Workshop @ Yukawa Institute

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Primordial non-Gaussianity from the DBI Galileons

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S.M, K. Koyama, arXiv:1009.0677 [hep-th]

S. Renaux-Petel, S.M, K. Koyama, in preparation

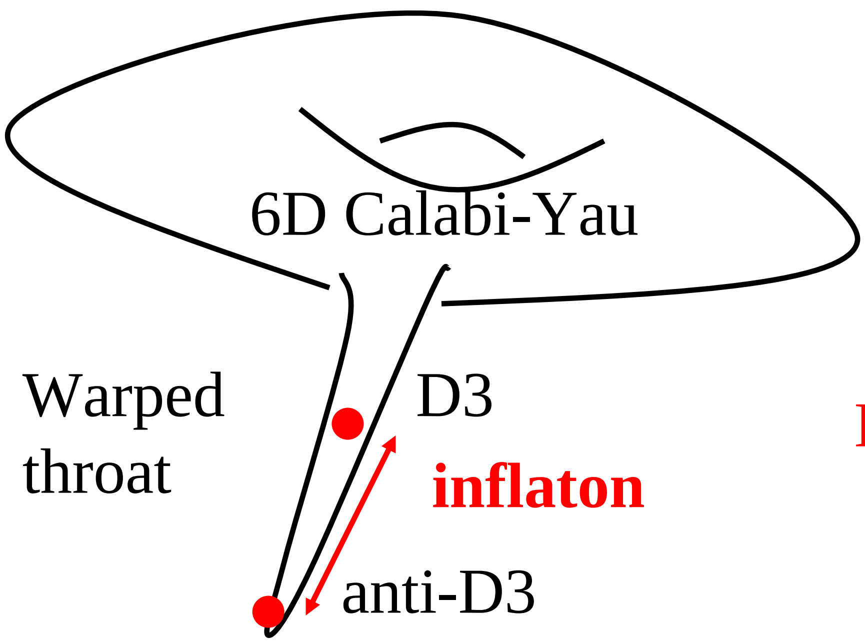
DBI inflation

Silverstein, Tong (2004)

Inflation is driven by a mobile D3-brane
with relativistic speed $\gamma = \dfrac{1}{\sqrt{1-\dot{\phi}^2/T(\phi)}}$

- Action

$$S = \int d^4\xi \sqrt{-g^{(4)}} \left[\underbrace{\boxed{-T(\phi)\sqrt{1+\partial^\mu\phi\partial_\mu\phi/T(\phi)} + T(\phi)}}_{\textbf{DBI part}} - V(\phi) \right]$$



$$d\phi = T_3^{1/2} d\rho, \quad T(\phi) = T_3^{1/2} h^4$$

ρ : radial position of the brane

Large equilateral non-Gaussianity

$$f_{\rm NL}^{equil} \propto \gamma^2 \propto c_s^{-2}$$

Galileon

Nicolis, Rattazzi, Trincherini (2008)

Deffayet, Esposito-Farese, Vikman (2009)

Self-accelerating universe without a ghost instability
from the terms consistent with Galileon symmetry

$\partial_\mu \pi \to \partial_\mu \pi + c_\mu$: Shift symmetry in gradient space

- Galileon invariant terms

$$\Pi^\mu_{\ \nu} \equiv \partial^\mu \partial_\nu \pi$$
$$[\Pi] \equiv \partial^\mu \partial_\mu \pi$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= -\frac{1}{2}\partial\pi\cdot\partial\pi \\
\mathcal{L}_3 &= -\frac{1}{2}[\Pi]\partial\pi\cdot\partial\pi \\
\mathcal{L}_4 &= -\frac{1}{4}([\Pi]^2\partial\pi\cdot\partial\pi - 2[\Pi]\partial\pi\cdot[\Pi]\cdot\partial\pi - [\Pi^2]\partial\pi\cdot\partial\pi + 2\partial\pi\cdot\Pi^2\cdot\partial\pi) \\
\mathcal{L}_5 &= -\frac{1}{5}([\Pi]^3\partial\pi\cdot\partial\pi - 3[\Pi]^2\partial\pi\cdot\Pi\cdot\partial\pi - 3[\Pi][\Pi^2]\partial\pi\cdot\partial\pi + 6[\Pi]\partial\pi\cdot\Pi^2\cdot\partial\pi \\
&\qquad +2[\Pi^3]\partial\pi\cdot\partial\pi + 3[\Pi^2]\partial\pi\cdot\Pi\cdot\partial\pi - 6\partial\pi\cdot\Pi^3\cdot\partial\pi)
\end{aligned}$$

