Future crossing of the phantom divide in *f*(*R*) gravity

Reference:

K. Bamba, C. Q. Geng and C. C. Lee, JCAP 1011, 001 (2010) [arXiv:1007.0482 [astro-ph.CO]].

Two-day workshop in the YITP molecule-type workshop "Cosmological Perturbation and Cosmic Microwave Background",

Panasonic Auditorium at Yukawa Institute for Theoretical Physics on 22nd March, 2011

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< Contents >

* We use the ordinary metric formalism, in which the connection is written by the differentiation of the metric.

I. Introduction

- Recent observations of Supernova (SN) Ia confirmed that the current expansion of the universe is accelerating.

  [Perlmutter *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999)]

  [Riess *et al*. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998)]

  [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)]

- There are two approaches to explain the current cosmic acceleration. **[Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)]** **[Tsujikawa, arXiv:1004.1493 [astro-ph.CO]]**

< Gravitational field equation >

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

Gravity **Matter**

$G_{\mu\nu}$: Einstein tensor

$T_{\mu\nu}$: Energy-momentum tensor

$$\kappa^2 \equiv 8\pi / {M_{\rm Pl}}^2$$

$M_{\rm Pl}$: Planck mass

(1) **General relativistic approach** ⟶ **Dark Energy**

(2) **Extension of gravitational theory**

(1) General relativistic approach

- **Cosmological constant**
- **Scalar fields**: **X matter**, **Quintessence**, **Phantom**, **K-essence**, **Tachyon**.
- **Fluid**: **Chaplygin gas**

$f(R)$: Arbitrary function of the Ricci scalar R

(2) Extension of gravitational theory

- ***f*(*R*) gravity** **[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]** **[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]**
- **Scalar-tensor theories** **[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]**
- **Ghost condensates** **[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]**
- **Higher-order curvature term**
- $f(\mathcal{G})$ **gravity**

$\mathcal{G}$: Gauss-Bonnet term

- **DGP braneworld scenario** **[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]**
- ***f*(*T*) gravity** **[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]** **[Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]**
- **Galileon gravity** **[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]**

T : torsion scalar

< Flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time >

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\boldsymbol{x}^2$$

$a(t)$: Scale factor

< Equation for $a(t)$ with a perfect fluid >

$$T_{\mu\nu} = \mathrm{diag}(\rho, P, P, P)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(1+3w)\rho$$

ρ : Energy density

P : Pressure

$\dot{} = \partial/\partial t$

$$w \equiv \frac{P}{\rho}$$

: Equation of state (EoS)

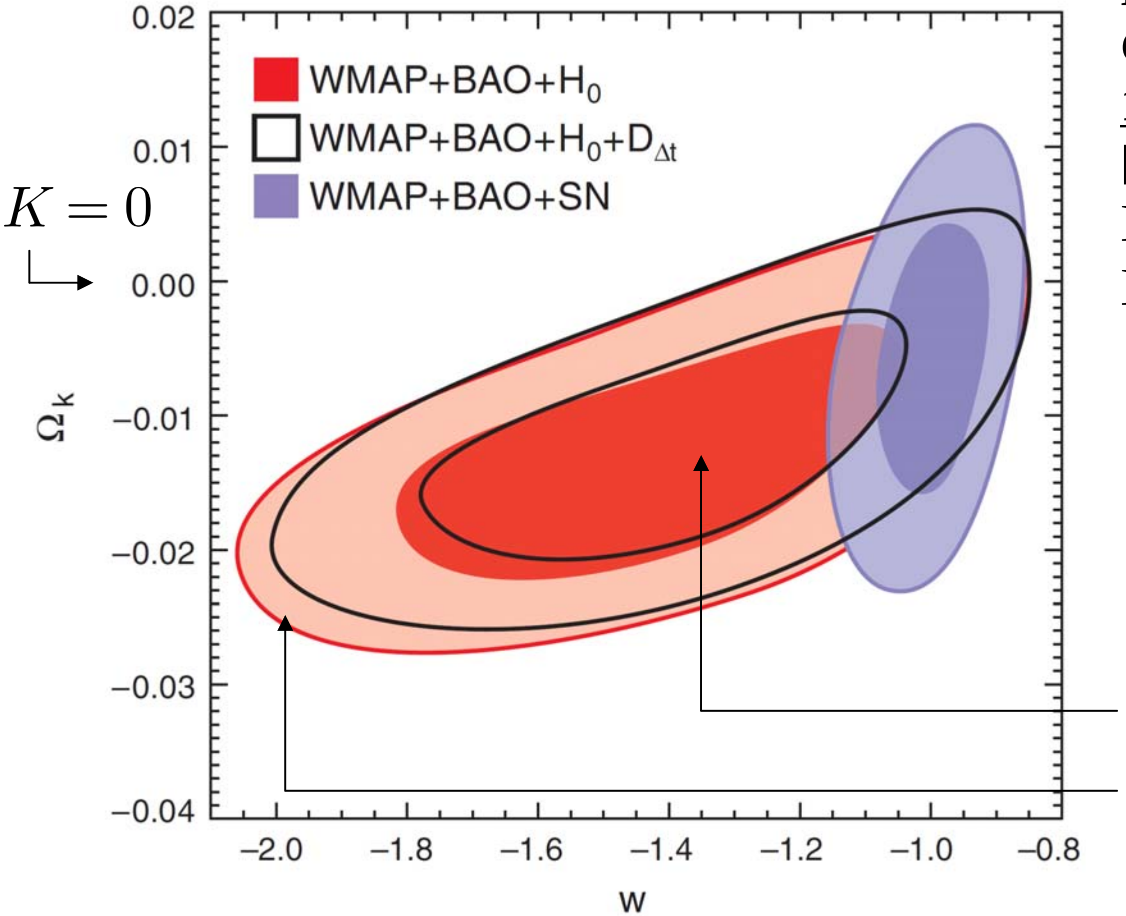
$\ddot{a} > 0$: **Accelerated expansion**

$$w < -\frac{1}{3}$$

: **Condition for accelerated expansion**

Cf. Cosmological constant $\Longrightarrow w = -1$

< 7-year WMAP data on the current value of w >



From [E. Komatsu *et al*. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]]].

Hubble constant (H_0) measurement

Baryon acoustic oscillation (BAO)

: Special pattern in the large-scale correlation function of Sloan Digital Sky Survey (SDSS) luminous red galaxies

$D_{\Delta t}$: Time delay distance

$$\Omega_K \equiv \frac{K}{(a_0 H_0)^2}$$

: Density parameter for the curvature

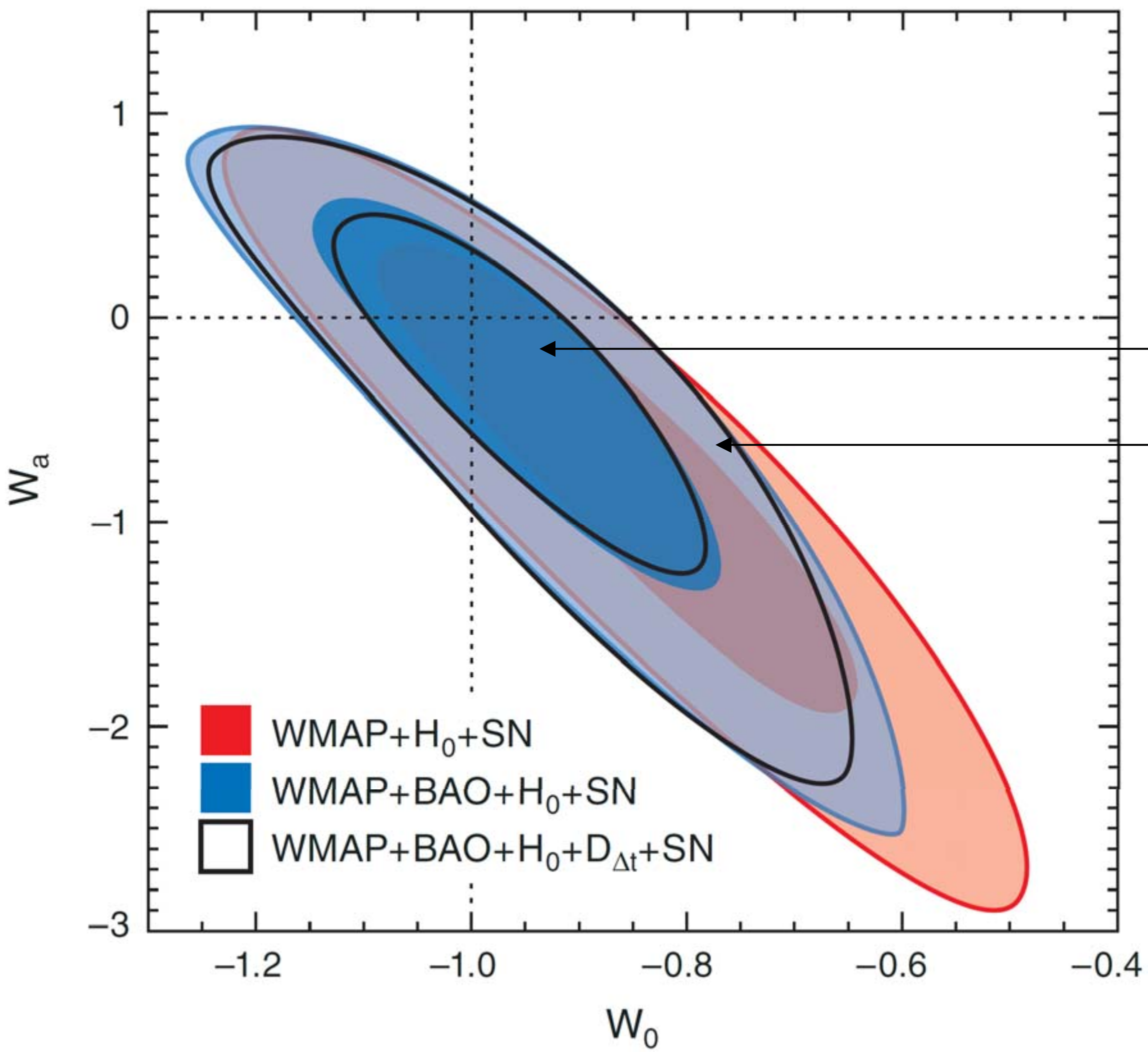
$K = 0$: Flat universe

- For the flat universe, constant w:

$$w = -1.10 \pm 0.14 \ (68\% \ \mathrm{CL})$$

(From $WMAP$ +BAO+H_0 .)

Cf. $\Omega_\Lambda = 0.725 \pm 0.016$ (68% CL)



From [E. Komatsu *et al*. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]]].

Time-dependent w

$$w(a) = w_0 + w_a (1 - a)$$

$$a = \frac{1}{1+z}$$

w_0 : Current value of w

z : Redshift

(From *WMAP*+BAO +H_0+SN.)

▪ For the flat universe, a variable EoS :

$$w_0 = -0.93 \pm 0.13\,,\ w_a = -0.41^{+0.72}_{-0.71} \quad (68\%\ \mathrm{CL})$$

< $f(R)$ gravity >

$$S = \int d^4x \sqrt{-g}\, \frac{f(R)}{2\kappa^2}$$

$f(R)$ gravity

Cf. $f(R) = R$: General Relativity

[Nojiri and Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [arXiv:hep-th/0601213]; arXiv:1011.0544 [gr-qc]]

[Capozziello and Francaviglia, Gen. Rel. Grav. 40, 357 (2008)]

[Sotiriou and Faraoni, Rev. Mod. Phys. 82, 451 (2010)]

[De Felice and Tsujikawa, Living Rev. Rel. 13, 3 (2010)]

< Conditions for the viability of $f(R)$ gravity >

(1) $f'(R) > 0$ $\longleftarrow$ **Positivity of the effective gravitational coupling**

$f'(R) \equiv df(R)/dR$ $\quad G_{\rm eff} = G/f'(R) > 0$ $\quad G$: Gravitational constant

(2) $f''(R) > 0$ $\longleftarrow$ **Stability condition:** $M^2 \approx 1/(3f''(R)) > 0$

$f''(R) \equiv d^2f(R)/dR^2$

[Dolgov and Kawasaki, Phys. Lett. B 573, 1 (2003)]

M : Mass of a new scalar degree of freedom ("scalaron") in the weak-field regime.

(3) $f(R) \to R - 2\Lambda$ for $R \gg R_0$. $\longleftarrow$ **Existence of a matter-dominated stage**

R_0 : Current curvature, Λ : Cosmological constant

(4) $0 < m \equiv Rf''(R)/f'(R) < 1$ $\longleftarrow$ **Stability of the late-time de Sitter point**

[Amendola, Gannouji, Polarski and Tsujikawa, Phys. Rev. D 75, 083504 (2007)]

Cf. For general relativity, $m = 0$.

[Amendola and Tsujikawa, Phys. Lett. B 660, 125 (2008)]

[Faraoni and Nadeau, Phys. Rev. D 75, 023501 (2007)]

(5) **Constraints from the violation of the equivalence principle**

$M = M(R)$ $\longleftarrow$ Scale-dependence : **"Chameleon mechanism"**

Cf. [Khoury and Weltman, Phys. Rev. D 69, 044026 (2004)]

(6) **Solar-system constraints**

[Chiba, Phys. Lett. B 575, 1 (2003)]

[Chiba, Smith and Erickcek, Phys. Rev. D 75, 124014 (2007)]

< Models of *f*(*R*) gravity (examples) >

(i) Hu-Sawicki model

[Hu and Sawicki, Phys. Rev. D 76, 064004 (2007)]

Cf. [Nojiri and Odintsov, Phys. Lett. B 657, 238 (2007); Phys. Rev. D 77, 026007 (2008)]

$$f_{\mathrm{HS}} = R - \frac{c_1 R_{\mathrm{HS}} \left(R/R_{\mathrm{HS}}\right)^p}{c_2 \left(R/R_{\mathrm{HS}}\right)^p + 1}$$

$c_1,\ c_2,\ p(>0),\ R_{\mathrm{HS}}(>0)$: Constant parameters

(ii) Starobinsky's model

[Starobinsky, JETP Lett. 86, 157 (2007)]

$$f_{\mathrm{S}} = R + \lambda R_{\mathrm{S}} \left[\left(1 + \frac{R^2}{R_{\mathrm{S}}^2} \right)^{-n} - 1 \right]$$

$\lambda(>0),\ n(>0),\ R_{\mathrm{S}}$: Constant parameters

(iii) Tsujikawa's model

[Tsujikawa, Phys. Rev. D 77, 023507 (2008)]

$$f_{\mathrm{T}} = R - \mu R_{\mathrm{T}} \tanh\left(\frac{R}{R_{\mathrm{T}}}\right)$$

$\mu(>0),\ R_{\mathrm{T}}(>0)$: Constant parameters

(iv) Exponential gravity model

[Cognola, Elizalde, Nojiri, Odintsov, Sebastiani and Zerbini, Phys. Rev. D 77, 046009 (2008)]

[Linder, Phys. Rev. D 80, 123528 (2009)]

$$f_{\mathrm{E}} = R - \beta R_{\mathrm{E}} \left(1 - e^{-R/R_{\mathrm{E}}}\right)$$

$\beta,\ R_{\mathrm{E}}$: Constant parameters

< Other model >

- **Appleby-Battye model** **[Appleby and Battye, Phys. Lett. B 654, 7 (2007)]**

$$f_{\mathrm{AB}}(R) = \frac{R}{2} + \frac{1}{2b_1}\log\left[\cosh(b_1 R) - \tanh(b_2)\sinh(b_1 R)\right]$$

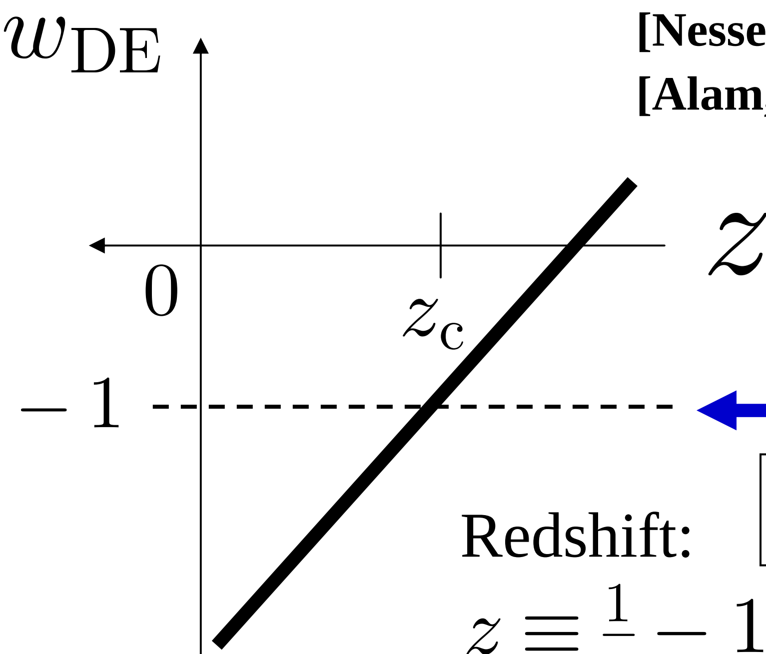
$$b_1(>0),\quad b_2$$

: Constant parameters

< Crossing of the phantom divide >

- Various observational data (SN, Cosmic microwave background radiation (CMB), BAO) imply that the effective EoS of dark energy $w_{\rm DE}$ may evolve from larger than -1 (non-phantom phase) to less than -1 (phantom phase). Namely, it crosses -1 (the crossing of the phantom divide).

