Neutrino masses from Planck scale

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+ work in progress

In collaboration with Alejandro Ibarra and Patrick Strobl (TUM)

Introduction

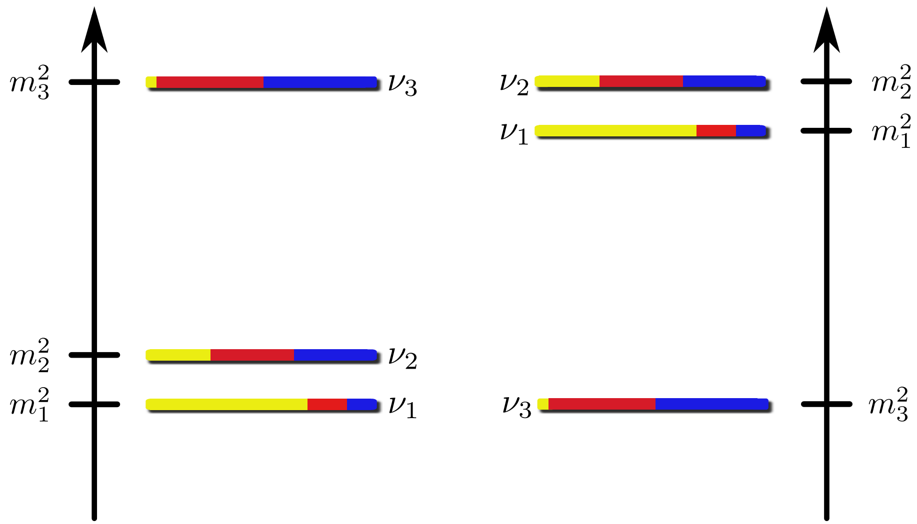
Neutrinos are massive. (massless in the Standard Model)

- Neutrino oscillation data $\Rightarrow \mathcal{O}(0.1)$ eV

	NH	IH
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
Δm^2_{21} [eV^2]	$7.50^{+0.19}_{-0.17}\times 10^{-5}$	$7.50^{+0.19}_{-0.17}\times 10^{-5}$
$\Delta m^2_{3\ell}$ [eV^2]	$2.524^{+0.039}_{-0.040}\times 10^{-3}$	$-2.514^{+0.038}_{-0.041}\times 10^{-3}$

Esteban et al. JHEP (2017)

- Very small masses of neutrinos and large mixing angles.
$\Rightarrow$ different mechanism of mass generation?



Neutrino mass generation

- There are many neutrino mass generation mechanisms.
- Seesaw mechanism (Type I, Type II, Type III...), inverse seesaw, linear seesaw, radiative generation etc.
  In Type I seesaw (simplest), three heavy right-handed neutrinos N_R are introduced.

$$\mathcal{L} = -\phi^\dagger \overline{\ell_L} y_\nu N_R - \frac{1}{2}\overline{N_R^c} M N_R + \text{h.c.}$$

$$\to -\overline{\nu_L} m_D N_R - \frac{1}{2}\overline{N_R^c} M N_R + \text{h.c.} \qquad m_D = y_\nu \langle \phi \rangle$$