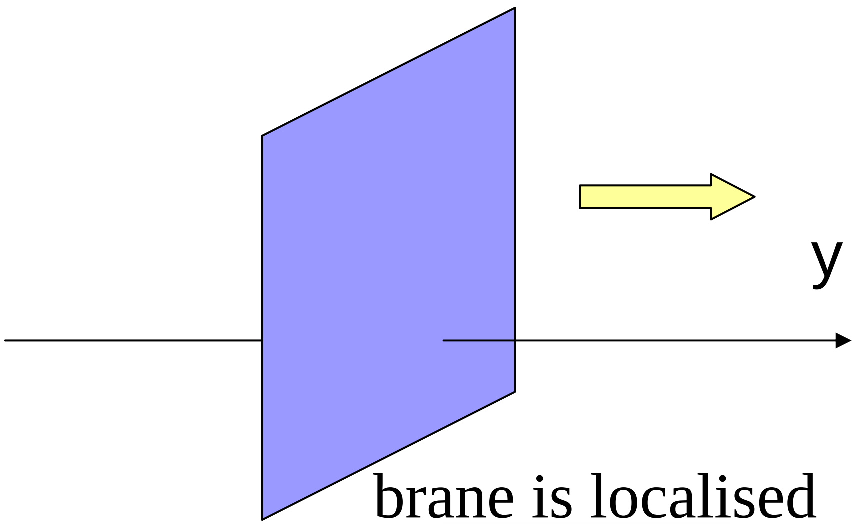
$\Rightarrow$ E.O.Ms are kept to be second order

DBI Galileon

de Rham, Tolley (2010)

DBI and Galileon are profoundly related through the higher dimensional picture with a brane

- Probe brane in 5D Minkowski bulk



brane is localised at $y = \pi(x^\mu)$

Induced metric of the brane

$$q_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

Extrinsic curvature of the brane

$$K_{\mu\nu} = -\gamma \partial_\mu \partial_\nu \pi$$

Lorentz factor $\gamma = \dfrac{1}{\sqrt{1+(\partial\pi)^2}}$

Requiring E.O.Ms are kept to be second order

$$\mathcal{L} = \sqrt{-q}\left(-\lambda - M_5^3 K + \frac{M^2}{2} R - \beta \frac{M^4}{M_5^3} \mathcal{K}_{\rm GB}\right)$$

DBI Galileon (cont.)

- Effective action

$$\mathcal{L} = \sqrt{-q}\left(\underline{-\lambda - M_5^3 K} + \frac{M^2}{2}R - \beta\frac{M^4}{M_5^3}\mathcal{K}_{\rm GB}\right)$$

taking non-relativistic limit $\gamma \to 1$

$$-\lambda\sqrt{1+(\partial\pi)^2}$$

DBI action

$$e^{-4\pi/\ell}\left(-\lambda\sqrt{1+e^{2\pi/\ell}(\partial\pi)^2}+\lambda\right)$$

If we start with AdS bulk

$$-\frac{\lambda}{2}(\partial\pi)^2 + \frac{M_5^3}{2}(\partial\pi)^2\,\Box\pi$$

Galileon $(\mathcal{L}_2 \;\text{ and }\; \mathcal{L}_3)$

Similarly, we can show

$$\mathcal{L}_R \longrightarrow \mathcal{L}_4$$

$$\mathcal{L}_{GB} \longrightarrow \mathcal{L}_5$$

in the non-relativistic limit

Natural unification of the DBI and Galileon !!

1. Motivations

2. Model and linear perturbations

3. Non-Gaussianity (Bispectrum)

4. Multi-field extensions

5. Conclusions

Model

Kobayashi, Yamaguchi, Yokoyama (2010)

see also Deffayet et al (2010)

- Action

$$\boxed{S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(\phi, X) - 2G(\phi, X)\Box\phi]}$$

- Minimally coupled to gravity

$$X = -(1/2)(\partial\phi)^2$$

- Including the DBI Galileons up to $\mathcal{L}_3$

cf. $P(\phi, X) = -f(\phi)^{-1}\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} + V(\phi)\,,\ \ G(\phi, X) = \dfrac{g(\phi)X}{1-2Xf(\phi)}$

This extension is done in a similar way as

DBI inflation $\longrightarrow$ K inflation

$-f(\phi)^{-1}\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} + V(\phi)$ $\qquad$ $P(\phi, X)$

Furthermore, we assume ``slow-varying"

$$\epsilon \equiv |\dot{H}|/H^2 \ll 1\ ,\ |\ddot{\phi}/(H\dot{\phi})| \ll 1\ ,\ |G_{,\phi}\dot{\phi}/(GH)| \ll 1$$

