Constraints on Neutrino Masses from Weak Lensing and Future CMB Lensing-Galaxy correlations

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- Constraints on Neutrino Masses
  From weak lensing (comic shear)
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  S. Saito, KI, A. Taruya, in progress
Cosmic Neutrinos

- Thermal Relic From the Big Bang
  Not directly observed yet
  Fermi distribution: \( f = \frac{1}{1 + e^{E/kT_\nu}} \)
  Present density (per flavor)
  \( n_\nu = 113 \text{cm}^{-3} \)

Small mass (sub-eV) makes significant contribution to the total mass density of the universe
Inputs from Particle Physics

✔ Neutrino oscillation

At least two flavors of neutrino are non-relativistic at present universe

Only mass (squared) difference

✔ Summary: $\sum m_\nu \gtrsim 0.056(0.095)\text{eV}$
Massive $\nu$ in the Background

- $\rho \propto a^{-4}$ (Relativistic)
- $\rho \propto a^{-3}$ (Non-Relativistic)

Matter-Radiation Equality in the past

Effect of Massive $\nu$ (1):

Mass of neutrinos delays the Matter-Radiation Equality, fixing matter density at present

Large ISW effect in the CMB spectrum
CMB angular spectrum

Larger 1st, 2nd peaks
**Massive $\nu$ in the perturbation**

- Large thermal (random) velocity
  
  $$v_{\text{th}} = \frac{<p>}{m} = \frac{3T_\nu}{m} \sim 150 \frac{a_0}{a} \left( \frac{1 \text{ eV}}{m} \right) \text{ km/s}$$

- $\nu$ s do not clump below free streaming scale
  
  $$k_{fs} \sim a(t)(v_{\text{th}}t)^{-1} \sim 0.82 \frac{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}}{(1+z)^2} \left( \frac{m}{1 \text{ eV}} \right) h \text{ Mpc}^{-1}$$

  $$k_{fs} > k_{nr} = 0.018 \sqrt{\Omega_m} \left( \frac{m}{1 \text{ eV}} \right)^{1/2} h \text{ Mpc}^{-1}$$

**Effect of Massive $\nu$ (2):**

Massive neutrinos do not contribute to the gravitational potential for $k > k_{nr}$

Smaller matter power at $k > k_{nr}$ at present
Evolution Examples

\[ k = 2.0 \times 10^{-3} \text{Mpc}^{-1} \]

\[ k = 0.2 \text{Mpc}^{-1} \]

\[ \delta(x) = \frac{\delta \rho(x)}{\rho} = \int d^3 k \delta(k) e^{i k \cdot x} \]

\[ \ddot{\delta} + 2H \dot{\delta} = 4\pi G (\rho_{\text{cdm}} \delta_{\text{cdm}} + \rho_{\nu} \delta_{\nu}) \quad \text{at large scales} \]

\[ \ddot{\delta} + 2H \dot{\delta} = 4\pi G (\rho_{\text{cdm}} \delta_{\text{cdm}} + \rho_{\nu} \delta_{\nu}) \quad \text{at small scales} \]

Friction by hubble expansion (\( \nu \) always contributes)

Fourier mode of density perturbation

horizon crossing

equality

\( \nu \) becomes massive

Friction by hubble expansion (\( \nu \) is negligible at small scale)
Matter Power Spectrum

$k_{nr}$

$P(k)$ vs. $k/h$ [Mpc$^{-1}$]

- $m_{\nu} = 0$ eV
- $m_{\nu} = 0.38$ eV
- galaxy clustering (SDSS)
Constraints from CMB alone

Larger $m_\nu$ can be compensated with larger $\Omega_M$ (and smaller $h$), giving the same matter-radiation equality.

CMB does not have any constraining power for

$$m_\nu \lesssim 0.5 \text{eV} \quad \left( \sum m_\nu \lesssim 1.5 \text{ eV} \right)$$

(Ichikawa et al., 2005)

To include the information from the matter power is critical to go below $O(1\text{eV})$ constraint
CFHTLS (Cosmic Shear)

- Canada-France-Hawaii Telescope Legacy Survey

- Imaging observation of galaxies of \( O(10^6) \)
- 450 nights over 5 yr
- 2 pt correlation
- Photometric redshift in the future
Merits of galaxy weak lensing

 Plenty of data will come out in the next decade
 Free from light-mass bias
 No constraints had been reported yet

 See, however,  Tereno et al., 0810.0555  Gong et al., 0810.3572

<table>
<thead>
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<th>telescope</th>
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<th>Depth (#/arcmin^2)</th>
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<td>SNAP (2013-)</td>
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Distant galaxies are distorted due to gravitational lensing effects by large scale structure.

