Primordial non-Gaussianity from the DBI Galileons

Shuntaro Mizuno (Portsmouth)


DBI inflation

Silverstein, Tong (2004)

Inflation is driven by a mobile D3-brane with relativistic speed

\[ \gamma = \frac{1}{\sqrt{1 - \dot{\phi}^2 / T(\phi)}} \]

- **Action**

\[
S = \int d^4 \xi \sqrt{-g^{(4)}} \left[ -T(\phi) \sqrt{1 + \partial^\mu \phi \partial_\mu \phi / T(\phi) + T(\phi)} - V(\phi) \right]
\]

**DBI part**

\[
d\phi = T_3^{1/2} d\rho, \quad T(\phi) = T_3^{1/2} h^4
\]

\[ \rho : \text{radial position of the brane} \]

Large equilateral non-Gaussianity

\[ f_{NL}^{\text{equil}} \propto \gamma^2 \propto c_s^{-2} \]
Galileon

Nicolis, Rattazzi, Trincherini (2008)
Deffayet, Esposito-Farese, Vikman (2009)

Self-accelerating universe without a ghost instability from the terms consistent with Galileon symmetry

\[ \partial_\mu \pi \rightarrow \partial_\mu \pi + c_\mu \quad : \text{Shift symmetry in gradient space} \]

- Galileon invariant terms

\[
\begin{align*}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= -\frac{1}{2} \partial \pi \cdot \partial \pi \\
\mathcal{L}_3 &= -\frac{1}{2} [\Pi] \partial \pi \cdot \partial \pi \\
\mathcal{L}_4 &= -\frac{1}{4} ([\Pi]^2 \partial \pi \cdot \partial \pi - 2[\Pi] \partial \pi \cdot [\Pi] \cdot \partial \pi - [\Pi^2] \partial \pi \cdot \partial \pi + 2\partial \pi \cdot \Pi^2 \cdot \partial \pi) \\
\mathcal{L}_5 &= -\frac{1}{5} ([\Pi]^3 \partial \pi \cdot \partial \pi - 3[\Pi]^2 \partial \pi \cdot \Pi \cdot \partial \pi - 3[\Pi][\Pi^2] \partial \pi \cdot \partial \pi + 6[\Pi] \partial \pi \cdot \Pi^2 \cdot \partial \pi \\
&\quad + 2[\Pi^3] \partial \pi \cdot \partial \pi + 3[\Pi^2] \partial \pi \cdot \Pi \cdot \partial \pi - 6\partial \pi \cdot \Pi^3 \cdot \partial \pi)
\end{align*}
\]

E.O.Ms are kept to be second order
DBI Galileon

de Rham, Tolley (2010)

DBI and Galileon are profoundly related through the higher dimensional picture with a brane

- Probe brane in 5D Minkowski bulk

Induced metric of the brane

\[ q_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi \]

Extrinsic curvature of the brane

\[ K_{\mu\nu} = -\gamma \partial_\mu \partial_\nu \pi \]

Lorentz factor

\[ \gamma = \frac{1}{\sqrt{1 + (\partial \pi)^2}} \]

Requiring E.O.Ms are kept to be second order

\[ \mathcal{L} = \sqrt{-q} \left( -\lambda - M_5^3 K + \frac{M^2}{2} R - \beta \frac{M^4}{M_5^3} K_{GB} \right) \]
Effective action

\[ \mathcal{L} = \sqrt{-q} \left( -\lambda - M_5^3 K + \frac{M^2}{2} R - \beta \frac{M^4}{M_5^3} \mathcal{K}_{GB} \right) \]

\[ -\lambda \sqrt{1 + (\partial \pi)^2} \]

DBI action

\[ e^{-4\pi/\ell} \left( -\lambda \sqrt{1 + e^{2\pi/\ell} (\partial \pi)^2 + \lambda} \right) \]

If we start with AdS bulk

Taking non-relativistic limit \( \gamma \to 1 \)

\[ -\frac{\lambda}{2} (\partial \pi)^2 + \frac{M_5^3}{2} (\partial \pi)^2 \square \pi \]

Galileon \( (\mathcal{L}_2 \text{ and } \mathcal{L}_3) \)

Similarly, we can show

\[ \mathcal{L}_R \to \mathcal{L}_4 \]

\[ \mathcal{L}_{GB} \to \mathcal{L}_5 \]

in the non-relativistic limit

Natural unification of the DBI and Galileon !!
1. Motivations
2. Model and linear perturbations
3. Non-Gaussianity (Bispectrum)
4. Multi-field extensions
5. Conclusions
Model