Predictions from linear perturbations

- power spectrum

Yamaguchi-san's talk

$$\boxed{\mathcal{P}_\zeta = \frac{1}{8\pi^2 M_{pl}^2}\frac{H^2}{c_s\tilde{\epsilon}}}$$

with $c_s^2 = \dfrac{P_{,X} + \underline{4\dot{\phi}HG_{,X}}}{P_{,X} + 2XP_{,XX} + \underline{6H\dot{\phi}(G_{,X} + XG_{,XX})}}$

$$\tilde{\epsilon} \quad = \quad \epsilon \quad + \quad \nu$$

cf. k-inflation $\left[(P_{,X} + \underline{4\dot{\phi}HG_{,X}}) = (P_{,X} + \underline{3\dot{\phi}HG_{,X}}) + (\underline{\dot{\phi}HG_{,X}})\right] \times \dfrac{X}{M_{pl}^2 H^2}$

- scalar spectral index

$$n_s - 1 = -2\epsilon - \underline{\tilde{\eta}} - s$$

with $\tilde{\eta} \equiv \dfrac{\dot{\tilde{\epsilon}}}{\tilde{\epsilon}H}, \quad s \equiv \dfrac{\dot{c}_s}{c_s H}$

$\Rightarrow$ $\epsilon, \quad \tilde{\eta}, \quad s$ very small for almost scale invariant power spectrum

- tensor to scalar ratio

$r = 16c_s\tilde{\epsilon}$ consistency relation $r = -8c_s n_T$ is broken!!

Bispectrum

- 3 rd order action $\phi(t,x)=\phi_0(t)+Q(t,x)$ (flat gauge)

$$S_3 = \int dtd^3x \frac{a^3}{\dot\phi}\left[\underline{C_1\dot Q^3} + \underline{\frac{C_2}{a^2}\dot Q\partial^i Q\partial_i Q} + \underline{\frac{C_3}{a^4 H}\partial^i Q\partial_i Q\partial^j\partial_j Q} + \underline{\frac{C_4}{a^2 H}\dot Q\partial^i \dot Q\partial_i Q}\right]$$

$$C_1 = \frac{2}{3}X^2P_{,XXX}+XP_{,XX}+2H\dot\phi X^2G_{,XXX}+5H\dot\phi XG_{,XX}+H\dot\phi G_{,X}\,,$$

$$C_2=-\left(XP_{,XX}+3H\dot\phi XG_{,XX}+H\dot\phi G_{,X}\right)\,,\qquad C_3=\frac{1}{2}H\dot\phi G_{,X}\,,\qquad C_4=2H\dot\phi G_{,X}+2H\dot\phi XG_{,XX}$$

in-in formalism

- bispectrum

$$\langle Q(\mathbf{k_1})Q(\mathbf{k_2})Q(\mathbf{k_3})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\frac{H^5}{(P_{,X}+4\dot\phi HG_{,X})^3\dot\phi}\frac{1}{\Pi_{i=1}^3 k_i^3}\underline{\mathcal{A}_\phi}$$

shape function
(momentum dep.)

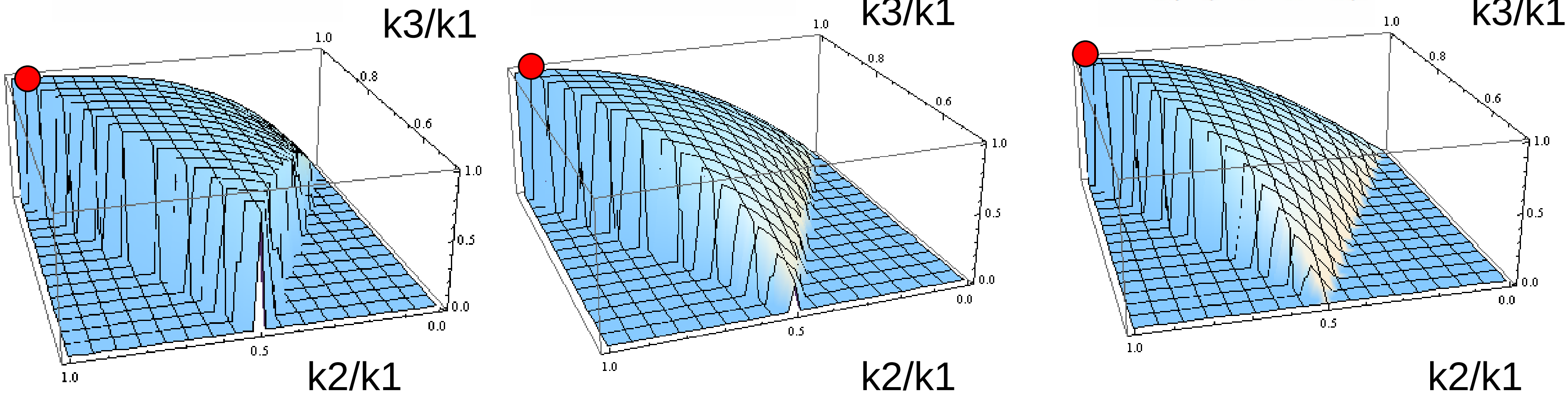
$$\mathcal{A}_\phi = 3\left(C_1-\frac{C_4}{c_s^2}\right)\underline{\mathcal{A}_1}+\frac{C_2}{2c_s^2}\underline{\mathcal{A}_2}+\frac{C_3}{c_s^4}\underline{\mathcal{A}_3}$$

