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Complex Langevin analysis of the spontaneous rotational symmetry breaking in the dimensionally-reduced super-Yang-Mills models

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1. Introduction

Difficulties in simulating complex partition functions.

$$Z = \int dA \exp(-S_0 + i\Gamma), \;\; Z_0 = \int dA e^{-S_0}$$

Sign problem:

The reweighting $\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. exp[O(N^2)]

$\langle * \rangle_0$ = (V.E.V. for the phase-quenched partition function Z_0)

2. The Euclidean IKKT model

IKKT model [N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

⇒Promising candidate for nonperturbative string theory

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_\mathrm{b}} + \underbrace{N\mathrm{tr}\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_\mathrm{f}}$$

- A_μ, Ψ_α ⇒N × N Hermitian traceless matrices.

$$\mu = 1,2,\cdots,D, \quad \alpha,\beta = \begin{cases} 1,2,3,4 & (D=6) \\ 1,2,\cdots,16 & (D=10) \end{cases}$$

- Eigenvalues of A_μ : spacetime coordinate⇒ $\mathcal{N}$=2 SUSY

- Originally defined in **D=10**. In the following, we consider the ***simplified Euclidean D=6 case.***

2. The Euclidean IKKT model

- Integrating out ψ yields det $\mathscr{M}$ in D=6 (Pf $\mathscr{M}$ in D=10)

- Det/Pf $\mathscr{M}$'s *complex phase* contributes to the Spontaneous Symmetry Breaking (SSB) of SO(D).

- Result of Gaussian Expansion Method (GEM)

[T.Aoyama, J.Nishimura, and T.Okubo, arXiv:1007.0883、J.Nishimura, T.Okubo and F.Sugino, arXiv:1108.1293]

SSB **SO(6)→SO(3)** (In D=10, too, SO(10)→SO(3))
Dynamical compactification to 3-dim spacetime.

$$\lambda_\mu (\lambda_1 \geqq \cdots \geqq \lambda_D) : \text{ eigenvalues of } T_\mu = \frac{1}{N}\mathrm{tr}(A_\mu A_\nu)$$

$$\rho_\mu = \frac{\langle \lambda_\mu \rangle}{\sum_{\nu=1}^{D} \langle \lambda_\nu \rangle} = \begin{cases} 0.30 & (\mu = 1,2,3) \\ 0.035 & (\mu = 4,5,6) \end{cases} \quad (D=6)$$

Complex Langevin Method (CLM)

⇒Solve the complex version of the Langevin equation.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

drift term

$$\frac{d(A_\mu)_{ij}}{dt} = -\left\{ \frac{dS_b}{d(A_\mu)_{ji}} - c_d \mathrm{Tr}\left(\mathscr{M}^{-1} \frac{d\mathscr{M}}{d(A_\mu)_{ji}} \right) \right\} + \eta_{\mu,ij}(t) \qquad c_d = \begin{cases} 1 & (D=6 \to \det \mathscr{M}) \\ \frac{1}{2} & (D=10 \to \mathrm{Pf}\mathscr{M}) \end{cases}$$

- A_μ : Hermitian→general complex traceless matrices.
- η_μ: Hermitian white noise obeying the probability distribution $\exp\left(-\frac{1}{4}\int \mathrm{tr}\eta^2(t)dt \right)$

3. Complex Langevin Method

SETSUDAI

CLM does not work when it encounters these problems:

(1) Excursion problem: A_μ is too far from Hermitian

⇒Gauge Cooling minimizes the Hermitian norm

$$\mathcal{N} = \frac{-1}{4N} \sum_{\mu=1}^{D} \mathrm{tr}[(A_\mu - A_\mu^\dagger)^2].$$

(2) Singular drift problem:

The drift term $dS/d(A_\mu)_{ji}$ diverges due to $\mathcal{M}$'s near-zero eigenvalues.

We trust CLM when the distribution p(u) of the drift norm

$$u = \sqrt{\frac{1}{DN^3} \sum_{\mu=1}^{D} \sum_{i,j=1}^{N} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2}$$

falls exponentially as $p(u) \propto e^{-au}$.

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

Look at the drift term ⇒ Get the drift of CLM!!

