Complex Langevin analysis of the spontaneous rotational symmetry breaking in the Euclidean type IIB matrix model

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Smart and Human

常翔学園

摂南大学

1. Introduction

Difficulties in simulating complex partition functions.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

Sign problem:

The reweighting $\langle \mathscr{O} \rangle = \frac{\langle \mathscr{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. $\exp[O(N^2)]$

$<*>_0$ = (V.E.V. for the phase-quenched partition function Z_0)

2. Euclidean type IIB matrix model

SETSUDAI

Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115] (a.k.a. "IKKT model")

$$Z = \int dA d\psi e^{-(S_{\mathrm{b}}+S_{\mathrm{f}})}$$

$$S_{\mathrm{b}} = -\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2, \quad S_{\mathrm{f}} = N\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]$$

- ***Euclidean*** case after Wick rotation $A_0 \to iA_{10}, \Gamma^0 \to -i\Gamma_{10}$.

⇒Path integral is finite without cutoff.

[W. Krauth, H. Nicolai and M. Staudacher, hep-th/9803117, P. Austing and J.F. Wheater, hep-th/0103059]

- A_μ, Ψ_α ⇒ $N \times N$ Hemitian traceless matrices.

  (μ=1,2,…,10, α,β=1,2,…,16)
- Eigenvalues of A_μ : spacetime coordinate⇒ $\mathcal{N}$=2 SUSY

2. Euclidean type IIB matrix model

$$Z=\int dA de^{-S_{\rm b}} \underbrace{\left(\int d\psi e^{-S_{\rm f}}\right)}_{={\rm Pf}\mathcal{M}=|{\rm Pf}\mathcal{M}|e^{i\Gamma}=e^{-S_{\rm f,eff}}} = \int dA \underbrace{e^{-S}}_{e^{-(S_{\rm b}+S_{\rm f,eff})}}$$

- ${\rm Pf}\mathcal{M}$'s *complex phase Γ* contributes to the Spontaneous Symmetry Breaking (SSB) of SO(10).

- Result of Gaussian Expansion Method (GEM)

[T.Aoyama, J.Nishimura, and T.Okubo, arXiv:1007.0883、J.Nishimura, T.Okubo and F.Sugino, arXiv:1108.1293]

SSB **SO(10)→SO(3).**
Dynamical compactification to 3-dim spacetime.

3. Complex Langevin Method

Complex Langevin Method (CLM)

⇒Solve the complex version of the Langevin equation.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_\mu)_{ij}}{dt} = -\boxed{\frac{\partial S}{\partial (A_\mu)_{ji}}} + \eta_{\mu,ij}(t) = -\left\{ \frac{\partial S_{\rm b}}{\partial (A_\mu)_{ji}} - \underbrace{\frac{1}{2}\mathrm{Tr}\left(\frac{\partial \mathscr{M}}{\partial (A_\mu)_{ji}} \mathscr{M}^{-1} \right)}_{=\partial \log \mathrm{Pf}\mathscr{M} / \partial (A_\mu)_{ji}} \right\} + \eta_{\mu,ij}(t)$$

drift term

- A_μ : Hermitian→general complex traceless matrices.

- η_μ: Hermitian-matrix white noise obeying the probability distribution $\exp\left(-\frac{1}{4}\int \mathrm{tr}\eta^2(t) dt \right)$

3. Complex Langevin Method

CLM does not work when it encounters these problems:

(1) Excursion problem: A_μ is too far from Hermitian

⇒Gauge Cooling minimizes the Hermitian norm

$$\mathcal{N} = \frac{-1}{10N} \sum_{\mu=1}^{10} \mathrm{tr}[(A_\mu - (A_\mu)^\dagger)^2]$$

(2) Singular drift problem:

The drift term $dS/d(A_\mu)_{ji}$ diverges due to $\mathcal{M}$'s near-zero eigenvalues.

We trust CLM when the distribution p(u) of the drift norm

$$u = \sqrt{\frac{1}{10N^3} \sum_{\mu=1}^{10} \sum_{i,j=1}^{N} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2}$$

falls exponentially as $p(u) \propto e^{-au}$.