appear in k-inflation (known to be almost equilateral)

Momentum dependence of Bispectrum

- shape functions

$\mathcal{A}_1/(k_1k_2k_3)$ $\mathcal{A}_2/(k_1k_2k_3)$ $\mathcal{A}_3/(k_1k_2k_3)$



For all shapes, signal is maximum at k1 ~ k2 ~ k3 ⟶ Equilateral type

- Correlation with equilateral shape

$\mathcal{A}_3$: better than 99.9% cf. $\mathcal{A}_1$: 93.6% $\mathcal{A}_2$: 99.5%

⇨ we can measure NG by an equilateral-type template!!

Observational constraint

- non-linear parameter for equilateral type NG

$$f_{\rm NL}^{equil} = \frac{10}{243} f^{(1)} - \frac{85}{81} f^{(2)} - \frac{65}{81} f^{(3)}$$

with $f^{(1)} = \frac{-3(C_1 c_s^2 - C_4)}{(P_{,X} + 4\dot{\phi} H G_{,X})}\,,\quad f^{(2)} = -\frac{C_2}{2(P_{,X} + 4\dot{\phi} H G_{,X})}\,,\quad f^{(3)} = -\frac{C_3}{(P_{,X} + 4\dot{\phi} H G_{,X}) c_s^2}$

$$C_1 = \frac{2}{3} X^2 P_{,XXX} + X P_{,XX} + 2H\dot{\phi} X^2 G_{,XXX} + 5H\dot{\phi} X G_{,XX} + H\dot{\phi} G_{,X}\,,$$

$$C_2 = -\left(X P_{,XX} + 3H\dot{\phi} X G_{,XX} + H\dot{\phi} G_{,X}\right)\,,\quad C_3 = \frac{1}{2} H\dot{\phi} G_{,X}\,,\quad C_4 = 2H\dot{\phi} G_{,X} + 2H\dot{\phi} X G_{,XX}$$

- Notice that $f_{\rm NL}^{\rm equil} \sim 0$ may not mean pure Gaussian.

  It may give orthogonal type non-Gaussianity.

- WMAP7 constraint

$$-214 < f_{\rm NL}^{equil} < 266 \qquad\qquad -410 < f_{\rm NL}^{\rm orthog} < 6$$

Summary of single field model

- DBI Galileon

$$P(\phi, X) = -f(\phi)^{-1}\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} + \underline{V(\phi)}, \quad G(\phi, X) = \frac{g(\phi)X}{1-2Xf(\phi)}$$

Assuming that the inflation is driven by the potential term V as in DBI inflation

non-linear parameter for equilateral type NG

$$\boxed{f_{\rm NL}^{equil} = -\frac{5}{324c_s^2}\frac{(21+546b_D+3776b_D^2+6048b_D^3)}{(1+4b_D)(1+12b_D)^2}}$$

with $b_D \equiv \dfrac{gH\dot{\phi}}{\sqrt{1-2fX}^3}$

$\longrightarrow$ $-0.16/c_s^2$ for $b_D \gg 1$ and $-0.32/c_s^2$ for $b_D \to 0$

$$f_{\rm NL}^{equil} \simeq -20\frac{(1-n_s)^2}{r^2}$$

$\longleftarrow$ almost indep. of b_D

- Related works for single field model

Yamaguchi — NG from $P(X,\phi) - G(X,\phi)\Box\phi$ up to sub-leading order

De Felice — NG from (further) generalized model

Naruko — Conservation of NL curvature perturbation

DBI Galileon with higher co-dimensions

- Induced metric

$$q_{\mu\nu} = g_{\mu\nu} + f(\phi^I) G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J \;, \quad I = 1, \ldots, N$$

