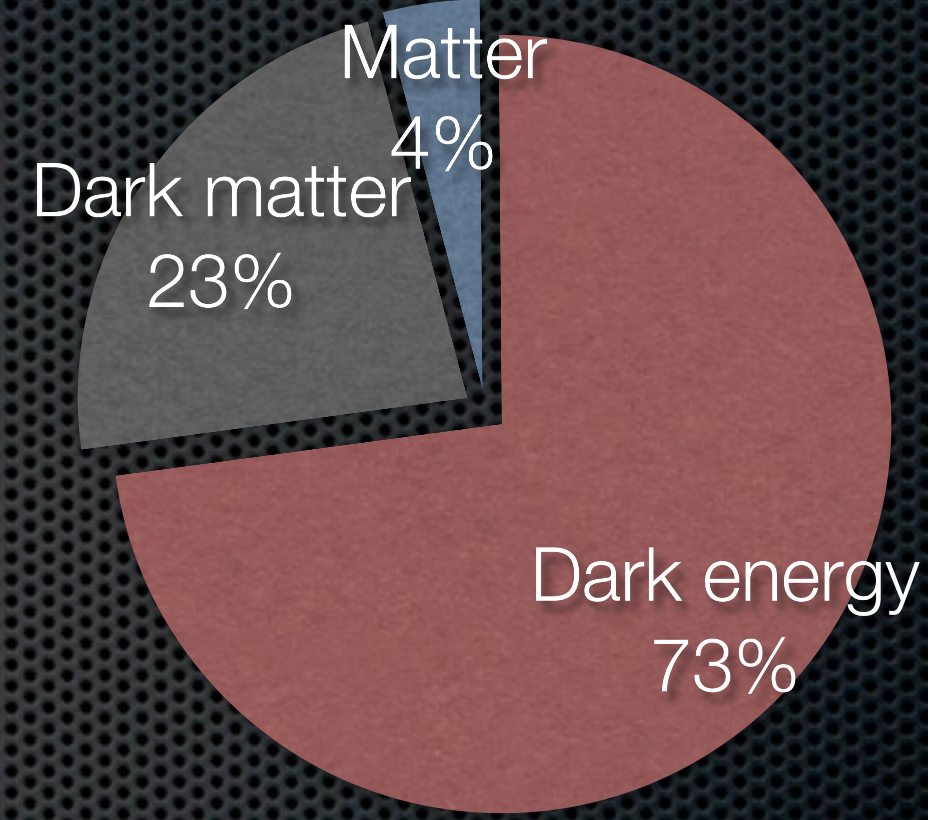
Today's talk

1. Introduction
2. Kinetic Gravity Braiding
3. Background expansion
4. Cosmological Perturbations
5. Summary

Introduction

What is the origin of an accelerated expansion of the universe?



Dark energy?

Modified gravity?

ΛCDM model

consistent with various observations
(type Ia supernovae, LSS, CMB, etc...)

→ Cosmological constant problem

Goal : To construct modified gravity model which is consistent with observations

2. Kinetic Gravity Braiding

Kinetic Gravity Braiding (KGB)

Deffayet, Pujolas, Sawicki, Vikman '10
Kobayashi, Yamaguchi, Yokoyama '10

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + K(X) - G(X)\Box\phi + \mathcal{L}_{\rm m}\right]$$

Galileon theory $X\Box\phi$ **Nicolis et al. '09** **Koyama, Silva '09**

K(X) and G(X) are arbitrary functions of the kinetic term

$$X = -g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi/2, \quad \Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$

- **Second-order differential equation**
- **No ghost instability**
- **Vainshtein Mechanism**

  → recover the GR behavior in high-density region

Kinetic Gravity Braiding (KGB)

Deffayet, Pujolas, Sawicki, Vikman '10

Lagrangian is invariant under a constant shift symmetry $\phi \to \phi + c$, then the scalar field satisfies

$$\dot{J}_0 + 3HJ_0 = 0$$

$$J_0 = \dot{\phi}\left(3\dot{\phi}G_X H - K_X\right) \propto 1/a^3$$

'Charge density' approaches zero as the universe expands

⬇

Attractor solution

Solution along the attractor $\dot{\phi} = K_X / 3G_X H$

Kinetic Gravity Braiding (KGB)