- Action

\[ S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ M_{\text{pl}}^2 R + 2P(\phi, X) - 2G(\phi, X) \Box \phi \right] \]

- Minimally coupled to gravity

- Including the DBI Galileons up to \( \mathcal{L}_3 \)

\[ X = -(1/2)(\partial \phi)^2 \]

\[ P(\phi, X) = -f(\phi)^{-1} \sqrt{1 - 2Xf(\phi) + f(\phi)^{-1}} + V(\phi), \quad G(\phi, X) = \frac{g(\phi)X}{1 - 2Xf(\phi)} \]

This extension is done in a similar way as

DBI inflation \( \longrightarrow \) K inflation

\[ -f(\phi)^{-1} \sqrt{1 - 2Xf(\phi)} + f(\phi)^{-1} + V(\phi) \quad \text{and} \quad P(\phi, X) \]

Furthermore, we assume "slow-varying"

\[ \epsilon \equiv |\dot{H}|/H^2 \ll 1, \quad |\dot{\phi}/(H\dot{\phi})| \ll 1, \quad |G_{,\phi}/(GH)| \ll 1 \]
Predictions from linear perturbations

- power spectrum

\[ P_\zeta = \frac{1}{8\pi^2 M^2_{pl}} \frac{H^2}{c_s \tilde{\epsilon}} \]

with

\[ c_s^2 = \frac{P_{,X} + 4\dot{\phi}HG_{,X}}{P_{,X} + 2XP_{,XX} + 6H\dot{\phi}(G_{,X} + XG_{,XX})} \]

\[ \tilde{\epsilon} = \epsilon + \nu \]

\[ \left( P_{,X} + 4\dot{\phi}HG_{,X} \right) = \left( P_{,X} + 3\dot{\phi}HG_{,X} \right) + \left( \dot{\phi}HG_{,X} \right) \times \frac{X}{M^2_{pl}H^2} \]

cf. k-inflation

- scalar spectral index

\[ n_s - 1 = -2\epsilon - \tilde{\eta} - s \]

with

\[ \tilde{\eta} \equiv \frac{\dot{\tilde{\epsilon}}}{\tilde{\epsilon}H}, \quad s \equiv \frac{\dot{c}_s}{c_s H} \]

\[ \epsilon, \tilde{\eta}, s \] very small for almost scale invariant power spectrum

- tensor to scalar ratio

\[ r = 16c_s \tilde{\epsilon} \]

consistency relation \[ r = -8c_s n_T \] is broken!!
Bispectrum

- 3rd order action

\[ \phi(t, x) = \phi_0(t) + Q(t, x) \] (flat gauge)

\[ S_3 = \int dt d^3x \frac{a^3}{\dot{\phi}} \left[ C_1 \dot{Q}^3 + \frac{C_2}{a^2} \dot{Q} \partial^i Q \partial_i Q + \frac{C_3}{a^4 H} \partial^i Q \partial_j Q \partial^j Q + \frac{C_4}{a^2 H} \dot{Q} \partial^i \dot{Q} \partial_i Q \right] \]

\[ C_1 = \frac{2}{3} X^2 P_{,XX} + XP_{,XX} + 2H \dot{\phi} X G_{,XX} + 5H \dot{\phi} X G_{,XX} + H \dot{\phi} G_{,X} , \]

\[ C_2 = - \left( XP_{,XX} + 3H \dot{\phi} X G_{,XX} + H \dot{\phi} G_{,X} \right) , \quad C_3 = \frac{1}{2} H \dot{\phi} G_{,X} , \quad C_4 = 2H \dot{\phi} G_{,X} + 2H \dot{\phi} X G_{,XX} \]

- bispectrum

\[ \langle Q(k_1)Q(k_2)Q(k_3) \rangle = \frac{(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3)}{(P_{,X} + 4\dot{\phi} HG_{,X})^3 \dot{\phi} \prod_{i=1}^{3} k_i^3 A_\phi} \frac{H^5}{1} \]

\[ A_\phi = 3 \left( C_1 - \frac{C_4}{C_3} \right) A_1 + \frac{C_2}{2c_s^2} A_2 + \frac{C_3}{c_s^4} A_3 \]

appear in k-inflation (known to be almost equilateral)
Momentum dependence of Bispectrum

- **shape functions**

\[ A_1/(k_1 k_2 k_3) \]
\[ A_2/(k_1 k_2 k_3) \]
\[ A_3/(k_1 k_2 k_3) \]