If we require the action to be scalar with respect to I, only tension and scalar curvature can be allowed!!

$$\mathcal{L} = \sqrt{-q}\left(-\lambda - \cancel{M_5^3 K} + \frac{M^2}{2} R - \cancel{\beta \frac{M^4}{M_5^3} \mathcal{K}_{\rm GB}} \right)$$

- Action

$$\boxed{S = \int d^4x \left[\sqrt{-q}\left(-\frac{1}{f(\phi^I)} + M^2 R[q] \right) + \sqrt{-g}\left(\frac{M_P^2}{2} R[g] + \frac{1}{f(\phi^I)} - V(\phi^I) \right) \right]}$$

Multi-field DBI inflation is recovered in the limit $M^2 \longrightarrow 0$

Derivation of the effective action

We need to rewrite every quantity with $q_{\mu\nu}$ in terms of the one with $g_{\mu\nu}$!!

The multi-field DBI action requires the introduction of the mixed kinetic terms:

$$X^{IJ} \equiv -\frac{1}{2}\partial_\mu \phi^I \partial^\mu \phi^J$$

- Determinant

$$\boxed{\sqrt{-q} = \sqrt{-g}\sqrt{\mathcal{D}}}$$

with $\mathcal{D} = 1 - 2fX_I^I + 4f^2 X_I^{[I} X_J^{J]} - 8f^3 X_I^{[I} X_J^J X_K^{K]} + 16f^4 X_I^{[I} X_J^J X_K^K X_L^{L]}$

- Inverse metric

$$\boxed{q^{\mu\nu} = Ag^{\mu\nu} - A_{IJ}\nabla^\mu \phi^I \nabla^\nu \phi^J}$$

with $A \equiv \frac{1}{\mathcal{D}}\left(1 - 2fX_I^I + 4f^2 X_I^{[I} X_J^{J]} - 8f^3 X_I^{[I} X_J^J X_K^{K]}\right)$

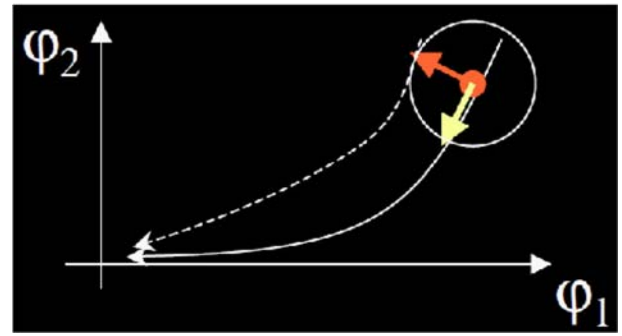
$$A_{IJ} \equiv \frac{f}{\mathcal{D}}\left((1 - 2fX_A^A + 4f^2 X_B^{[B} X_C^{C]})G_{IJ} + 2f(1 - 2fX_K^K)X_{IJ} + 4f^2 X_{IK} X_J^K\right)$$

$\Rightarrow$ Effective action with respect to $g_{\mu\nu}$

Second-order action

- Work at leading order in a slow-varying approximation $\frac{\dot{Y}}{HY} \ll 1$
- Adopting a flat gauge $\phi^I = \phi_0^I + Q^I$
- Using adiabatic and entropy decomposition

Q_σ Q_{se}



$$S_{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\frac{\dot{Q}_\sigma^2}{c_D^3}\left(1-6\alpha(3-2c_D^2)\right) - \frac{(\partial Q_\sigma)^2}{c_D a^2}\left(1-2\alpha(5-2c_D^2)\right) + \frac{1-6\alpha}{c_D}\left(\dot{Q}_{se}^2 - c_D^2\frac{(\partial Q_{se})^2}{a^2}\right)\right)$$

with $\alpha \equiv \frac{fH^2M^2}{c_D^2}$, $c_D \equiv \sqrt{1-f\dot{\sigma}^2}$, $\dot{\sigma} \equiv \sqrt{G_{IJ}\dot{\phi}_0^I\dot{\phi}_0^J}$