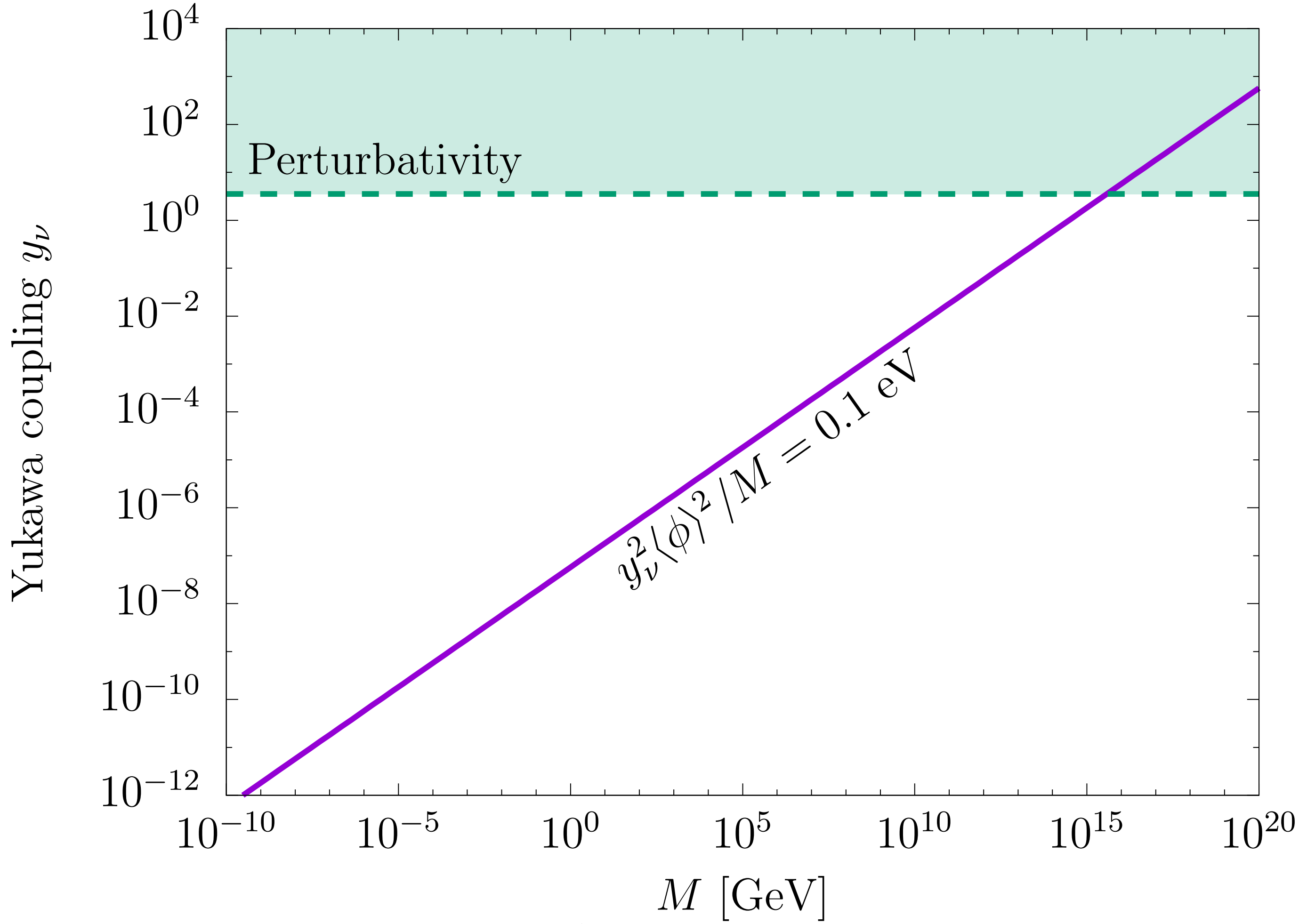
Mass matrix $\nu_L \quad N_R^c$

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \to \quad m_\nu \approx -m_D M^{-1} m_D^T + \cdots$$

(if $m_D \ll M$)

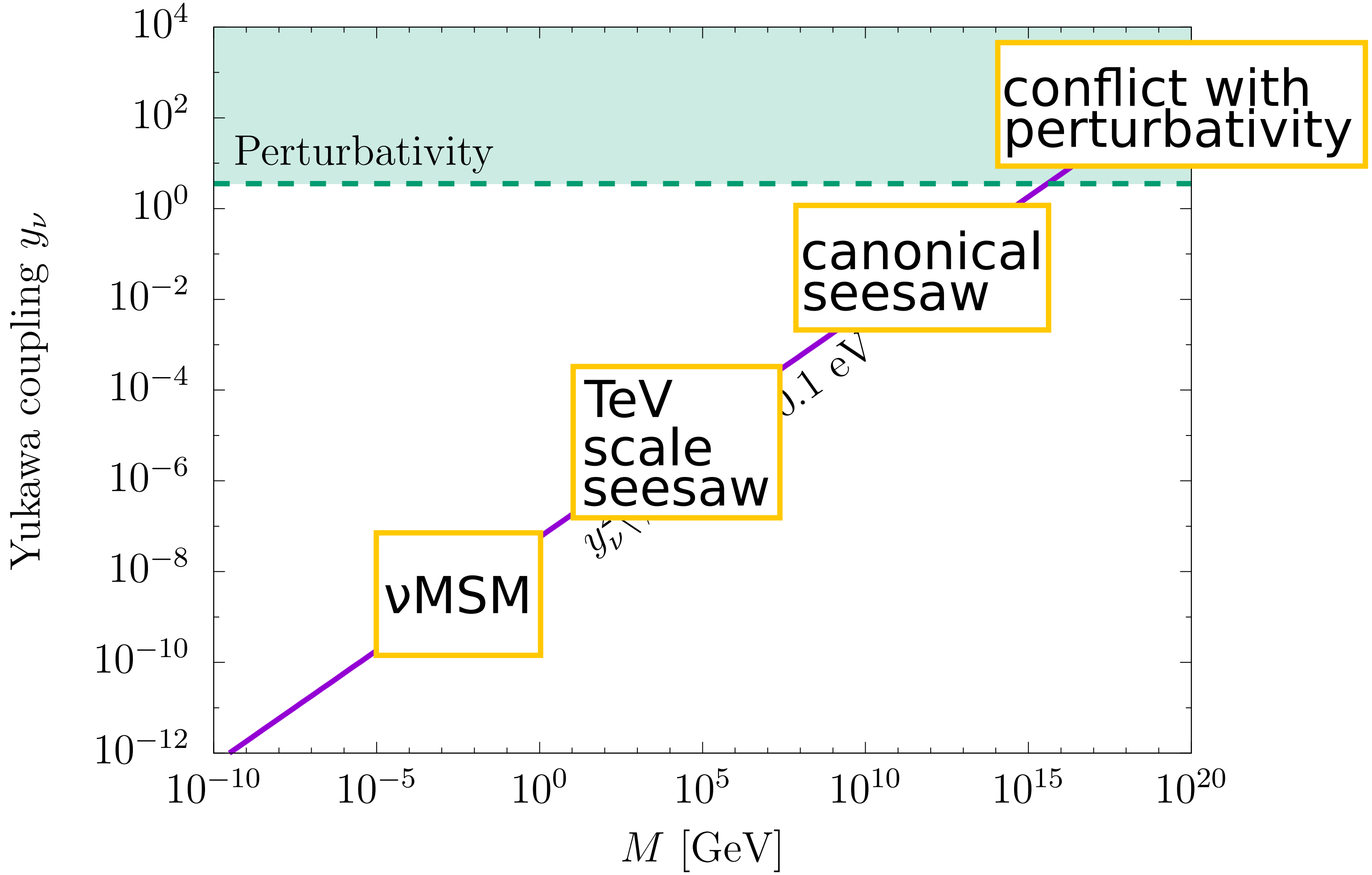
- Rough picture $\quad m_\nu \sim \dfrac{y_\nu^2 \langle \phi \rangle^2}{M} \sim 0.1\ \text{eV}$