For example...

$$K(X) = -X$$

$$G(X) = M_{\mathrm{Pl}} \left(\frac{r_c^2}{M_{\mathrm{Pl}}^2} X \right)^n$$

Kimura, Yamamoto '10

$$r_c \sim H_0^{-1}$$

n : Model parameter

For n=1, Galileon Model

Deffayet, Pujolas, Sawicki, Vikman '10

3. Background Expansion

Background Expansion

Friedmann Equation

$$\left(\frac{H}{H_0}\right)^2 = (1-\Omega_0)\left(\frac{H}{H_0}\right)^{-\frac{2}{2n-1}} + \Omega_0 a^{-3}$$

=Dvali-Turner model (2003)

Phenomenologically modified Friedmann equation

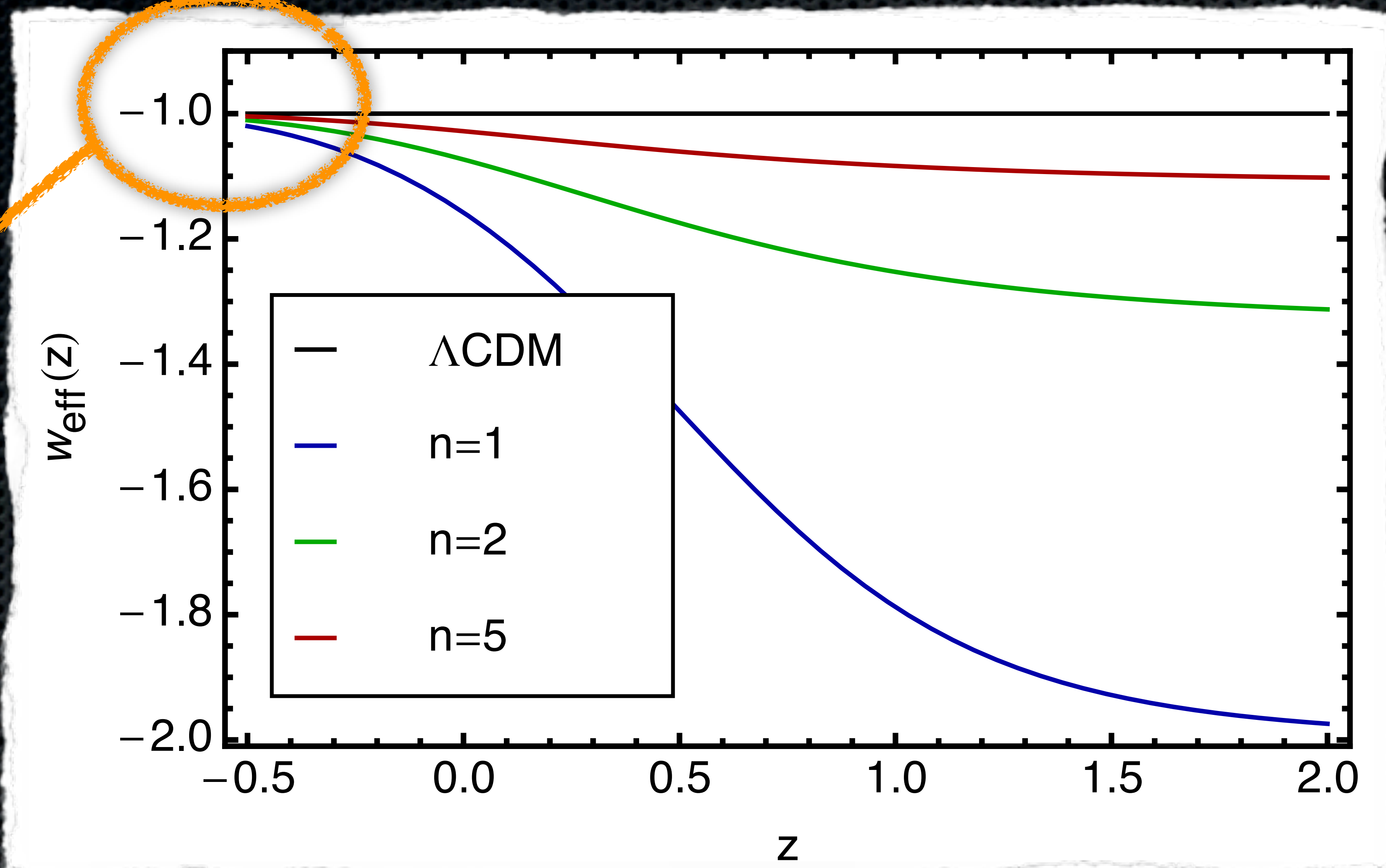
For n>100

$$\left(\frac{H}{H_0}\right)^2 \simeq 1-\Omega_0+\Omega_0 a^{-3}$$