[Alam, Sahni and Starobinsky, JCAP 0406, 008 (2004)]
[Nesseris and Perivolaropoulos, JCAP 0701, 018 (2007)]
[Alam, Sahni and Starobinsky, JCAP 0702, 011 (2007)]



Redshift: $z \equiv \frac{1}{a} - 1$

$z_{\rm c}$: Red shift at the crossing of the phantom divide

(a) $w_{\rm DE} > -1$

Non-phantom phase

(b) $w_{\rm DE} = -1$

Crossing of the phantom divide

(c) $w_{\rm DE} < -1$

Phantom phase

- **It is known that in several viable $f(R)$ gravity models, the crossing of the phantom divide can occur in the past.**

(i) Hu-Sawicki model

[Hu and Sawicki, Phys. Rev. D 76, 064004 (2007)]

[Martinelli, Melchiorri and Amendola, Phys. Rev. D 79, 123516 (2009)]

Cf. [Nozari and Azizi, arXiv:0909.0351 [gr-qc]]

(ii) Starobinsky's model

[Motohashi, Starobinsky and Yokoyama, Prog. Theor. Phys. 123, 887 (2010); Prog. Theor. Phys. 124, 541 (2010)]

(iv) Exponential gravity model

[Linder, Phys. Rev. D 80, 123528 (2009)]

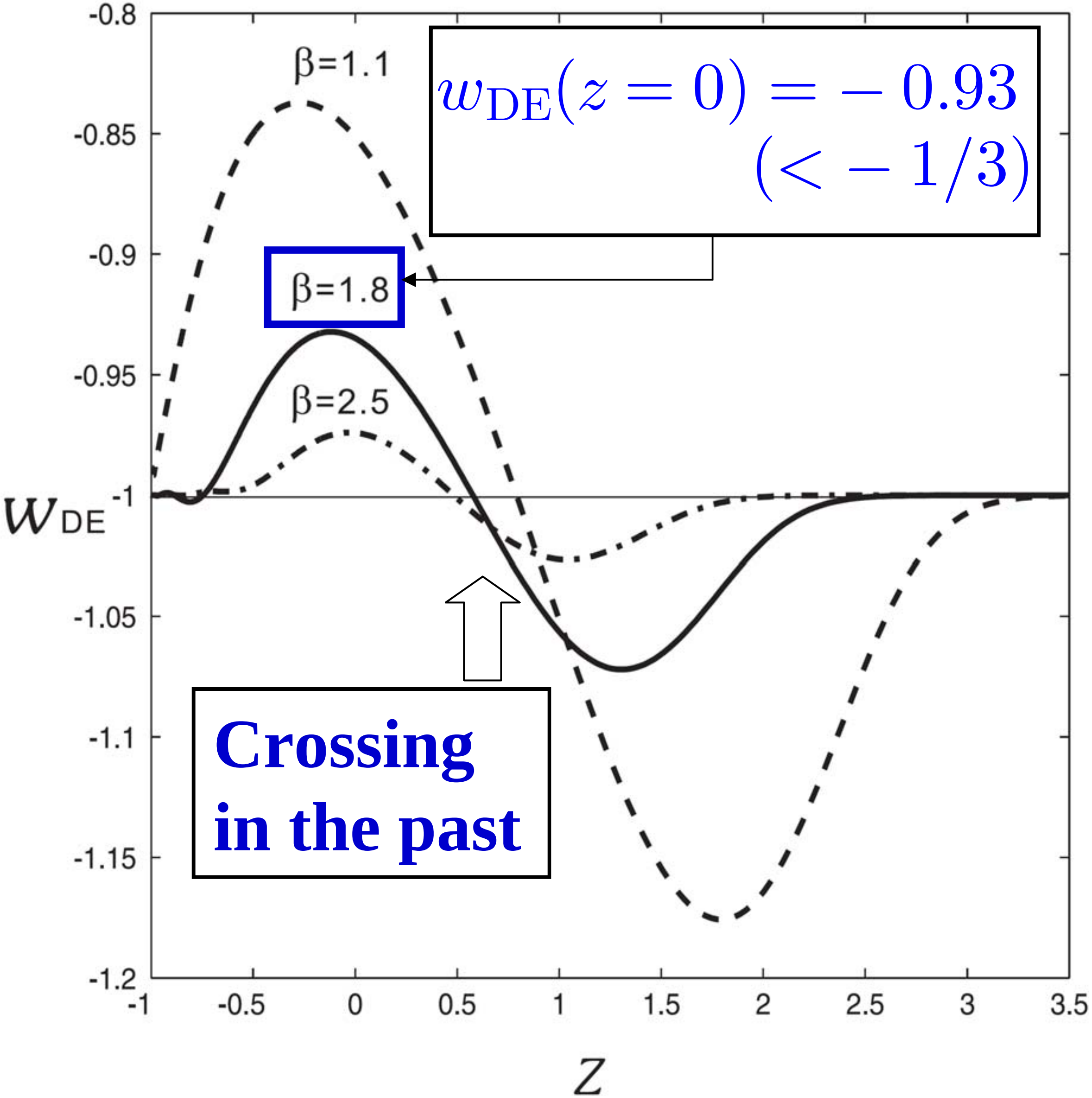
[KB, Geng and Lee, JCAP 1008, 021 (2010) arXiv:1005.4574 [astro-ph.CO]]

- **Appleby-Battye model**

[Appleby, Battye and Starobinsky, JCAP 1006, 005 (2010)]

< Cosmological evolution of $w_{\rm DE}$ in the exponential gravity model >

From [KB, Geng and Lee, JCAP 1008, 021 (2010)].



$$f_{\rm E}(R) = R - \beta R_{\rm E}\left(1 - e^{-R/R_{\rm E}}\right)$$

$$w_{\rm DE} = -1$$

Crossing of the phantom divide

$$\beta R_{\rm E} \simeq 18 H_0^2 \Omega_{\rm m}^{(0)}$$

We explicitly demonstrate that the future crossings of the phantom divide line $w_{\rm DE} = -1$ are the generic feature in the existing viable *f*(*R*) gravity models.

- Recent related study on the future crossings of the phantom divide:

  [Motohashi, Starobinsky and Yokoyama, arXiv:1101.0744 [astro-ph.CO]]

II. Cosmological evolution in *f*(*R*) gravity

< Action >

$$g = \det(g_{\mu\nu})$$

$$I = \int d^4x \sqrt{-g} \frac{f(R)}{2\kappa^2} + I_{\mathrm{matter}}(g_{\mu\nu}, \Upsilon_{\mathrm{matter}})$$

$g_{\mu\nu}$: Metric tensor

f : Arbitrary function of R

I_{matter} : Action of matter

$\Upsilon_{\mathrm{matter}}$: Matter fields

< Gravitational field equation >

$$F G_{\mu\nu} = \kappa^2 T^{(\mathrm{matter})}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (FR - f) + \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F$$

$$F(R) \equiv df(R)/dR$$

$T^{(\mathrm{matter})}_{\mu\nu}$: Energy-momentum tensor of all perfect fluids of matter

$$G_{\mu\nu} = R_{\mu\nu} - (1/2)\, g_{\mu\nu} R$$

$R_{\mu\nu}$: Ricci tensor

∇_μ : Covariant derivative operator

$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$: Covariant d'Alembertian

< Flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time >

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2$$

$a(t)$: Scale factor

→ Gravitational field equations in the FLRW background:

$$3FH^2 = \kappa^2 \rho_{\rm M} + \frac{1}{2}(FR - f) - 3H\dot{F}$$

$$-2F\dot{H} = \kappa^2 (\rho_{\rm M} + P_{\rm M}) + \ddot{F} - H\dot{F}$$

$\dot{} = \partial/\partial t$

$H = \dot{a}/a$: Hubble parameter

$\rho_{\rm M}$ and $P_{\rm M}$: Energy density and pressure of all perfect fluids of matter, respectively,

< Analysis method >

[Hu and Sawicki, Phys. Rev. D 76, 064004 (2007)]

$$H^2 - (F-1)\left(H\frac{dH}{d\ln a} + H^2\right) + \frac{1}{6}(f - R) + H^2 F' \frac{dR}{d\ln a} = \frac{\kappa^2 \rho_{\rm M}}{3} \quad (1)$$

- Ricci scalar:

$$R = 12H^2 + 6H\frac{dH}{d\ln a} \quad (2)$$

$'$ (prime): Derivative with respect to R

→ We solve Equations (1) and (2) by introducing the following variables:

$$y_H \equiv \frac{\rho_{\mathrm{DE}}}{\rho_{\mathrm{m}}^{(0)}} = \frac{H^2}{\bar{m}^2} - a^{-3} - \chi a^{-4}$$

$$y_R = \frac{R}{\bar{m}^2} - 3a^{-3}$$

$$\bar{m}^2 \equiv \frac{\kappa^2 \rho_{\mathrm{m}}^{(0)}}{3}$$

$$\chi \equiv \frac{\rho_{\mathrm{r}}^{(0)}}{\rho_{\mathrm{m}}^{(0)}} \simeq 3.1 \times 10^{-4}$$

- '(0)'denotes the present values.

ρ_{DE} : Energy density of dark energy

ρ_{m} : Energy density of non-relativistic matter (cold dark matter and baryon)

ρ_{r} : Energy density of radiation

⇒

$$\frac{dy_H}{d\ln a} = \frac{y_R}{3} - 4y_H \qquad (3)$$

$$\frac{dy_R}{d\ln a} = 9a^{-3} - \frac{1}{y_H + a^{-3} + \chi a^{-4}} \frac{1}{\bar{m}^2 F'} \times \left[y_H - (F-1)\left(\frac{1}{6}y_R - y_H - \frac{1}{2}a^{-3} - \chi a^{-4}\right) + \frac{1}{6}\frac{f-R}{\bar{m}^2}\right] \qquad (4)$$

→ Combining Equations (3) and (4), we obtain

$$\frac{d^2 y_H}{d(\ln a)^2} + J_1 \frac{d y_H}{d \ln a} + J_2 y_H + J_3 = 0$$

: **Equation for y_H**

$$J_1 = 4 + \frac{1}{y_H + a^{-3} + \chi a^{-4}} \frac{1-F}{6\bar{m}^2 F'}$$

$$J_2 = \frac{1}{y_H + a^{-3} + \chi a^{-4}} \frac{2-F}{3\bar{m}^2 F'}$$

$$J_3 = -3a^{-3} - \frac{(1-F)\left(a^{-3} + 2\chi a^{-4}\right) + (R-f)/\left(3\bar{m}^2\right)}{y_H + a^{-3} + \chi a^{-4}} \frac{1}{6\bar{m}^2 F'}$$

< Equation of state for (the component corresponding to) dark energy >

$$w_{\mathrm{DE}} \equiv \frac{P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}}$$

$$\rho_{\mathrm{DE}} = \frac{1}{\kappa^2}\left[\frac{1}{2}\left(FR - f\right) - 3H\dot{F} + 3\left(1 - F\right)H^2\right]$$

$$P_{\mathrm{DE}} = \frac{1}{\kappa^2}\left[-\frac{1}{2}\left(FR - f\right) + \ddot{F} + 2H\dot{F} - \left(1 - F\right)\left(2\dot{H} + 3H^2\right)\right]$$

< Continuity equation for dark energy >

$$\dot{\rho}_{\mathrm{DE}} + 3H\left(1 + w_{\mathrm{DE}}\right)\rho_{\mathrm{DE}} = 0$$

$$\Rightarrow \quad w_{\mathrm{DE}} = -1 - \frac{1}{3}\frac{1}{y_H}\frac{dy_H}{d\ln a}$$

< Bekenstein-Hawking entropy on the apparent horizon in the flat FLRW background >

$$S = \frac{\pi}{GH^2}$$

$S = \frac{A}{4G}$: Bekenstein-Hawking entropy

$A = 4\pi \tilde{r}_A^2$: Area of the apparent horizon

$\tilde{r} = 1/H$: Radius of the apparent horizon in the flat FLRW space-time

- It has been shown that it is possible to obtain a picture of equilibrium thermodynamics on the apparent horizon in the FLRW background for *f*(*R*) gravity due to a suitable redefinition of an energy momentum tensor of the "dark" component that respects a local energy conservation.

[KB, Geng and Tsujikawa, Phys. Lett. B 688, 101 (2010)]

⇨ In this picture, the horizon entropy is simply expressed as $S = \pi / \left(GH^2\right)$.

III. Future crossing of the phantom divide

(i) Hu-Sawicki model

[Hu and Sawicki, Phys. Rev. D 76, 064004 (2007)]

$$f_{\rm HS} = R - \frac{c_1 R_{\rm HS} \left(R/R_{\rm HS}\right)^p}{c_2 \left(R/R_{\rm HS}\right)^p + 1}$$

$$p = 1$$
$$c_1 = 1\,, \quad c_2 = 1$$

(ii) Starobinsky's model

[Starobinsky, JETP Lett. 86, 157 (2007)]

$$f_{\rm S} = R + \lambda R_{\rm S} \left[\left(1 + \frac{R^2}{R_{\rm S}^2}\right)^{-n} - 1 \right]$$

$$n = 2$$
$$\lambda = 1.5$$

(iii) Tsujikawa's model

[Tsujikawa, Phys. Rev. D 77, 023507 (2008)]

$$f_{\rm T} = R - \mu R_{\rm T} \tanh\left(\frac{R}{R_{\rm T}}\right)$$

$$\mu = 1$$

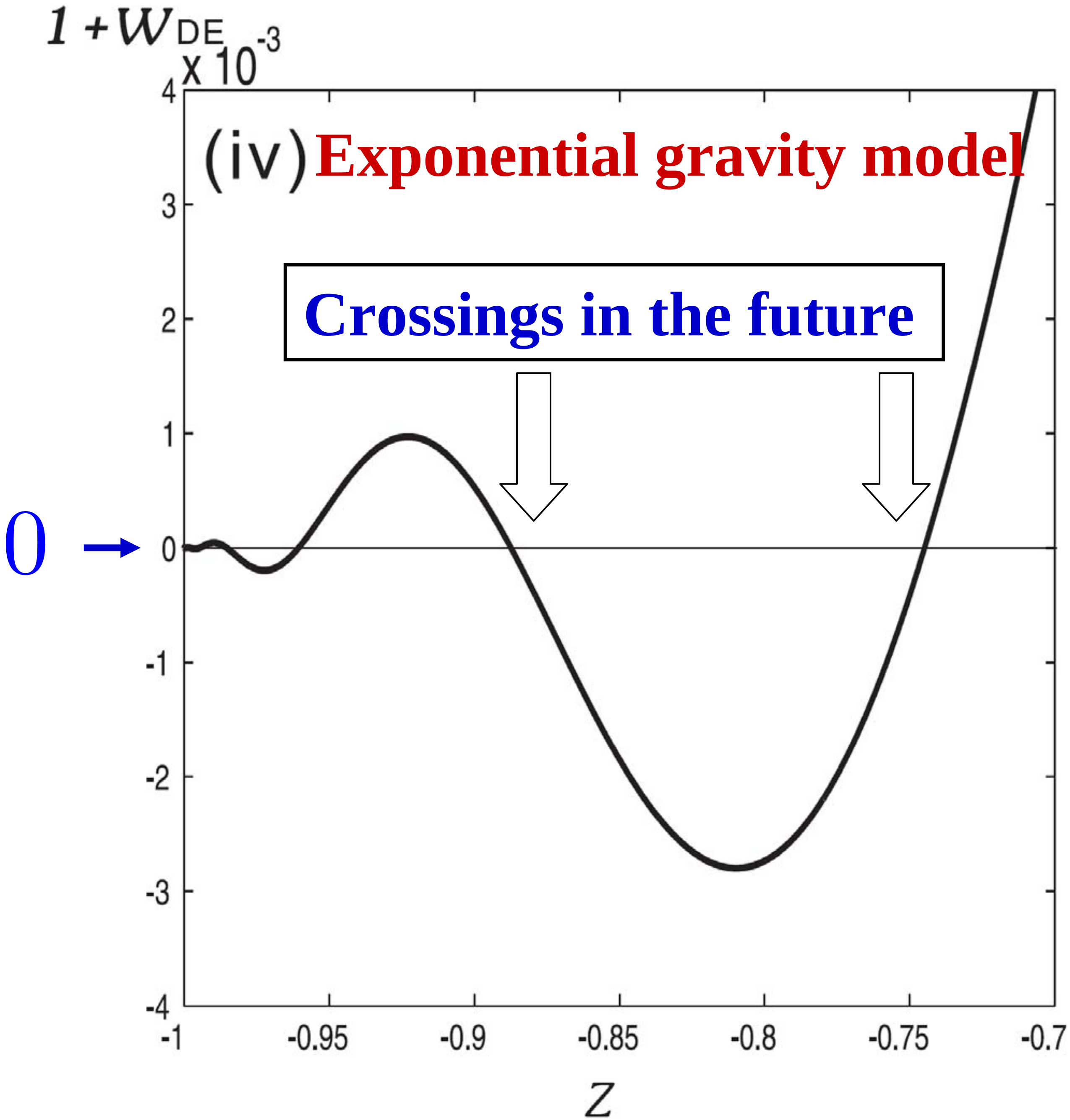
(iv) Exponential gravity model

[Cognola, Elizalde, Nojiri, Odintsov, Sebastiani and Zerbini, Phys. Rev. D 77, 046009 (2008)]
[Linder, Phys. Rev. D 80, 123528 (2009)]

