High-energy effects on the spectra of cosmological perturbations in braneworld cosmology

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Takashi Hiramatsu\textsuperscript{A} and Kazuya Koyama\textsuperscript{B}

\textsuperscript{A} Department of Physics, University of Tokyo, Japan
\textsuperscript{B} Institute of Cosmology and Gravitation, Portsmouth University, United Kingdom
Our Interests

- Cosmological perturbation probes the early universe.
  - Scalar type
    - CMB (COBE, WMAP, Planck, …)
    - LSS, Galaxy distribution (2df, SDSS, …)
  - Tensor type
    - Relic gravitational waves (LISA, DECIGO/BBO, …)

http://universe.nasa.gov/
Our Interests

- Extra dimensions
  - M-/superstring theories suggest 11/10 dimensions.
  - Braneworld
    - Matter is confined to a hypersurface.
    - Randall-Sundrum (RS) type
Our works

Goal

Cosmological perturbation in Randall-Sundrum II model

- To understand high-energy effects on the spectra of CPs.
  - Gravity escaping from brane into bulk (Kaluza-Klein modes)
  - Quadratic term in the Friedmann equation

\[
H^2 = \frac{\kappa_4^2}{3} \rho + \frac{\kappa_5^4}{36} \rho^2
\]

2\text{nd term dominates when} \quad \rho^{1/4} \geq 4.1 \text{TeV} \left( \frac{\ell}{0.1 \text{mm}} \right)^{-1/2}

- To obtain observational evidence of extra-dimensions from spectra of the perturbations

Method

Directly solve the evolution equations (partial differential eqs) without any approximations by pseudo-spectral method.
**Tensor Perturbations**

- GWs are not coupling to matter on brane. \( (\delta \pi^T \approx 0) \)
  \( \Rightarrow \) relatively easy to obtain the primordial spectrum of GWs.

- GWs are affected by 5D gravity near horizon crossing.

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**Inflationary Gravitational Waves**
- Langlois, Maartens, Wands, PLB(2000)
- Gorbunov, Rubakov, Sibiryakov, JHEP(2001)
- Kobayashi, Tanaka, JCAP(2004); PRD(2005)

**GWs in high-energy RD regime**
- TH,Koyama,Taruya PLB578(2004)269
- TH,Koyama,Taruya PLB609(2005)133
- TH, PRD73(2006)084008

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*The two effects cancel each other!*
**Scalar Perturbations**

- Density fluctuation (matter) is coupled to metric (gravity) in inflationary epoch.

In order to validate approximations and make observed spectra, it is required to solve the coupled equations directly.

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**Inflation**

- **ρ²**
- **RD**
- **RD/MD**

**Large scale fluctuation**

- **Horizon crossing**

**Interesting scale**

\[ \leq 10^{-15} \text{ Hz} \]

- WMAP, Planck, etc.

*Slow-roll approximation etc.*

- Koyama, Mizuno, Wands, JCAP(2005)
- de Rham, PRD(2005)
**Numerical Simulation – background**

- Hawkins-Lidsey’s brane inflation model

\[ V(\phi) \propto \cosh^2 \left( \frac{\sqrt{2\pi} C}{M_4} \phi \right) \]

- Non-trivial evolution of inflaton fluctuation
  - KK-mode excitation
  - Modified Friedmann equation

\[ a_0(t) \propto t^{1/2} \]

\[ \delta g_{\mu\nu} \]

\[ \delta \phi \]

\[ \text{brane} \]
**Numerical Simulation – gravity**

- **Evolution of metric** (GN+5D longitudinal gauge)

\[ ds^2 = -n^2 (1 + 2A)dt^2 + a^2 (1 + 2\mathcal{R})d\vec{x}^2 + (1 + 2A_{yy})dy^2 + nA_y dy dt \]

5D perturbed Einstein equation

\[ ^{(5)} \delta G_{AB} + \Lambda_5 ^{(5)} g_{AB} = 0 \]