Background expansion is identical to the ΛCDM Model

→ This model is consistent with the observations such as type Ia supernovae

Effective equation of state $w_{\rm eff} \equiv p_\phi/\rho_\phi$



$w_{\rm eff} \simeq -1$ (Scalar field dominated era)

$\boxed{\textbf{de-Sitter expansion}}$

The Condition for the Avoidance of Ghost Instability

Quadratic action $\qquad \phi(t) \to \phi(t) + \delta\phi(t, \mathbf{x})$

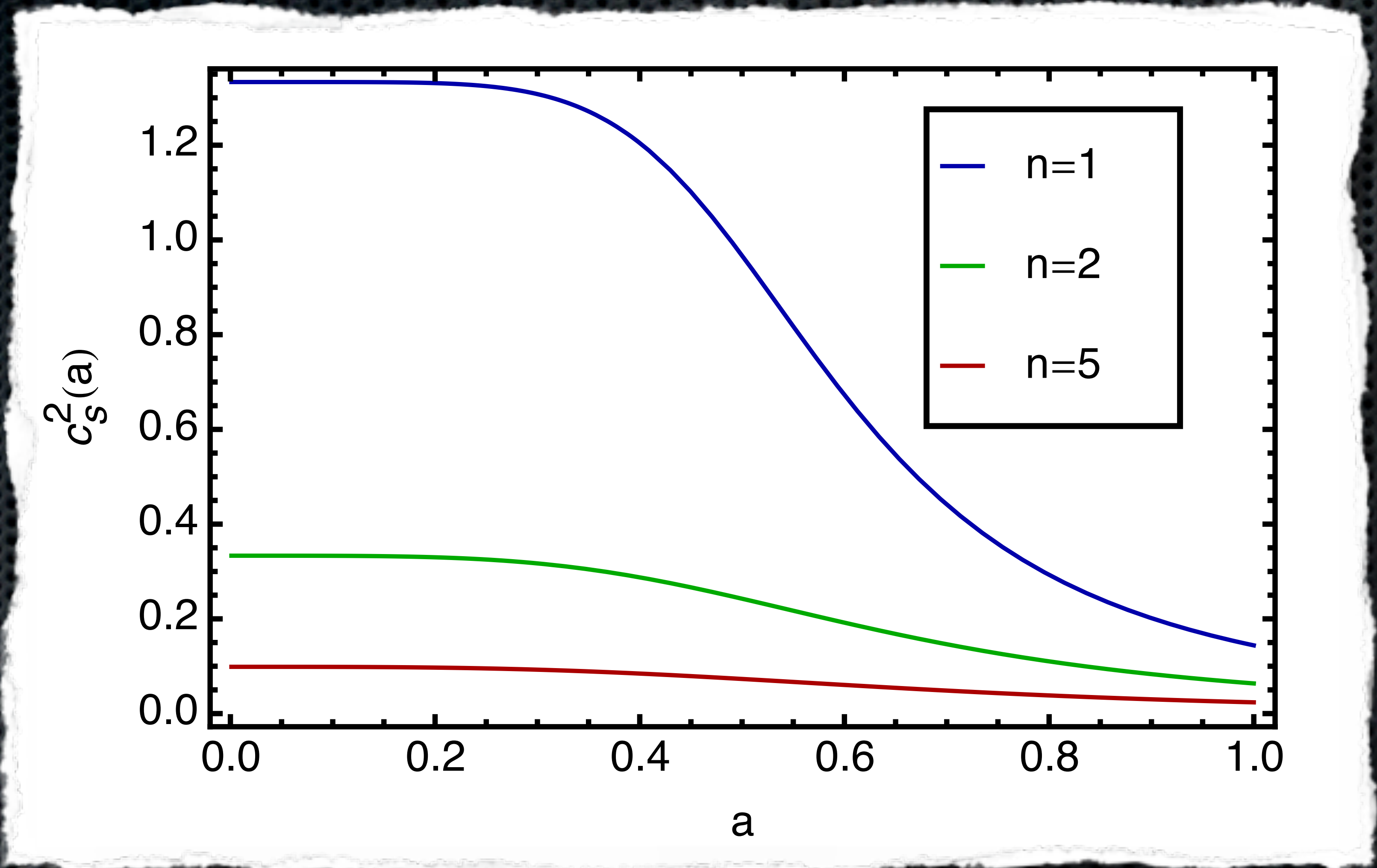
$$\delta S^{(2)} = \frac{1}{2}\int d^4x\sqrt{-g}\kappa(a)\left[\dot{\delta\phi}^2 - \frac{c_s^2(a)}{a^2}(\partial_i\delta\phi)^2\right]$$

