A parametrisation of the growth index in GR with a non-trivial vacuum energy and Fisher matrix analysis

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Work in progress in collaboration with
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Outline

Tracking down Dark Energy

- Accelerated phase of the Universe $\Longrightarrow$ Many theoretical models of DE
- "Easiest" solution: ΛCDM with $w = -1$ but it could also be $w = w(a)$
- Need to devise strategies to distinguish between models $\Longrightarrow$ measuring the expansion history is not enough!
- The growth rate of structures can distinguish between models with similar expansion histories
- Large scale structure measurements can measure $G(a) = \delta_m/a$ and $\mathbf{f(a)} = \frac{d\ln\delta}{d\ln a} = \Omega_m(a)^{\gamma}$

Measuring γ

Weak lensing

- Deflection angle determined by $\phi + \psi \Longrightarrow$ information about matter density distribution
- Include different models of DE $\Longrightarrow \begin{cases} k^2\phi = -4\pi G Q(k,a)\bar{\rho}_m \delta_m \\ \psi = (1+\eta(k,a))\phi \end{cases}$
- If $Q \neq 1$ and $\eta \neq 0$ evolution of matter perturbations changes $\Rightarrow$ modified growth of linear perturbations parametrised by γ

$$P_{ij}(\ell) = H_0^3 \int_0^\infty \frac{dz}{E(z)} W_i(z) W_j(z) P_{nl}\left[P_l\left(\frac{H_0 \ell}{r(z)}, z\right)\right]$$

Measuring γ

Galaxy power spectrum

- Power spectrum analysis of galaxy redshift surveys containing acoustic peaks can measure cosmological parameters
- Amplitude of matter power spectrum rescaled by $G(z)$

$$P_{obs}(z,k) = \frac{D_{Ar}^2(z)H(z)}{D_A^2(z)H_r(z)} G^2(z) b(z)^2 \left(1+\beta\mu^2\right)^2 P_{0r}(k) + P_{shot}(z)$$

- Observation of galaxy power spectra affected by:
  - *Bias factor*: galaxy overdensity traces matter distribution through *bias*
  - *Redshift space distortions*: we only measure P_{gal} in redshift space, which is distorted compared to real space
  - *Reference cosmology*: takes into account difference in comoving volumes in 2 different cosmologies
  - *Shot noise*: Poissonian-like noise from the number count of galaxies in the survey volume

Perturbation equations

- Linear perturbation equations for a general fluid with $w = p/\rho$

$$\delta' = 3\,(1+w)\,\Phi' - \frac{V}{Ha^2} - 3\frac{1}{a}\left(\frac{\delta p}{\rho} - w\delta\right)\,,$$

$$V' = -\,(1-3w)\,\frac{V}{a} + \frac{k^2}{Ha^2}\frac{\delta p}{\rho} + (1+w)\,\frac{k^2}{Ha^2}\Psi - (1+w)\,\frac{k^2}{Ha^2}\sigma$$

Ma, Bertschinger 1995

where $\delta = \delta\rho/\rho$ and $V = ik_j T^j_0/\rho$

- Interested in evolution of matter density field $\Longrightarrow$ $\delta p = w = \sigma = 0$
- Master equation for small scales (assuming no DE perturbations):

$$\delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{3}{2a^2}\frac{\Omega_{\rm M}^0 a^{-3}}{E^2}\delta = 0\,,$$

- Solution for constant w (growing mode) Lee, Ng 2010

$$\delta(a) = C_1\, a\; {}_2F_1\left(\frac{w-1}{2w}, \frac{-1}{3w}, 1 - \frac{5}{6w}; -\frac{\Omega_{\rm DE}^0}{\Omega_{\rm M}^0}\, a^{-3w}\right)$$

From δ to γ

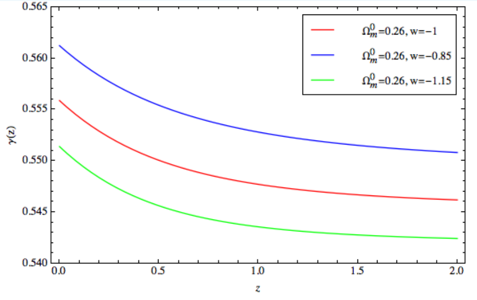
- Steps to build the growth index
  - Logarithmic derivative:

$$f(a) \equiv \frac{d\log\delta}{d\log a} = 1 - \frac{3(w-1)}{6w-5}\frac{\Omega^0_{\rm DE}}{\Omega^0_{\rm M}}\, a^{-3w}\, \frac{{}_2F_1\left[\frac{3w-1}{2w}, \frac{3w-1}{3w}, \frac{12w-5}{6w}; -\frac{\Omega^0_{\rm DE}}{\Omega^0_{\rm M}}\, a^{-3w}\right]}{{}_2F_1\left[\frac{w-1}{2w}, \frac{-1}{3w}, \frac{6w-5}{6w}; -\frac{\Omega^0_{\rm DE}}{\Omega^0_{\rm M}}\, a^{-3w}\right]},$$