http://www.astro.uni-bonn.de/~webiaef/research/lensing/lenses-3e.shtml
Signal – correlated shear

Weak lensing produces:
(a) [Diagram showing correlation]
(b) [Diagram showing correlation]

Does not produce:
(c) [Diagram indicating absence]
Observable: ellipticity (Shear)

- Distortions can be described as mapping between the source plane (s) and image plane (i)

\[ \delta x^S_i = A_{ij} \delta x^I_j \]

\[ A_{ij} = \begin{pmatrix} 1 - \kappa - \gamma & -\gamma \\ -\gamma & 1 - \kappa + \gamma \end{pmatrix} \]

- Convergence \( \kappa \) is difficult to be measured. Shear \( \gamma \) is easier.
Link between theory and observation (Munshi et al., phys. Rep, 2008)

Observation: shear 2 pt correlation function

\[ \xi(\theta) = \left\langle \gamma(\vec{r})\gamma^* (\vec{r} + \vec{\theta}) \right\rangle \quad \gamma = \gamma_1 + i\gamma_2 \]

Theory: the lensing power spectrum

\[ P_\kappa(\ell) = \frac{9}{4} \Omega_m^2 \left( \frac{H_0}{c} \right)^4 \int \frac{d\chi}{a^2(\chi)} \, P_\delta \left( \frac{\ell}{\chi}; \chi \right) \times \left[ \int_{\chi}^{\chi_{lim}} d\chi' n(\chi') \frac{\chi' - \chi}{\chi'} \right]^2 \]

These two are related through the relation

\[ \xi(\theta) = \int \frac{\ell d\ell}{2\pi} P_\kappa(\ell) J_0(\ell \theta) \]
We need non-linear $P(k)$!

- Linear theory breaks down when $<\delta^2> \sim k^3 P(k) \sim 1$
- Fitting formula calibrated with LCDM N-body simulation
  \[ \text{NL} P(k, z) = f(\text{L} P(k, z)) \]
  e.g., Smith et al., (2003)

Power of density fluctuation

- $m_\nu = 0$ eV
- $m_\nu = 1.1$ eV

Lensing power spectrum

- $\kappa$
Non-linear effect

- Baryon and CDM go into non-linear regime while neutrino stays almost linear (see, however, Shoji&Komatsu?)
- So, we model $P(k)$ as

$$P(k)^{\text{tot}} = \left[ f_{\nu} \sqrt{P^L_{\nu}(k)} + (f_b + f_c) \sqrt{P^L_{b+c}(k)} \right]^2$$

$$\frac{P^{\text{massive}} - P^{\text{massless}}}{P^{\text{massless}}}$$

This should be checked by N-body simulations including massive neutrinos.

(S. Saito et al., PRL, 2008)
Non-linear perturbation in neutrinos (Shoji&Komatsu, 0903.2669)

- Difference is < 0.5% for $\sum m_\nu = 0.64$ eV
- Negligible difference for the current data quality
Shear correlation function

Data points:
CFHTLS result from Fu et al., 2008

Error bars:
diagonal terms in the inverse of the covariance matrix that includes contributions from the shot noise of intrinsic galaxy ellipticities and the Gaussian and non-Gaussian sample variances (Takada and Jain, 2008)

Lines:
(black) best fit model to WL only
(red) best fit model to ALL data sets
(blue) massive neutrino model with the other parameters fixed to the same values as in the red dotted line
Systematic issue 1: in the covariance matrix

- First three principal modes contributing to S/N
- Different non-linear models give different weightings
Systematic issue 2: in the signal estimation

- Halo fit underestimates the power at small scales
Systematics3: in the galaxy distribution

\[ n(z) = \frac{\beta}{\sigma_s \Gamma \left( \frac{1 + \alpha}{\beta} \right)} \left( \frac{z}{z_s} \right)^\alpha \exp \left[ - \left( \frac{z}{z_s} \right)^\beta \right], \]

\[ \alpha = 0.838, \beta = 3.43 \]

\[ z_c = 1.172 \pm 0.026 \]

RESULT 1:
WMAP5 vs. WMAP5+WL

WMAP5 -- $m_\nu < 1.2$ eV

WMAP5+CFHTLS -- $m_\nu < 1.1$ eV

✓ The limit does not improve due to degeneracy (as in the CMB)
RESULT2: WMAP5+WL+BAO+SN

- BAO+SNe can break the main degeneracy between $\Omega_M$ and $\Sigma m_\nu$ and tighten the limit on $\Sigma m_\nu$

\[
\sum m_\nu < 1.2 \text{ eV}
\]

\[
+\text{BAO+SNe}
\]

\[
\sum m_\nu < 0.76 \text{ eV}
\]

\[
+\text{BAO+SNe+WL}
\]

\[
\sum m_\nu < 0.54 \text{ eV}
\]
Comparison with related works
Tereno et al., 0810.0555