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

Look at the drift term ⇒ Get the drift of CLM!!

3. Complex Langevin Method

Mass deformation [Y. Ito and J. Nishimura, arXiv:1609.04501]

- SO(D) symmetry breaking term $\Delta S_{\mathrm{b}} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu \mathrm{tr}(A_\mu)^2$

  Here, we take m_μ=(0.5,0.5,1,2,4,8,8,8,8,8)

Order parameters for SSB of SO(10): $\lambda_\mu = \mathrm{Re}\left\{\frac{1}{N}\mathrm{tr}(A_\mu)^2\right\}$

- Fermionic mass term: $\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}\left(\bar{\psi}_\alpha (i\Gamma_8 \Gamma_9^\dagger \Gamma_{10})_{\alpha\beta}\psi_\beta\right)$

  Avoids the singular eigenvalue distribution of $\mathscr{M}$.

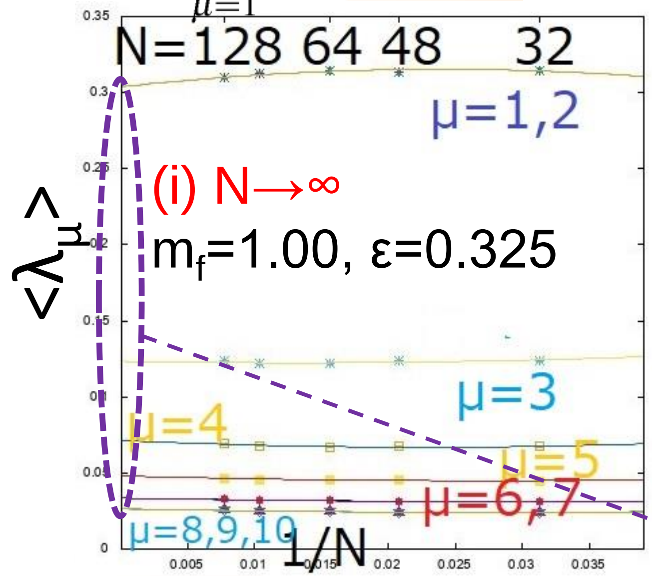
Extrapolation (i) N→∞ ⇒(ii)ε_m→0 ⇒ (iii) m_f→0.

