Monte Carlo Studies of the GWW Phase Transition in Large-N Gauge Theories

(arXiv:0711.nnnn)

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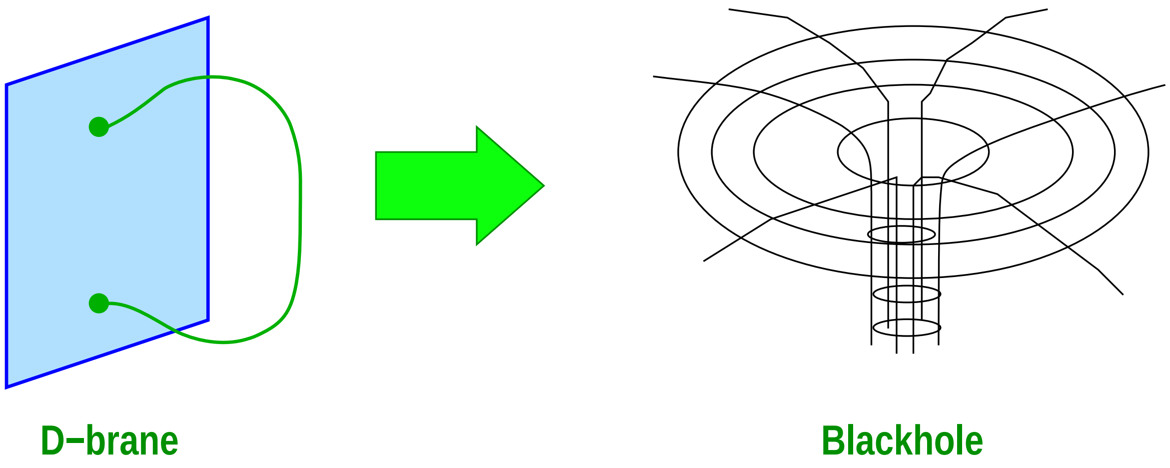
Collaboration with Pallab Basu and Spenta R. Wadia

Contents

1 Introduction

AdS/CFT correspondence: J. M. Maldacena, hep-th/9711200

duality between type IIB superstring on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM gauge theory.



- **Nonperturbative studies of superstring.**
- **Quantum description of the blackhole.**

Thermodynamic aspects of quantum gravity in AdS spacetime.

S. W. Hawking and D. N. Page, Commun. Math. Phys. 87, 577 (1983).

AdS spacetime allows two Schwarzschild blackhole solutions.

- **Small black hole (SBH): Horizon radius smaller than AdS.**
  Negative specific heat. Unstable.

- **Big black hole (BBH): horizon radius comparable to AdS. Positive specific heat.**

First-order phase transition between the AdS space and the BBH.

Hawking-Page transition corresponds to a large-N deconfinement transition in the gauge theory at strong coupling. E. Witten, hep-th/9803131.

Studies of blackhole physics on the SYM side.

2 The model

Zero-mode action of the bosonic sector of $\mathcal{N}=4$ SYM on S^3 at finite temperature.

$$Z = \int dM_\mu dA e^{-S}, \quad \text{where}$$
$$S = N \int_0^\beta dt \left\{ \text{tr} \sum_{\mu=1}^{D} (D_t M_\mu(t))^2 - \frac{\lambda}{2} \text{tr} \sum_{\mu,\nu=1}^{D} [M_\mu(t), M_\nu(t)]^2 + m^2 \text{tr} \sum_{\mu=1}^{D} M_\mu^2(t) \right\}.$$

- $D_t M_\mu(t) = \partial_t M_\mu(t) - i[A, M_\mu(t)]$: covariant derivative

  (A = zero mode of the time component of the gauge field on S^3)

- $M_\mu(t)$: SO(6) scalar fields ($\mu, \nu, \cdots = 1, 2, \cdots, D$, here $D = 6$)

- U(N) gauge symmetry : $M_\mu(t) \rightarrow g(t) M_\mu(t) g^\dagger(t)$, $A \rightarrow g(t) A g^\dagger(t) + i g(t) \frac{dg(t)^\dagger}{dt}$

- $\beta = 1/T$: inverse temperature

  Periodic boundary condition : $A(t+\beta) = A(t)$, $M_\mu(t+\beta) = M_\mu(t)$.

