Monte Carlo simulations of a supersymmetric matrix model of dynamical compactification in nonperturbative string theory

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arXiv:1211.0950,1303.XXXX

1. Introduction

IKKT model (IIB matrix model)
⇒Promising candidate for nonperturbative formulation of superstring theory

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

- A_μ, ψ_α ⇒N × N Hermitian matrices.
- In the following, we study the Euclidean 6dim version of the IKKT model as a toy model (μ,ν=1,2,3,4,5,6)
- SO(6) rotational symmetry.
- Eigenvalues of A_μ⇒spacetime coordinate.
- Presence of N=2 supersymmetry.

How does our 4dim spacetime emerge from superstring theory?

We studied Spontaneous Breakdown of the SO(6) rotational symmetry.
Dynamical compactification of spacetime.

Complex phase of the fermion determinant
⇒important for *SO(6) symmetry breakdown*.

full partition function Z:

$$Z = \int dA d\psi d\bar{\psi} e^{-S} = \int dA e^{-S_B} \underbrace{(\det \mathcal{M})}_{=\int d\psi d\bar{\psi} e^{-S_F}} = \int dA e^{-S_0} e^{i\Gamma}$$

phase-quenched partition function Z_0:

$$Z_0 = \int dA e^{-S_B} |\det \mathcal{M}| = \int dA e^{-S_0}$$

Fermion determinant *detM is complex*.

2. Summary of Gaussian Expansion Method (GEM) results

Order parameter for SO(6) symmetry breakdown.

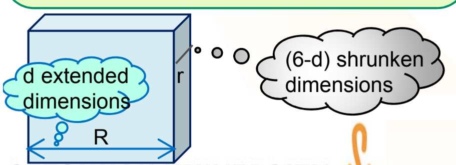
$$\lambda_n (n=1,2,\cdots,6): \text{eigenvalues of } T_{\mu\nu} = \frac{1}{N}\mathrm{tr}(A_\mu A_\nu)$$
$$(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_6)$$

$$\langle\lambda_1\rangle = \cdots = \langle\lambda_d\rangle (=R^2) \gg \langle\lambda_{d+1}\rangle = \cdots = \langle\lambda_6\rangle (=r^2)$$

Extended d dim and shrunken (6-d) dim at N→∞.
Symmetry breakdown of SO(6) to SO(d).
[<*> = (V.E.V. of full model Z)]

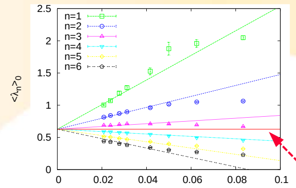
Results of Gaussian Expansion Method
[T. Aoyama, J. Nishimura and T. Okubo, arXiv:1007.0883, J. Nishimura, T. Okubo and F. Sugino, arXiv:1108.1293]

- Universal "compactification scale" $r^2 \cong 0.223$ independent of d.
- Constant volume property: Volume $V = R^d r^{6-d} = l^6$ ⇒ independent of d. Phase-Quenched model Z_0 ⇒ $l^2 \cong 0.627$.
- Free energy takes minimum at d=3 ⇒Symmetry breakdown of SO(6) to SO(3).



3. Monte Carlo simulations

Simulation of phase-quenched model Z_0



$\langle\lambda_1\rangle_0 = \langle\lambda_2\rangle_0 = \cdots = \langle\lambda_6\rangle_0 \simeq 0.6$ at large N.
[$\langle * \rangle_0$ = (V.E.V. of phase quenched model Z_0)]
No breakdown of SO(6) symmetry.
Consistent with GEM result $\langle\lambda_n\rangle_0 \cong 0.627$.

Simulation of full model Z via factorization method

[K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041, K.N. Anagnostopoulos, T. Azuma and J. Nishimura, arXiv:1009.4504,1108.1534]

An approach to remove the overlap problem.

Discrepancy of a distribution function between phase quenched model Z_0 and full model Z.

Factorization property of the distribution function.
$\tilde{\lambda}_n \stackrel{\text{def}}{=} \lambda_n / \langle\lambda_n\rangle_0$ ⇒effect of complex phase.

$$\rho(x_1,\cdots,x_6) \stackrel{\text{def}}{=} \left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\rangle$$
$$= \underbrace{\frac{1}{\langle e^{i\Gamma}\rangle_0}}_{=1/C} \times \underbrace{\left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\rangle_0}_{=\rho^{(0)}(x_1,\cdots,x_6)} \times \underbrace{\frac{\left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) e^{i\Gamma} \right\rangle_0}{\left\langle \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\rangle_0}}_{=w(x_1,\cdots,x_6)}$$
$$w(x_1,\cdots,x_6) = \langle e^{i\Gamma} \rangle_{x_1,\cdots,x_6},$$
$$\langle * \rangle_{x_1,\cdots,x_6} = \left\{ \text{V.E.V. for } Z_{x_1,\cdots,x_6} = \int dA e^{-S_0} \prod_{k=1}^{6} \delta(x_k - \tilde{\lambda}_k) \right\}.$$

Simulation of the constrained system $Z_{x_1,\ldots,x_6}$
⇒sample the region important for the full model Z.