  - Gamma parameter: $f(a) = \Omega^{\gamma}_{\rm M}(a)$
    - For a non-redshift dependent γ: $\gamma = \frac{3(w-1)}{6w-5} = \frac{6}{11} \simeq 0.545$ $(w=-1)$

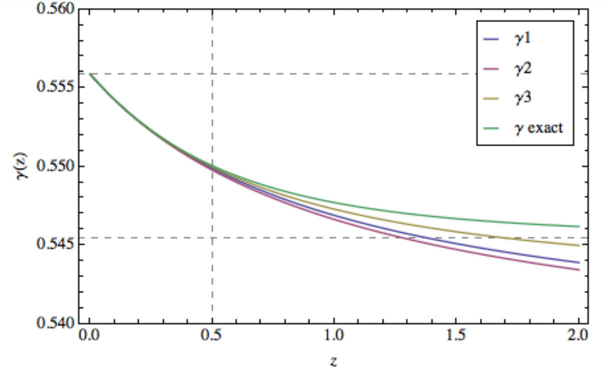
Linder, Cahn 2008

    - Redshift dependent γ



Parametrisation of γ

- If γ depends on redshift, usual parametrisation: $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{1+z}$
- Is this a good parametrisation?



Varying the equation of state

- Usual parametrisation of $w(a) = w_0 + w_1(1-a)$
- Matter density parameter:
$\Omega_{\rm M}(a) = \left(1 + \frac{\Omega^0_{\rm DE}}{\Omega^0_{\rm M}} a^{-3(w_0+w_1)}\, e^{-3w_1(1-a)}\right)^{-1}$
- Density contrast for slowly varying $w(a)$

$$\delta(a) = a\ {}_2F_1\left[\frac{w(a)-1}{2w(a)}\left(1-\frac{3w_1}{3w(a)-1}\right), \frac{-1}{3w(a)}\left(1+\frac{3w_1}{3w(a)-1}\right), \frac{6w(a)-5}{6w(a)} - \frac{w_1}{w(a)}; -\frac{\Omega^0_{\rm DE}}{\Omega^0_{\rm M}} a^{-3(w_0+w_1)}\, e^{-3w_1(1-a)}\right]$$

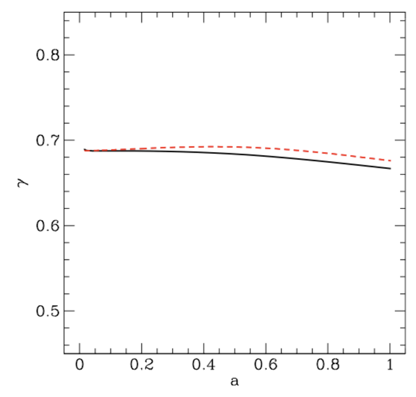
AB, García-Bellido, Sapone, in preparation

γ for DGP

- Differential equation for density contrast in DGP

$$k^2\Phi = -4\pi G\left(1-\frac{1}{3\beta}\right)\bar{\rho}_m\delta_m, \qquad \beta = 1-\frac{2(Hr_c)^2}{2Hr_c-1}$$

- Approximation for γ: $\gamma = \frac{7+5\Omega_M(a)+7\Omega_M^2(a)+3\Omega_M^3(a)}{(1+\Omega_M^2(a))(11+5\Omega_M(a))}$ Linder, Cahn 2008



Amendola, Kunz, Sapone 2008

γ for f(R)

- Perturbation equations in Fourier space: Scale dependent!

$$\ddot{\delta}_m + \left(2H + \frac{\dot{F}}{2F}\right)\dot{\delta}_m - \frac{\rho_m}{2F}\delta_m = \frac{1}{2F}\left[\left(-6H^2 + \frac{k^2}{a^2}\right)\delta F + 3H\delta\dot{F} + 3\delta\ddot{F}\right]$$