- Static and diagonal gauge: $A = \frac{1}{\beta}\text{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$, where $\{\alpha_a\} \in [-\pi, \pi]$.

  Gauge fixing term : $S_{\text{g.f.}} = -\sum_{a,b=1,a\neq b}^{N} \log \sin |(\alpha_a - \alpha_b)/2|$.

3 Gross-Witten-Wadia phase transition

Effective action of the SYM theory on S^3 at finite temperature

$\rightarrow$ Described by **Polyakov line U**.

Phase structure of the YM theory and **blackhole states in supergravity**.

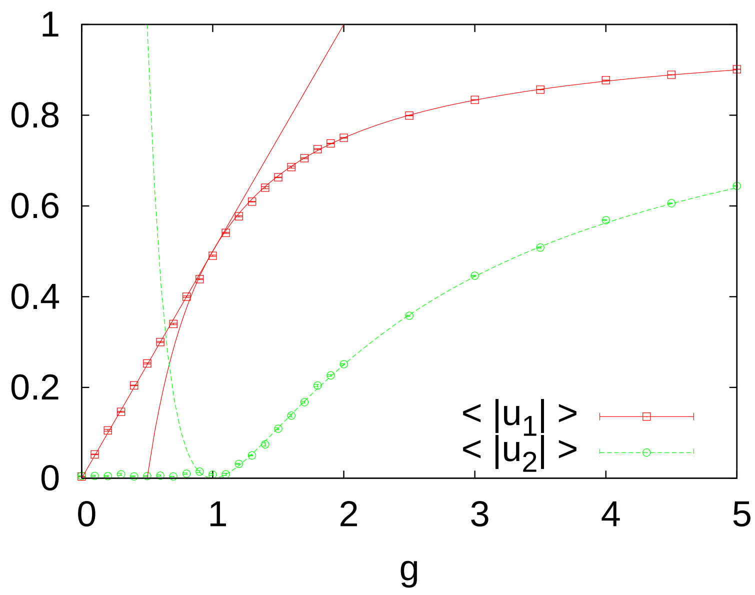
L. Alvarez-Gaume, C. Gomez, H. Liu and S.R. Wadia hep-th/0502227

Gross-Witten-Wadia (GWW) third-order phase transition of the partition function

$$Z_g = \int dU \exp\left(\frac{Ng}{2}(\text{tr}\, U + \text{tr}\, U^\dagger)\right),$$

$$\langle|u_1|\rangle = \begin{cases} \frac{g}{2}, & (g < 1) \\ 1 - \frac{1}{2g} & (g > 1). \end{cases}$$

$$\langle|u_2|\rangle = \begin{cases} 0, & (g < 1) \\ 1 - \frac{2}{g} + \frac{1}{g^2} & (g > 1). \end{cases}$$



MC simulation for $N = 128$, $u_n = \frac{1}{N}\text{tr}\, U^n$,

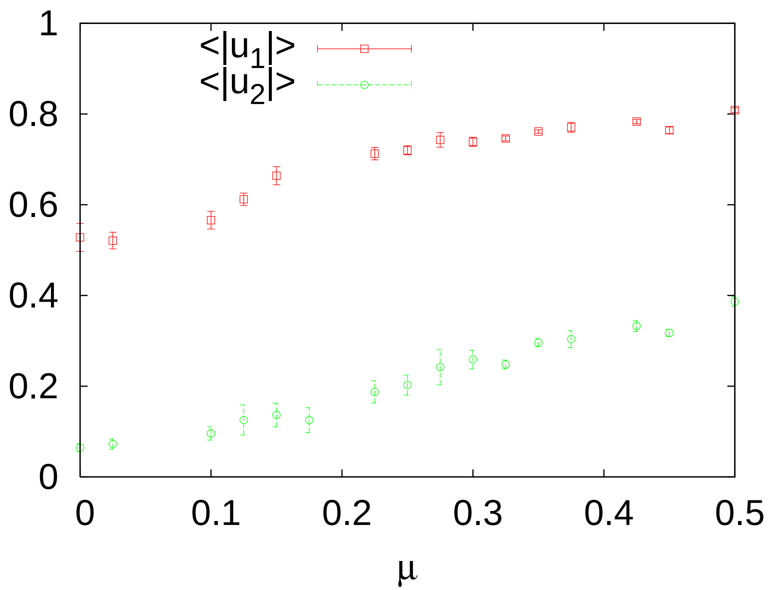
- $g < 1$: no gap on unit circle.
- $g > 1$: a system has a gap.

