# Electric Dipole Moments of Charged Leptons with Sterile Fermions

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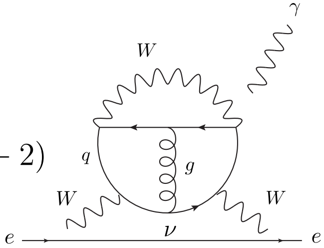
# Introduction

- Electric Dipole Moments (EDMs) are CP violating observables and sensitive to new physics.
- In the SM, the CKM matrix is only source of CP violation.
  $\rightarrow$ Charged lepton EDMs are induced at four-loop level.

$$|d_e|/e \sim \frac{\alpha_W^3 \alpha_s m_e}{246(4\pi)^4 m_W^2} J_{CP} \sim 10^{-38}\ \mathrm{cm}$$

- Current bounds of charged lepton EDMs

$$|d_e|/e < 8.7 \times 10^{-29}\ \mathrm{cm} \quad \text{(ACME)}$$
$$|d_\mu|/e < 1.9 \times 10^{-19}\ \mathrm{cm} \quad \text{(Muon } g-2\text{)}$$
$$|d_\tau|/e < 4.5 \times 10^{-17}\ \mathrm{cm} \quad \text{(Belle)}$$




$\rightarrow$ new physics enhances theoretical prediction of EDMs.

# Introduction

- Adding sterile fermions to the SM is an economical extension. (motivatied by neutrino masses and mixings, Dark matter, baryon asymmetry of the universe)
  - Neutrino masses $\to$ seesaw, inverse seesaw etc
  - Dark matter $\to$ the lightest sterile neutrino (keV scale)
  - Baryon asymmetry $\to$ leptogenesis

  $\to$ Consider the models extended with sterile fermions.
- In the extended models, new CP violating phases are introduced in lepton sector (necessary).

  $\to$ charged lepton EDMs.

$$\mathcal{L}_{\mathrm{eff}} = -\frac{d_\ell}{2}\overline{\ell} i\sigma^{\mu\nu}\gamma_5 \ell F_{\mu\nu}$$

EDMs are computed in the effective models and inverse seesaw models.

# The Effective Models

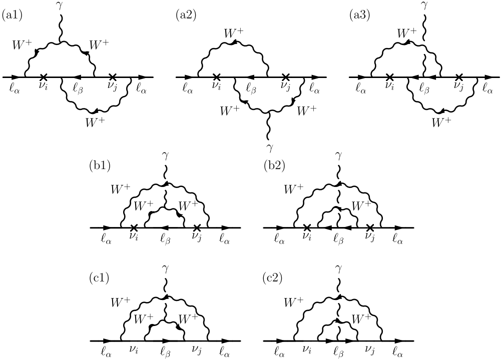
# The Effective Models

- The SM plus $N$ sterile fermions.
- The interactions are given by

$$\mathcal{L} = -\frac{g_2}{\sqrt{2}} U_{\alpha i} W_\mu^- \overline{\ell_\alpha} \gamma^\mu P_L \nu_i - \frac{g_2}{\sqrt{2}} U_{\alpha i} H^- \overline{\ell_\alpha} \left( \frac{m_\alpha}{m_W} P_L - \frac{m_i}{m_W} P_R \right) \nu_i + \text{H.c.}$$

  in Feynman-'t Hooft gauge.
- We do not fix neutrino mass generation mechanism.
  (Ex. Type-I seesaw, Inverse seesaw, Linear seesaw)
  $\rightarrow$ the mixing matrix $U_{\alpha i}$ and the neutrino mass $m_i$ are assumed to be independent.
- Parametrization of $U$
  For $N = 1$ case, $3$ Dirac, $3$ Majorana phases, $U = 4 \times 4$ matrix.
  For $N = 2$ case, $6$ Dirac, $4$ Majorana phases, $U = 5 \times 5$ matrix.

# Diagrams contributing to EDM




- Leading contribution comes from two-loop.
- Totally 44 diagrams exist in Feynman-'t Hooft gauge.
- Analytical result is obtained by using FeynCalc.
- EDMs are numerically evaluated.