$$f_{\rm E} = R - \beta R_{\rm E} \left(1 - e^{-R/R_{\rm E}}\right)$$

$$\beta = 1.8$$

< Future evolutions of $1+w_{\rm DE}$ as functions of z >



$$1+w_{\rm DE}$$

$1+w_{\rm DE}=0$

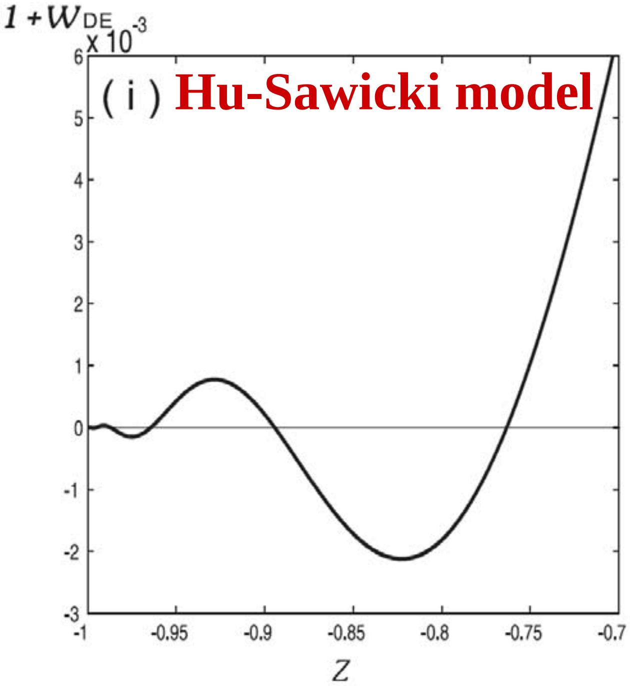
Crossing of the phantom divide

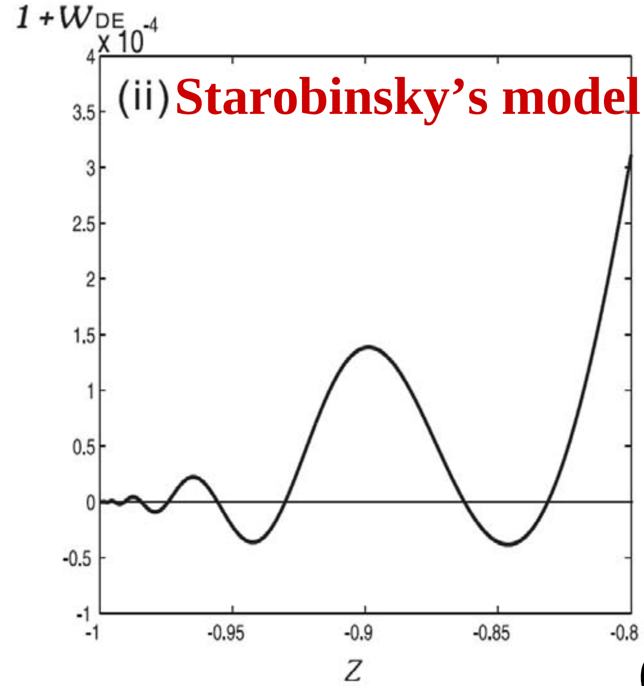
Redshift:

$$z \equiv \frac{1}{a} - 1$$

($z<0$: Future)

$1 + w_{\mathrm{DE}}$

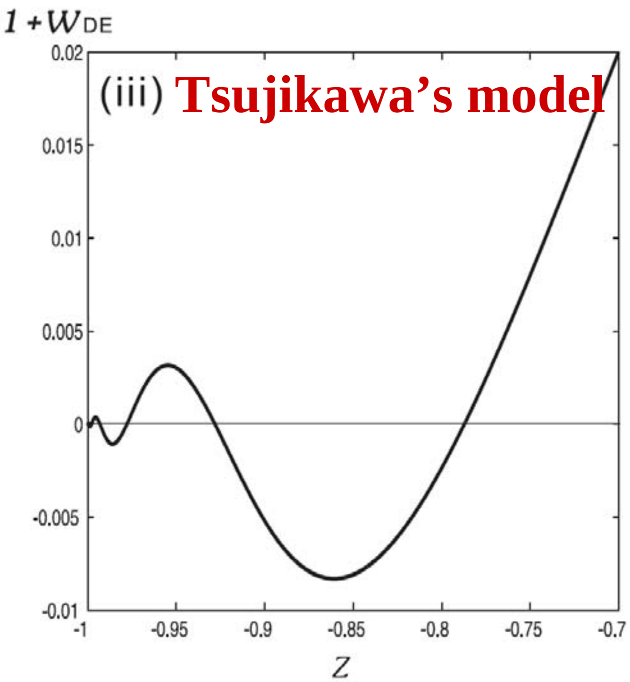




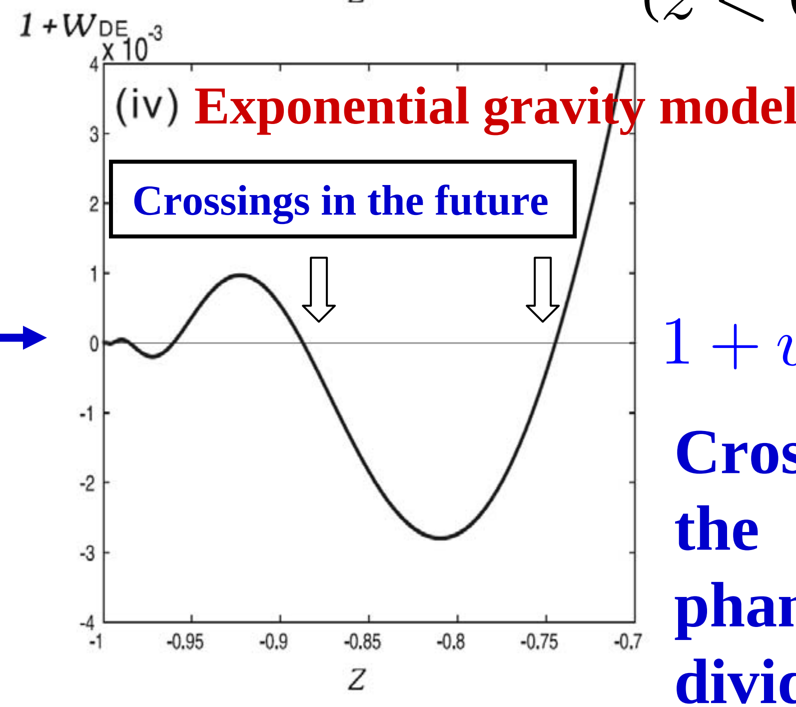
Redshift:

$$z \equiv \frac{1}{a} - 1$$

($z < 0$: Future)



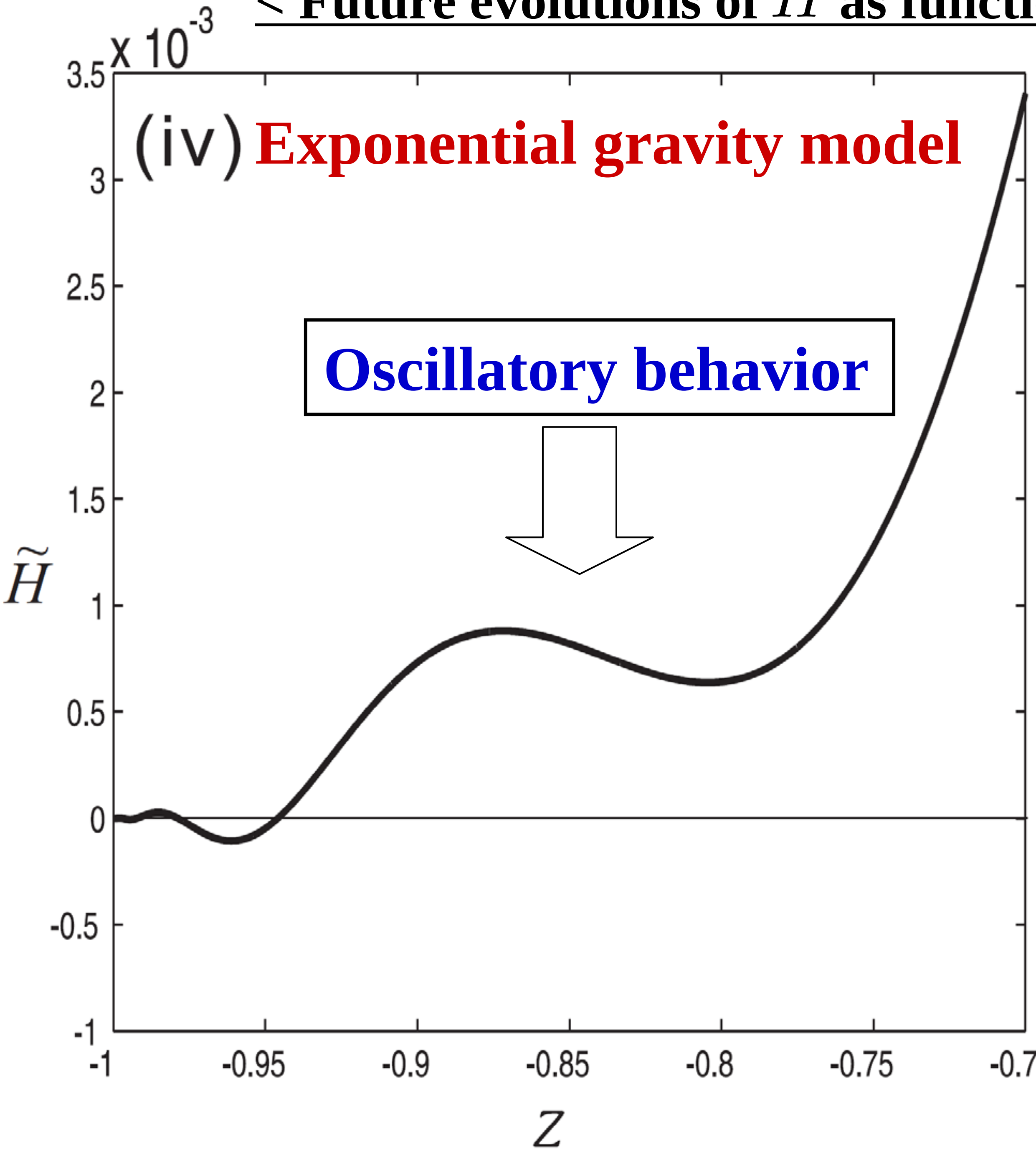
$0 \longrightarrow$



$1 + w_{\mathrm{DE}} = 0$

Crossing of the phantom divide

< Future evolutions of H as functions of z >



$$\tilde{\bar{H}} \equiv \bar{H} - \bar{H}_{\mathrm{f}}$$

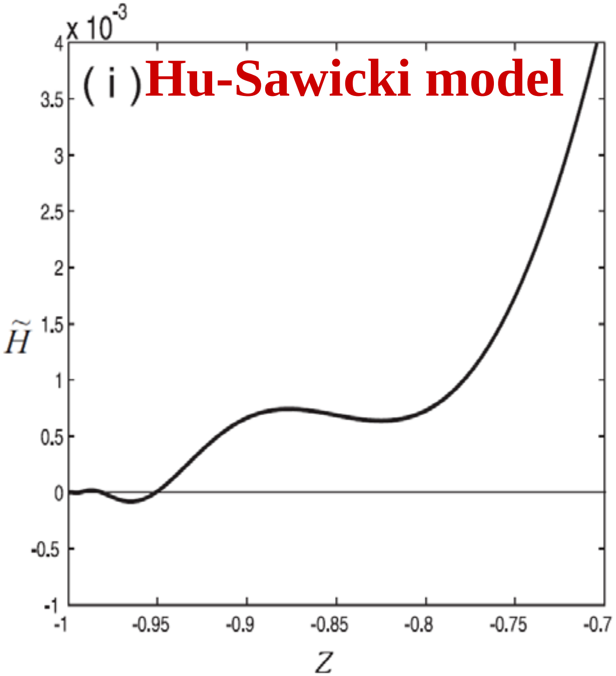
$$\bar{H} \equiv H/H_0$$

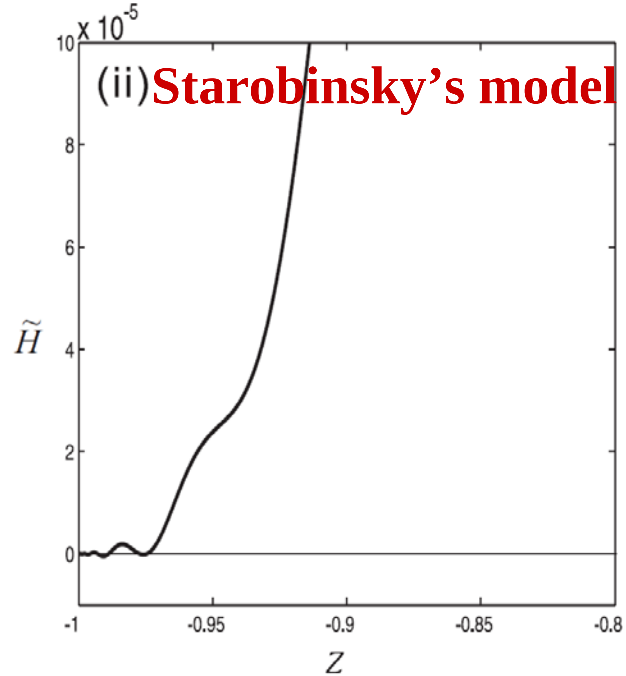
$$\bar{H}_{\mathrm{f}} \equiv \frac{H(z=-1)}{H_0}$$

: ‘f’ denotes the value at the final stage $z = -1$.