Saddle point of the gauge field for our model:

$$S' = S + N\mu\beta(\mathrm{tr}\, U + \mathrm{tr}\, U^{\dagger}).$$

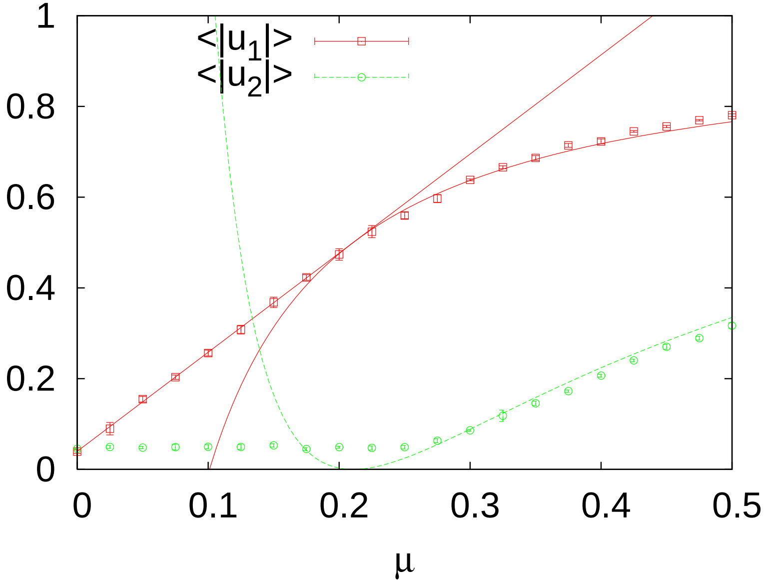
$D = 2$ case

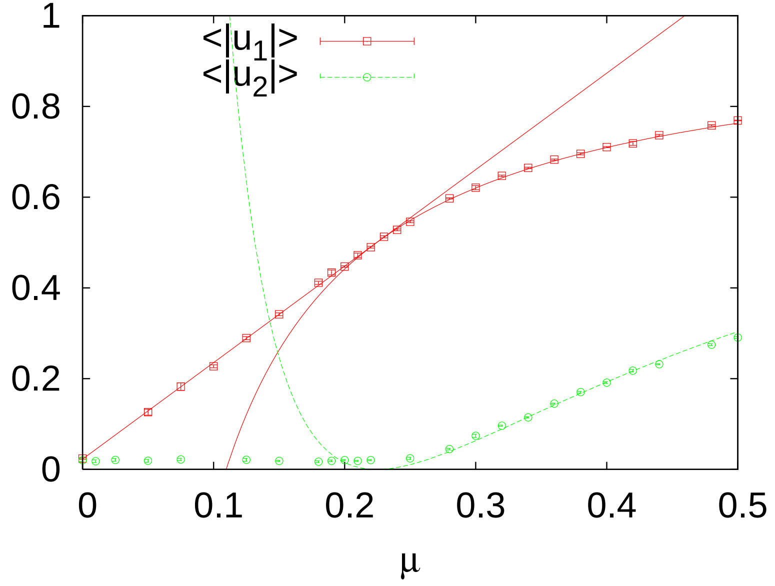
High-temperature $(\beta = 0.5)$ case $(\lambda = m = 1.0,\ N = 16)$.



No GWW-type phase transition.

Low-temperature ($\beta = 2.0$) case ($\lambda = m = 1.0$, $N = 16$ (left), $N = 48$ (right)).





$$\langle |u_1| \rangle = \begin{cases} \frac{p_1 \mu}{2} + q_1, & (\langle |u_1| \rangle < \frac{1}{2}), \quad p_1 = 4.369 \pm 0.033, \ q_1 = 0.039 \pm 0.001, \quad (N = 16, D = 2), \\ \frac{1 + p_1 \mu_c}{2} - \frac{p_1 \mu_c^2}{2\mu}, & (\langle |u_1| \rangle > \frac{1}{2}), \quad p_1 = 4.253 \pm 0.019, \ q_1 = 0.022 \pm 0.0017, \quad (N = 48, D = 2). \end{cases}$$