$$\kappa(a) = 2n - \Omega_{\mathrm{m}}$$ > 0 (No ghost)

$$c_s^2(a) = \frac{5n\Omega_{\mathrm{m}} - \Omega_{\mathrm{m}}^2}{3(2n - \Omega_{\mathrm{m}})^2}$$ > 0 (No instability)

Ghost instability can be avoided if n > 1/2

Sound Speed of Perturbations of the Scalar Field



$$c_s^2 \propto 1/n$$

The sound speed of perturbations become zero for n=∞

This property is important in cosmological perturbations

4. Cosmological Perturbation

Linear Cosmological Perturbations

Quasi-static & Sub-horizon approximation

$$\mathcal{O}(k^2 c_s^2/a^2) \gg \mathcal{O}(H^2)$$

Density perturbations obeys

$$\ddot{\delta} + 2H\dot{\delta} \simeq 4\pi G_{\rm eff}\,\rho\delta$$

The effective gravitational constant

$$G_{\rm eff} = G\left[1 + 4\pi G\frac{G_X^2\dot{\phi}^4}{\beta(a)}\right]$$

However, this approximation no longer works for n>10 because the speed of sound approaches zero.

Linear Cosmological Perturbations

Einstein equations

$$2M_{\rm Pl}^2\left[-3H(\dot\Phi-H\Psi)+\frac{1}{a^2}\nabla^2\Phi\right]$$
$$=-K_X\delta X-G_X\left(3\dot\phi^3\dot\Phi-12H\dot\phi^3\Psi+9H\dot\phi^2\dot{\delta\phi}-\frac{\dot\phi^2}{a^2}\nabla^2\delta\phi\right)$$
$$-3G_{XX}H\dot\phi^3\delta X-\delta\rho$$

$$2M_{\rm Pl}^2\left(\dot\Phi-H\Psi\right)=-K_X\dot\phi\delta\phi-G_X\dot\phi^2\left(\dot\phi\Psi-\dot{\delta\phi}+3H\delta\phi\right)+\delta q$$

$$2M_{\rm Pl}^2\left[(3H^2+2\dot H)\Psi+H\dot\Psi-\ddot\Phi-3H\dot\Phi\right]$$
$$=K_X\delta X+G_X\left(\dot\phi^3\dot\Psi-\dot\phi^2\ddot{\delta\phi}+4\dot\phi^2\ddot\phi\Psi-2\dot\phi\ddot\phi\dot{\delta\phi}\right)-G_{XX}\dot\phi^2\ddot\phi\delta X$$