For all shapes, signal is maximum at \( k_1 \sim k_2 \sim k_3 \) → **Equilateral type**

- **Correlation with equilateral shape**

\[ A_3 : \text{better than } 99.9\% \]
\[ \text{cf. } A_1: 93.6\% \quad A_2: 99.5\% \]

→ we can measure NG by an equilateral-type template!!
Observational constraint

- non-linear parameter for equilaterial type NG

\[
f_{\text{NL}}^{\text{equil}} = \frac{10}{243} f^{(1)} - \frac{85}{81} f^{(2)} - \frac{65}{81} f^{(3)}
\]

with

\[
f^{(1)} = \frac{-3(C_3c^2 - C_4)}{(P_{,X} + 4\phi H G_{,X})}, \quad f^{(2)} = \frac{C_2}{2(P_{,X} + 4\phi H G_{,X})}, \quad f^{(3)} = \frac{-C_3}{(P_{,X} + 4\phi H G_{,X})c^2}
\]

\[
C_1 = \frac{2}{3}X^2P_{,XXX} + XP_{,XX} + 2H\phi X^2G_{,XXX} + 5H\phi X G_{,XX} + H\phi G_{,X},
\]

\[
C_2 = -\left(XP_{,XX} + 3H\phi XG_{,XX} + H\phi G_{,X}\right), \quad C_3 = \frac{1}{2}H\phi G_{,X}, \quad C_4 = 2H\phi G_{,X} + 2H\phi X G_{,XX}
\]

- Notice that \( f_{\text{NL}}^{\text{equil}} \sim 0 \) may not mean pure Gaussian.
  It may give orthogonal type non-Gaussianity.

- WMAP7 constraint

\[
-214 < f_{\text{NL}}^{\text{equil}} < 266
\]

\[
-410 < f_{\text{NL}}^{\text{orthog}} < 6
\]
Summary of single field model

- **DBI Galileon**

\[
P(\phi, X) = -f(\phi)^{-1}\sqrt{1 - 2Xf(\phi)} + f(\phi)^{-1} + V(\phi), \quad G(\phi, X) = \frac{g(\phi)X}{1 - 2Xf(\phi)}
\]

Assuming that the inflation is driven by the potential term \( V \) as in DBI inflation.

Non-linear parameter for equilateral type NG

\[
f_{\text{NL}}^{\text{equil}} = \frac{-5}{324c_s^2} \frac{(21 + 546b_D + 3776b_D^2 + 6048b_D^3)}{(1 + 4b_D)(1 + 12b_D)^2}
\]

with \( b_D = \frac{gH\dot{\phi}}{\sqrt{1 - 2fX^3}} \)

\[
f_{\text{NL}}^{\text{equil}} \approx -\frac{0.16}{c_s^2} \quad \text{for} \quad b_D \gg 1 \quad \text{and} \quad -\frac{0.32}{c_s^2} \quad \text{for} \quad b_D \rightarrow 0
\]

- **Related works for single field model**

  - **Yamaguchi**  NG from \( P(X, \phi) - G(X, \phi)\Box\phi \) up to sub-leading order
  - **De Felice**  NG from (further) generalized model
  - **Naruko**  Conservation of NL curvature perturbation
DBI Galileon with higher co-dimensions

- Induced metric

\[ q_{\mu \nu} = g_{\mu \nu} + f(\phi^I) G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J, \quad I = 1, \ldots, N \]

If we require the action to be scalar with respect to I, only tension and scalar curvature can be allowed!!

\[ \mathcal{L} = \sqrt{-q} \left( -\lambda - \frac{M^2}{2} K + \frac{M^2}{2} R - \beta \frac{M^4}{M^2_3} K_{\text{GB}} \right) \]

- Action

\[ S = \int d^4x \left[ \sqrt{-q} \left( -\frac{1}{f(\phi^I)} + M^2 R[q] \right) + \sqrt{-g} \left( \frac{M^2_P}{2} R[g] + \frac{1}{f(\phi^I)} - V(\phi^I) \right) \right] \]

Multi-field DBI inflation is recovered in the limit \( M^2 \to 0 \)
Derivation of the effective action

We need to rewrite every quantity with $q_{\mu\nu}$ in terms of the one with $g_{\mu\nu}$!!