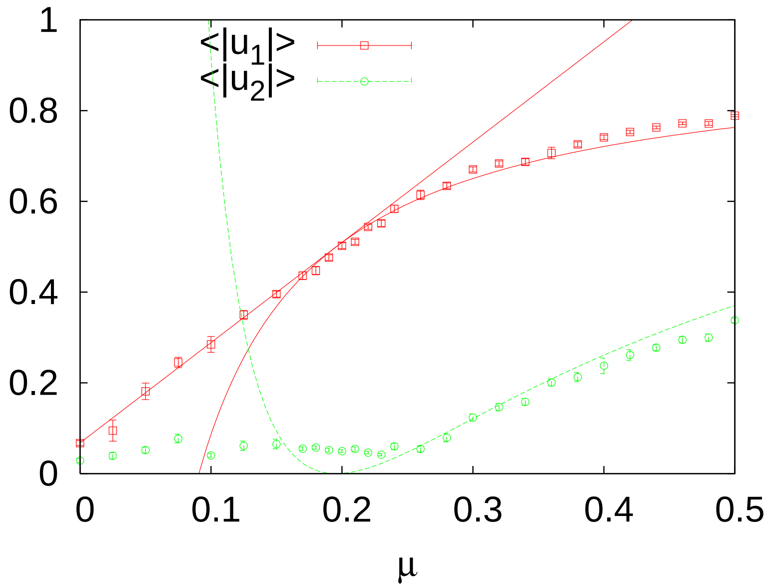
$\langle |u_1| \rangle$ and $\frac{d\langle |u_1| \rangle}{d\mu}$ are continuous at critical point $\mu_c = \frac{1 - 2q_1}{p_1}$ ($\frac{d^2 \langle |u_1| \rangle}{d\mu^2}$ is discontinuous).

GWW-type third order phase transition.

$$\langle |u_2| \rangle = 1 - \frac{2\mu_c}{\mu} + \frac{\mu_c^2}{\mu^2}, \quad (\mu > \mu_c).$$

$D = 6$ case

Low-temperature $(\beta = 2.0)$ case $(\lambda = m = 1.0,\ N = 16)$.



$$p_1 = 4.421 \pm 0.086, \quad q_1 = 0.068 \pm 0.004,$$

The result is insensitive to the dimensionality.

4 SO(6) R-symmetry breaking

Supergravity-side analysis of the 10-dim small black hole (SBH) in $AdS_5 \times S^5$..

L. Alvarez-Gaume, P. Basu, M. Marino and S.R. Wadia, hep-th/0605041

Metric of 10-dim SBH in $AdS_5 \times S^5$ → not symmetric under SO(6) rotation of S^5.

⇕

Corresponding saddle point → transform under SO(6) R-symmetry group.

Supergravity analysis : SO(6) R-symmetry is not spontaneously broken.

How about in the gauge theory side?

⇒ Monte Carlo simulation of the gauge theory.

Order parameter for the SO(6) R-symmetry breaking

Eigenvalue $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_6$ of integrated "momentum of inertia"

$$I_{\mu\nu} = \frac{1}{N} \int_0^{\beta} dt \mathrm{tr}\, M_\mu(t) M_\nu(t).$$

(in analogy to the IKKT model's case) hep-th/9811220, 0003208, 0005147, 0104260, 0108041, 0108070, 0402194

Large-N extrapolation of the VEV $\langle \lambda_\mu \rangle$.

Large-N limit : Eigenvalues $\langle \lambda_\mu \rangle$ converge to the same value.

SO(6) R-symmetry is not broken.