$$\Psi+\Phi=0$$

Linear Cosmological Perturbations

Scalar field equation

$$
\begin{aligned}
&-K_X\left[3\dot{\phi}\dot{\Phi}-\dot{\phi}\dot{\Psi}-2(\ddot{\phi}+3H\dot{\phi})\Psi+\ddot{\delta\phi}+3H\dot{\delta\phi}-\frac{1}{a^2}\nabla^2\delta\phi\right]\\
&-3G_{XXX}H\dot{\phi}^3\ddot{\phi}\delta X-G_X\Bigg[3\dot{\phi}^2\ddot{\Phi}+6(\ddot{\phi}+3H\dot{\phi})\dot{\phi}\dot{\Phi}-9H\dot{\phi}^2\dot{\Psi}\\
&-12\left\{(\dot{H}+3H^2)\dot{\phi}^2+2H\dot{\phi}\ddot{\phi}\right\}\Psi-\frac{\dot{\phi}^2}{a^2}\nabla^2\Psi+6H\dot{\phi}\ddot{\delta\phi}\\
&+6\left\{H\ddot{\phi}+(\dot{H}+3H^2)\dot{\phi}\right\}\dot{\delta\phi}-\frac{2}{a^2}(\ddot{\phi}+2H\dot{\phi})\nabla^2\delta\phi\Bigg]\\
&-G_{XX}\Bigg[3\dot{\phi}^3\ddot{\phi}\dot{\Phi}-3H\dot{\phi}^4\dot{\Psi}-3\left\{8H\dot{\phi}^3\ddot{\phi}+(\dot{H}+3H^2)\dot{\phi}^4\right\}\Psi\\
&+3H\dot{\phi}^3\ddot{\delta\phi}+3\left\{5H\dot{\phi}^2\ddot{\phi}+(\dot{H}+3H^2)\dot{\phi}^3\right\}\dot{\delta\phi}-\frac{\dot{\phi}^2\ddot{\phi}}{a^2}\nabla^2\delta\phi\Bigg]=0
\end{aligned}
$$

For n>10, we need to solve these equations...