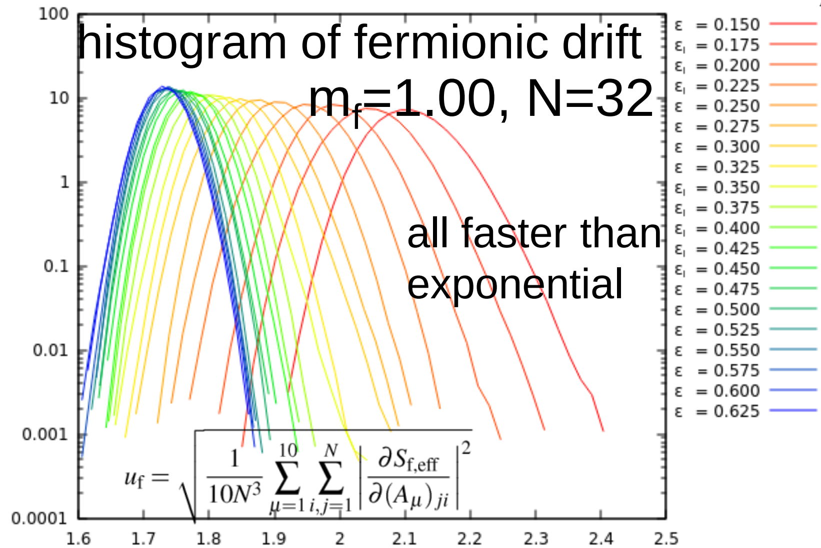
4. Result

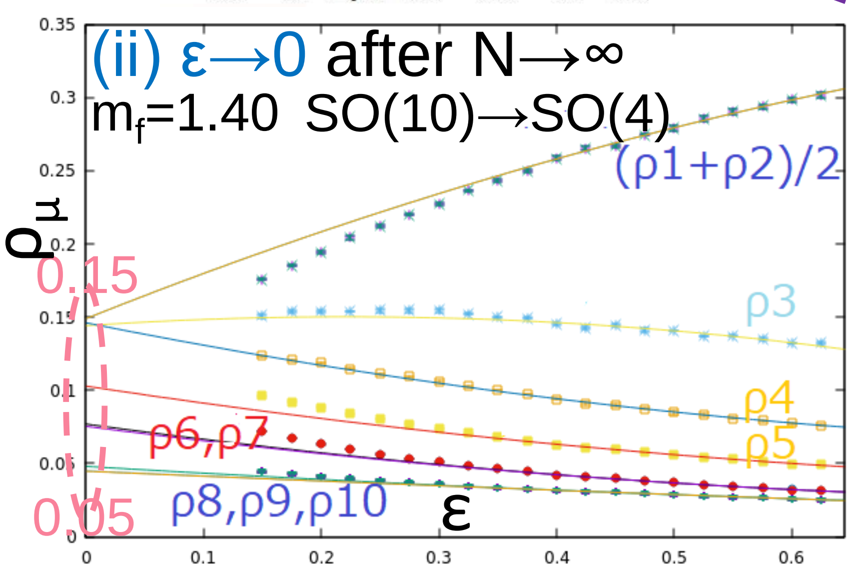
$$\Delta S_{\mathrm{b}} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu \mathrm{tr}(A_\mu)^2$$

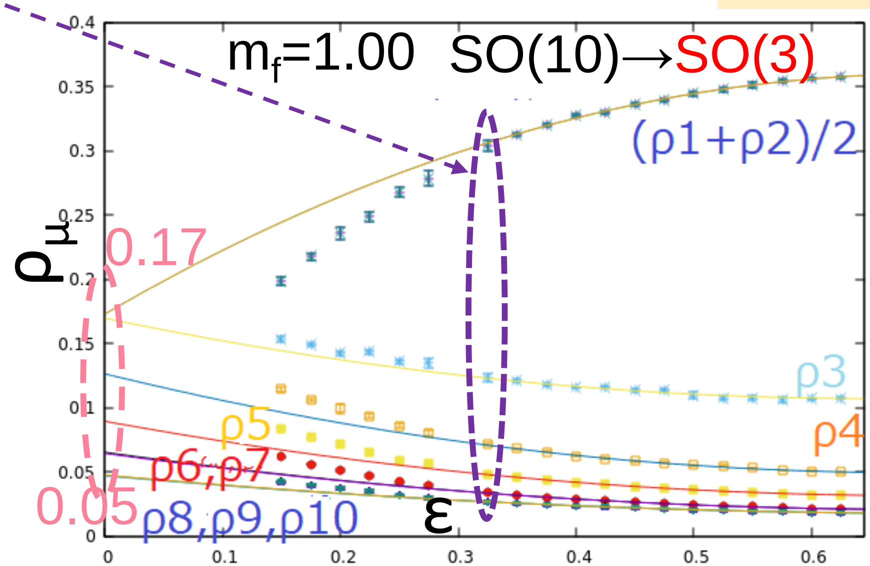
$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}\left(\bar{\Psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\Psi_\beta\right)$$

$$\rho_\mu(\varepsilon,m_{\mathrm{f}})=\frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\mathrm{f}}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\mathrm{f}}}}$$







