Monte Carlo studies of the six-dimensional IKKT model

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1. Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model) ⇒ Promising candidate for constructive definition of superstring theory.

- Evidences for spontaneous breakdown of SO(10) → SO(4).
- Complex fermion determinant:
  - Crucial for rotational symmetry breaking.
  - Difficulty of Monte Carlo simulation.

2. 6d IKKT matrix model

Toy model for studying rotational symmetry breaking.

\[ S = -\frac{N}{4} \text{tr} [\gamma_\mu \gamma_\nu A_{\mu\nu}] + \text{N} \text{tr} \bar{\psi}(\gamma^\mu A_\mu \psi) . \]

- \( A_\mu \) (6d vector) and \( \psi \) (6d Weyl spinor) are \( N \times N \) matrices.
- SO(6) rotational symmetry and SL(N) gauge symmetry.
- Presence of \( N = 2 \) supersymmetry.
- \( Z = \int dA e^{-S} (\text{det} M) = \int dA e^{-S_0} e^{tr}. \) CPU cost is \( O(N^6) \).

4d → det \( M \) is real positive 6d and 10d → det \( M \) is complex.

Complex phase is important in SO(6) breakdown.

- Previous works on this model:
  - Simulation of phase-quenched 6d and 10d IKKT model ⇒ no symmetry breakdown of SO(6) (and SO(10)).
  - T. Aoyama, J. Nishimura and T. Okubo
    - Observable for probing dimensionality: \( T_{\mu\nu} = \frac{1}{N} \text{tr} (A_\mu A_\nu) \).

\( \lambda_\alpha \) (\( n = 1, 2, \ldots, 6 \)) : eigenvalues of \( T_{\mu\nu} \) \((\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_6)\)
- At large \( N \), \( (\lambda_1, 2, 3) \gg (\lambda_4, 5, 6) \)
- \( (\lambda_6) = 0.18 \) → finite extent of extra dimension.

3. Monte Carlo simulation

Factorization method

Numerical approach to the complex action problem.

Distribution function

\[ \rho_{\alpha, \nu}(x) \equiv \langle \delta(x - \hat{\lambda}_\alpha) \rangle = \frac{1}{C} \rho_{\alpha, \nu}^{(0)}(x) w_{\alpha}(x) , \]

\( \hat{\lambda}_\alpha = \lambda_\alpha / \langle \lambda_\alpha \rangle_0 \), \( C = \langle \cos \Gamma \rangle_0 \).

\[ \rho_{\alpha, \nu}^{(0)}(x) \equiv \langle \delta(x - \hat{\lambda}_\alpha) \rangle_0 . \]

\( \langle \rangle_0 = \langle \text{V.E.V. for the phase-quenched model} \rangle \)

no symmetry breakdown of SO(6) for \( Z_0 = \int dA e^{-S_0} \).

\( \hat{\lambda}_\alpha = \lambda_\alpha / \langle \lambda_\alpha \rangle_0 \neq 1 \) → SO(6)'s breakdown.

\[ w_{\alpha}(x) = \langle \cos \Gamma \rangle_\alpha, \]

\( \langle \rangle_\alpha = \langle \text{V.E.V. for the partition function} Z_{\alpha, x} \rangle \)

\[ Z_{\alpha, x} = \int dA e^{-S_0} \delta(x - \hat{\lambda}_\alpha) . \]

Resolution of the overlap problem:
The system is forced to visit the configurations where \( \rho_{\alpha}(x) \) is important.

The position of the peak \( x_p \) for the distribution function \( \rho_{\alpha}(x) \):

\[ 0 = \frac{\partial}{\partial x} \log \rho_{\alpha}(x) = \frac{f_{\alpha}^{(0)}(x) - \langle \lambda_\alpha \rangle_0 V'(\langle \lambda_\alpha \rangle_0 x)}{\langle \lambda_\alpha \rangle_0} , \]

where \( f_{\alpha}^{(0)}(x) \equiv \frac{\partial}{\partial x} \log \rho_{\alpha}^{(0)}(x) . \)

Monte Carlo evaluation of \( \langle \hat{\lambda}_\alpha \rangle \)

\[ w_{\alpha}(x) > 0 \Rightarrow \langle \hat{\lambda}_\alpha \rangle \text{ is the minimum of } F_{\alpha}(x) . \]

We solve \( F_{\alpha}(x) = 0 \), namely \( \frac{1}{N^2} f_{\alpha}^{(0)}(x) = \frac{d}{dx} \left( \frac{1}{N^2} \log w_{\alpha}(x) \right) . \)

Both \( \frac{1}{N^2} \log w_{\alpha}(x) \) and \( \frac{1}{N^2} f_{\alpha}^{(0)}(x) \) scale at large \( N \) as

\[ \frac{1}{N^2} \log w_{\alpha}(x) \propto \Phi_{\alpha}(x), \]

\[ \frac{1}{N^2} f_{\alpha}^{(0)}(x) \propto F_{\alpha}(x). \]

Behavior of \( \frac{1}{N^2} \log w_{\alpha}(x) \)

Expected power behaviors:

\[ \frac{1}{N^2} \log w_{\alpha}(x) \propto \left\{ \begin{array}{ll}
\epsilon_n x^2 & (x < 1, n = 1, 2, \ldots, 6) \\
\text{const.} & (x \geq 1, n = 1, 2, \ldots, 5)
\end{array} \right. \]

Behavior of \( \frac{1}{N^2} f_{\alpha}^{(0)}(x) \)

Small \( x \Rightarrow (7 - n) \) directions are shrunk.

- \( n = 2, \ldots, 6 \): \( \rho_{\alpha}^{(0)}(x) \sim (\sqrt{2}) n^2 (7 - n) \)
  - \( n = 1 \): Eigenvalues of \( A_\mu \) are collapsed to zero.

\( \Rightarrow \) Add the effect of fermionic determinant (polynomial of \( A_\mu \) with degree \( 4N^2 \)):

\[ \rho_{\alpha}^{(0)}(x) \sim (\sqrt{2}) (n+4) n^2 \]

\[ \frac{1}{N^2} f_{\alpha}^{(0)}(x) \propto \left\{ \begin{array}{ll}
\tau + 4(t+1) & (x < 1) \\
0 & (x > 1)
\end{array} \right. \]

Double-peak structure for \( n = 2, 3, 4, 5 \).

Symmetry breakdown of SO(6) to SO(3) in Gaussian Expansion Method.

\( z_0 \) for \( n = 4 \) ⇒ extent of extra dimension.

\[ \frac{1}{N^2} f_{\alpha}^{(0)}(x) \text{ scales at small } x \text{ in 6d full IKKT model.} \]

(in contrast to \( \frac{1}{N^2} f_{\alpha}^{(0)}(x) \)'s scaling in 6d one-loop model).

⇒ hard-core potential at large \( N \) (\( z_0 \) is finite).

Consistent with Gaussian Expansion Method.

4. Conclusion

Monte Carlo simulation of 6d IKKT model.

Can we understand the emergence of spacetime?

- Extent of the extra dimension → finite
  - Consistent with Gaussian Expansion Method.

Future works:

- Analysis of large-\( z \) regime.
- Comparison of the free energy.