$$H_0 = 71 \mathrm{km/s/Mpc}$$

: Present value of the Hubble parameter

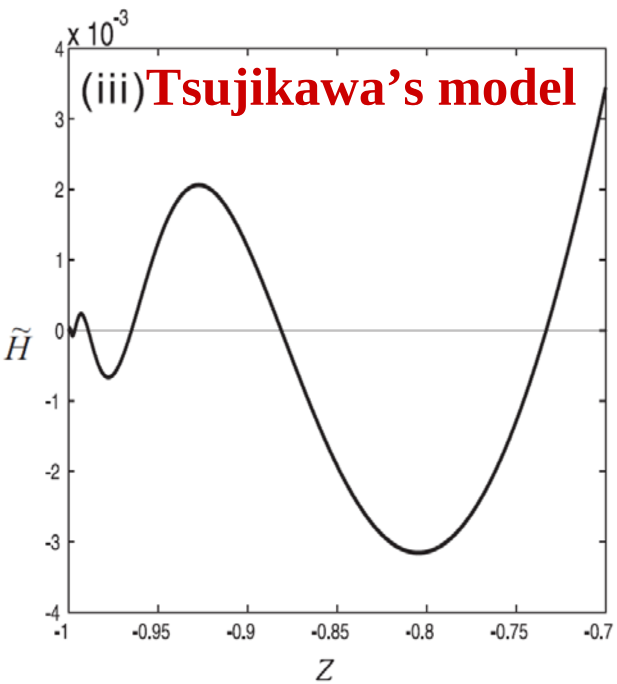


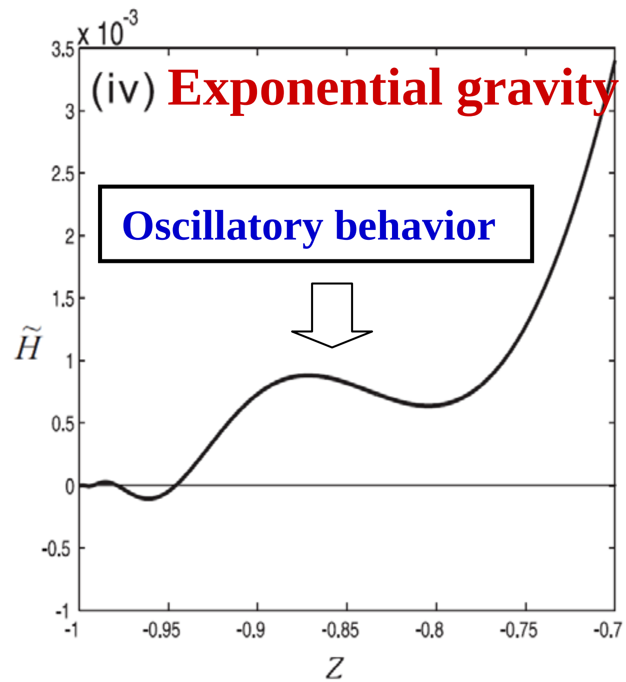


$$\tilde{H} \equiv \bar{H} - \bar{H}_{\mathrm{f}}$$

$$\bar{H} \equiv H/H_0$$

$$\bar{H}_{\mathrm{f}} \equiv \frac{H(z=-1)}{H_0}$$

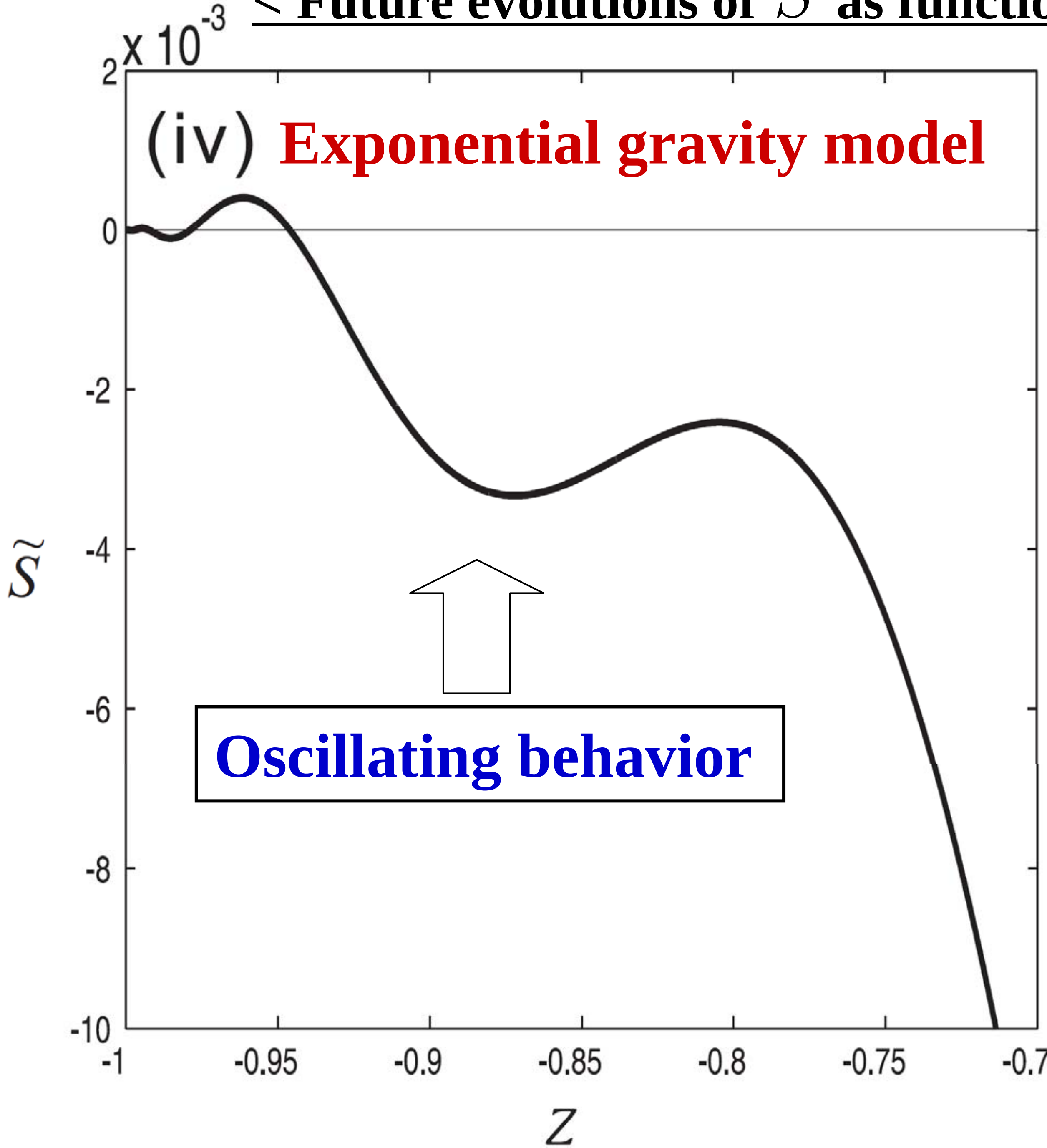




$H_0 = 71\mathrm{km/s/Mpc}$

: Present value of the Hubble parameter

< Future evolutions of S as functions of z >



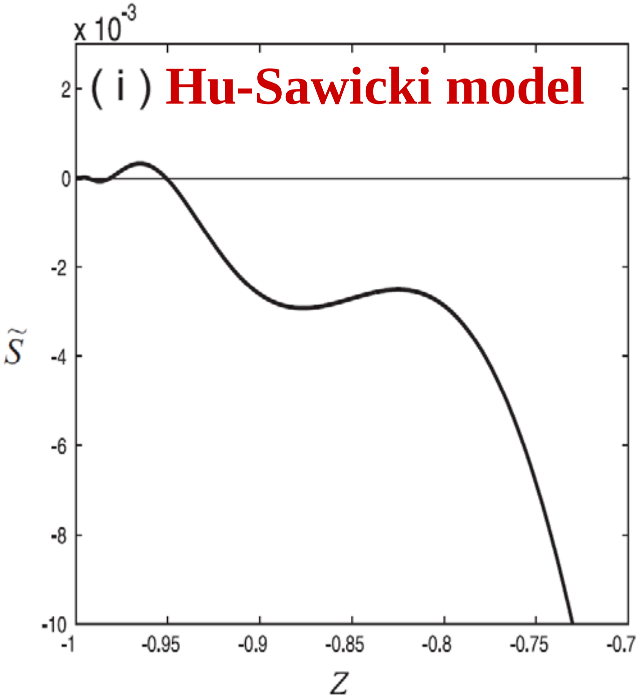
$$\tilde{S} \equiv \bar{S} - \bar{S}_{\mathrm{f}}$$

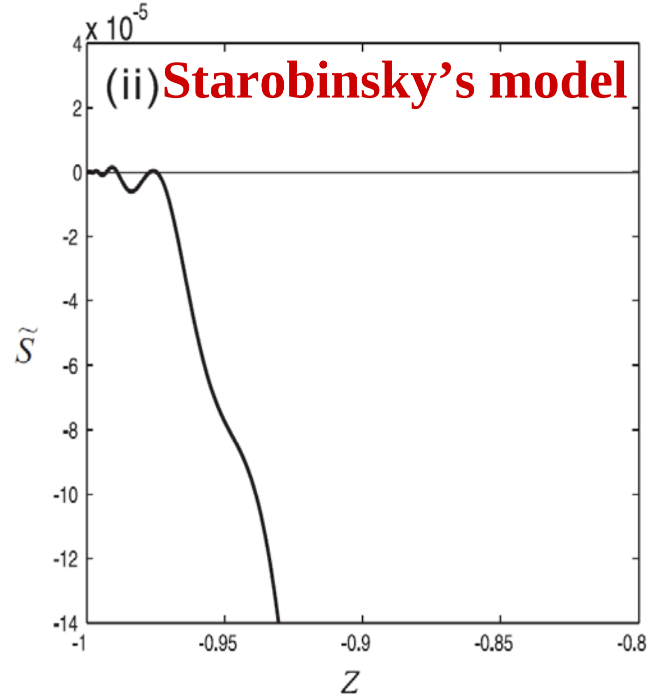
$$\bar{S} \equiv S/S_0$$

$$\bar{S}_{\mathrm{f}} \equiv \frac{S(z=-1)}{S_0}$$

$$S_0 = \pi/\left(G H_0^2\right)$$

: Present value of the horizon entropy

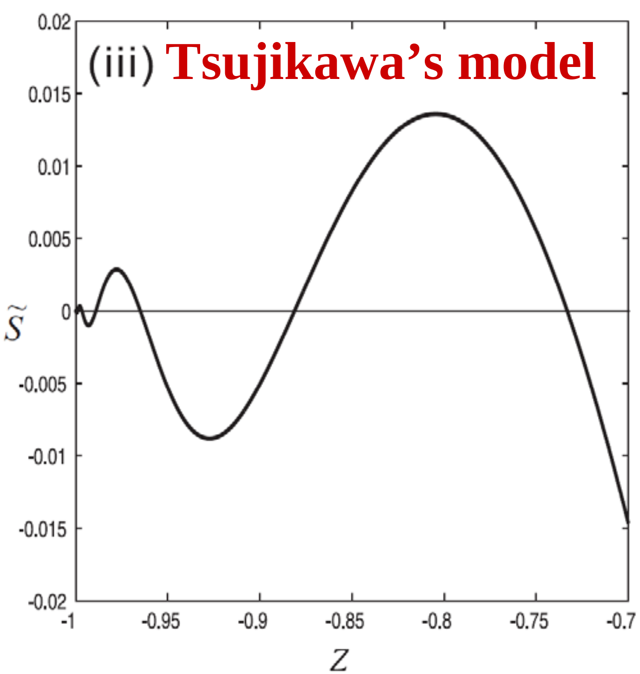


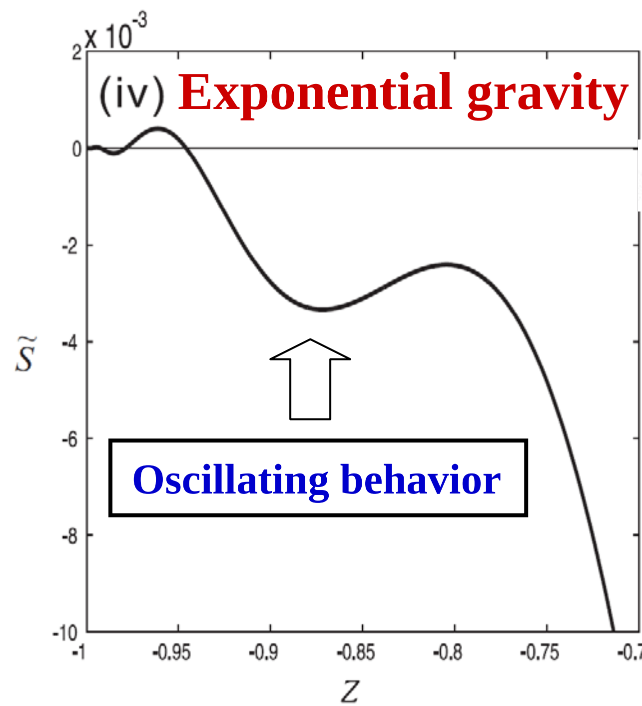


$$\tilde{S} \equiv \bar{S} - \bar{S}_{\mathrm{f}}$$

$$\bar{S} \equiv S/S_0$$

$$\bar{S}_{\mathrm{f}} \equiv \frac{S(z=-1)}{S_0}$$





$$S_0 = \pi / \left(G H_0^2\right)$$

: Present value of the horizon entropy

- In the future ($-1 \leq z \lesssim -0.74$), the crossings of the phantom divide are the generic feature for all the existing viable *f*(*R*) models.
- As z decreases ($-1 \leq z \lesssim -0.90$), dark energy becomes much more dominant over non-relativistic matter ($\Xi = \Omega_{\mathrm{m}}/\Omega_{\mathrm{DE}} \lesssim 10^{-5}$).

< Effective equation of state for the universe >

$$w_{\mathrm{eff}} \equiv -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = \frac{P_{\mathrm{tot}}}{\rho_{\mathrm{tot}}}$$

$$\rho_{\mathrm{tot}} \equiv \rho_{\mathrm{DE}} + \rho_{\mathrm{m}} + \rho_{\mathrm{r}}$$

: Total energy density of the universe

$$P_{\mathrm{tot}} \equiv P_{\mathrm{DE}} + P_{\mathrm{m}} + P_{\mathrm{r}}$$

: Total pressure of the universe

P_{DE} : Pressure of dark energy

P_{m} : Pressure of non-relativistic matter (cold dark matter and baryon)

P_{r} : Pressure of radiation

$$\Rightarrow \quad w_{\mathrm{DE}} \approx w_{\mathrm{eff}}$$

$$w_{\rm eff} = -1 - \frac{2\dot{H}}{3H^2}$$

(a) $\dot{H} < 0 \Longrightarrow w_{\rm eff} > -1$ **Non-phantom phase**

(b) $\dot{H} = 0 \Longrightarrow w_{\rm eff} = -1$ **Crossing of the phantom divide**

(c) $\dot{H} > 0 \Longrightarrow w_{\rm eff} < -1$ **Phantom phase**

- The physical reason why the crossing of the phantom divide appears in the farther future ($-1 \leq z \lesssim -0.90$) is that **the sign of $\dot{H}$ changes from negative to positive due to the dominance of dark energy over non-relativistic matter.**
- As $w_{\rm DE} \approx w_{\rm eff}$ in the farther future, $w_{\rm DE}$ oscillates around the phantom divide line $w_{\rm DE} = -1$ because **the sign of $\dot{H}$ changes and consequently multiple crossings can be realized.**

- **Since $S \propto H^{-2}$, the oscillating behavior of S comes from that of H.**

⇨ However, it should be emphasized that although S decreases in some regions, the second law of thermodynamics in *f*(*R*) gravity can be always satisfied.

⟶ This is because S is the cosmological horizon entropy and it is not the total entropy of the universe including the entropy of generic matter.

Cf. It has been shown that the second law of thermodynamics can be verified in both phantom and non-phantom phases for the same temperature of the universe outside and inside the apparent horizon.

[KB and Geng, JCAP 1006, 014 (2010)]

IV. Summary

- We have explicitly shown that **the future crossings of the phantom divide are the generic feature in the existing viable $f(R)$ gravity models.**

- We have also illustrated that **the cosmological horizon entropy oscillates with time due to the oscillatory behavior of the Hubble parameter.**

- The new cosmological ingredient obtained in this study is that **in the future the sign of $\dot{H}$ changes from negative to positive due to the dominance of dark energy over non-relativistic matter.**

  ⇨ **This is a common physical phenomena to the existing viable $f(R)$ models and thus it is one of the peculiar properties of $f(R)$ gravity models characterizing the deviation from the Λ CDM model.**

Backup Slides A

< *f*(*R*) gravity >

$$S = \int d^4x \sqrt{-g}\, \frac{f(R)}{2\kappa^2}$$

$f(R)$ gravity

$f(R) = R$: General Relativity

[Nojiri and Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [arXiv:hep-th/0601213]; arXiv:1011.0544 [gr-qc]]

[Capozziello and Francaviglia, Gen. Rel. Grav. 40, 357 (2008)]

[Sotiriou and Faraoni, Rev. Mod. Phys. 82, 451 (2010)]

[De Felice and Tsujikawa, Living Rev. Rel. 13, 3 (2010)]

< Gravitational field equation >

$$f'(R) = df(R)/dR$$

$$f'(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\Box f'(R) - \nabla_\mu\nabla_\nu f'(R) = 0$$

$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$: Covariant d'Alembertian

∇_μ : Covariant derivative operator

- In the flat FLRW background, gravitational field equations read

No. BS-A2

$$H^2 = \frac{\kappa^2}{3f'(R)}\rho_{\rm eff} \,, \quad \dot{H} = -\frac{\kappa^2}{2f'(R)}\left(\rho_{\rm eff} + P_{\rm eff}\right)$$

$\rho_{\rm eff}$, $P_{\rm eff}$: Effective energy density and pressure from the term $f(R) - R$

$$\rho_{\rm eff} = \frac{1}{\kappa^2}\left[\frac{1}{2}\left(-f(R) + Rf'(R)\right) - 3H\dot{R}f''(R)\right]$$

$$P_{\rm eff} = \frac{1}{\kappa^2}\left[\frac{1}{2}\left(f(R) - Rf'(R)\right) + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)\right]$$

$$\rightarrow \quad w_{\rm eff} = \frac{P_{\rm eff}}{\rho_{\rm eff}} = \frac{\left(f(R) - Rf'(R)\right)/2 + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)}{\left(-f(R) + Rf'(R)\right)/2 - 3H\dot{R}f''(R)}$$

- Example: $f(R) = R - \frac{\mu^{2(n+1)}}{R^n}$ **[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]**

μ : Mass scale, n : Constant

Second term become important as R decreases.

$\Rightarrow$ $a \propto t^q$, $\quad q = \frac{(2n+1)(n+1)}{n+2}$

$$w_{\rm eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$$

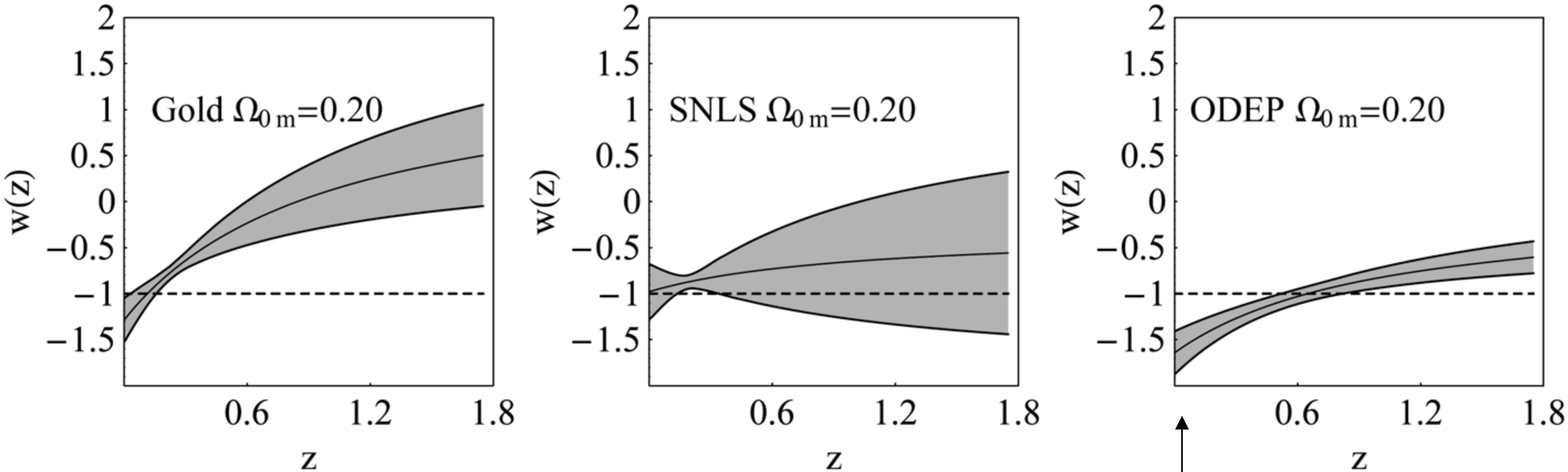
If $q > 1$, accelerated expansion can be realized.