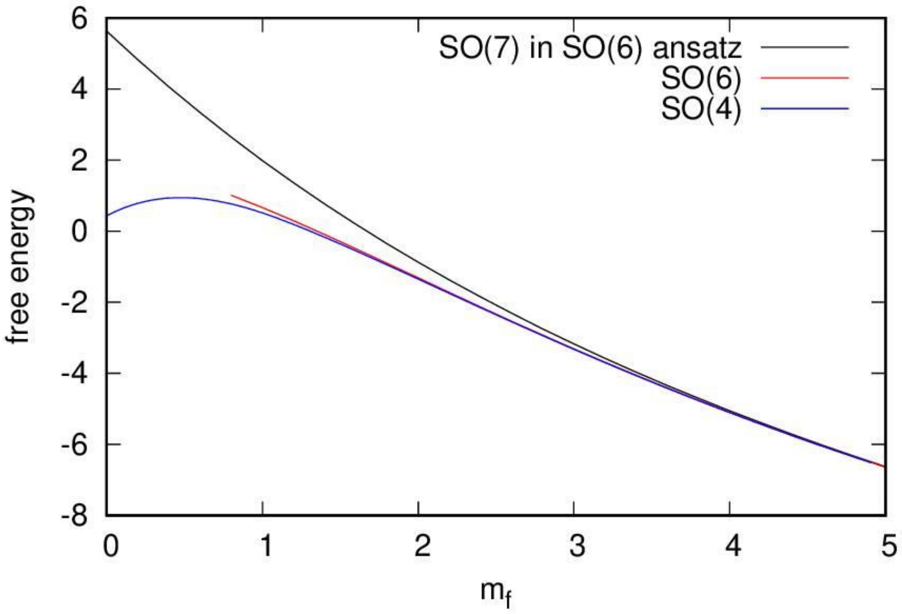


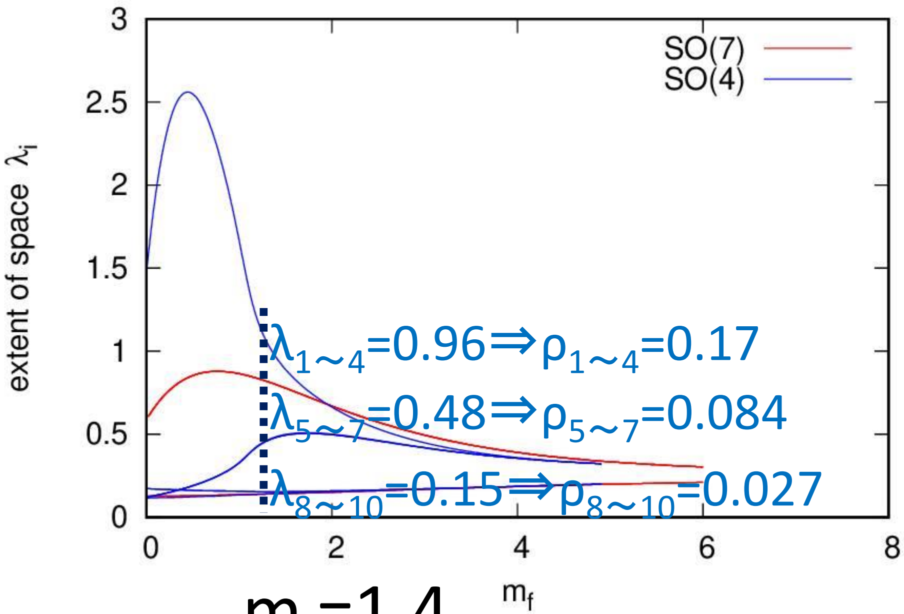
4. Result

$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}\left(\bar{\psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\psi_\beta\right)$$ (no $\Delta S_{\mathrm{b}} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu \mathrm{tr}(A_\mu)^2$ term)

GEM result for $m_f>0$ at 3 loop solutions of SO(4) and SO(6) ansatz

$$\rho_\mu(\varepsilon, m_{\mathrm{f}}) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\mathrm{f}}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\mathrm{f}}}}$$





$m_f=1.4$

5. Summary

Dynamical compactification of the spacetime in the Euclidean IKKT model.

"Complex Langevin Method"
⇒trend of SSB SO(10)→SO(3).

Future works

Test various ideas

- Reweighting method [J. Bloch, arXiv:1701.00986]
- Other deformations than the mass deformation [Y. Ito, J. Nishimura, arXiv:1710.07929]