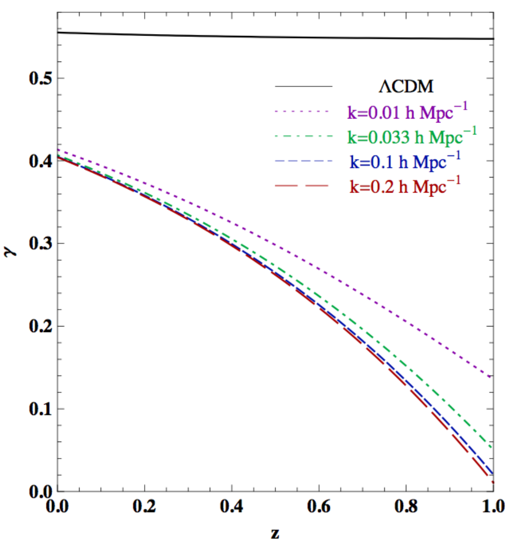
$$\delta\ddot{F} + 3H\delta\dot{F} + \left(\frac{k^2}{a^2} + \frac{f_{,R}}{3f_{,RR}} - \frac{R}{3}\right)\delta F = \frac{1}{3}\rho_m\delta_m + \dot{F}\dot{\delta}_m$$

Tsujikawa, 2008

- Small scale approximation $(k^2/a^2 \gg H^2)$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m \simeq 0, \qquad G_{\text{eff}} \equiv \frac{G}{F}\frac{4\frac{k^2}{a^2} + \frac{f_{,R}}{f_{,RR}}}{3\frac{k^2}{a^2} + \frac{f_{,R}}{f_{,RR}}}$$

γ for f(R)



Tsujikawa, Gannouji, Moraes, Polarski 2009

Survey specifications and fiducial model

- Survey specifications
  - Photometric survey in the range $0.5 < z < 2.1$ divided in $\Delta z = 0.2$ bins. Survey area of 20000 deg^2
  - Galaxy number density per redshift bin Laureijs et al. 2009

z	$n_1(z) \times 10^{-3}$	$n_2(z) \times 10^{-3}$
$0.5 - 0.7$	4.69	3.56
$0.7 - 0.9$	3.33	2.42
$0.9 - 1.1$	2.57	1.81
$1.1 - 1.3$	2.10	1.44
$1.3 - 1.5$	1.52	0.99
$1.5 - 1.7$	0.92	0.55
$1.7 - 1.9$	0.54	0.29
$1.9 - 2.1$	0.31	0.15

- Fiducial model
  - Cosmological parameters: ΛCDM model with WMAP-7yr data.
  - Dark energy parameters: $w_0 = -0.96$, $w_1 = 0$, $\gamma = 0.545$ and γ as a free parameter

Baryon Acoustic Oscillations

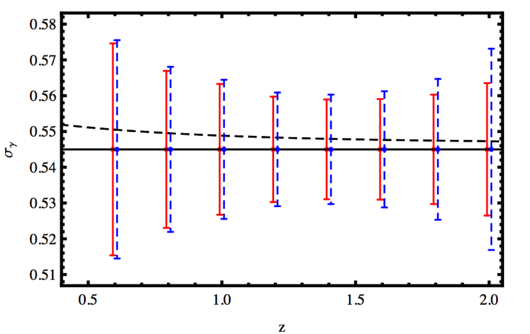
- Evaluate F_{ij} for the following parameters:

	Parameters	
1	total matter density	$\omega_m = \Omega_{m_0} h^2$
2	total baryon density	$\omega_b = \Omega_{b_0} h^2$
3	optical thickness	τ
4	spectral index	n_s
5	matter density today	Ω_{m_0}
	For each redshift bin	
6	Hubble parameter	$\log H(z)$
7	Angular diameter distance	$\log D_A(z)$
8	Growth factor	$\log G(z)$
9	z-distortion	$\log \beta(z)$
10	shot noise	P_s

$\Rightarrow$ project into dark energy parameters: w_0, w_1 and γ

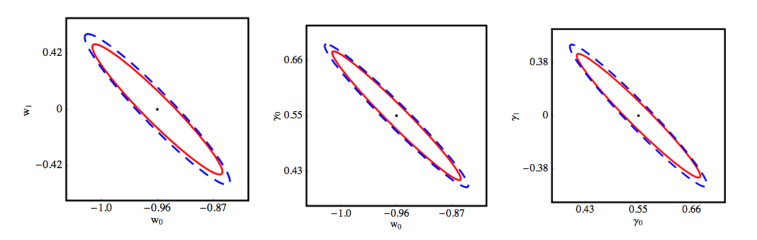
Baryon Acoustic Oscillations

- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin



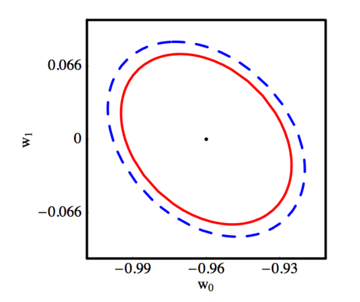
Baryon Acoustic Oscillations

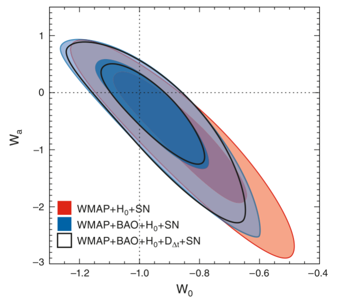
- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin
  - γ free parameter parametrised by γ_0 and γ_1 but equal for all redshift bins