Minimum of the free energy $\mathcal{F} = -\frac{1}{N^2}\log\rho(x_1,\cdots,x_6)$
⇒solve $\frac{1}{N^2}\frac{\partial}{\partial x_n}\log\rho^{(0)}(x_1,\cdots,x_6) = -\frac{\partial}{\partial x_n}\frac{1}{N^2}\log w(x_1,\cdots,x_6)$

Monte Carlo evaluation of $\langle\tilde{\lambda}_n\rangle$

- We study the SO(d) symmetric vacua (d=2,3,4,5) $x_1 = \ldots = x_d > 1 > x_{d+1}, \ldots, x_6$
- The large eigenvalues $\lambda_1, \ldots \lambda_d$ do not affect much the fluctuation of the phase.
- The smaller eigenvalues $\lambda_{d+1}, \ldots, \lambda_6$ tend to acquire the same value.

⇒We constrain only λ_{d+1}.

Small eigenvalue $\langle\tilde{\lambda}_{d+1}\rangle_{\mathrm{SO}(d)}$ in the SO(d) symmetric vacuum ⇒solve the equation for n=d+1 at x<1.

$$\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx}\frac{1}{N^2}\log w_n(x) \quad \text{where}$$
$$f_n^{(0)}(x) \stackrel{\text{def}}{=} \frac{d}{dx}\log \underbrace{\langle\delta(x - \tilde{\lambda}_n)\rangle_0}_{=\rho_n^{(0)}(x)}, \quad w_n(x) = \langle e^{i\Gamma}\rangle_{n,x}$$
$$\langle * \rangle_{n,x} = \left\{ \text{V.E.V. for } Z_{n,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_n) \right\}$$

Scaling properties in the large-N limit:

$$\frac{f_n^{(0)}(x)}{N^2} = \begin{cases} O(N^0) & x < 1 \\ O\left(\frac{1}{N}\right) & x \geq 1 \end{cases} \qquad \frac{1}{N^2}\log w_n(x) = O(N^0)$$

in contrast to the one-loop approximation of the 6d IKKT model, where $\frac{f_n^{(0)}(x)}{N^2} = O\left(\frac{1}{N}\right)$ for all x.

[K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041]

Existence of hardcore potential at small x.

Results of the simulation

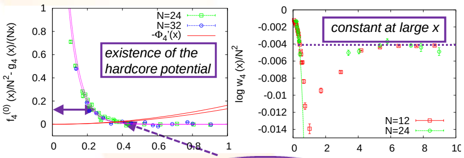
At x≅1⇒ $f_n^{(0)}(x)/N$ scales at large N.
Ansatz: $\frac{x}{N} f_n^{(0)}(x) \simeq g_n(x) = a_{1,n}(x-1) + a_{2,n}(x-1)^2$

At x<1⇒ $f_n^{(0)}(x)/N^2$ scales at large N.
Ansatz: $\frac{1}{N^2} f_n^{(0)}(x) - \frac{g_n(x)}{Nx} = p_n e^{-q_n x}$

The phase $w_n(x)$ behaves as

$$\Phi_n(x) = \lim_{n \to +\infty} \frac{1}{N^2}\log w_n(x) \propto \begin{cases} x^{7-n} & (x<1) \\ \text{const.} & (x>1) \end{cases}$$

Result for n=4:



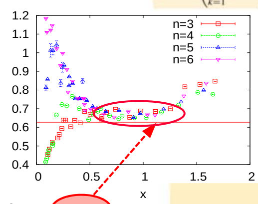
The solution lies at $\bar{x}_n = 0.35(1)$

n	n=3	n=4	n=5	n=6
$\bar{x}_n$	0.31(3)	0.35(1)	0.33(4)	0.38(7)

$\bar{x}_n \cong 0.35$⇒Universal compactification scale.
Consistent with GEM $\langle\tilde{\lambda}_k\rangle = \frac{\langle\lambda_k\rangle}{\langle\lambda_k\rangle_0} = \frac{r^2}{l^2} \simeq \frac{0.223}{0.627} \simeq 0.355\cdots$

Constant volume property

Geometric mean $L_n^2(x) = \left(\prod_{k=1}^{6}\langle\lambda_k\rangle_{n,x}\right)^{1/6}$ for n=3,4,5,6.



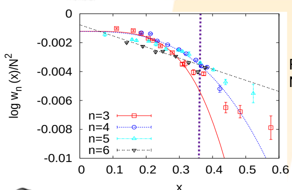
Result for N=32, n=3,4,5,6

$L_n^2(x) \cong 0.627$ within 0.5<x<1.
Consistent with GEM $V = R^d r^{6-d} \simeq (\sqrt{0.627})^6$

Comparison of the free energy

Free energy of the SO(d) vacua

$$\mathcal{F}_{\mathrm{SO}(d)} = \int_{\bar{x}_n}^{1} \frac{1}{N^2} f_n^{(0)}(x) dx - \frac{1}{N^2}\log w_n(\bar{x}_n), \text{ where } n = d+1$$



Result for N=24, n=3,4,5,6

$\mathcal{F}_{\mathrm{SO}(d)}$ are close to each other for d=2,3,4,5.
⇒difficult to determine the true vacuum.

4. Conclusion

We studied the dynamical compactification of the spacetime via Euclidean 6dim version of the IKKT model.

Monte Carlo simulation via factorization method
⇒We have obtained the results consistent with GEM:

- Universal "compactification" scale.
- Constant volume property.
- Comparison of the free energy for SO(d) symmetric vacua is subtle.