- No ghosts for $\alpha < \frac{1}{9-6c_D^2}$
- different sound speeds

cf. Arroja, SM, Koyama, 2008

Third-order action

$$S^{\rm eff}_{(3)} = \int {\rm d}t\,{\rm d}^3x\,a^3\frac{f\dot\sigma}{2c_D^5}\Big[\left(1-6\alpha\left(5-2c_D^2-4\lambda^2+\lambda^4\right)\right)\dot Q_\sigma^3 \\ -c_D^2\left(1-2\alpha\left(9-2c_D^2-3\lambda^2\right)\right)\dot Q_\sigma\frac{(\partial Q_\sigma)^2}{a^2} \\ +c_D^2\dot Q_\sigma\dot Q_{se}^2+c_D^4\dot Q_\sigma\frac{(\partial Q_{se})^2}{a^2}-2c_D^4(1-4\alpha)\frac{\partial Q_\sigma\partial Q_{se}}{a^2}\dot Q_{se} \\ -3\alpha c_D^4\left(\frac{1}{\lambda^2}-1\right)\dot Q_{se}^2\frac{\partial^2 Q_\sigma}{Ha^2}+\alpha c_D^6\left(\frac{1}{\lambda^2}-1\right)\frac{(\partial Q_{se})^2}{a^2}\frac{\partial^2 Q_\sigma}{Ha^2}\Big]$$

with $\lambda \equiv \dfrac{c_{s,en}}{c_{s,ad}} = \sqrt{\dfrac{1-6\alpha(3-2c_D^2)}{1-2\alpha(5-2c_D^2)}}$

$$\zeta \simeq \mathcal{A}_\sigma Q_{\sigma *} + \underline{\mathcal{A}_{se}} Q_{se*}\,, \quad \mathcal{A}_\sigma = \left(\frac{H}{\dot\sigma}\right)_*$$

transfer from
entropy perturbation

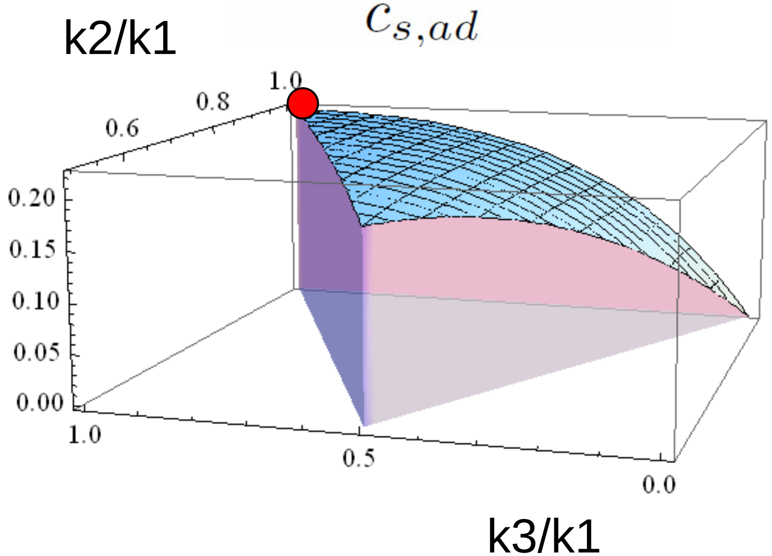
- Bispectrum of curvature perturbation

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = \mathcal{A}_\sigma^3\underbrace{\langle Q_\sigma(\mathbf{k}_1)Q_\sigma(\mathbf{k}_2)Q_\sigma(\mathbf{k}_3)\rangle}_{\text{(almost) equilateral}} + \mathcal{A}_\sigma\mathcal{A}_{se}^2\underbrace{\langle Q_\sigma(\mathbf{k}_1)Q_{se}(\mathbf{k}_2)Q_{se}(\mathbf{k}_3)\rangle}_{\text{new shape}} + \text{perm.}$$

Multi-field non-Gaussianity (new-effect)

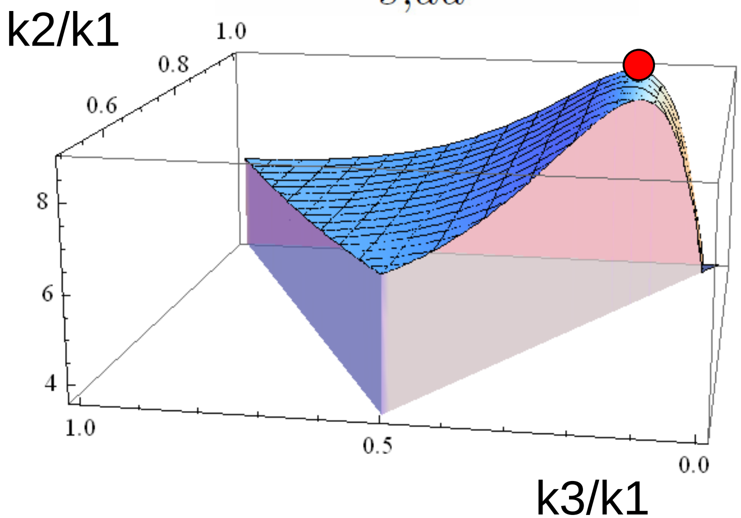
- Bispectrum shape generated by $\dot{Q}_{se}^2 \partial^2 Q_\sigma$

$$\lambda \equiv \frac{c_{s,en}}{c_{s,ad}} = 1.0$$



``Equilateral”

$$\lambda \equiv \frac{c_{s,en}}{c_{s,ad}} = 0.1$$



``local”

For smaller λ , location of the maximum changes from Equilateral to local !!