Baryon Acoustic Oscillations

- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin
  - γ free parameter parametrised by γ_0 and γ_1 but equal for all redshift bins
  - γ dependent on dark energy parameters w_0 and w_1





Komatsu et al. 2011

Galaxy Power Spectrum

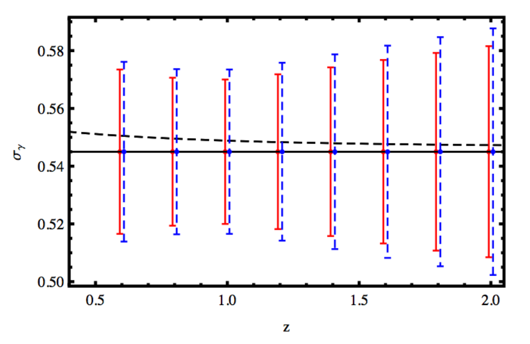
- Evaluate F_{ij} for the following parameters:

	Parameters	
1	total matter density	$\omega_m = \Omega_{m_0} h^2$
2	total baryon density	$\omega_b = \Omega_{b_0} h^2$
3	optical thickness	τ
4	spectral index	n_s
5	matter density today	Ω_{m_0}
6	equation of state parameter	w_0
7	equation of state parameter	w_1
	For each redshift bin	
8	growth index	γ
9	shot noise	P_s

$\Rightarrow$ get directly the errors for dark energy parameters w_0, w_1 and γ

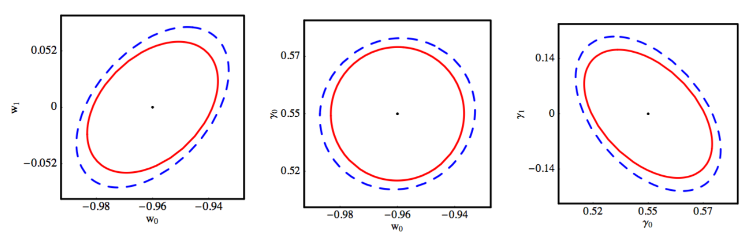
Galaxy Power Spectrum

- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin



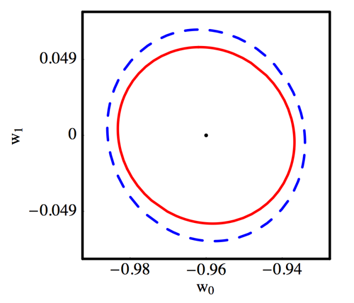
Galaxy Power Spectrum

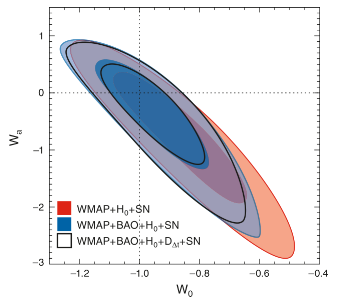
- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin
  - γ free parameter parametrised by γ_0 and γ_1 but equal for all redshift bins



Galaxy Power Spectrum

- We consider 3 cases for γ
  - γ free parameter, independent in each redshift bin
  - γ free parameter parametrised by γ_0 and γ_1 but equal for all redshift bins
  - γ dependent on dark energy parameters w_0 and w_1





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Weak Lensing

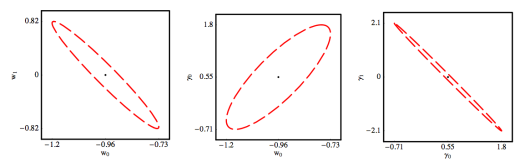
- Evaluate F_{ij} for the following parameters:

	Parameters	
1	total matter density	$\omega_m = \Omega_{m_0} h^2$
2	total baryon density	$\omega_b = \Omega_{b_0} h^2$
3	optical thickness	τ
4	spectral index	n_s
5	matter density today	Ω_{m_0}
6	equation of state parameter	w_0
7	equation of state parameter	w_1
	For each redshift bin	
8	growth index	γ
9	rms of the density perturbations at $8h^{-1}$Mpc	σ_8