Seesaw mechanism



- Ex.1: $y_\nu \sim \mathcal{O}(1)$ for $M = 10^{14}$ GeV
- Ex.2: $y_\nu \sim \mathcal{O}(10^{-7})$ for $M = 1$ GeV

Seesaw mechanism



- Cannot directly correlate neutrino mass scale and Planck scale.

Neutrino masses from Planck scale

A simple case

- Two right-handed neutrinos and one lepton doublet.

$$\mathcal{L} = -Y_i \tilde{H} \overline{L} N_j - \frac{M_i}{2} \overline{N_i^c} N_i + \text{H.c.}$$

- Assumption: hierarchical right-handed neutrino masses at Planck scale.

$$M \approx \begin{pmatrix} 0 & 0 \\ 0 & M_2 \end{pmatrix}, \quad \rightarrow \quad M(\mu) = \begin{pmatrix} M_1(\mu) & 0 \\ 0 & M_2(\mu) \end{pmatrix}$$

$M_1 \ll M_2 \sim M_P$
at Planck scale

at lower energy scale

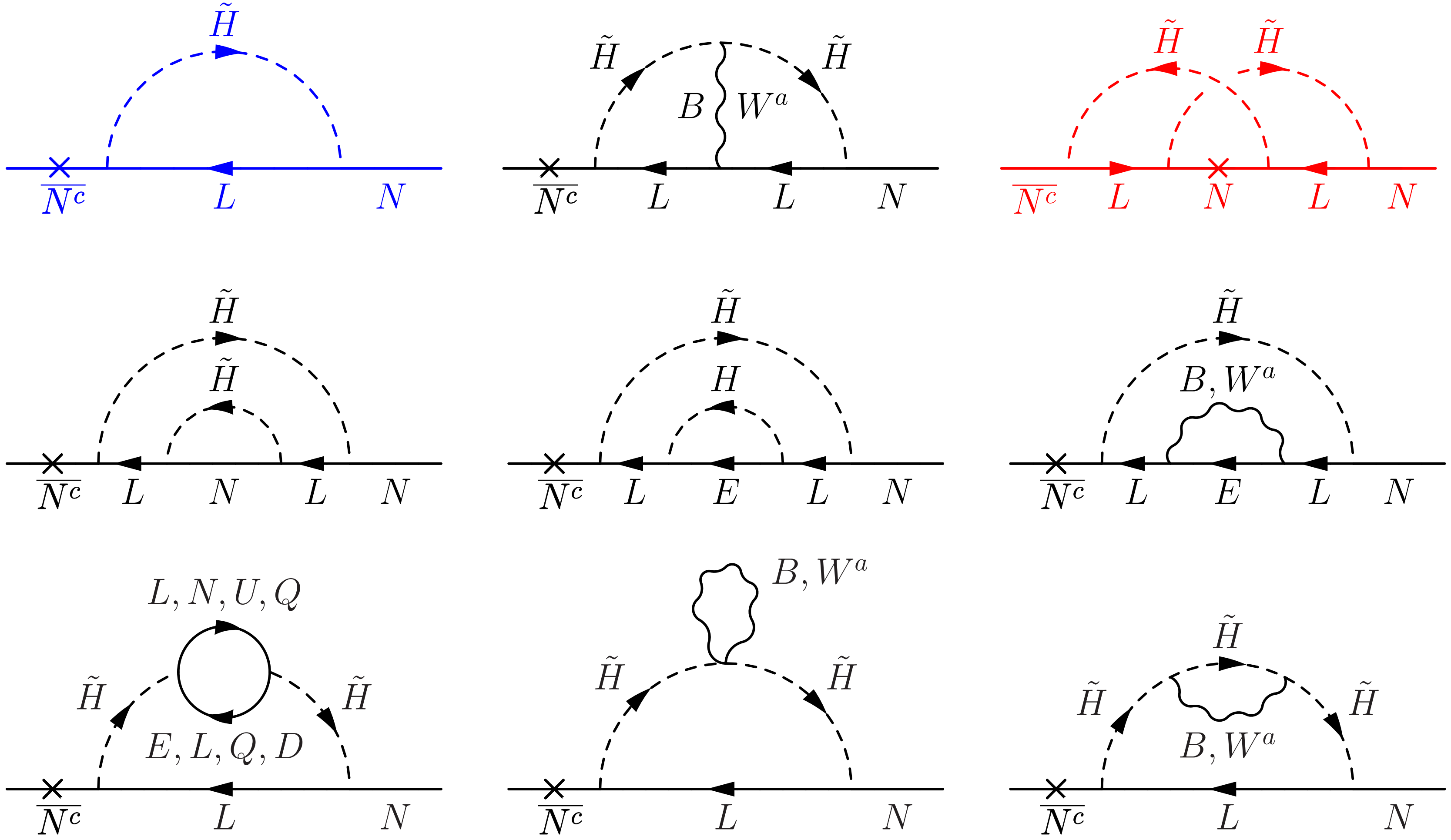
- Right-handed Majorana neutrino mass matrix may be generated via gravitational interactions. $\leftarrow$ No flavor discrimination