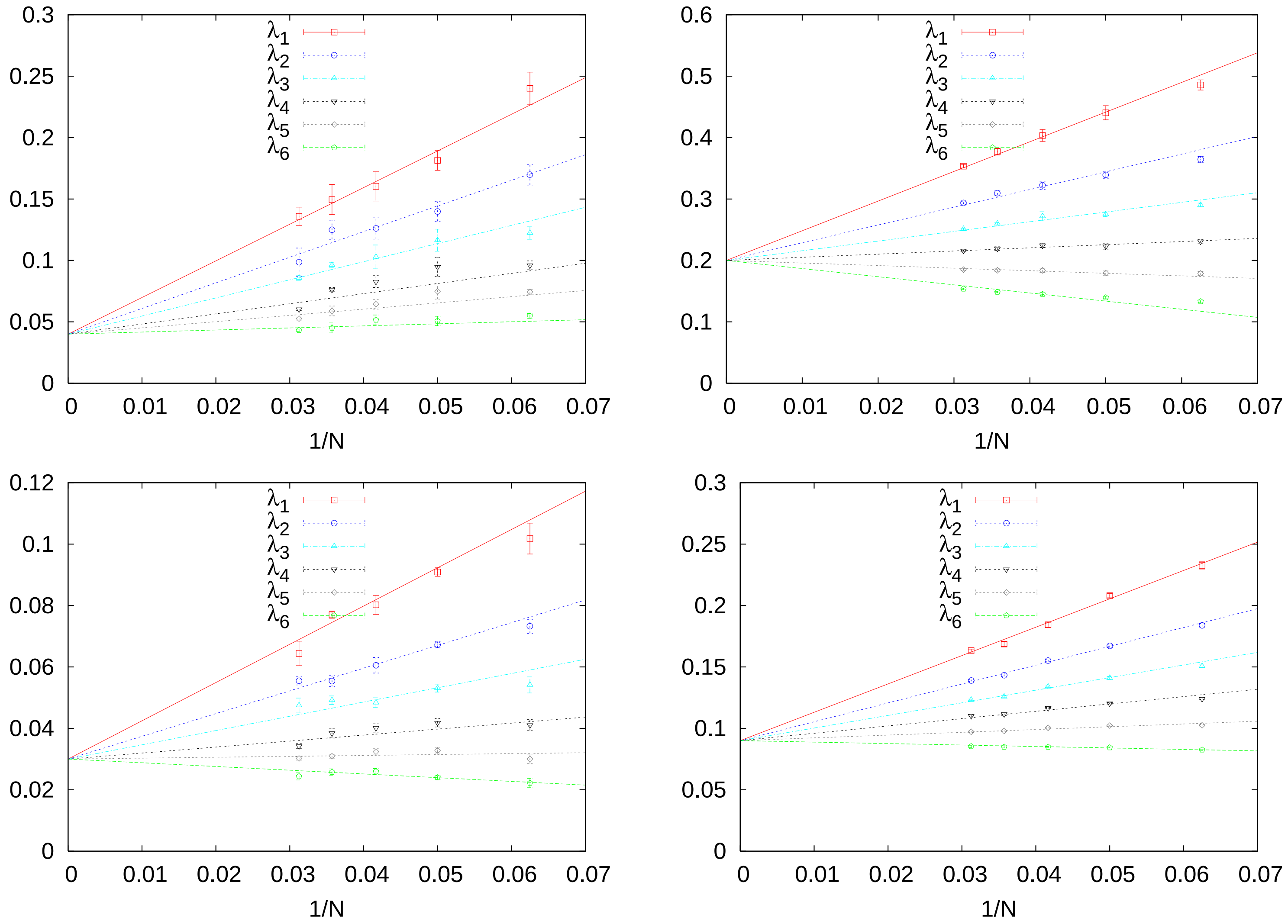
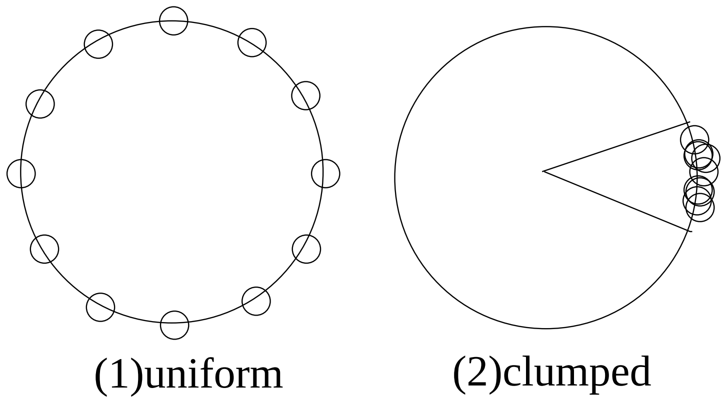


Figure 1: $(\beta, \lambda) = (0.1, 0.1)$ (upper left), $(\beta, \lambda) = (1.0, 0.1)$ (upper right), $(\beta, \lambda) = (0.1, 1.0)$ (lower left), $(\beta, \lambda) = (1.0, 1.0)$ (lower right), for $N = 16, 20, 24, 28, 32$, $m = 1.0$.