Cosmological Perturbations for n=∞

In the limit of n=∞, the leading term of the galileon field equation becomes

$$\delta\dot{X} + 3H\delta X = 0$$

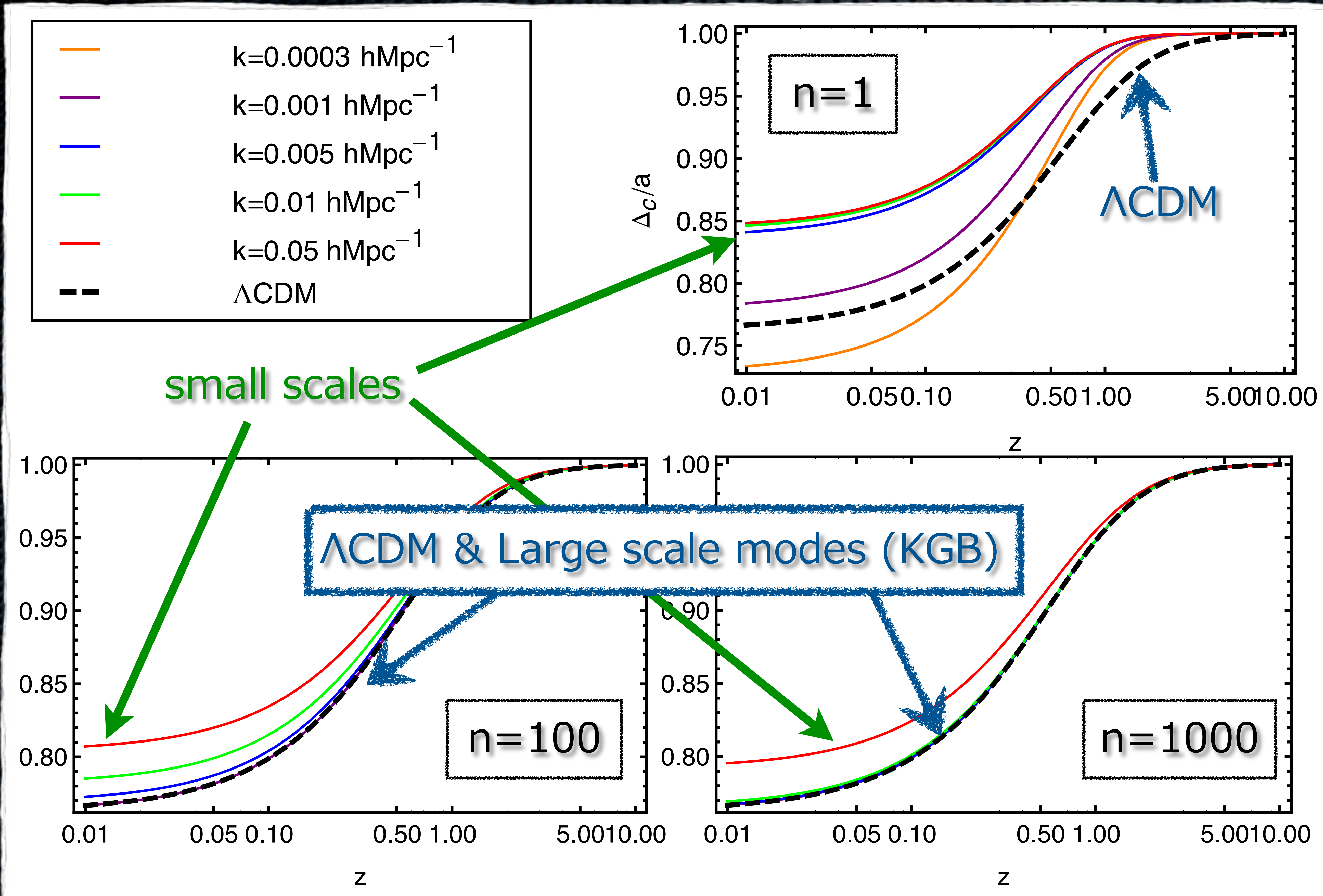
$$\delta X = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\Psi$$

The solution is given by

$$\delta X = \mathrm{Const}/a^3 \quad \rightarrow \quad \delta X = 0$$

The (i,j) component of the Einstein equation reduces to that of the ΛCDM model. Therefore, **the evolution of the gravitational potential is the same as the ΛCDM model.**

Growth Rate of Density Perturbations



Linear Cosmological Perturbations

In linear perturbation theory,

1. **The growth of long wavelength modes for large n in KGB approaches that in the ΛCDM model because the speed of sound of the perturbed scalar field is very small.**

2. **In the limit for $n=\infty$, the growth of density perturbations for all wavelength modes is the same as that in the ΛCDM model**

For $n=\infty$, the evolution of the background expansion and the linear perturbations of KGB are equivalent to those of the ΛCDM model.

Initial Condition of $\delta\phi$

In the previous slide, we adopt the following initial condition,

$$\delta\phi_i = 0, \quad \delta\dot{\phi}_i = 0$$

However, these initial conditions might not be realistic in our universe.