$$M = M_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \text{Mass eigenvalues} = 0, 2M_0$$

E. K. Akhmedov et al., PRL (1992), V. Berezinsky et al., JHEP (2005)

Renormalization Group Equation for M

- M_1 is generated by radiative effects.
  $\Rightarrow$ Renormalization group equation (RGE) for M.



Renormalization Group Equation for M

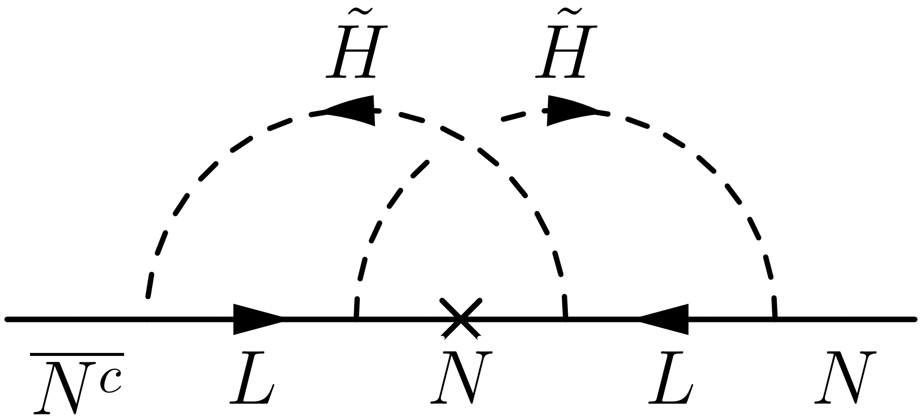
- At 1-loop, only one diagram contributes

$$\beta_M^{\text{1-loop}} = \frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y^\dagger Y\right)^T M + M\left(Y^\dagger Y\right)\right]$$

- At 2-loop, there are many contributions

$$\beta_M^{\text{2-loop}} = \frac{dM}{dt} = \frac{4}{(4\pi)^4}\left(Y^\dagger Y\right)^T M\left(Y^\dagger Y\right) + \cdots$$

Rank increasing diagram



the other diagrams do not increase rank of M.

Renormalization Group Equation for M

Full beta function (can be done by SARAH https://sarah.hepforge.org/)

$$\frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y^\dagger Y\right)^T M + M\left(Y^\dagger Y\right)\right] + \frac{4}{(4\pi)^4}\left(Y^\dagger Y\right)^T M\left(Y^\dagger Y\right)$$
$$+\frac{1}{(4\pi)^4}\left[\frac{17}{8}\left(g_Y^2 + 3g_2^2\right)\left(Y^\dagger Y\right) - \frac{1}{4}Y^\dagger Y Y^\dagger Y - \frac{1}{4}Y^\dagger Y_e Y_e^\dagger Y\right.$$
$$\left. -\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y^\dagger Y + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y^\dagger Y\right)\right]^T M$$
$$+\frac{1}{(4\pi)^4}M\left[\frac{17}{8}\left(g_Y^2 + 3g_2^2\right)\left(Y^\dagger Y\right) - \frac{1}{4}Y^\dagger Y Y^\dagger Y - \frac{1}{4}Y^\dagger Y_e Y_e^\dagger Y\right.$$
$$\left. -\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y^\dagger Y + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y^\dagger Y\right)\right]$$

- We include only M and Y for simplicity.
  The other contributions give small corrections.