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{4(4\pi)^4 m_W^2} \sum_\beta \sum_{i,j} \sqrt{x_i x_j} \Big[ J^M_{ij\alpha\beta} I_M(x_i, x_j) + J^D_{ij\alpha\beta} I_D(x_i, x_j) \Big]$$

where $\alpha, \beta = e, \mu, \tau$ and $i, j = 1-4$ or $1-5$, $x_i \equiv m_i^2/m_W^2$,

$$J^M_{ij\alpha\beta} \equiv \mathrm{Im}\left(U_{\alpha j} U_{\beta j} U^*_{\beta i} U^*_{\alpha i}\right), \quad J^D_{ij\alpha\beta} \equiv \mathrm{Im}\left(U_{\alpha j} U^*_{\beta j} U_{\beta i} U^*_{\alpha i}\right)$$

# $N = 1$ case

- $I_{M,D}$ can be expanded by $x_i, x_j$ for $i, j = 1, 2, 3$. ($x_i \equiv m_i^2/m_W^2 \ll 1$, $x_4 \not\ll 1$)
- $\sum_{i=1}^{3} \sqrt{x_i} J^M_{i4\alpha\beta} = 0$ for $N = 1$ due to $\sum_{i=1}^{4} U^*_{\alpha i} m_i U^*_{\beta i} = m_{\alpha\beta} = 0$.

EDM $\rightarrow$

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2} \sum_\beta \sum_{i=1}^{3} \sqrt{x_i x_4} \left[ J^M_{i4\alpha\beta} x_i \frac{\partial I_M}{\partial x_i}(0, x_4) + J^D_{i4\alpha\beta} I_D(0, x_4) \right]$$

$$\approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2} \sum_\beta \sum_{i=1}^{3} \sqrt{x_i x_4} \left[ J^D_{i4\alpha\beta} I_D(0, x_4) \right].$$

Taking $I_D(0, x_4) \sim x_4 \sim 1$ and $x_i \sim 10^{-24}$ $(i = 1 - 3)$,

$\rightarrow |d_e|/e \lesssim 10^{-39}$ cm. The predicted EDM is very suppressed.

# $N = 2$ case

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2}\sqrt{x_4 x_5}\Big[J_\alpha^M I_M(x_4, x_5) + J_\alpha^D I_D(x_4, x_5)\Big]$$

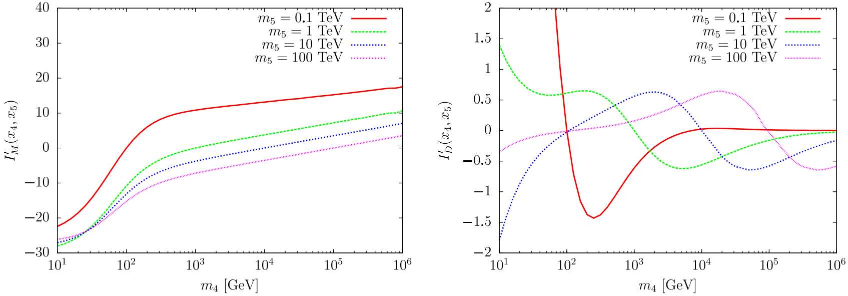
where $J_\alpha^{M/D} = \sum_\beta J_{45\alpha\beta}^{M/D}$.

EDMs can potentially be large enough to be detected.

- Only heavy states give a dominant contribution.
- At least two sterile fermions are needed to induce sizable EDMs.
- For $N$ sterile fermions, EDMs are expected to increase with the factor $N(N-1)/2$.

# Numerical Computations

## Loop functions $I_M$, $I_D$

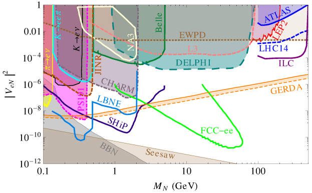


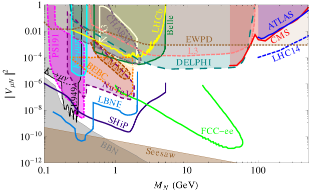

- When $x_i, x_j \gg 1$, $I_M(x_i, x_j) \gg I_D(x_i, x_j)$
- $I_D \gg I_M$ if $x_i, x_j \ll 1$, but the predicted EDMs are too small. $\rightarrow$ although $I_{M/D}$ are anti-symmetric, $I_{M/D}(0,0)$ is almost constant.

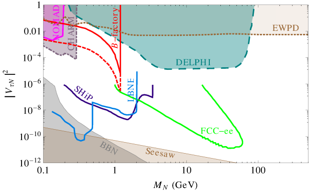
# Constraints

- Neutrino oscillation data
- Lepton Flavour Violation
  $\mathrm{Br}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$
  $\mathrm{Br}(\mu \to e\bar{e}e) \leq 1.0 \times 10^{-12}$
- Direct collider production
  $pp \to W^{\pm *} \to \ell^{\pm}\nu_i \to \ell^{\pm}\ell^{\pm}jj$ at LHC
  $e^+e^- \to \nu_i\nu_j^* \to \nu_i e^{\pm}W^{\mp}$ at ILC
- Electroweak precision data
  $W$ boson decay, $Z$ invisible decay,
  Lepton flavor universality of meson decays,
  Non-unitarity of the mixing matrix $U_{\alpha i}$
- Perturbative unitarity bound