The multi-field DBI action requires the introduction of the mixed kinetic terms:

$$X^{I,J} \equiv -\frac{1}{2} \partial_\mu \phi^I \partial_\nu \phi^J$$

- **Determinant**

$$\sqrt{-q} = \sqrt{g} \sqrt{\mathcal{D}}$$

with

$$\mathcal{D} = 1 - 2fX^I_I + 4f^2 X^I_I X^J_J - 8f^3 X^I_I X^J_J X^K_K + 16f^4 X^I_I X^J_J X^K_K X^L_L$$

- **Inverse metric**

$$q^{\mu\nu} = Ag^{\mu\nu} - A_{IJ} \nabla^\mu \phi^I \nabla^\nu \phi^J$$

with

$$A \equiv \frac{1}{\mathcal{D}} \left( 1 - 2fX^I_I + 4f^2 X^I_I X^J_J - 8f^3 X^I_I X^J_J X^K_K \right)$$

$$A_{IJ} \equiv \frac{f}{\mathcal{D}} \left( (1 - 2fX^A_A + 4f^2 X^B_B X^C_C)G_{IJ} + 2f(1 - 2fX^K_K)X_{IJ} + 4f^2 X_{IK}X^K_J \right)$$

Effective action with respect to $g_{\mu\nu}$
Second-order action

- Work at leading order in a slow-varying approximation \( \frac{\dot{Y}}{HY} \ll 1 \)
- Adopting a flat gauge \( \phi^I = \phi_0^I + Q^I \)
- Using adiabatic and entropy decomposition
  \( Q_\sigma \) \quad \( Q_{se} \)

\[
S_{(2)} = \frac{1}{2} \int dt \, d^3 x \, a^3 \left( \frac{\dot{Q}_\sigma^2}{c_D^2} \left( 1 - 6\alpha(3 - 2c_D^2) \right) - \frac{(\partial Q_\sigma)^2}{c_D a^2} \left( 1 - 2\alpha(5 - 2c_D^2) \right) + \frac{1 - 6\alpha}{c_D} \left( \dot{Q}_{se}^2 - c_D^2 \frac{(\partial Q_{se})^2}{a^2} \right) \right)
\]

with \( \alpha \equiv \frac{f H^2 M^2}{c_D^2} \), \( c_D \equiv \sqrt{1 - f \dot{\sigma}^2} \), \( \dot{\sigma} \equiv \sqrt{G_{IJ} \dot{\phi}_0^I \dot{\phi}_0^J} \)

- No ghosts for \( \alpha < \frac{1}{9 - 6c_D^2} \)
- different sound speeds \( \text{cf. Arroja, SM, Koyama, 2008} \)
Third-order action

\[
S_{(3)}^{\text{eff}} = \int dt \, d^3x \, a^3 \frac{f \dot{\sigma}}{2c_D^2} \left[ (1 - 6\alpha (5 - 2c_D^2 - 4\lambda^2 + \lambda^4)) \dot{Q}_\sigma^3 
- c_D^2 (1 - 2\alpha (9 - 2c_D^2 - 3\lambda^2)) \dot{Q}_\sigma \frac{(\partial Q_\sigma)^2}{a^2} 
+ c_D^2 \dot{Q}_\sigma \dot{Q}_{se}^2 + c_D^4 \dot{Q}_\sigma \frac{(\partial Q_{se})^2}{a^2} - 2c_D^4 (1 - 4\alpha) \frac{\partial Q_\sigma \partial Q_{se}}{a^2} \dot{Q}_{se} 
- 3\alpha c_D^4 \left( \frac{1}{\lambda^2} - 1 \right) \dot{Q}_{se}^2 \frac{a^2}{H a^2} + \alpha c_D^6 \left( \frac{1}{\lambda^2} - 1 \right) \frac{(\partial Q_{se})^2}{a^2} \frac{\partial^2 Q_\sigma}{H a^2} \right]
\]

with \( \lambda \equiv \frac{c_{s,\text{en}}}{c_{s,\text{ad}}} = \sqrt{\frac{1 - 6\alpha (3 - 2c_D^2)}{1 - 2\alpha (5 - 2c_D^2)}} \)

\[
\zeta \simeq A_\sigma Q_\sigma^* + A_{se} Q_{se}^* , \quad A_\sigma = \left( \frac{H}{\dot{\sigma}} \right)^* 
\]

- Bispectrum of curvature perturbation

\[
\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = A_\sigma^3 \langle Q_\sigma(k_1) Q_\sigma(k_2) Q_\sigma(k_3) \rangle + A_\sigma A_{se}^2 \langle Q_\sigma(k_1) Q_{se}(k_2) Q_{se}(k_3) \rangle + \text{perm.}
\]

(almost) equilateral new shape
Multi-field non-Gaussianity (new-effect)

- Bispectrum shape generated by $\dot{Q}_{se}^2 \partial^2 Q_{\sigma}$

\[ \lambda \equiv \frac{c_{s, en}}{c_{s, ad}} = 1.0 \]
\[ \lambda \equiv \frac{c_{s, en}}{c_{s, ad}} = 0.1 \]

``Equilateral''

``local''

For smaller $\lambda$, location of the maximum changes from Equilateral to local!!