After simplification

$$\rightarrow \frac{dM}{dt} \approx \frac{1}{(4\pi)^2}\left[\left(Y^\dagger Y\right)^T M + M\left(Y^\dagger Y\right)\right] + \frac{4}{(4\pi)^4}\left(Y^\dagger Y\right)^T M \left(Y^\dagger Y\right)$$

$$-\frac{1}{4(4\pi)^4}\left(Y^\dagger Y Y^\dagger Y\right)^T M - \frac{1}{4(4\pi)^4} M\left(Y^\dagger Y Y^\dagger Y\right)$$

$$= \left(1 - \frac{1}{4}P^T\right)P^T M + MP\left(1 - \frac{1}{4}P\right) + 4P^T M P$$

where $P \equiv \frac{1}{(4\pi)^2} Y^\dagger Y = \frac{1}{(4\pi)^2}\begin{pmatrix} Y_1^2 & Y_1 Y_2 \\ Y_1 Y_2 & Y_2^2 \end{pmatrix}$.

- Picard iterative integration:

$$M(\mu) = M(M_P) + \left(\beta|_{\mu=M_P}\right)\log\left(\frac{\mu}{M_P}\right) + \cdots$$

at leading order

Analytic solution

$$M(\mu) \approx \begin{pmatrix} 0 & 0 \\ 0 & M_2 \end{pmatrix} + M_2 \begin{pmatrix} 4P_{12}^2 & P_{12} + 4P_{12}P_{22} \\ P_{12} + 4P_{12}P_{22} & 2P_{22} + 4P_{22}^2 \end{pmatrix} \log\left(\frac{\mu}{M_P}\right)$$

- Diagnalize $M(\mu)$

$$M_2(\mu) \approx M_2,$$

$$M_1(\mu) \approx 4M_2P_{12}^2 \log\left(\frac{\mu}{M_P}\right) = \frac{4M_2Y_1^2Y_2^2}{(4\pi)^4}\log\left(\frac{\mu}{M_P}\right),$$

· M_1 is proportional to M_2. $M_1 \sim 10^{14}$ GeV if $Y_1 \sim Y_2 \sim \mathcal{O}(1)$

· small neutrino mass scale is obtained by seesaw mechanism

$$m_\nu = -v^2 Y M^{-1} Y^T = -v^2\left(\frac{Y_1^2}{M_1} + \frac{Y_2^2}{M_2}\right) \approx -\frac{v^2(4\pi)^4}{4M_2Y_2^2\log(\mu/M_P)}$$

$$= 0.05\text{ eV}\left(\frac{0.6}{Y_2}\right)^2\left(\frac{1.2\times 10^{19}\text{ GeV}}{M_2}\right)$$

Realistic case

- Three right-handed neutrinos and lepton doublets.

$$\mathcal{L} = -Y_{ij}\tilde{H}\overline{L_i}N_j - \frac{M_i}{2}\overline{N_i^c}N_i + \text{H.c.}$$

- Assumption: hierarchical mass eigenvalues at Planck scale.

$$M \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad \rightarrow \quad M(\mu) = \begin{pmatrix} M_1(\mu) & 0 & 0 \\ 0 & M_2(\mu) & 0 \\ 0 & 0 & M_3(\mu) \end{pmatrix}$$

$M_1 \ll M_2 \ll M_3$
at Planck scale

at low energy scale

M_1 and M_2 are generated by radiative effects?

Analytic solution

- Diagnalize $M(\mu)$ assumption: $M_1 \ll M_2 \ll M_3$ at $\mu = M_P$

1. If radiative corrections are dominant,

$$M_3(\mu) \approx M_3,$$

$$M_2(\mu) \approx 4M_3 \left(P_{31}^2 + P_{32}^2\right) \log\left(\frac{\mu}{M_P}\right),$$

$$M_1(\mu) \approx 8M_3 \frac{\left[P_{31}P_{32}\left(P_{11} - P_{22}\right) - P_{21}\left(P_{31}^2 - P_{32}^2\right)\right]^2}{P_{31}^2 + P_{32}^2} \log^2\left(\frac{\mu}{M_P}\right)$$

· All masses are proportional to M_3.
· $M_1(\mu)$ is order of four-loop.