Detailed analysis of the isocurvature-induced shape is under way.
two-parameters (α , c_D) family of shape

Conclusions

- Cosmological perturbations and Primordial non-Gaussianity from models with Galileon-like terms

  - DBI Galileon

    Obtained by a motion of a probe brane with induced gravity and recovers DBI and Galileon as special limits

- Single field model

$$\boxed{S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(\phi, X) - 2G(\phi, X)\Box\phi]}$$

⇨ template for the equilateral NG, $f_{\rm NL}^{equil}$, works well

(other information is necessary to distinguish with other equilateral type models)

- Multi-field model

$$\boxed{S = \int d^4x\left[\sqrt{-q}\left(-\frac{1}{f(\phi^I)} + M^2 R[q]\right) + \sqrt{-g}\left(\frac{M_P^2}{2}R[g] + \frac{1}{f(\phi^I)} - V(\phi^I)\right)\right]}$$

⇨ new shapes interpolating between equilateral shape and local shape

Background

- Friedmann eq.

$$3M_{pl}^2H^2 = 2P_{,X}X - P + \underline{6G_{,X}H\dot{\phi}X - \cancel{2G_{,\phi}X}}$$

- Field eq.

$$P_{,X}(\cancel{\ddot{\phi}} + 3H\dot{\phi}) + \cancel{2P_{,XX}X\ddot{\phi}} + \cancel{2P_{,X\phi}X} - P_{,\phi} - \underline{\cancel{2G_{,\phi}(\ddot{\phi} + 3H\dot{\phi}) - 2G_{,X\phi}X(\ddot{\phi} - 3H\dot{\phi})}}$$
$$\underline{+6G_{,X}[(\cancel{HX})^{\cdot} + 3H^2X] - \cancel{2G_{,\phi\phi} + 6G_{,XX}HX\dot{X}}} = 0\,.$$

new terms

vanish at slow-varying

We are interested in inflationary background

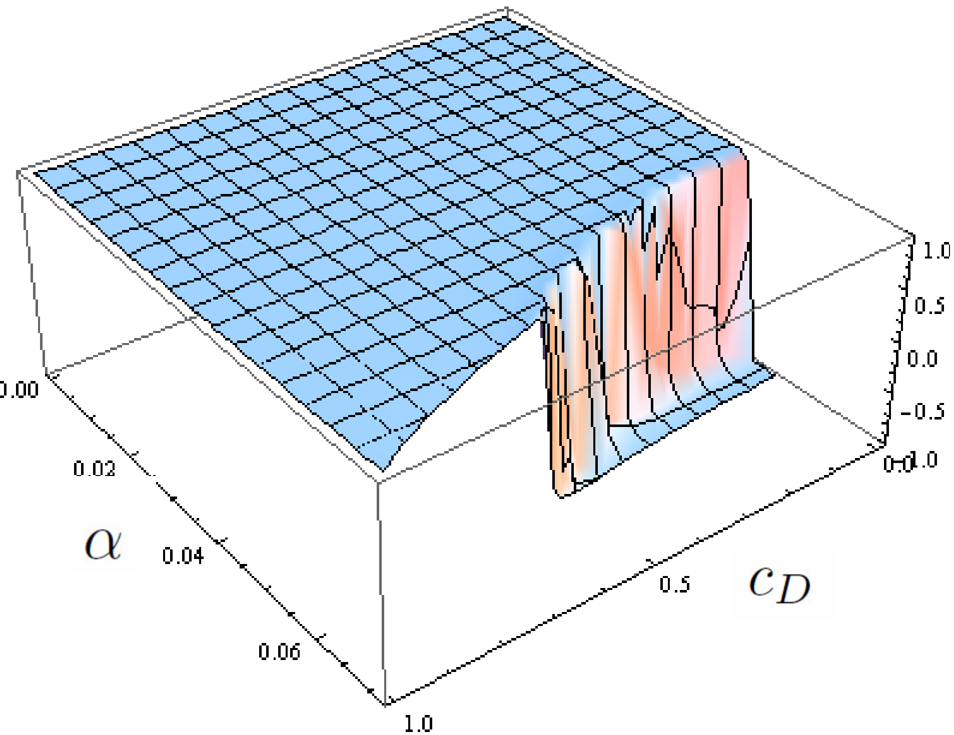
⇒ assuming $|\epsilon| \ll 1$, $|\ddot{\phi}/(H\dot{\phi})| \ll 1$, $|G_{,\phi}\dot{\phi}/(GH)| \ll 1$