3. Complex Langevin Method

Mass deformation [Y. Ito and J. Nishimura, arXiv:1609.04501]

- SO(D) symmetry breaking term $\Delta S_{\rm b} = N\frac{\varepsilon_{\rm m}}{2}\sum_{\mu=1}^{D} m_\mu {\rm tr}(A_\mu)^2.$

  Here, we take $m_\mu=(0.5,0.5,1,2,4,8)$

Order parameters for SSB of SO(D): $\lambda_\mu = {\rm Re}\left\{\frac{1}{N}{\rm tr}(A_\mu)^2\right\}$

- Fermionic mass term: $\Delta S_{\rm f} = N m_{\rm f} {\rm tr}(\bar{\psi}_\alpha (\Gamma_D)_{\alpha\beta}\psi_\beta)$

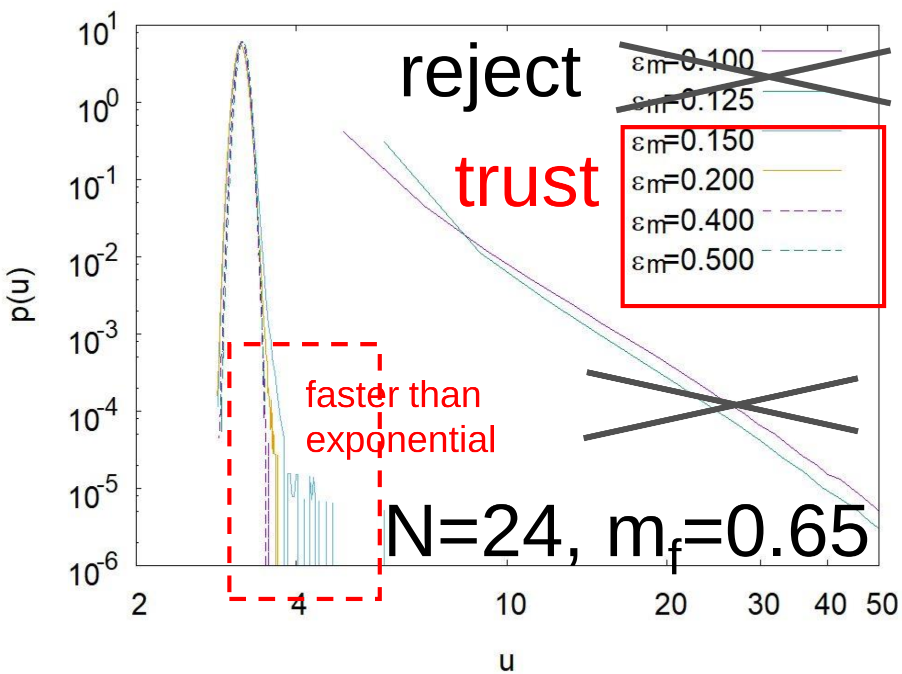
  Avoids the singular eigenvalue distribution of $\mathscr{M}$.

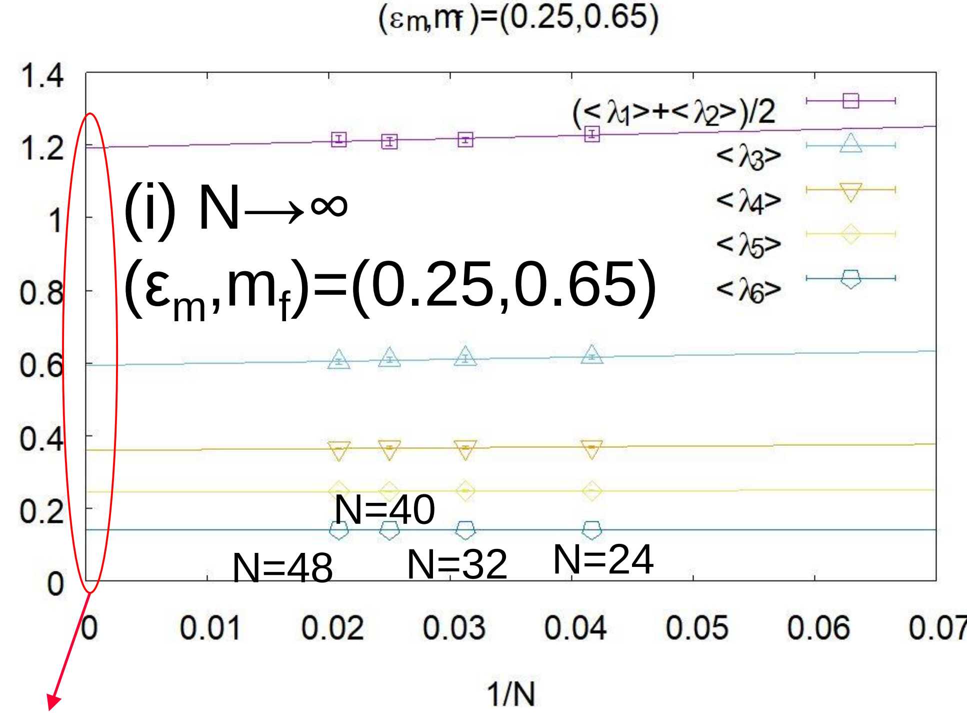
Extrapolation (i) $N\to\infty$ ⇒(ii)$\varepsilon_{\rm m}\to 0$ ⇒ (iii) $m_{\rm f}\to 0$.