2. If tree contribution M_1 is dominant,

$$M_1(\mu) \approx \frac{M_2 P_{31}^2 + M_1 P_{32}^2}{P_{31}^2 + P_{32}^2}$$

Analytic solution

- Why $M_1(\mu)$ is order of four loop?
  If RGE is expressed in terms of M_i (eigenvalues) and U:

Re diag: $$\frac{dM_i}{dt} = 2M_i\mathrm{Re}\hat{P}_{ii} + 4\sum_k M_k\mathrm{Re}\left(\hat{P}^2_{ki}\right),$$

Im diag: $$-2M_i\mathrm{Im}\left(U^\dagger\frac{dU}{dt}\right)_{ii} = 4\sum_k M_k\mathrm{Im}\left(\hat{P}^2_{ki}\right),$$

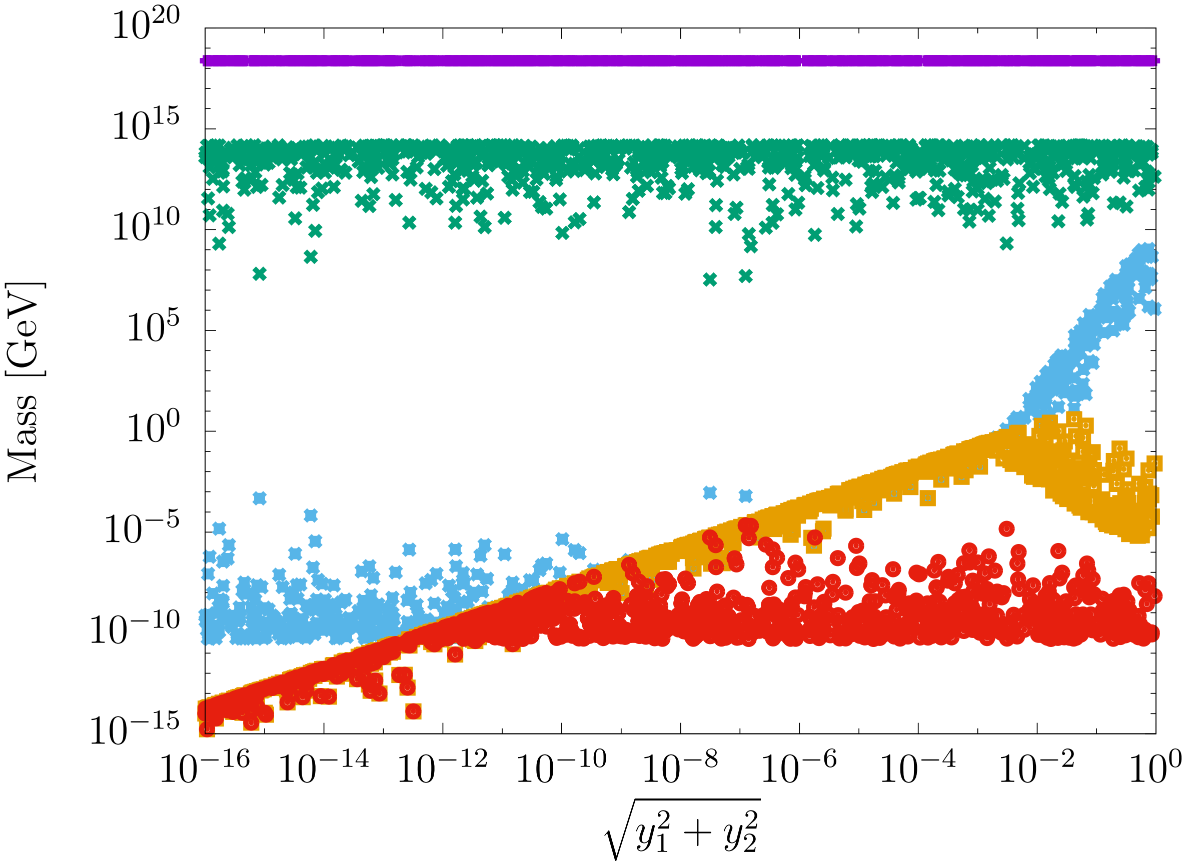
Re non-diag: $$\left(M_j - M_i\right)\mathrm{Re}\left(U^\dagger\frac{dU}{dt}\right)_{ij} = \left(M_i + M_j\right)\mathrm{Re}\hat{P}_{ij} + 4\sum_k M_k\mathrm{Re}\left(\hat{P}_{ki}\hat{P}_{kj}\right),$$

Im non-diag: $$-\left(M_j + M_i\right)\mathrm{Im}\left(U^\dagger\frac{dU}{dt}\right)_{ij} = \left(M_i - M_j\right)\mathrm{Re}\hat{P}_{ij} + 4\sum_k M_k\mathrm{Im}\left(\hat{P}_{ki}\hat{P}_{kj}\right),$$

where $\hat{P} = U^\dagger P U$ and $U^T M U = \mathrm{diag}(M_1, M_2, M_3)$.