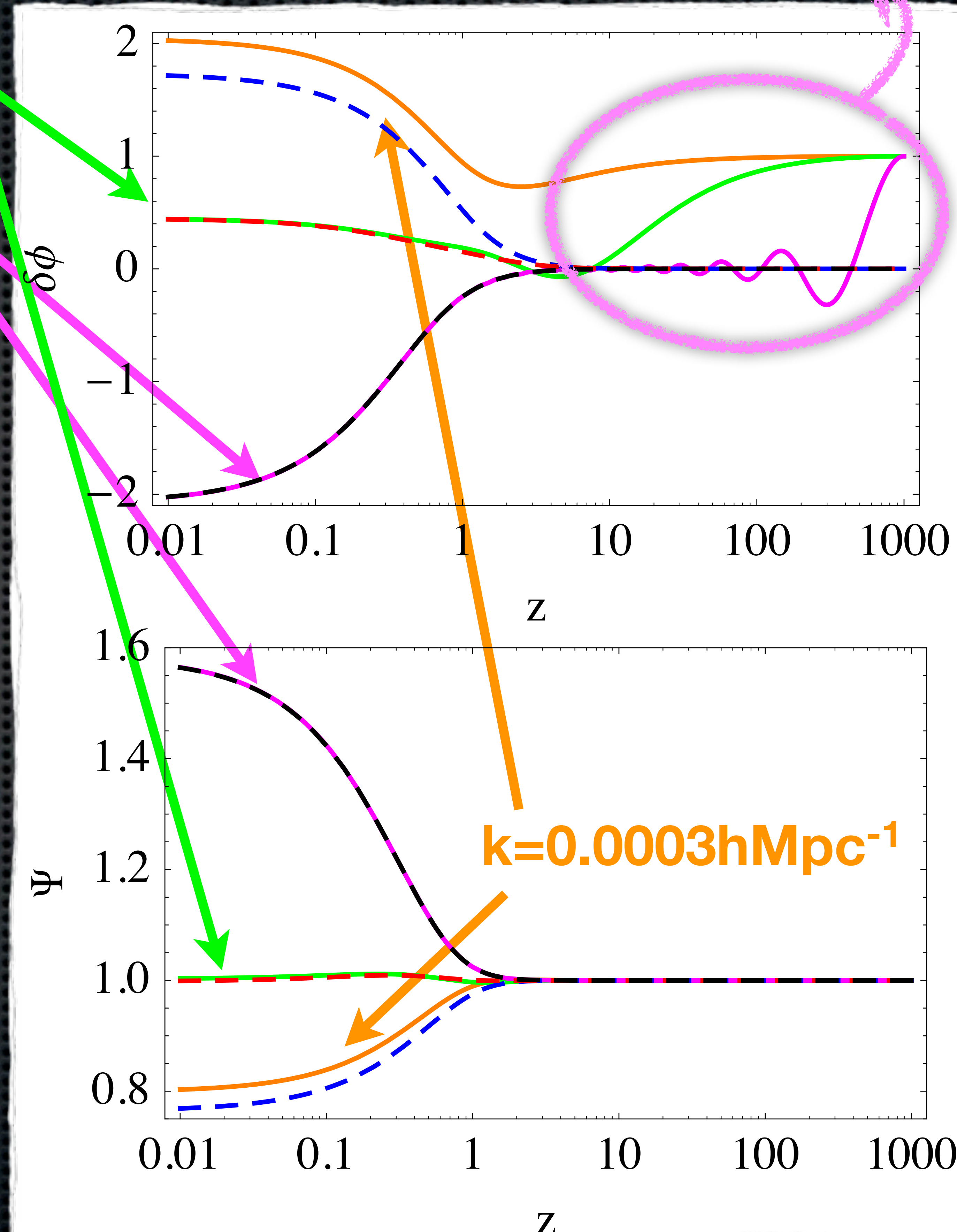
Initial Condition of δφ

Dashed lines

$$\delta\phi_i/M_{\rm Pl} = 0$$

Solid lines

$$\delta\phi_i/M_{\rm Pl} = \Psi_i$$



As long as

$$\delta\phi_i/M_{\rm Pl} \lesssim \Psi_i$$

$$\dot{\delta\phi}_i/M_{\rm Pl}H_i \lesssim \Psi_i$$

the choices of the initial condition does not alter the results.