(For $n = 1$, $q = 2$ and $w_{\rm eff} = -2/3$.)

< Data fitting of $w(z)$ (1) >

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

From [Nesseris and L. Perivolaropoulos, JCAP 0701, 018 (2007)].



SN gold data set

[Riess *et al*. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004)]

SNLS data set

[Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)]

Shaded region shows 1σ error.

Cosmic microwave background radiation (CMB) data

[Spergel *et al*. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007)]

+ SDSS baryon acoustic peak (BAO) data

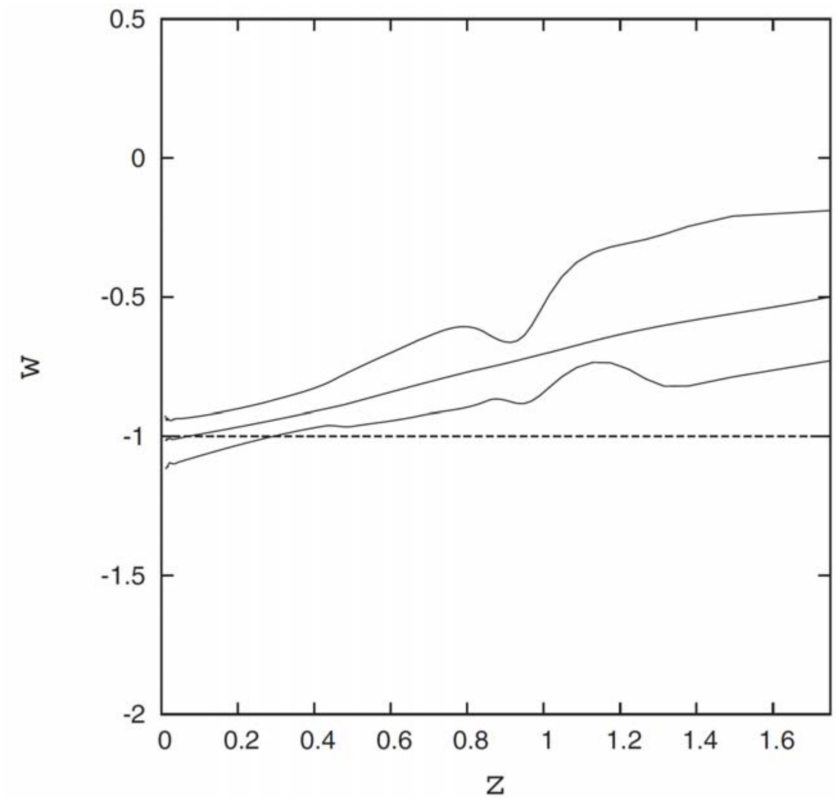
[Eisenstein *et al*. [SDSS Collaboration], Astrophys. J. 633, 560 (2005)]

< Data fitting of $w(z)$ (2) >

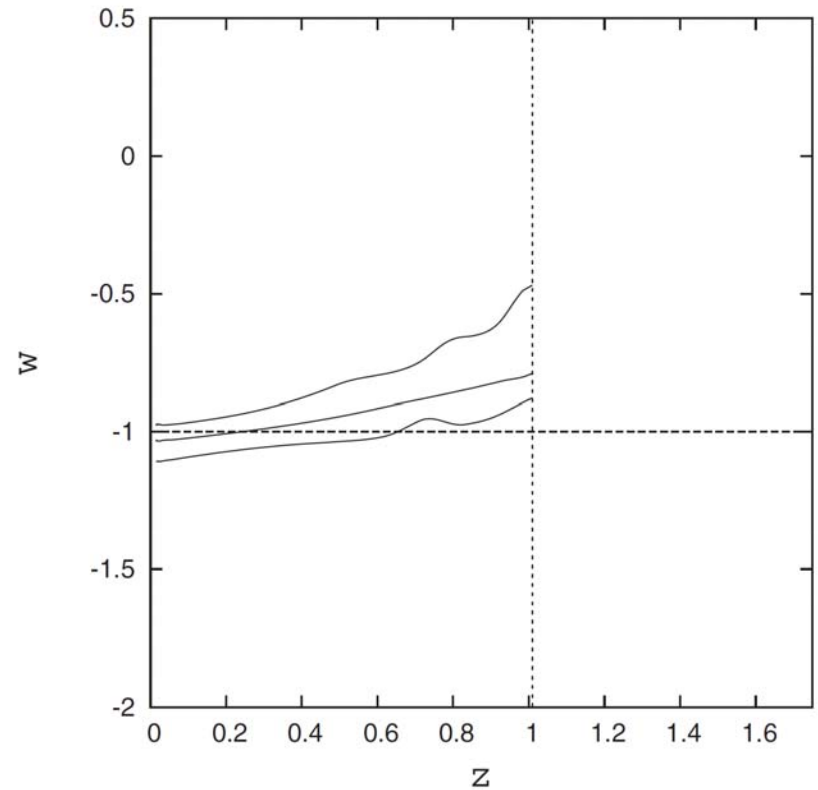
$$w(x) = \frac{(2x/3)\,\mathrm{d}\ln H/\mathrm{d}x - 1}{1 - (H_0/H)^2\Omega_{0m}x^3}$$

$$x = 1 + z$$

From [Alam, Sahni and Starobinsky, JCAP 0702, 011 (2007)].



SN gold data set+CMB+BAO



SNLS data set+CMB+BAO

- $\Omega_{0\mathrm{m}} = 0.28 \pm 0.03$
- 2σ **confidence level.**

Appendix A

(1) General relativistic approach

- **Cosmological constant**
- **Scalar field : X matter, Quintessence** ← Canonical field

[Chiba, Sugiyama and Nakamura, Mon. Not. Roy. Astron. Soc. 289, L5 (1997)]
[Caldwell, Dave and Steinhardt, Phys. Rev. Lett. 80, 1582 (1998)]

Cf. Pioneering work: [Fujii, Phys. Rev. D 26, 2580 (1982)]

Phantom ← Wrong sign kinetic term

[Caldwell, Phys. Lett. B 545, 23 (2002)]

K-essence ← Non canonical kinetic term

[Chiba, Okabe and Yamaguchi, Phys. Rev. D 62, 023511 (2000)]
[Armendariz-Picon, Mukhanov and Steinhardt, Phys. Rev. Lett. 85, 4438 (2000)]

Tachyon ← String theories

[Padmanabhan, Phys. Rev. D 66, 021301 (2002)]

- **Chaplygin gas** ← $p = -A/\rho$

$A > 0$: Constant
ρ : Energy density
p : Pressure

[Kamenshchik, Moschella and Pasquier, Phys. Lett. B 511, 265 (2001)]

(2) Extension of gravitational theory

Cf. Application to inflation:
[Starobinsky, Phys. Lett. B 91, 99 (1980)]

- ***f*(*R*) gravity**

$f(R)$: Arbitrary function of the Ricci scalar R

[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

$f_i(\phi)$ $(i = 1, 2)$: Arbitrary function of a scalar field ϕ

- **Scalar-tensor theories** $\longleftarrow f_1(\phi)R$

[Boisseau, Esposito-Farese, Polarski and Starobinsky, Phys. Rev. Lett. 85, 2236 (2000)]

[Gannouji, Polarski, Ranquet and Starobinsky, JCAP 0609, 016 (2006)]

- **Ghost condensates** **[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]**

- **Higher-order curvature term**

Gauss-Bonnet term with a coupling to a scalar field: $f_2(\phi)\mathcal{G}$

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$R_{\mu\nu}$: Ricci curvature tensor

[Nojiri, Odintsov and Sasaki, Phys. Rev. D 71, 123509 (2005)]

$R_{\mu\nu\rho\sigma}$: Riemann tensor

- **$f(\mathcal{G})$ gravity** $\longleftarrow \frac{R}{2\kappa^2} + f(\mathcal{G})$ $\quad \kappa^2 \equiv 8\pi G$

[Nojiri and Odintsov, Phys. Lett. B 631, 1 (2005)]

G : Gravitational constant

- **DGP braneworld scenario**

  [Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]

  [Deffayet, Dvali and Gabadadze, Phys. Rev. D 65, 044023 (2002)]

- ***f(T)* gravity** : Extended teleparallel Lagrangian density described by the torsion scalar T.

  [Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

  [Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]

  - "Teleparallelism" : One could use the Weitzenböck connection, which has no curvature but torsion, rather than the curvature defined by the Levi-Civita connection.

    [Hayashi and Shirafuji, Phys. Rev. D 19, 3524 (1979) [Addendum-ibid. D 24, 3312 (1982)]]

- **Galileon gravity** **[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]**

  Recent review: [Tsujikawa, Lect. Notes Phys. 800, 99 (2010)]

$\Box\phi(\partial^\mu\phi\partial_\mu\phi)$ ⟵ Longitudinal graviton (i.e. a branebending mode ϕ)

- The equations of motion are invariant under the Galilean shift: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$

⇨ One can keep the equations of motion up to the second-order.

→ This property is welcome to avoid the appearance of an extra degree of freedom associated with ghosts.

$\Box$: Covariant d'Alembertian

< Flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time >

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\boldsymbol{x}^2$$

$a(t)$: Scale factor

< Equation for $a(t)$ with a perfect fluid >

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p)$$

ρ : Energy density, p : Pressure

$\dot{} = \partial/\partial t$

- If $\rho + 3p < 0 \longrightarrow \ddot{a} > 0$: **Accelerated expansion**

< Equation of state (EoS) w >

$$w \equiv \frac{p}{\rho}$$

Condition for accelerated expansion : $w < -\frac{1}{3}$

Cf. Cosmological constant $\Longrightarrow w = -1$

- **Continuity equation**

$$\dot{\rho} + 3H(1+w)\rho = 0 \longrightarrow a \propto t^{\frac{2}{3(1+w)}}, \quad \rho \propto a^{-3(1+w)}$$

$H = \dot{a}/a$: Hubble parameter

< Canonical scalar field >

$$S_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$g = \det(g_{\mu\nu})$

ϕ : Scalar field

$V(\phi)$: Potential of ϕ

- For a homogeneous scalar field $\phi = \phi(t)$:

$\longrightarrow$ $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

$\Rightarrow$ $$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}$$

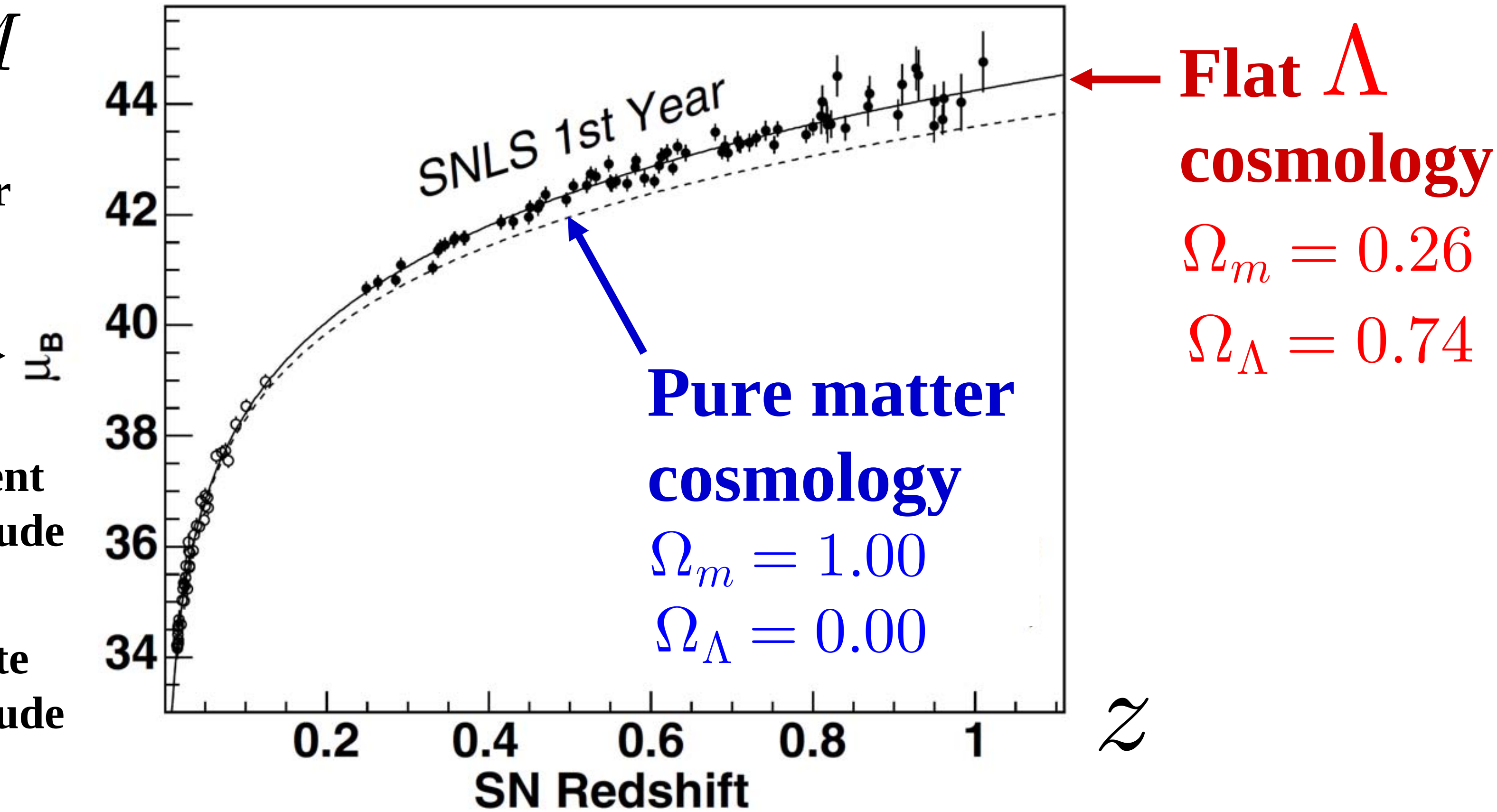
If $\dot{\phi}^2 \ll V(\phi)$, $\underline{w_\phi \approx -1}$.

$\longrightarrow$ **Accelerated expansion can be realized.**

< SNLS data >

$m - M$

: Distance estimator



m : Apparent magnitude

M : Absolute magnitude

From [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)].

$$\frac{1}{H_0^2}\frac{\ddot{a}}{a} = -\frac{\Omega_m}{2}(1+z)^3 + \Omega_\Lambda$$

$a_0 = 1$

$$\Omega_m \equiv \frac{\kappa^2 \rho(t_0)}{3H_0^2}$$: Density parameter for matter

$$1 + z = \frac{a_0}{a},$$ z : Redshift

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$$: Density parameter for Λ

"0" denotes quantities at the present time t_0 .

< Conditions for the viability of *f*(*R*) gravity >

(1) $f'(R) > 0$

- **Positivity of the effective gravitational coupling**

$G_{\rm eff} = G_0 / f'(R) > 0$ G_0 : Gravitational constant

(The graviton is not a ghost.)

(2) $f''(R) > 0$ **[Dolgov and Kawasaki, Phys. Lett. B 573, 1 (2003)]**

- **Stability condition:** $M^2 \approx 1/(3f''(R)) > 0$

M : **Mass of a new scalar degree of freedom (called the "scalaron") in the weak-field regime.**

(The scalaron is not a tachyon.)

(3) $f(R) \to R - 2\Lambda \quad \text{for} \quad R \gg R_0$

R_0 : Current curvature

Λ : Cosmological constant

- **Realization of the ΛCDM-like behavior in the large curvature regime**

↑ Standard cosmology [Λ + Cold dark matter (CDM)]

(4) Solar system constraints

$f(R)$ gravity ⟺ **Brans-Dicke theory with $\omega_{\rm BD} = 0$**

Equivalent

$\omega_{\rm BD}$: Brans-Dicke parameter

[Bertotti, Iess and Tortora, Nature 425, 374 (2003).] $\longrightarrow$ Observational constraint: $|\omega_{\rm BD}| > 40000$

[Chiba, Phys. Lett. B 575, 1 (2003)]

[Erickcek, Smith and Kamionkowski, Phys. Rev. D 74, 121501 (2006)]

[Chiba, Smith and Erickcek, Phys. Rev. D 75, 124014 (2007)]

- However, if the mass of the scalar degree of freedom M is large, namely, the scalar becomes short-ranged, it has no effect at Solar System scales.
- $M = M(R)$ $\leftarrow$ Scale-dependence: "**Chameleon mechanism**"

Cf. [Khoury and Weltman, Phys. Rev. D 69, 044026 (2004)]

⇨ The scalar degree of freedom may acquire a large effective mass at terrestrial and Solar System scales, shielding it from experiments performed there.