- If $M_1 \sim M_2 \sim 0$ at Planck scale $\to$ fixed point condtions
  $\mathrm{Im}\left(\hat{P}^2_{21}\right) = 0$, $\mathrm{Im}\left(\hat{P}^2_{31}\right) = 0$, $\mathrm{Im}\left(\hat{P}^2_{32}\right) = 0$, $\hat{P}_{31}\hat{P}_{32} = 0$,
  $M_1(\mu) = 4M_3\mathrm{Re}\left(\hat{P}^2_{31}\right)\log\left(\frac{\mu}{M_P}\right) \to 0$ at $\mathcal{O}(P^2)$ order

Numerical results (radiatively generated M_1)



- Parametrization

$$Y = V_L Y_D V_R^\dagger$$

$$P = \frac{1}{(4\pi)^2} V_R Y_D^2 V_R^\dagger$$

- At Planck scale

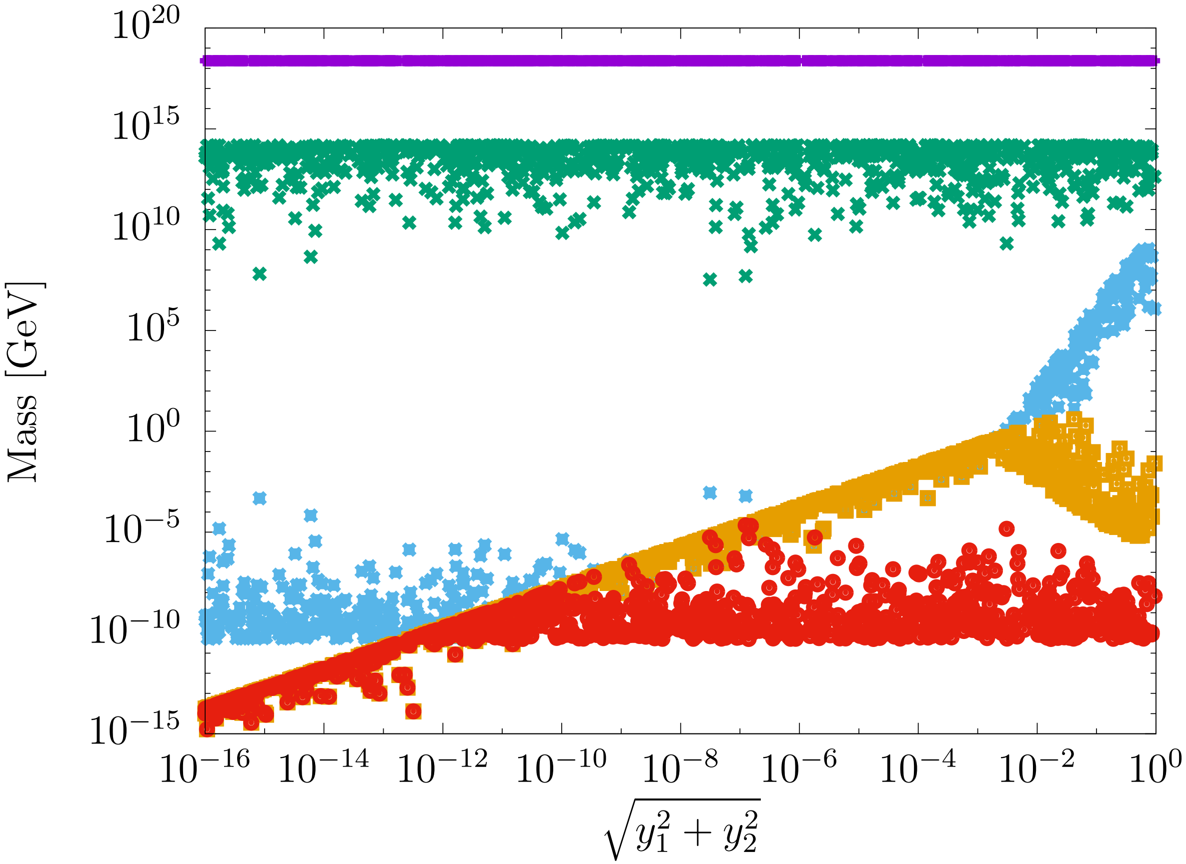
$$M = \mathrm{diag}\,(0, 0, M_P)$$

$$Y_D = \mathrm{diag}\,(y_1, y_2, 1)$$

- Run RGE

- Numerically diagonalize 6×6 mass matix $\begin{pmatrix} 0 & Yv \\ Y^T v & M \end{pmatrix}$ at lower energy scale
- 6 mass eigenvalues are obatained

Numerical results (radiatively generated M_1)