The same data for CMB and WL, but different data for BAO and SN
Their covariance for WL data might be different from ours

Red: WMAP5
Pink: WMAP5+WL
Green: WMAP5+BAO+SN
Blue: ALL

$\sum m_\nu < 0.3 \text{ eV}$
Comparison with related works
Gong et al., 0810.3572

\[
\sum m_\nu < 0.8\text{eV}
\]
When $w=\pm 1$ is allowed

Red: WMAP5 only
Blue: WMAP+BAO+SN
Green: ALL

$$\sum m_\nu < 0.63 \text{ eV}$$
Summary

- Massive neutrinos play important roles in cosmic structure formation
  Diminish the power at small scales
- Non-linearity is important for estimating both the signal and error
- Weak lensing data from CFHTLS are used to put constraints on neutrino masses along with CMB BAO and SNe

$$\sum m_\nu < 1.2 \text{ eV} \quad \text{(WMAP5)} \quad \sum m_\nu < 0.54 \text{ eV} \quad \text{(ALL)}$$
Lensing of the CMB

CMB lensing
- deflection field
- lensing potential

\[ C^\phi_\ell = \ell (\ell + 1) C^\phi_\ell \]

\[ C^\phi_\ell = 4\pi \int \frac{dk}{k} P_R(k) \left[ 2 \int_0^{\chi^*} d\chi T_{\Psi}(k; \eta_0 - \chi) j_\ell(k\chi) \left( \frac{\chi^* - \chi}{\chi^* + \chi} \right)^2 \right] \]

Graphs showing linear and nonlinear effects with labeled regions and percentages.
Galaxy survey in the near future

- Galaxy number count with HSC
  \[ \delta_g(\hat{n}) = \int dz b(z)\mathcal{N}(z)\delta_m(\chi \hat{n}; z) \quad \text{where} \quad \mathcal{N}(z) = \frac{3z^2}{2z_*^3} \exp \left[ - \left( \frac{z}{z_*} \right)^{1.5} \right] \]
  \[ z_m = 1.41 z_* = 0.9 \]

- Cross power spectrum
  \[ C_{\ell}^{\phi g} = 4\pi \int \frac{dk}{k} \mathcal{P}_\mathcal{R}(k)\Delta_\ell^\phi(k)\Delta_\ell^g(k) \quad \text{where} \quad \Delta_\ell^g(k) = \int dz b(z)\mathcal{N}(z)j_\ell(k\chi)T_m(z) \]

- Cosmological parameters (favored in WMAP)
  1. \( h = 0.734 \)
  2. \( \Delta_\phi^2 = 2.35 \times 10^{-9} \)
  3. \( w = -1 \)
  4. \( f_\nu = 0.01 \quad \leftrightarrow \quad \sum m_\nu = 0.12 \text{eV} \)
  5. \( n_s = 0.961 \)
  6. \( n_{\text{run}} = 0 \)
  7. \( \Omega_b h^2 = 0.0223 \)
  8. \( \Omega_m h^2 = 0.126 \)
  9. \( \tau = 0.091 \)
  10. \( b = 1 \)

Planck x HSC 1
Fisher matrix formalism

\[ F_{\alpha\beta} = \sum_{\ell,XY} \frac{\partial C_\ell^X}{\partial p_\alpha} (\Xi_\ell)^{-1}_{XY} \frac{\partial C_\ell^Y}{\partial p_\alpha} \] \[ \sigma(p_\alpha) = \sqrt{(F^{-1})_{\alpha\alpha}} \]

where \( \{X, Y\} = \{TT, EE, TE, gg, \phi\phi, \phi g, T\phi, Tg\} \)
- unlensed CMB
- CMB lensing
- ISW

10 free parameters \( \{h, w_0, f_\nu, \Delta^2_R, n_s, n_{run}, \Omega_b h^2, \Omega_m h^2, \tau, b\} \)

For simplicity, we assume
- constant e.o.s of dark energy
- constant linear bias (b=1), but marginalized over

Planck x HSC 3
Importance of the non-linear regime

If we have theory at Nonlinear regime, (l < 1000) constraints may be significantly improved.
Summary (very preliminary)

- $1\sigma$ marginalized error ellipse

1. CMB + CMB Lens
2. CMB + Galaxy
3. CMB + Galaxy + CMB Lens

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td>$\sigma(\sum m_\nu)$</td>
<td>0.14 eV</td>
<td>0.14 eV</td>
<td>0.084 eV</td>
</tr>
<tr>
<td>$\sigma(w)$</td>
<td>0.098</td>
<td>0.103</td>
<td>0.052</td>
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