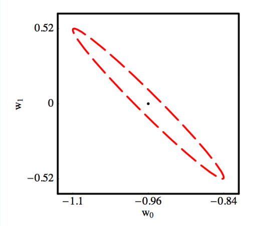
$\Rightarrow$ get directly the errors for dark energy parameters w_0, w_1 and γ

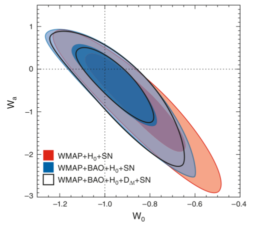
Weak Lensing

- We consider 2 cases for γ
  - γ free parameter parametrised by γ_0 and γ_1 but equal for all redshift bins



  - γ dependent on dark energy parameters w_0 and w_1





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Conclusions and outlook

- It is important to study and constrain γ to distinguish between DE models
- We found an analytic expression for γ for a slowly varying equation of state $w = w(a)$
- We found that the Fisher matrix analysis for future surveys such as Euclid tell us we will be able to distinguish between DE models
- Future work (already in progress!)
  - Study the growth index for other theoretical models and use the forecasts to see if it will be possible to rule them out
  - Study the scale dependence of the growth index
  - Study the role of DE perturbations in the growth of matter
  - Add the forecasts from different observables to reduce errors

Thank you for listening!

Fisher matrix formalism

- Observed galaxy power spectrum:

$$P_{obs}(z,k) = \frac{D_{Ar}^2(z)H(z)}{D_A^2(z)H_r(z)} G^2(z) b(z)^2 \left(1+\beta\mu^2\right)^2 P_{0r}(k) + P_{shot}(z)$$

Seo, Eisenstein 2003

  - r: Values assumed for the reference cosmology
  - μ: direction cosine within the survey
  - P_{0r}: present matter power spectrum for the fiducial cosmology (CAMB output)
  - $\beta(z)$: encodes distortion induced by redshift. $\beta(z) = \frac{\Omega_m^\gamma(z)}{b}$
  - $(1+\beta\mu^2)$: accounts *only* for linear distortion in redshift space
  - Bias b: We only see baryons, so we assume $\delta_b = b\delta_m$. Assumed to be scale independent on large scales. We use $b = \sqrt{1+z}$
  - $P_{shot}(z)$: Shot noise assumed Poissonian $P_{shot} \sim \frac{1}{nP_{gal}}$

Fisher matrix formalism

- Total galaxy power spectrum including the errors on redshift:

$$P(z;k) = P_{obs}(z;k)\, e^{k^2\mu^2\sigma_r^2}$$

where $\sigma_r = \frac{\delta z}{H(z)}$ and $\delta z = 0.001(1+z)$.

- Fisher matrix elements (assuming Gaussian likelihood)

$$F_{ij} = 2\pi \int_{k_{min}}^{k_{max}} \frac{\partial \log P(k)}{\partial \theta_i} \frac{\partial \log P(k)}{\partial \theta_j} \cdot V_{eff} \cdot \frac{k^2}{8\pi^3} \cdot \mathrm{d}k$$

where $V_{eff} = \int \left[\frac{n(\vec{r})P(k,\mu)}{n(\vec{r})P(k,\mu)+1}\right]^2 \mathrm{d}\vec{r} = \left[\frac{n(\vec{r})P(k,\mu)}{n(\vec{r})P(k,\mu)+1}\right]^2 V_{survey}$ and $\mu = \vec{k} \cdot \widehat{r}/k$

- The $k_{max}(z)$ is evaluated at z of the corresponding bin chosen to avoid non-linearity problems both in spectrum and bias $\Longrightarrow$ we choose values from $0.11h/\mathrm{Mpc}$ for small z bins to $0.25h/\mathrm{Mpc}$ for the highest z bins.

Weak Lensing

- The convergence weak lensing spectrum

$$P_{ij}(\ell) = H_0^3 \int_0^\infty \frac{dz}{E(z)} W_i(z) W_j(z) P_{nl}\left[P_l\left(\frac{H_0 \ell}{r(z)}, z\right)\right]$$

Hu, Jain 2004

- Fisher matrix elements

$$F_{\alpha\beta} = f_{sky} \sum_\ell \frac{(2\ell+1)\Delta\ell}{2} \partial(P_{ij})_{,\alpha} C_{jk}^{-1} \partial(P_{km})_{,\beta} C_{mi}^{-1}$$

where $C_{jk} = P_{jk} + \delta_{jk} \langle \gamma_{int}^2 \rangle n_j^{-1}$