slow-varying parameter

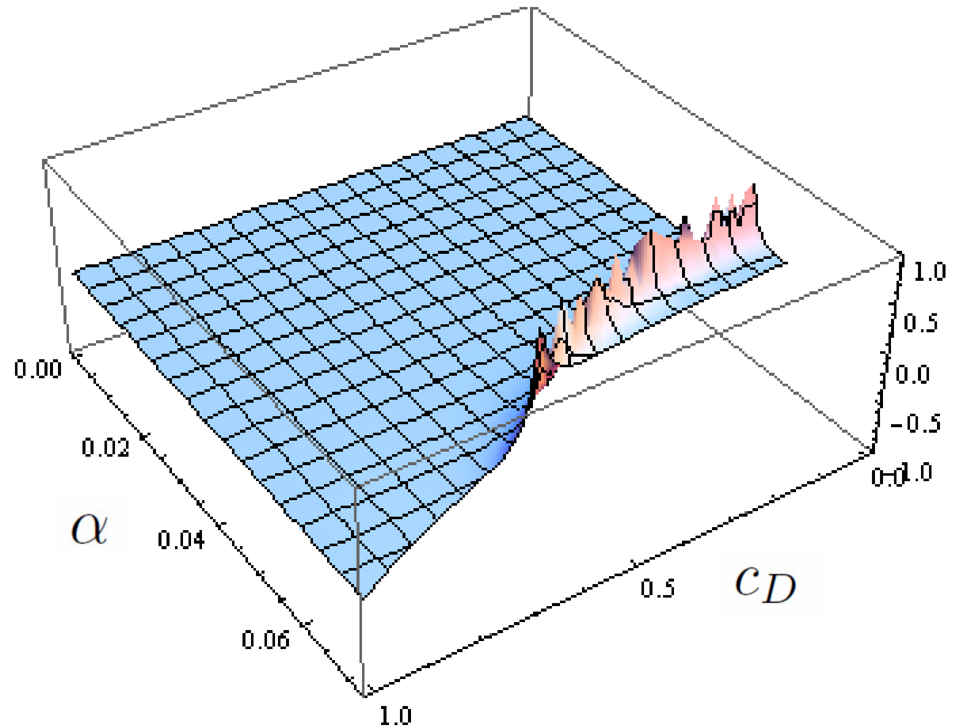
$$\boxed{\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{XP_{,X} + 3G_{,X}H\dot{\phi}X}{M_{pl}^2H^2}}$$

Shape correlation (adiabatic-induced bispectrum)

- vs Equilateral shape



- vs Orthogonal shape



For the parameter region without correlation with the equilateral shape, correlation with the orthogonal shape becomes high!!

Modifed Friedmann equations

$$\left(1+\frac{M^2}{c_D^3 M_P^2}\right)3H^2M_P^2 = V + \frac{1}{f}\left(\frac{1}{c_D}-1\right)$$

$$\left(1+\frac{M^2}{c_D M_P^2}\right)M_P^2\dot{H} = -\frac{\dot{\sigma}^2}{2c_D} + \frac{M^2}{2c_D}\left[3H^2\left(\frac{1}{c_D^2}-1\right)+2H\frac{\dot{c}_D}{c_D}\right]$$

1. Tends to violate the null energy condition
2. Gives a $\mathcal{O}(1)$ (or greater) contribution to

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$ for generic values of M

Inflation in the relativistic regime ?

A critical dimensionless quantity that sets the strength of the induced gravity effects: $\alpha \equiv \frac{f H^2 M^2}{c_D^2}$

Conditions for achieving a phase of quasi de-Sitter acceleration in the 'relativistic regime' without ghosts ?

$$\epsilon \ll 1$$
$$c_D^2 \ll 1$$
$$\alpha \lesssim \frac{1}{9}$$

Barring cancellations in $\epsilon = \epsilon_{\text{kinetic}} - \epsilon_{\text{induced}}$:

- * Usual DBI condition (potential domination) $V \gg \frac{1}{f c_D}$

- * Requirement on the new mass scale $\frac{M^2}{c_D^3 M_P^2} \ll 1$