arXiv:1502.06541

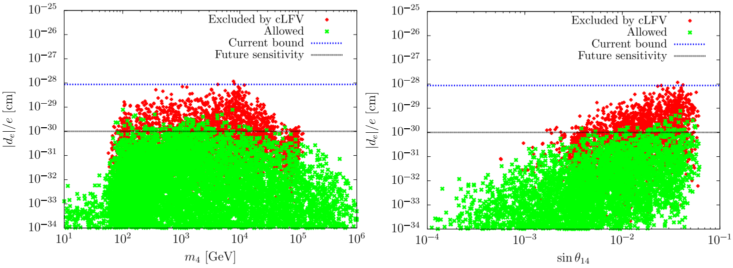


# Numerical Computations

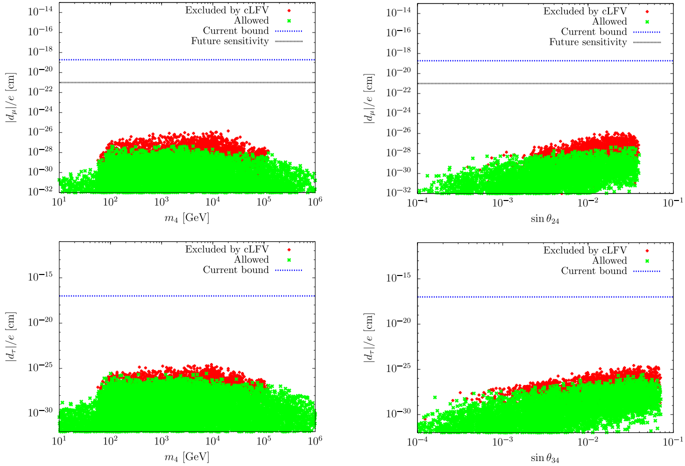
## Electron EDM




- Upper region is constrained by cLFV.
- Right upper region is cut by perturbative unitarity bound.
- Below $\mathcal{O}(100)$ GeV region is constrained by electroweak precision data.
- All the green points are lower than current bound. But can reach to future prospect (ACME).

# Numerical Computations

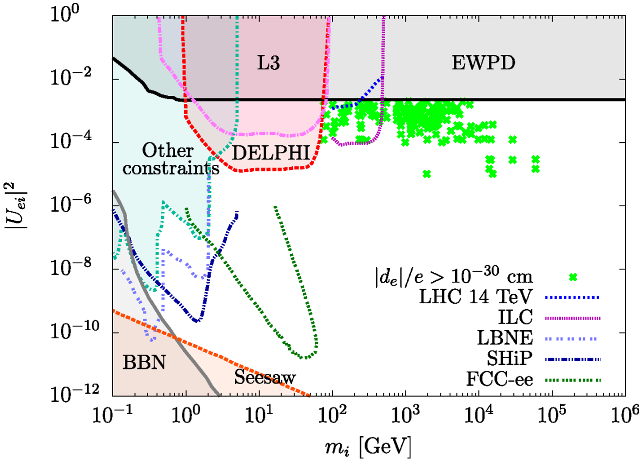
## Muon and tau EDM




- Predicted values are much below the future prospects.

# Numerical Computations

In $m_i - |U_{ei}|^2$ plane for electron




- Colored regions are already excluded.
- Green points: $|d_e|/e \gtrsim 10^{-30}$ cm.
- Some parameter space can be tested by ILC.

# Inverse Seesaw Models

# Inverse Seesaw Models

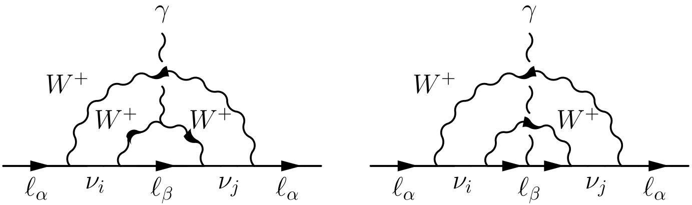
- The SM is extended with some right-handed neutrinos ($N^c$) and singlet fermions ($s$).
- At least two pairs of $N^c$ and $s$ are needed to fit the neutrino oscillation data. $\to$ minimal model Abada and Lucente, arXiv:1401.1507

$\to$ $(7 \times 7)$ Neutrino Mass matrix: $\mathcal{L} = -n_L^T C^{-1} M n_L$

$$M = \begin{pmatrix} 0 & 0 & 0 & d_{11} & d_{12} & 0 & 0 \\ 0 & 0 & 0 & d_{21} & d_{22} & 0 & 0 \\ 0 & 0 & 0 & d_{31} & d_{32} & 0 & 0 \\ d_{11} & d_{21} & d_{31} & m_{11} & m_{12} & n_{11} & n_{12} \\ d_{12} & d_{22} & d_{32} & m_{12} & m_{22} & n_{21} & n_{22} \\ 0 & 0 & 0 & n_{11} & n_{21} & \mu_{11} & \mu_{12} \\ 0 & 0 & 0 & n_{12} & n_{22} & \mu_{12} & \mu_{22} \end{pmatrix} \begin{matrix} \nu_L^1 \\ \nu_L^2 \\ \nu_L^3 \\ N_1^c \\ N_2^c \\ s_1 \\ s_2 \end{matrix}$$