(5) Existence of a matter-dominated stage and that of a late-time cosmic acceleration

- **Combing local gravity constraints, it is shown that $m \equiv Rf''(R)/f'(R)$ has to be several orders of magnitude smaller than unity.**

m quantifies the deviation from the Λ CDM model.

[Amendola, Gannouji, Polarski and Tsujikawa, Phys. Rev. D 75, 083504 (2007)]

[Amendola and Tsujikawa, Phys. Lett. B 660, 125 (2008)]

(6) Stability of the de Sitter space

$$\frac{(f_{\rm d}')^2 - 2f_{\rm d}f_{\rm d}''}{f_{\rm d}'f_{\rm d}''} > 0$$

$f_{\rm d} = f(R_{\rm d})$

$R_{\rm d}$: Constant curvature in the de Sitter space

- **Linear stability of the inhomogeneous perturbations in the de Sitter space** **[Faraoni and Nadeau, Phys. Rev. D 75, 023501 (2007)]**

Cf. $R_{\rm d} = 2f_{\rm d}/f_{\rm d}'$ $\Longrightarrow$ $m < 1$

< Other models >

- **Appleby-Battye model** **[Appleby and Battye, Phys. Lett. B 654, 7 (2007)]**

$$f_{\mathrm{AB}}(R) = \frac{R}{2} + \frac{1}{2b_1}\log\left[\cosh(b_1 R) - \tanh(b_2)\sinh(b_1 R)\right]$$

$b_1(>0), \quad b_2$: Constant parameters

Cf. Power-law model **[Amendola, Gannouji, Polarski and Tsujikawa, Phys. Rev. D 75, 083504 (2007)]**

[Li and Barrow, Phys. Rev. D 75, 084010 (2007)]

$$f(R) = R - \mu R^v$$

$\mu(>0)$: Constant parameter

$0 < v < 10^{-10}$: Constant parameter (close to 0)

[Capozziello and Tsujikawa, Phys. Rev. D 77, 107501 (2008)]

$$S = \frac{A}{4G}$$: **Bekenstein-Hawking horizon entropy in the Einstein gravity**

$$\hat{S} = \frac{FA}{4G}$$: **Wald entropy in modified gravity theories**

$A = 4\pi\tilde{r}_A^2$: Area of the apparent horizon

- **Wald introduced a horizon entropy $\hat{S}$ associated with a Noether charge in the context of modified gravity theories.**
- **The Wald entropy $\hat{S}$ is a local quantity defined in terms of quantities on the bifurcate Killing horizon.**

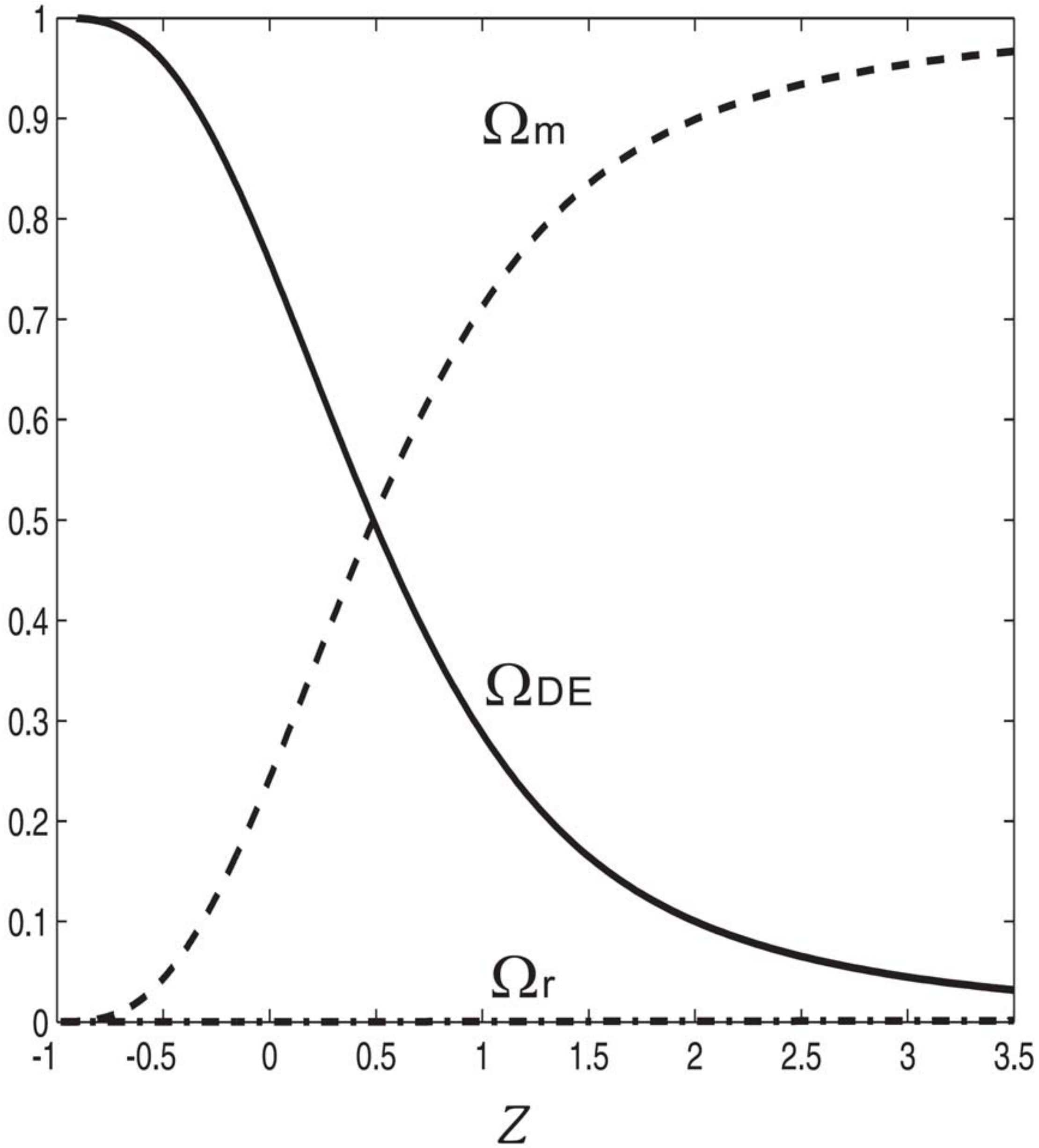
  → **More specifically, it depends on the variation of the Lagrangian density of gravitational theories with respect to the Riemann tensor.**

⇨ **This is equivalent to $\hat{S} = A/(4G_{\mathrm{eff}})$.**

$G_{\mathrm{eff}} = G/F$: Effective gravitational coupling

< Cosmological evolutions of $\Omega_{\rm DE}$, $\Omega_{\rm m}$ and $\Omega_{\rm r}$ in the exponential gravity model >

From [KB, Geng and Lee, JCAP 1008, 021 (2010)].



$$f_{\rm E}(R) =$$
$$R - \beta R_{\rm E} \left(1 - e^{-R/R_{\rm E}}\right)$$

$$\beta = 1.8$$

$$\beta R_{\rm E} \simeq 18 H_0^2 \Omega_{\rm m}^{(0)}$$

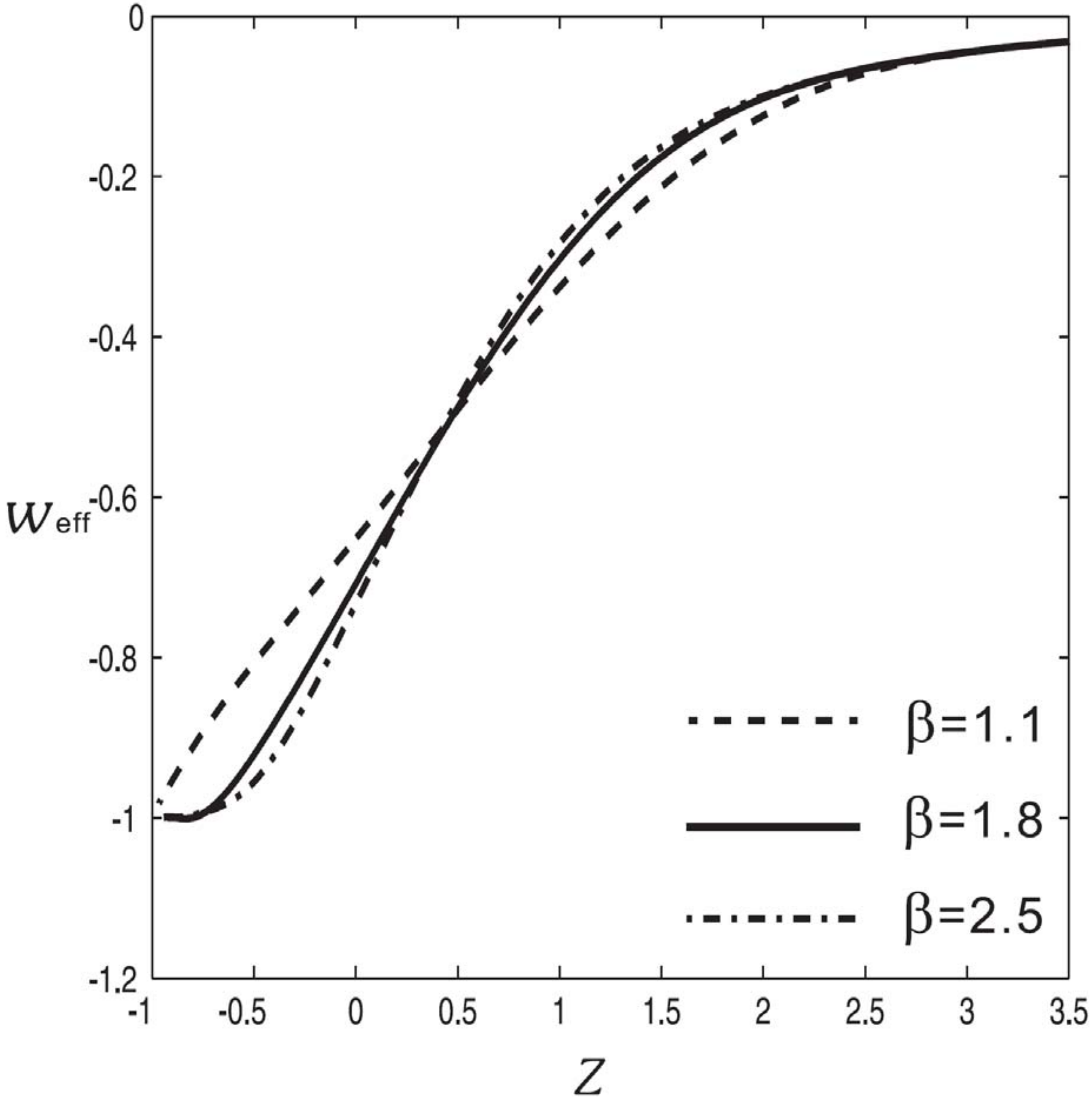
$$\Omega_{\rm DE} \equiv \rho_{\rm DE}/\rho_{\rm crit}^{(0)}$$

$$\Omega_{\rm m} \equiv \rho_{\rm m}/\rho_{\rm crit}^{(0)}$$

$$\Omega_{\rm r} \equiv \rho_{\rm r}/\rho_{\rm crit}^{(0)}$$

$$\rho_{\rm crit}^{(0)} = 3H_0^2/\kappa^2$$

< Cosmological evolution of w_{eff} in the exponential gravity model >



From [KB, Geng and Lee, JCAP 1008, 021 (2010)].

$$f_{\mathrm{E}}(R) =$$
$$R - \beta R_{\mathrm{E}} \left(1 - e^{-R/R_{\mathrm{E}}}\right)$$

$$\beta R_{\mathrm{E}} \simeq 18 H_0^2 \Omega_{\mathrm{m}}^{(0)}$$

< Remarks >

(a) The qualitative results do not strongly depend on the values of the parameters in each model.

(b) The evolutions of the Wald entropy $\hat{S}$ are similar to S in the models of (i)-(iv).

Cf. [KB, Geng and Tsujikawa, Phys. Lett. B 688, 101 (2010)]

[KB, Geng and Lee, JCAP 1008, 021 (2010)]

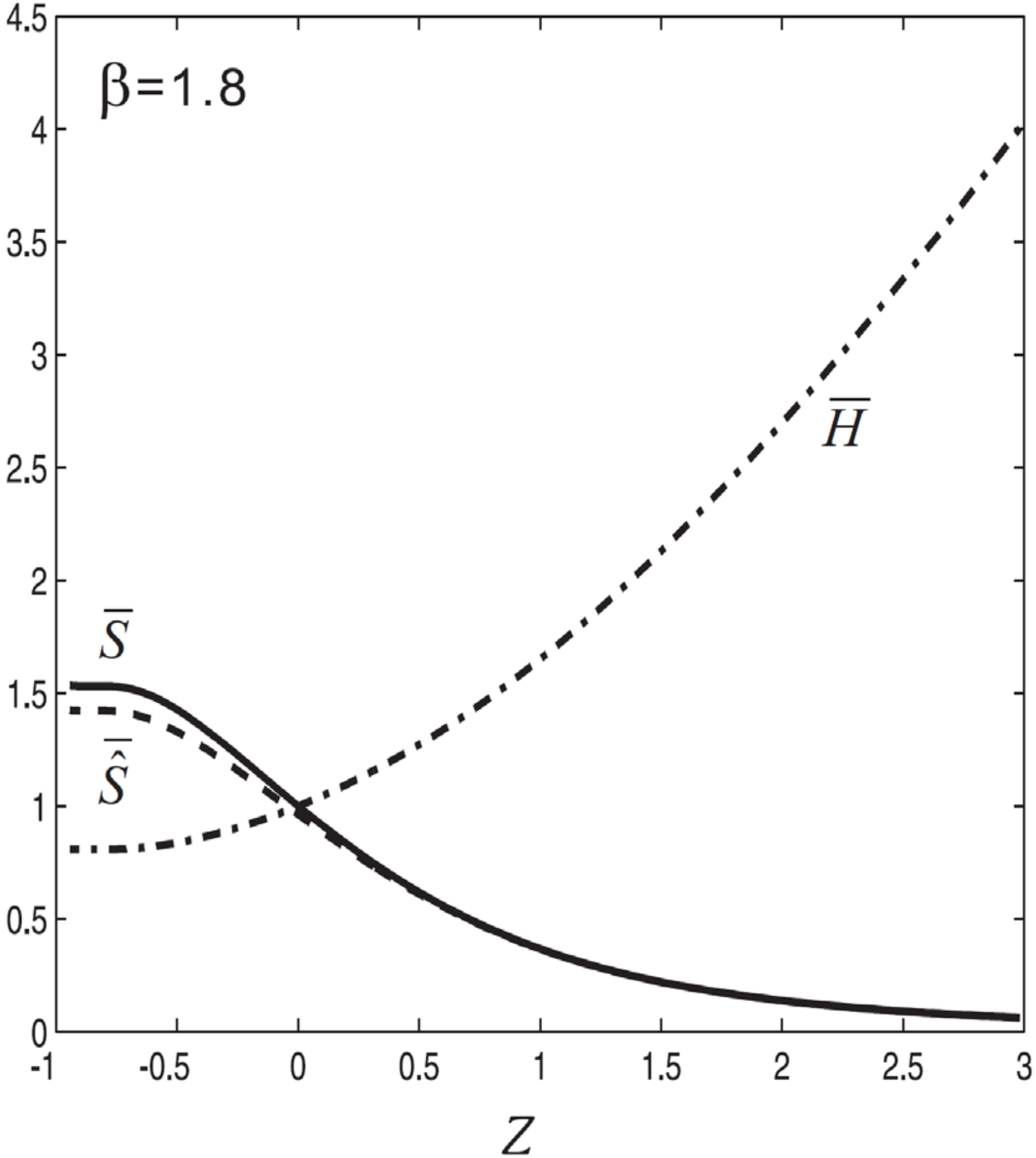
$$S = \frac{A}{4G}$$: **Bekenstein-Hawking horizon entropy in the Einstein gravity**

$$\hat{S} = \frac{F(R)A}{4G}$$: **Wald entropy in modified gravity theories including *f*(*R*) gravity**

(c) The numerical results in the Appleby-Battye model are similar to those in the Starobinsky model of (ii).

