Evolution of Gravitational Wave Background from Inflationary Brane-world

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Introduction

- Gravitational Wave Background generated in inflationary epoch as quantum fluctuations
  - GWs in the range of $10^{-18} \sim 10^{9} \text{Hz}$ (solid line is 4D theory)
  - a probe of the extremely early Universe

We may DIRECTLY see the extra dimensions.
• How does the GWBs behave in the Universe with extra dimensions?

GW Bs crossing the horizon at the high energy epoch

✓ Non-standard expansion law

Dashed line ——. This effect enhances the amplitude above the critical frequency.

✓ Excitation of KK-modes

The KK-modes escaping from our brane may damp the amplitude.

✓ Curvature scattering

The KK-modes re-entering our brane may enhance the amplitude.

etc…

Solving the Wave equation, and investigating the behavior on the Friedmann brane on Randall-Sundrum II model
Strategy

Non-separable wave equation → Cannot solve analytically

• Numerical Simulation
  – Spectrum Method (Tchebychev Collocation Method)
    1) good resolution near the boundaries
    2) adaptable to the complicated boundary conditions

• Low-Energy Expansion
  – for interpretation of the numerical results
Basic Equations

- **Randall-Sundrum II (RS2)**

Bintruy et al. (2000)

Gaussian Normal Coordinate

\[ ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \delta_{ij} dx^i dx^j + dy^2 \]

**warp factor**

\[ a(t, y) = a_0(t) \left( e^{\mu y} - \frac{\rho(t)}{\lambda} \sinh \mu y \right) \]

**lapse function**

\[ n(t, y) = e^{\mu y} + (2 + 3\omega) \frac{\rho(t)}{\lambda} \sinh \mu y \]

**scale factor**

\[ a_0(\tau) = (\tau^2 + 2c\tau - 2c)^{1/4} \]

**energy**

\[ \rho(\tau) = (\tau^2 + 2c\tau - 2c)^{-1} \]

Non-standard evolution

- \[ H^2 = \left( \frac{\dot{a}_0}{a_0} \right)^2 - \frac{k}{3} \rho(t) \left( 1 + \frac{\rho(t)}{2\lambda} \right) \]

- \[ \dot{\rho}(t) = -3(1 + \omega) \frac{\dot{a}_0}{a_0} \rho(t) \]

- \[ w = \frac{1}{3} \text{ (RD)} \]

- \[ c = \sqrt{1 + \frac{c_\infty}{2} - 1} \]

- \[ \tau = \frac{t}{\ell_*} \]
• Wave equation (radiation dominated brane)

\[ ds^2 = -n^2(t, y)dt^2 + a^2(t, y)(\delta_{ij} + \mathcal{E}^{TT}_{ij} dx^i dx^j + dy^2) \]

First order

Gravitational Waves

\[
\frac{\partial^2 E}{\partial t^2} + \left( 3 \frac{\dot{a}}{a} - \frac{n}{n} \right) \frac{\partial E}{\partial t} + \frac{n^2}{a^2} k^2 E - n^2 \left\{ \frac{\partial^2 E}{\partial y^2} + \left( 3 \frac{a'}{a} + \frac{n'}{n} \right) \frac{\partial E}{\partial y} \right\} = 0
\]

• Boundary Conditions

Langlois et al. (2000)

\[
\frac{n^\mu}{x^\mu} \left. \frac{\partial E}{\partial x^\mu} \right|_{y=y_{\text{reg}}(\tau)} = 0
\]

\[
y_{\text{reg}}(\tau) \equiv \bigcirc y_c(\tau)
\]

Coordinate singularity

\[ a(\tau, y_c) = () \]
• parameters

\[ \epsilon_\ast \]

\[ \text{energy} \ \epsilon(t) \equiv \frac{\mu(t)}{\lambda} \text{ at Horizon cross time } t = t_\ast \]

At the time, \[ k = a_\ast H_\ast = \mu \sqrt{2\epsilon_\ast \left(1 + \frac{\epsilon_\ast}{2}\right)} \]

\[ \gamma \]

\[ \text{Position of regulator brane } y_{\text{reg}}(t) = \gamma y_c(t) \ (0 < \gamma < 1) \]

• results

I.C. : de Sitter 0-mode \[ H(t, y) = \text{const.}, \ k = a_\ast H_\ast \]

• KK-mode is excited
• on the brane : smaller amplitude than 4D theory
• the effects are notable when \( \epsilon_\ast \) is large (i.e. small scale)
Numerical Results

I.C: de Sitter 0-mode

KK-mode

Waveform on the brane

Snapshots at t=constant
damping factor = ratio of amplitude 5D/4D.
Low-Energy Expansion

- **Gradient Expansion**

\[
\begin{align*}
E &= E_0 + E_1 + E_2 + \cdots \quad \epsilon \ll 1 \\
\alpha_0 &= \alpha_0^{(0)} + \alpha_0^{(1)} + \cdots \\
H &= H_0 + H_1 + \cdots
\end{align*}
\]

- Expand the equation order by order.
- Impose the (static) boundary condition
- ODEs with regard to \( E_0 \)

The numerical result with moving boundary is consistent to the low energy approximation with static boundary
• Effective Wave Equation on Brane

\[ B(y_0) = \frac{\mu y_0}{e^{2\mu y_0}} \]

\[
\frac{d^2 E_0}{dt^2} + 3 \left( 1 + \frac{H_1}{H_0} + 4\frac{\rho(t)}{\lambda} |1 - 2B(y_0)| \right) H_0 \frac{dE_0}{dt} \\
+ \left( \frac{k}{a_0^{(0)}} \right)^2 \left( 1 - 2\frac{a_0^{(1)}}{a_0^{(0)}} + 4\frac{\rho(t)}{\lambda} |1 - 2B(y_0)| \right) E_0 = 0
\]

Kanno, Soda (2002)
Kobayashi, Koyama (2002)

non-standard cosmological evolution
non-separable metric
KK-mode excitation
Summary

**KKK-mode excitation due to the non-separable metric suppresses the amplitude of GWB.**

The remaining problems are ...
- in the case of $\epsilon_* > 1$ (e.g. curvature scattering)
- effects of regulator brane (boundary conditions, trajectory ...etc.)

Enable us to quantitatively predict GWB spectrum.