$\to U^T M U = \mathrm{diag}(m_1, m_2, \cdots, m_7)$, $\quad m_4 \approx m_5$ and $m_6 \approx m_7$

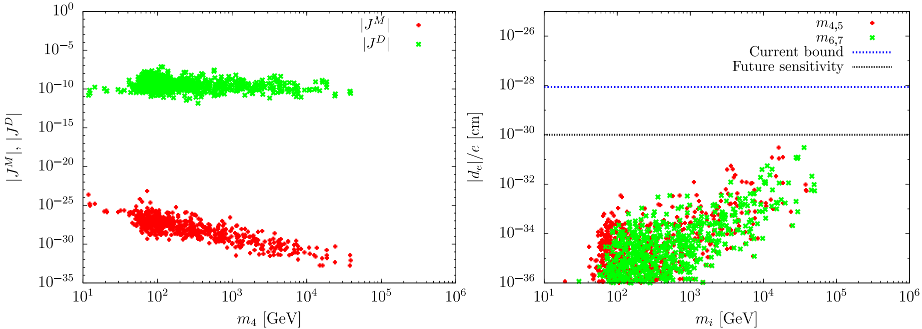
# EDM in Minimal Inverse Seesaw Model




- The EDM formula: $d_\ell \approx -\frac{g_2^4 e m_\ell}{2(4\pi)^4 m_W^2} J^D I'_D\,(x_4, x_6)$
- Pairs of $N^c$ and $s$ make pseudo-Dirac fermions ($m_4 \approx m_5$ and $m_6 \approx m_7$) $\rightarrow$ Dirac contribution is dominant.
- The current experimental bound:

$$|d_e|/e \leq 8.7 \times 10^{-29}\ \mathrm{cm}$$

# Numerical Computations




- All the relevant constraints are imposed.
- Roughly $|J^D_{\text{max}}| \sim 10^{-5} \times \left(\frac{\text{GeV}}{m_4}\right)$
- Majorana contribution $(J^M)$ is very suppressed.
- The maximum value of the prediction is a few factor below the future prospect.

# Beyond the minimal model

So far, we considered two pairs of sterile fermions. If more fermions are added, how much is the EDM enhanced?

- For $(N, N)$ model, electron EDM is roughly given by

$$|d_e^{(N,N)}| \sim \frac{g_2^4 e m_e}{2(4\pi)^4 m_W^2} \left|4N(N-1)\mathrm{Im}\left(U^4\right) I_D'\right|.$$

- But the LFV constraint becomes also strongrer at the same time,

$$\mathrm{Br}(\mu \to e\gamma) \sim \frac{\sqrt{2} G_F^2 m_\mu^5}{\Gamma_\mu} (2N)^2 |U|^4 \leq 4.2 \times 10^{-13}$$

- Eventually enhancement factor compared to the minimal model is given by

$$\left|\frac{d_e^{(N,N)}}{d_e^{(2,2)}}\right| \lesssim 2\left(1 - \frac{1}{N}\right) \to 2 \text{ (max)}$$

# Resonant leptogenesis

- Correlation with another CP violating observable (Baryon Asymmetry in the universe)
- Heavy sterile states are degenerate.
- Baryon asymmetry can be generated by (resonant) leptogenesis in the models.

The out-of-equilibrium condition $\Gamma_{\nu_4} < H(T)|_{T=m_4}$

and the decay width of $\nu_4$ is given by $\Gamma_{\nu_4} = \frac{g_2^2 m_4^3}{16\pi m_W^2} \sum_{\ell} |U_{\ell i}|^2$.

$$\rightarrow \quad \sum_{\ell} |U_{\ell i}|^2 \sim 10^{-15} \left(\frac{1\ \mathrm{TeV}}{m_4}\right) \left(\frac{g_*}{100}\right)^{1/2}$$

$\rightarrow$ Too small to get sizable EDMs.

# Summary

1. We computed charged lepton EDMs at two-loop level in the effective models and inverse seesaw models.
2. At least 2 sterile fermions are needed to get sizable electron EDM which is testable by future experiments.
3. In inverse seesaw models, Dirac contribution is dominant since pairs of sterile fermions make pseudo-Dirac states.
4. Maximum EDMs can be obtained if sterile fermion masses are 100 GeV − 10 TeV.