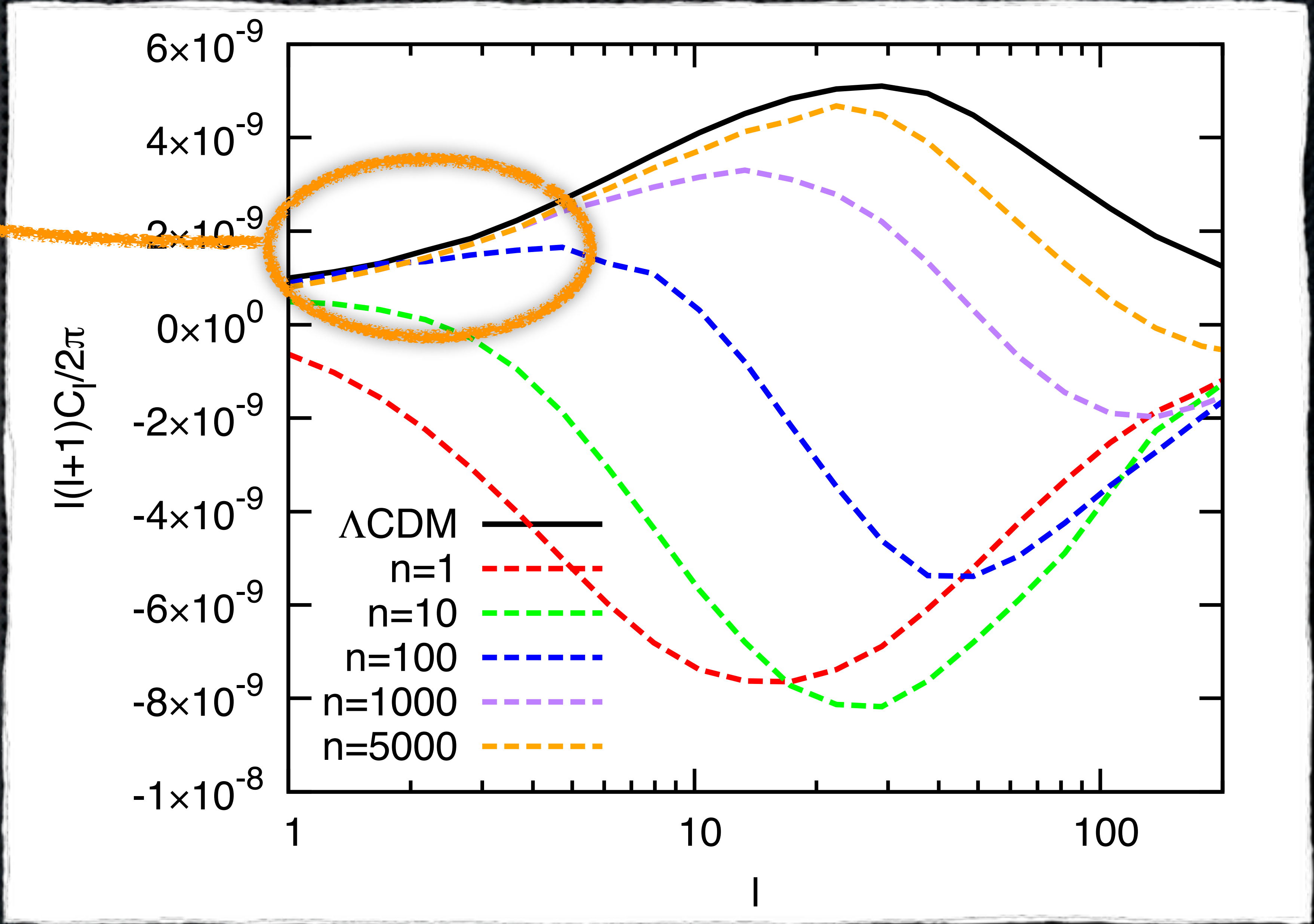
Current work

Integrated Sachs-Wolfe Effect

Cross-correlation of Galaxy distribution and CMB

$$\left\langle \frac{\Delta T(\vec{\gamma})}{T} \frac{\Delta N_g(\vec{\gamma}')}{N} \right\rangle = \frac{1}{4\pi} \sum_l (2l+1) C_l \mathcal{P}_l(\mu)$$

Large n & large scales



It is possible to constrain the model parameter

cf. Kobayashi (RESCEU)

Summary

- Kinetic gravity braiding (KGB) model
  - → No ghost-like instability
  - → Self-accelerating solution
  - → Vainshtein Mechanism
- $G(X) \propto X^n$ model
  - → Galileon model when $n=1$
  - → ΛCDM model when $n=\infty$
  - → Consistent with observations

Thank you!