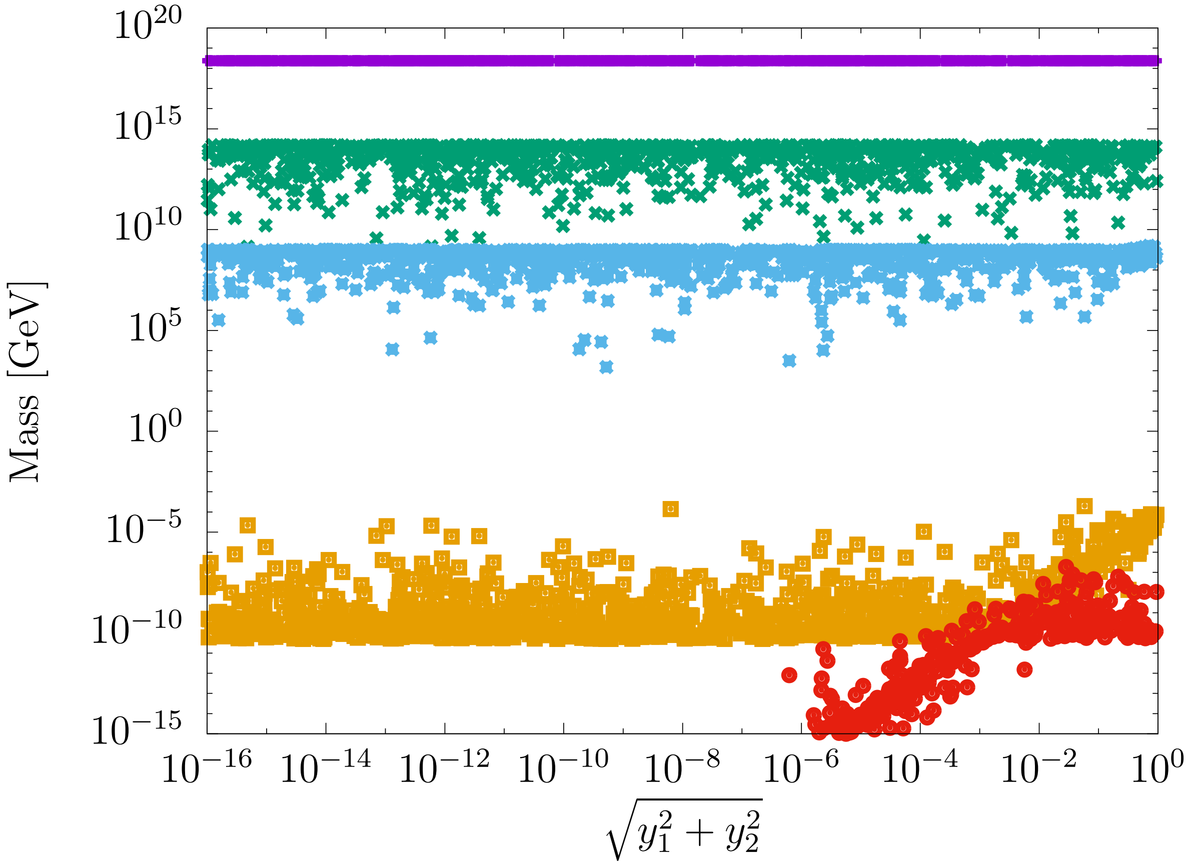
- Heaviest N_3: $M_3(\mu) \sim M_P$
- 2nd heaviest N_2:

$$M_2(\mu) \sim \frac{y_3^4 M_P}{(4\pi)^4} \times \text{mixing}^4 = 10^{10} \sim 10^{14}\ \mathrm{GeV}$$

- Lightest ν_1: $m_1(\mu) \sim 0$ suppressed by heaviest N_3

- 2nd lightest ν_2 can be $\sim 0.1\ \mathrm{eV}$ via seesaw with 2nd heaviest N_2.
- The other two states (ν_3, N_1) are degenerate if $\sqrt{y_1^2 + y_2^2} \lesssim 10^{-2}$
- Seesaw with N_1 does not work.

Numerical results (tree M_1)



If M is not exactly democratic,

- At Planck scale

$$M = \mathrm{diag}\left(0, 10^9\ \mathrm{GeV}, M_P\right)$$

$$Y_D = \mathrm{diag}\left(y_1, y_2, 1\right)$$

- Heaviest, 2nd heaviest, lightest are same with previous case.
- Lightest N_1: tree M_1

- Seesaw with N_2 and N_1 works if $10^{-4} \lesssim \sqrt{y_1^2 + y_2^2} \lesssim 10^{-2}$

Summary

1. If right-handed neutrino masses are hierarchical at Planck scale (high energy scale), radiative corrections may dominate right-handed neutrino masses at low energy scale.
2. Without introduing new energy scale, one of small neutrino masses was naturally generated from Planck scale by seesaw mechanism with N_2.

Outlook

1. This framework leads predictive phenomenology because number of parameters are reduced.
   Ex. application to leptogenesis, models with hierarchical mass spectrum.