Detailed analysis of the isocurvature-induced shape is under way. Two-parameters $(\alpha, c_D)$ family of shape
Conclusions

• Cosmological perturbations and Primordial non-Gaussianity from models with Galileon-like terms
  - DBI Galileon
    Obtained by a motion of a probe brane with induced gravity and recovers DBI and Galileon as special limits

• Single field model
  \[
  S = \frac{1}{2} \int d^4x \sqrt{-g}
  \left[ M_{pl}^2 R + 2P(\phi, X) - 2G(\phi, X)\Box\phi \right]
  \]
  template for the equilateral NG, \( f_{\text{NL}}^{\text{equil}} \), works well
  (other information is necessary to distinguish with other equilateral type models)

• Multi-field model
  \[
  S = \int d^4x \left[ \sqrt{-q} \left( -\frac{1}{f(\phi^I)} + M^2 R[q] \right) + \sqrt{-g} \left( \frac{M_{pl}^2 R[g]}{2} + \frac{1}{f(\phi^I)} - V(\phi^I) \right) \right]
  \]
  new shapes interpolating between equilateral shape and local shape
Background

- Friedmann eq.
  \[ 3M_{\text{pl}}^2 H^2 = 2P_{,X}X - P + 6G_{,X}H\dot{\phi}X - 2G_{,\phi}X \]

- Field eq.
  \[ P_{,X}(\dddot{\phi} + 3H\dot{\phi}) + 2P_{,XX}X\dddot{\phi} + 2P_{,X\phi}X - P_{,\phi} - 2G_{,\phi}(\dddot{\phi} + 3H\dot{\phi}) - 2G_{,X\phi}X(\dddot{\phi} - 3H\dot{\phi}) + 6G_{,X}[HX' + 3H^2X] - 2G_{,\phi\phi} + 6G_{,XX}HX\dot{X} = 0. \]

We are interested in inflationary background assuming \(|\epsilon| \ll 1, |\dddot{\phi}/(H\dot{\phi})| \ll 1, |G_{,\phi\phi}/(GH)| \ll 1\) slow-varying parameter

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{XP_{,X} + 3G_{,X}H\phi X}{M_{\text{pl}}^2 H^2} \]
Shape correlation (adiabatic-induced bispectrum)

- vs Equilateral shape
- vs Orthogonal shape

For the parameter region without correlation with the equilateral shape, correlation with the orthogonal shape becomes high!!
Modifed Friedmann equations

\[ \left(1 + \frac{M^2}{c_D^3 M_P^2}\right) 3H^2 M_P^2 = V + \frac{1}{f} \left( \frac{1}{c_D} - 1 \right) \]

\[ \left(1 + \frac{M^2}{c_D M_P^2}\right) M_P^2 \dot{H} = -\frac{\dot{\sigma}^2}{2c_D} + \frac{M^2}{2c_D} \left[ 3H^2 \left( \frac{1}{c_D^2} - 1 \right) \right] + 2H \frac{\dot{c}_D}{c_D} \]

1. Tends to violate the null energy condition
2. Gives a \( \mathcal{O}(1) \) (or greater) contribution to

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} \] for generic values of \( M \)
Inflation in the relativistic regime?

A critical dimensionless quantity that sets the strength of the induced gravity effects:
\[ \alpha \equiv \frac{fH^2M^2}{c^2D} \]

Conditions for achieving a phase of quasi de-Sitter acceleration in the ‘relativistic regime’ without ghosts:
\[ \epsilon \ll 1 \]
\[ c_D^2 \ll 1 \]
\[ \alpha \lesssim \frac{1}{9} \]

Barring cancellations in:
\[ \epsilon = \epsilon_{\text{kinetic}} - \epsilon_{\text{induced}} \]

- Usual DBI condition (potential domination)
\[ V \gg \frac{1}{fc_D} \]
\[ \frac{M^2}{c_D^3M_P^2} \ll 1 \]

- Requirement on the new mass scale