Specific vacuum configurations of the gauge field



- Uniform distrobution : $\alpha_a = \frac{\pi}{N}(2a - N)$ $(a = 1, 2, \cdots, N)$

  $\rightarrow \langle \mathrm{tr}\, U^n \rangle = 0$ for any $n \neq 0 \rightarrow$ Corresponds to $AdS_5 \times S^5$ geometry.

  E. Witten hep-th/9803131

- Clumped distribution : blackhole state

U: zero mode of the Polyakov line on S^3:

$$u_n = \frac{1}{N} \mathrm{tr}\, U^n = \frac{1}{N} \sum_{a=1}^{N} e^{i\alpha_a}, \quad U = P \exp\left(i \int_0^\beta dt A(t) \right).$$

SO(6) R-symmetry is not broken in these cases, too.

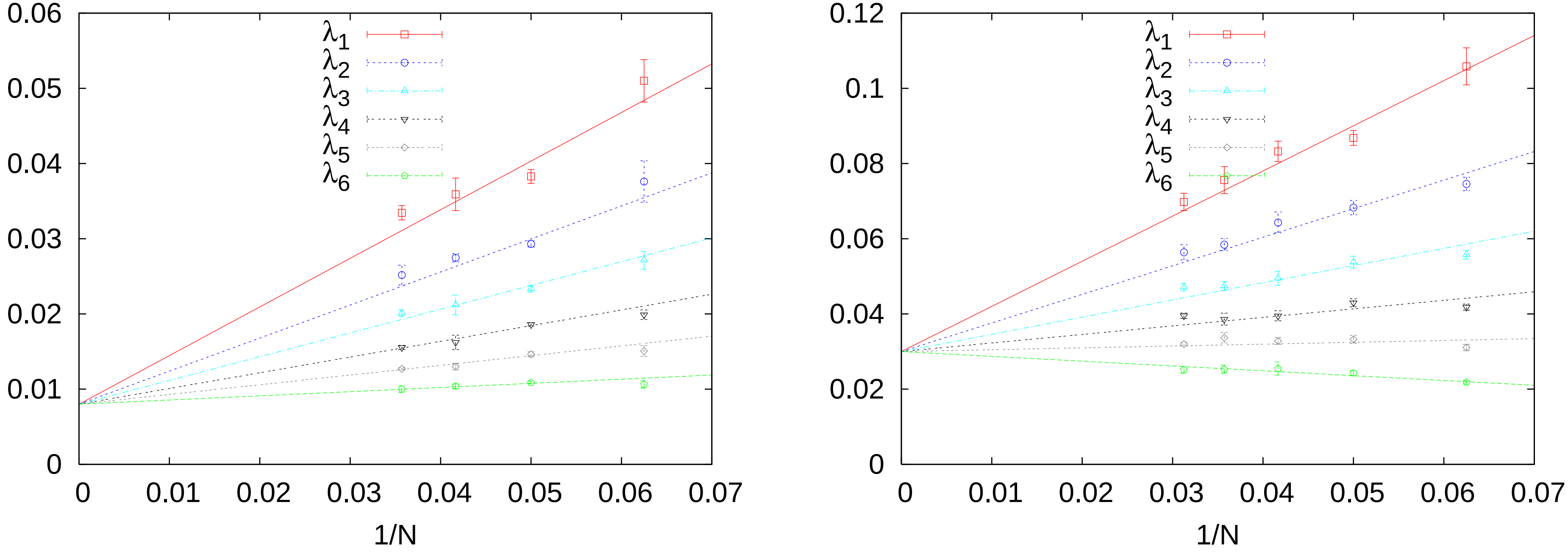


Figure 2: Uniform (left) and clumped (right) distribution for $\beta = 0.1$, $\lambda = 1.0$, $m = 1.0$.

5 Conclusion

Zero mode effective action of the $\mathcal{N} = 4$ SYM theory on S^3.

- **Gross-Witten-Wadia (GWW) type third-order phase transition of the matrix model.**
- **SO(6) R-symmetry of the Yang-Mills theory is unbroken.**