**Master equation**

\[ -\left( \frac{1}{na^3} \dot{\Omega} \right)' + \left( \frac{n}{a^3} \Omega' \right)' + \left( \frac{\mu^2}{a^2} + \frac{\nabla^2}{a^2} \right) \frac{n}{a^3} \Omega = 0 \]
**Numerical Simulation – matter**

- Evolution of density perturbation

\[
\text{Inflaton fluctuation } \delta \phi \quad \implies \quad \text{Mukhanov-Sasaki variable } \quad Q = \delta \phi - \frac{\dot{\phi}}{H} R
\]

Conservation law \( \nabla \mu \delta T_{\mu \nu} = 0 \)

**Mukhanov-Sasaki equation**

\[
\ddot{Q} + 3H \dot{Q} + \frac{k^2}{a^2} Q + \left\{ \frac{\ddot{H}}{H} - 2 \frac{\dot{H}}{H} \frac{V'}{\dot{\phi}} - 2 \left( \frac{\ddot{H}}{H} \right)^2 + V'' \right\} Q = J(\Omega)
\]

If \( J(\Omega) = 0 \), evolution eq. in 4D theory is recovered.

Mukhanov, JETP(1988)*
**Numerical Simulation – junction**

- Junction (Boundary) condition

Matter is coupling to gravity on brane

Perturbed effective Einstein equation

\[
(4) \delta G_{\mu\nu} = \kappa_4^2 \delta T_{\mu\nu} + \kappa_5^4 \delta \Pi_{\mu\nu} - \delta E_{\mu\nu}
\]

(Projection of 5D Einstein onto the brane)

**Junction condition**

\[
\kappa_5^2 a_0 \dot{\phi}^2 \left( \frac{H}{\dot{\phi}} \right) = - \kappa^2 \left\{ \frac{\kappa_5^2 \phi^2}{a_0^2} \left( \dot{\Omega} - H\Omega \right) + H \left( \Omega' - \frac{a'}{a} \Omega \right) \right\}_b
\]
**Numerical Simulation – initial**

- **Initial condition**

  ... should be determined from the quantum theory of the coupled system --- *TOO DIFFICULT!* (work in progress)

As the first step, here we focus only on evolution of perturbations in inflation epoch.

\[ \Omega(t_{in}, y) = \dot{\Omega}(t_{in}, y) = 0 \]

\[ Q(t_{in}) = 1 \quad \text{Junction condition} \quad \dot{Q}(t_{in}) = ... \]

The initial time is fixed as

\[ t_{in} = 40 / \mu \]
**Results – comoving curvature perturbation**

**Definition**

\[ R_c \equiv \frac{H}{\dot{\phi}} Q \left( = \mathcal{R} + \frac{H}{\rho + p} V \right) \]

In 4D theory,

\[ R_c = \text{const.} \quad k \ll aH \]

\[ R_c \propto \frac{1}{a_0} \quad k \gg aH \]

\( \mathcal{R}_c \) has roughly same feature as one predicted in 4D.
**Results – comoving curvature perturbation**

The amplitude is more suppressed than 4D one.
Results – master variable

Intlaton fluctuation $Q$ excites bulk metric perturbation $\Omega$
Results – scale dependence

The extra suppression depends on the scale of perturbations.

We assumed same initial condition for all scales at fixed initial time.

Smaller scale perturbations stay on sub-horizon scales for a longer time.

More significant suppression.
Conclusion & Discussion

Classical evolution of inflaton perturbations

- Numerical simulation of the coupled system
  - Hawkins-Lidsey model (analytic background solutions)
  - Inflaton perturbation → Mukhanov-Sasaki variable $Q$ (equation)
  - Bulk metric perturbation → master variable $\Omega$ (equation)
  - An ad-hoc initial condition ($\Omega$ is initially vanished)
  - Junction condition

- Results
  - Curvature perturbation becomes constant at superhorizon scale.
  - Small scale curvature perturbation is more suppressed
    \[ \rightarrow \begin{cases} 
    \text{Impossible to neglect gravity even on small scale} \\
    Q \text{ excites the bulk metric perturbations} 
  \end{cases} \]

Next task: In order to obtain ICs without ambiguity, quantise the coupled system
  → CMB/Galaxy observations; application to DGP model