< Cosmological evolutions of $\bar{S}$, $\bar{\hat{S}}$ and $\bar{H}$ in the exponential gravity model >

From [KB, Geng and Lee, JCAP 1008, 021 (2010)].



$$f_{\mathrm{E}}(R) = R - \beta R_{\mathrm{E}} \left(1 - e^{-R/R_{\mathrm{E}}}\right)$$

$$\beta R_{\mathrm{E}} \simeq 18 H_0^2 \Omega_{\mathrm{m}}^{(0)}$$

$$\bar{S} = S/S_0$$

$$\bar{\hat{S}} = \hat{S}/S_0$$

$$\bar{H} = H/H_0$$

< Numerical results >

Models of (i), (ii), (iii) and (iv)

- $w_{\mathrm{DE}}(z=0) =$ -0.92, -0.97, -0.92 and -0.93
- $(\bar{H}_{\mathrm{f}}, \bar{S}_{\mathrm{f}}) = (0.80, 1.6),\ (0.85, 1.4),\ (0.78, 1.7)$ and $(0.81, 1.5)$
- $(z_{\mathrm{cross}}, z_{\mathrm{p}}) =$
  (-0.76, -0.82), (-0.83, -0.98), (-0.79, -0.80) and (-0.74, -0.80)

z_{cross} : **Value of z at the first future crossing of the phantom divide**

z_{p} : **Value of z at the first sign change of $\dot{H}$ from negative to positive**

- $(\Xi(z = z_{\mathrm{cross}}), \Xi(z = z_{\mathrm{p}})) =$
  $(5.2 \times 10^{-3}, 2.1 \times 10^{-3})$, $(1.7 \times 10^{-3}, 4.8 \times 10^{-6})$,
  $(4.1 \times 10^{-3}, 3.1 \times 10^{-3})$ and $(6.2 \times 10^{-3}, 2.8 \times 10^{-3})$

$$\Xi \equiv \Omega_{\mathrm{m}}/\Omega_{\mathrm{DE}}$$

$$\Omega_{\mathrm{m}} \equiv \rho_{\mathrm{m}}/\rho_{\mathrm{crit}}^{(0)}\,, \quad \Omega_{\mathrm{DE}} \equiv \rho_{\mathrm{DE}}/\rho_{\mathrm{crit}}^{(0)}$$

< Second law of thermodynamics in equilibrium description >

[KB and Geng, JCAP 1006, 014 (2010)]

We consider the same temperature of the universe outside and inside the apparent horizon.

< Gibbs equation >

$$T\mathrm{d}S_{\mathrm{t}} = \mathrm{d}\left(\rho_{\mathrm{t}} V\right) + P_{\mathrm{t}}\mathrm{d}V = V\mathrm{d}\rho_{\mathrm{t}} + \left(\rho_{\mathrm{t}} + P_{\mathrm{t}}\right)\mathrm{d}V$$

< Second law of thermodynamics >

$$\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{\mathrm{d}S_{\mathrm{t}}}{\mathrm{d}t} \geq 0$$

$$R = 6\left(\dot{H} + 2H^2 + K/a^2\right)$$

< Condition >

$$\frac{12\pi}{G}\frac{H\left(\dot{H} - K/a^2\right)^2}{\left(H^2 + K/a^2\right)^2}\frac{1}{R} \geq 0$$

→ **As long as $R > 0$, the second law of thermodynamics can be met in both non-phantom ($\dot{H} < 0$, $w_{\mathrm{eff}} > -1$) and phantom ($\dot{H} > 0$, $w_{\mathrm{eff}} < -1$) phases.**

Cf. [Gong, Wang and Wang, JCAP 0701, 024 (2007)]

[Jamil, Saridakis and Setare, Phys. Rev. D 81, 023007 (2010)]

< Second law of thermodynamics >

[KB and Geng, JCAP 1006, 014 (2010)]

We assume the same temperature between the outside and inside of the apparent horizon.

< Gibbs equation >

$\hat{S}_{\rm t}$: Entropy of total energy inside the horizon

$$T{\rm d}\hat{S}_{\rm t} = {\rm d}\left(\hat{\rho}_{\rm t} V\right) + \hat{P}_{\rm t}{\rm d}V = V{\rm d}\hat{\rho}_{\rm t} + \left(\hat{\rho}_{\rm t} + \hat{P}_{\rm t}\right){\rm d}V$$

< Second law of thermodynamics >

$$\frac{{\rm d}\hat{S}}{{\rm d}t} + \frac{{\rm d}\left({\rm d}_i\hat{S}\right)}{{\rm d}t} + \frac{{\rm d}\hat{S}_{\rm t}}{{\rm d}t} \geq 0$$

[Wu, Wang, Yang and Zhang, Class. Quant. Grav. 25, 235018 (2008)]

< Condition >

$$J \equiv \left(\dot{H} - \frac{K}{a^2}\right)^2 F \geq 0$$

$F > 0$ because $G_{\rm eff} = G/F > 0$.

→ **The second law of thermodynamics in *f(R)* gravity can be satisfied in phantom ($\dot{H} > 0$, $w_{\rm eff} < -1$) as well as non-phantom ($\dot{H} < 0$, $w_{\rm eff} > -1$) phases.**

$w_{\rm eff} = -1 - 2\dot{H}/\left(3H^2\right)$: Effective equation of state (EoS)

< Initial conditions >

Models of (i), (ii), (iii) and (iv)

- $z_0 = 8.0,\ 8.0,\ 3.0 \text{ and } 3.5$

→ We have taken the initial conditions at $z = z_0$, so that $RF'(z = z_0) \sim 10^{-13}$ with $F' = dF/dR$, to ensure that they can be all close enough to the ΛCDM model with $RF' = 0$.

→ In order to save the calculation time, the different values of z_0 mainly reflect the forms of the models, i.e., the power-law types of (i) and (ii) and the exponential ones of (iii) and (iv).

- At $z = z_0$, $w_{\mathrm{DE}} = -1$.

- $\gamma R_c \simeq 18 H_0^2 \Omega_m^{(0)}$

Models of (i), (ii), (iii) and (iv)

$(\gamma, R_c) = (c_1, R_{HS}),\ (\lambda, R_S),\ (\mu, R_T)$ and (β, R_E)

- $\Omega_m^{(0)} \equiv \rho_m^{(0)} / \rho_{crit}^{(0)} = 0.26$

[E. Komatsu *et al*. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]]]

In the high z regime ($z \simeq z_0$), $R/R_c \gg 1$, in which *f*(*R*) gravity has to be very close to the ΛCDM model.

- $\gamma R_c \simeq 2\Lambda = 2\kappa^2 \left(\rho_{DE}/\rho_m^{(0)}\right) \left(\rho_m^{(0)}/\rho_{crit}^{(0)}\right) \rho_{crit}^{(0)} = 6 \left(\rho_{DE}/\rho_m^{(0)}\right) H_0^2 \Omega_m^{(0)}$
- $\left(\rho_{DE}/\rho_m^{(0)}\right) = 3.0$

- $y_H\left(z=z_0\right) \simeq \Omega_{\mathrm{DE}}^{(0)}/\Omega_{\mathrm{m}}^{(0)} \simeq 3.0$ $\quad y_H \equiv \rho_{\mathrm{DE}}/\rho_{\mathrm{m}}^{(0)}$

- By examining the cosmological evolutions of y_H and w_{DE} as functions of the redshift z for the models, we have found that $y_H\left(z=0\right)$ is close to its initial value of $y_H\left(z=z_0\right)$.

- This is because in the higher z regime, the universe is in the phantom phase ($w_{\mathrm{DE}} < -1$) and therefore, ρ_{DE} and y_H increase (since $y_H \propto \rho_{\mathrm{DE}}$), whereas in the lower z regime, the universe is in the non-phantom (quintessence) phase ($w_{\mathrm{DE}} > -1$) and hence they decrease.

  ⇨ Consequently, the above two effects cancel out.

- Our results are not sensitive to the initial values of z_0 and $y_H\left(z = z_0\right)$.

- The initial condition of $dy_H/d\ln a\left(z = z_0\right) = 0$ is due to that the *f*(*R*) gravity models at $z = z_0$ should be very close to the ΛCDM model, in which $dy_H/d\ln a = 0$.

$$\rho_{\mathrm{DE}}^{(0)}/\rho_{\mathrm{m}}^{(0)} = \left(1-\Omega_{\mathrm{m}}^{(0)}\right)/\Omega_{\mathrm{m}}^{(0)} = 2.85$$

$$R_{\mathrm{c}} \simeq 6\gamma^{-1} y_H\,(z = z_0)\bar{m}^2$$

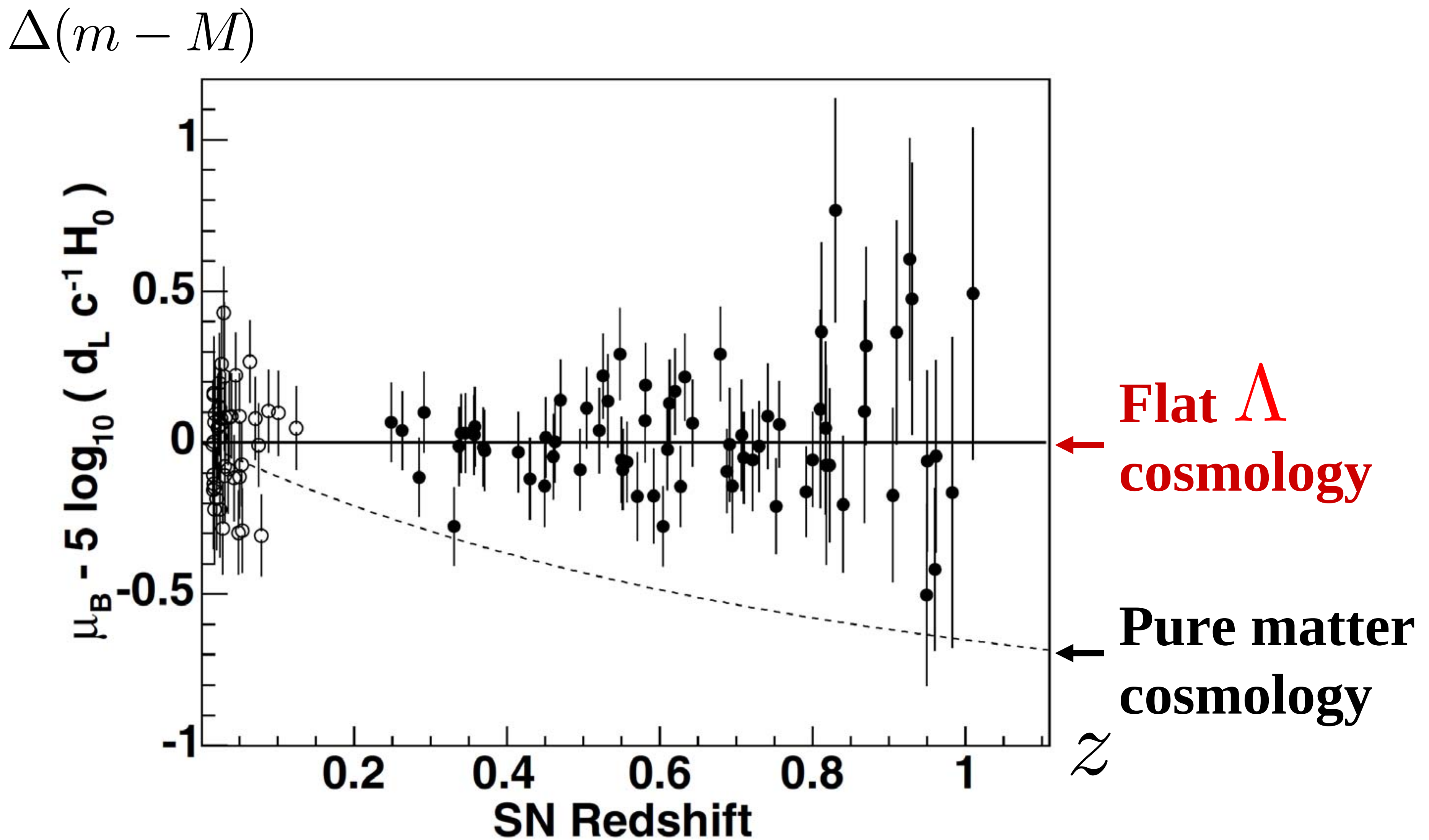
$$\bar{m}^2 = H_0^2 \Omega_{\mathrm{m}}^{(0)}$$

$$y_H\,(z = z_0) = 2.72$$

$$y_H\,(z = 0) = 2.85$$

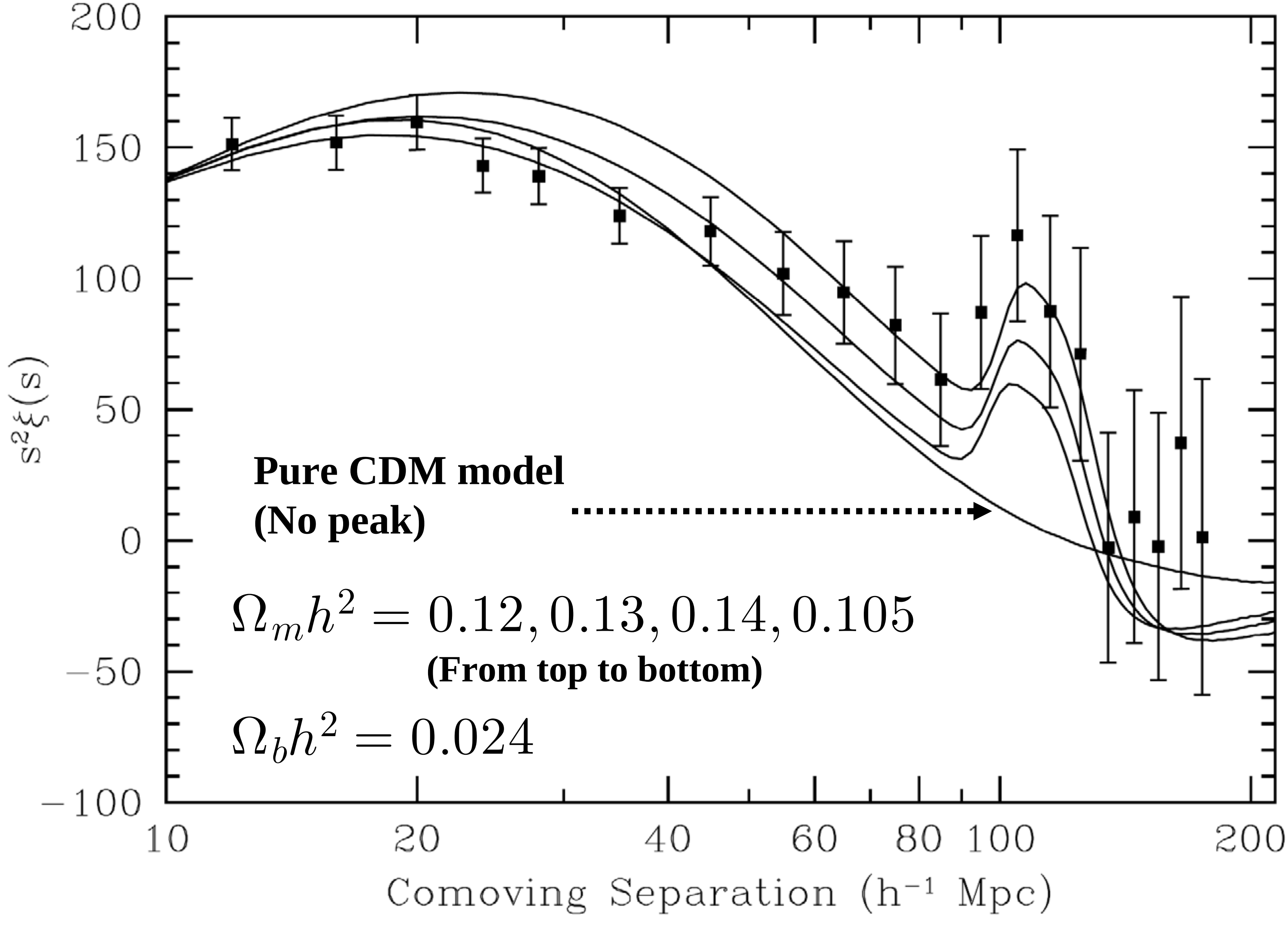
Backup Slides B

< Residuals for the best fit to a flat Λ cosmology >



From [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)]

< Baryon acoustic oscillation (BAO) >



From [Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. 633, 560 (2005)]

< 5-year WMAP data on w >

[Komatsu *et al*. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009), arXiv:0803.0547 [astro-ph]]

- For the flat universe, constant w : (From WMAP+BAO+SN)

$$-0.14 < 1 + w < 0.12 \quad (95\% \text{ CL})$$

Baryon acoustic oscillation (BAO) : Special pattern in the large-scale correlation function of Sloan Digital Sky Survey (SDSS) luminous red galaxies

- For a variable EoS :

$$-0.33 < 1 + w_0 < 0.21 \quad (95\% \text{ CL}) \longleftarrow z_{\text{trans}} = 10$$

$$w(a) = \frac{a\tilde{w}(a)}{a + a_{\text{trans}}} - \frac{a_{\text{trans}}}{a + a_{\text{trans}}}$$

$a < a_{\text{trans}}$: Dark energy density tends to a constant value

$$\tilde{w}(a) = \tilde{w}_0 + (1 - a)\tilde{w}_a \quad w_0 = w(a = 1)$$

Cf. Dark Energy : $\Omega_\Lambda = 0.726 \pm 0.015$

Dark Matter : $\Omega_c = 0.228 \pm 0.013$

Baryon : $\Omega_b = 0.0456 \pm 0.0015$ (68% CL)

$$\Omega_i \equiv \frac{\kappa^2 \rho_i^{(0)}}{3H_0^2} = \frac{\rho_i^{(0)}}{\rho_{\mathrm{c}}^{(0)}}$$

$$i = \Lambda, c, b$$

$\rho_{\mathrm{c}}^{(0)}$: Critical density

- In the flat FLRW background, gravitational field equations read

$$H^2 = \frac{\kappa^2}{3}\rho_{\rm eff}\,, \quad \dot{H} = -\frac{\kappa^2}{2}\left(\rho_{\rm eff} + p_{\rm eff}\right)$$

$\rho_{\rm eff}$, $p_{\rm eff}$: Effective energy density and pressure from the term $f(R) - R$

$$\rho_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(-f(R) + Rf'(R)\right) - 3H\dot{R}f''(R)\right]$$

$$p_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(f(R) - Rf'(R)\right) + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)\right]$$

$$\rightarrow \quad w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{\left(f(R) - Rf'(R)\right)/2 + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)}{\left(-f(R) + Rf'(R)\right)/2 - 3H\dot{R}f''(R)}$$

- Example : $f(R) \propto R^n \ (n \neq 1)$

$$\longrightarrow \quad a \propto t^q, \qquad q = \frac{-2n^2+3n-1}{n-2}$$

$$w_{\rm eff} = -\frac{6n^2-7n-1}{6n^2-9n+3}$$

[Capozziello, Carloni and Troisi, Recent Res. Dev. Astron. Astrophys. 1, 625 (2003)]

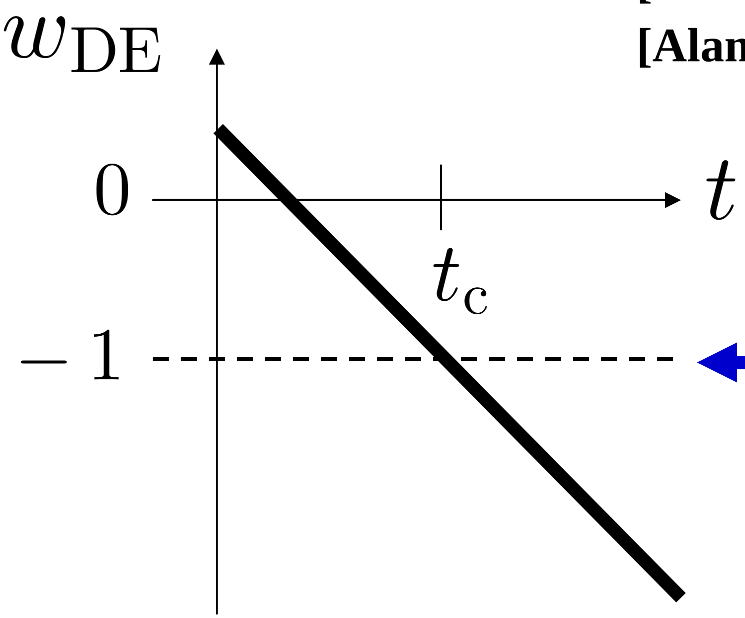
If $q > 1$, accelerated expansion can be realized.

(For $n = 3/2$ or $n = -1$, $q = 2$ and $w_{\rm eff} = -2/3$.)

< Crossing of the phantom divide >

- Various observational data (SN, Cosmic microwave background radiation (CMB), BAO) imply that the effective EoS of dark energy $w_{\rm DE}$ may evolve from larger than -1 (non-phantom phase) to less than -1 (phantom phase). Namely, it crosses -1 (the crossing of the phantom divide).

[Alam, Sahni and Starobinsky, JCAP 0406, 008 (2004)]
[Nesseris and Perivolaropoulos, JCAP 0701, 018 (2007)]
[Alam, Sahni and Starobinsky, JCAP 0702, 011 (2007)]



(a) $w_{\rm DE} > -1$

Non-phantom phase

(b) $w_{\rm DE} = -1$

Crossing of the phantom divide

(c) $w_{\rm DE} < -1$

Phantom phase

$t_{\rm c}$: Time of the crossing of the phantom divide

Appendix B

(7) Free of curvature singularities

[Frolov, Phys. Rev. Lett. 101, 061103 (2008)]

- **Existence of relativistic stars**

[Kobayashi and Maeda, Phys. Rev. D 78, 064019 (2008)]

[Kobayashi and Maeda, Phys. Rev. D 79, 024009 (2009)]

< Model with satisfying the condition (7) >

$$F_{\mathrm{MJWQ}}(R) = R - \alpha R_* \ln\left(1 + \frac{R}{R_*}\right)$$

$\alpha > 0, \quad R_* > 0$: Constants

[Miranda, Joras, Waga and Quartin, Phys. Rev. Lett. 102, 221101 (2009)]

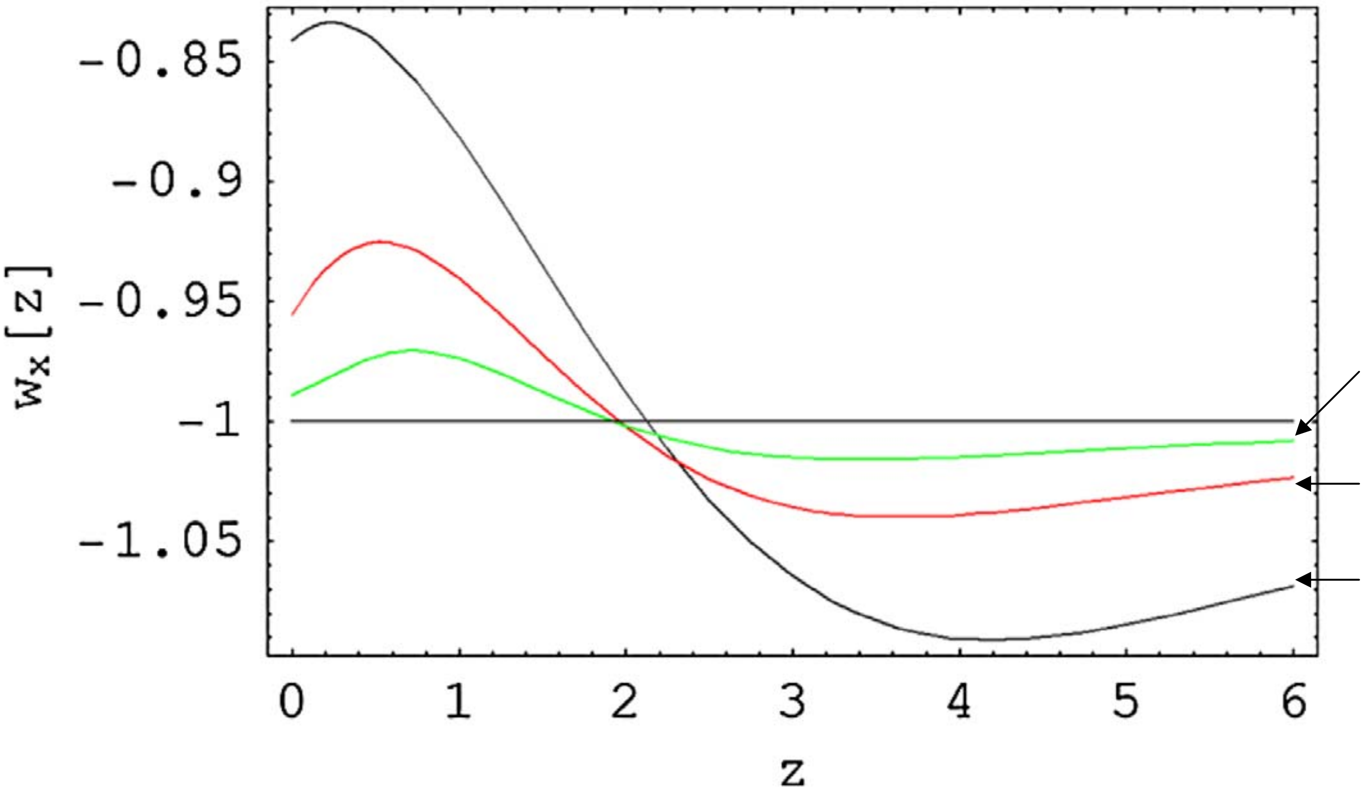
Cf. [de la Cruz-Dombriz, Dobado and Maroto, Phys. Rev. Lett. 103, 179001 (2009)]

< Recent work >

[Martinelli, Melchiorri and Amendola, Phys. Rev. D 79, 123516 (2009)]

Cf. [Nozari and Azizi, Phys. Lett. B 680, 205 (2009)]

- It has been shown that in the Hu-Sawicki model, the transition from the phantom phase to the non-phamtom one can also occur.



From [Martinelli, Melchiorri and Amendola, Phys. Rev. D 79, 123516 (2009)]

$n = 1$

$F'(R_0) = -0.01$

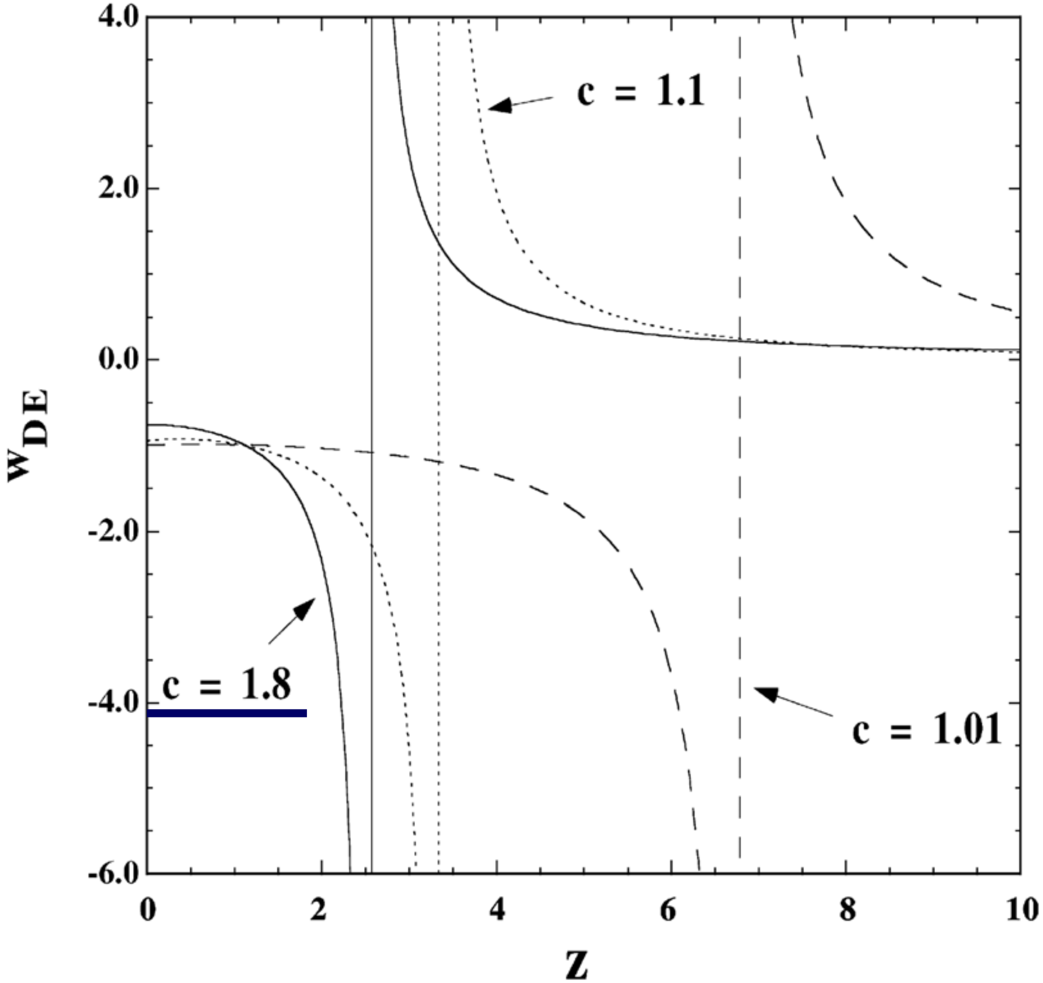
$F'(R_0) = -0.03$

$F'(R_0) = -0.1$

$\Omega_m = 0.24$

We reconstruct an explicit model of $F(R)$ gravity with realizing the crossing of the phantom divide.

< Preceding work >



From [Amendola and Tsujikawa, Phys. Lett. B 660, 125 (2008)]

$$F(R) = \left(R^{1/c} - \Lambda\right)^c$$

$c, \ \Lambda$: Constants

- Example: $c = 1.8$

Phantom phase

⇩

Non-phantom phase

(5) Existence of a matter-dominated stage and that of a late-time cosmic acceleration

$\longrightarrow$ **Analysis of $m(r)$ curve on the (r, m) plane**

$$m \equiv RF''(R)/F'(R), \qquad r \equiv -RF'(R)/F(R)$$

- **Presence of a matter-dominated stage**

$$m(r) \approx +0 \text{ and } \frac{dm}{dr} > -1 \text{ at } r \approx -1$$

- **Presence of a late-time acceleration**

(i) $m(r) = -r - 1, \quad \frac{\sqrt{3}-1}{2} < m \le 1$ and $\frac{dm}{dr} < -1$

(ii) $0 < m \le 1$ at $r = -2$

- **Combing local gravity constraints, we obtain**

$$m(r) = C(-r-1)^p \text{ with } p > 1 \text{ as } r \to -1.$$

$C > 0, \ p$: Constants

[Amendola, Gannouji, Polarski and Tsujikawa, Phys. Rev. D 75, 083504 (2007)]
[Amendola and Tsujikawa, Phys. Lett. B 660, 125 (2008)]

- For most observational probes (except the SNLS data), a low $\Omega_{0\mathrm{m}}$ prior $(0.2 < \Omega_{0\mathrm{m}} < 0.25)$ leads to an increased probability (mild trend) for the crossing of the phantom divide.

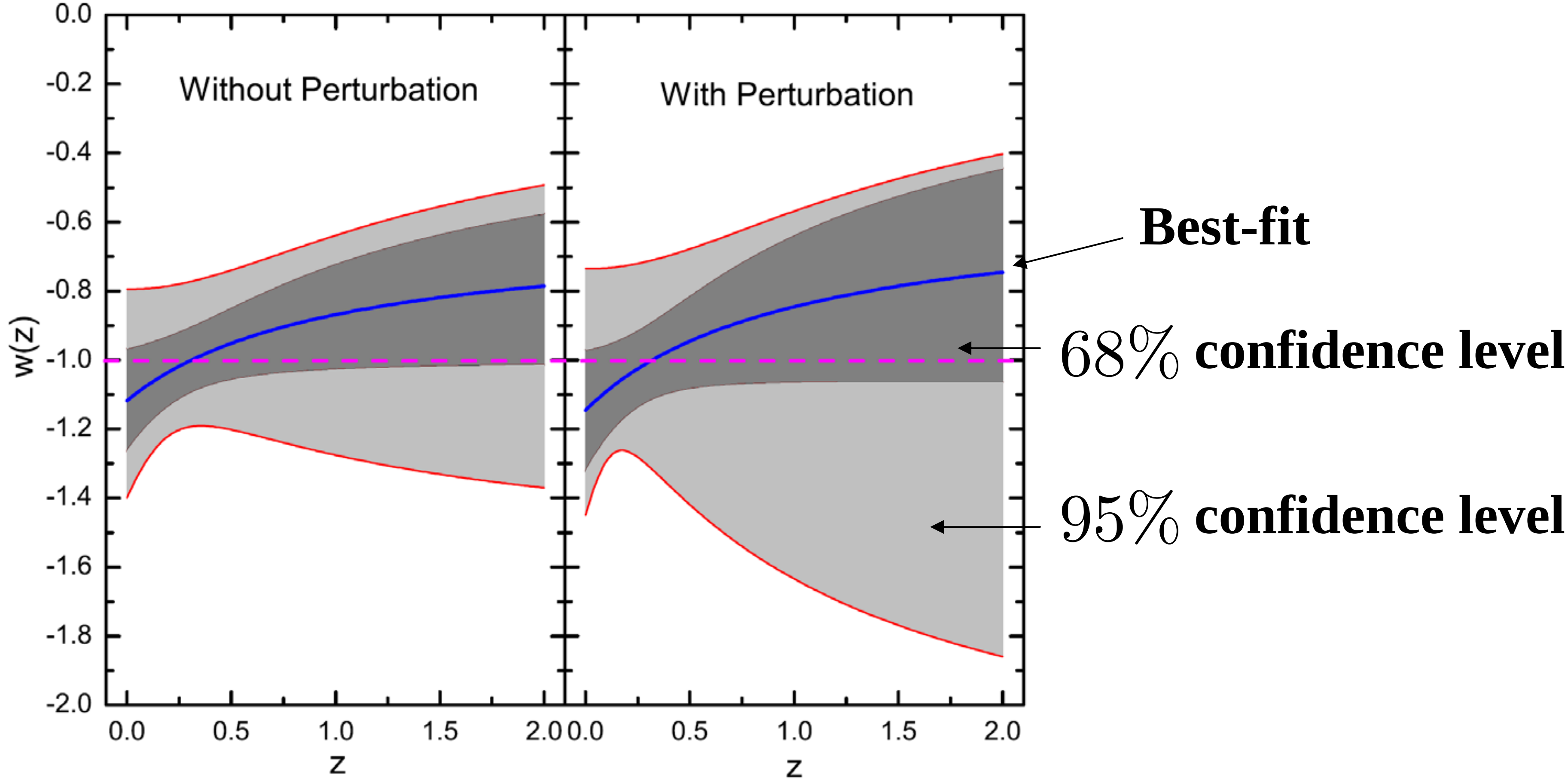
$\Omega_{0\mathrm{m}}$: Current density parameter of matter

[Nesseris and L. Perivolaropoulos, JCAP 0701, 018 (2007)]

< Data fitting of $w(z)$ (3) >

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

From [Zhao, Xia, Feng and Zhang, Int. J. Mod. Phys. D 16, 1229 (2007) [arXiv:astro-ph/0603621]]



157 “gold” SN Ia data set+WMAP 3-year data+SDSS with/without dark energy perturbations.