4. Result

$$\Delta S_b = N\frac{\varepsilon_m}{2}\sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2. \quad \Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_D)_{\alpha\beta}\psi_\beta) \ (D=6)$$

$u = \sqrt{\frac{1}{DN^3}\sum_{\mu=1}^{D}\sum_{i,j=1}^{N}\left|\frac{\partial S}{\partial (A_\mu)_{ji}}\right|^2}$'s distribution p(u) (log-log)





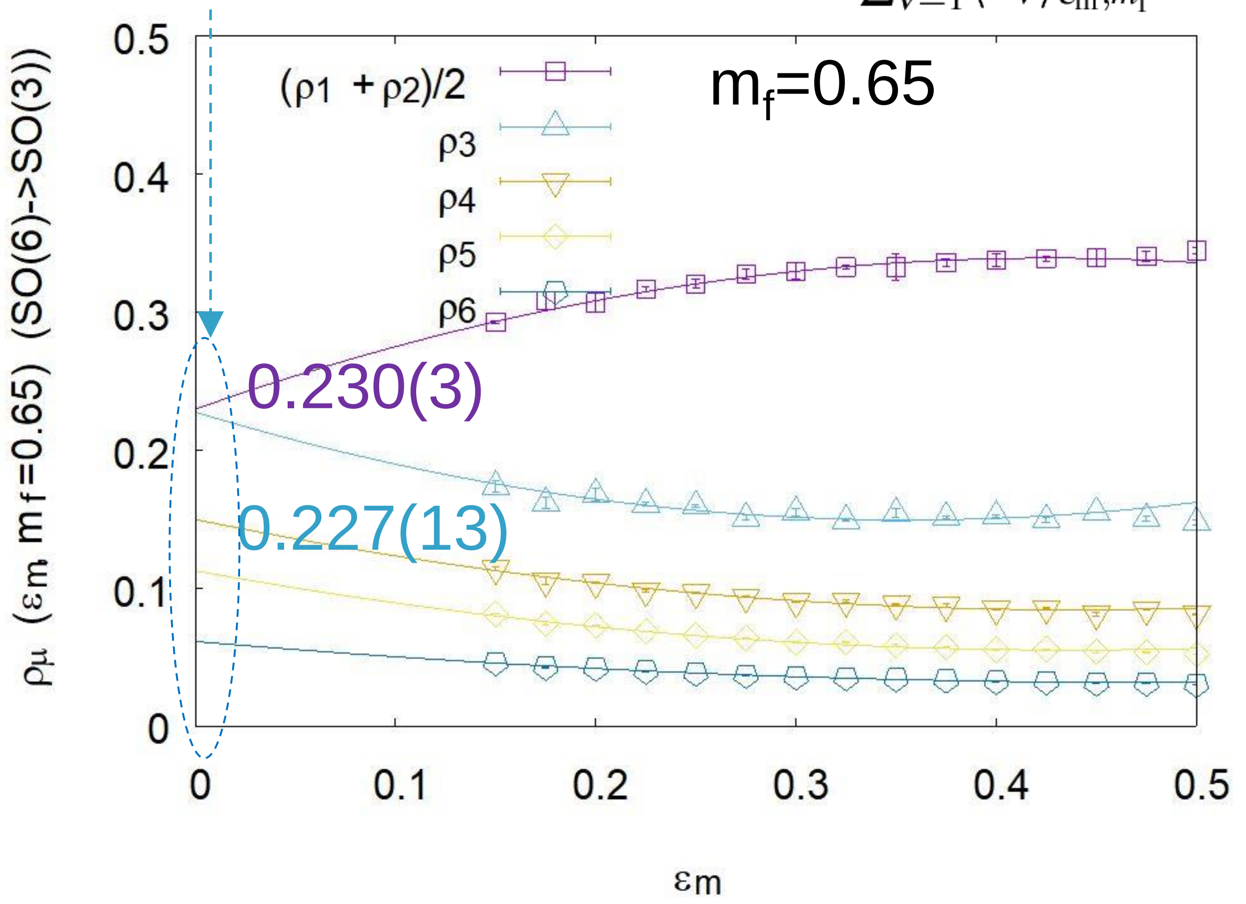
$\langle \lambda_\mu \rangle_{\varepsilon_m, m_f}$ at large N

4. Result

$$\Delta S_b = N\frac{\varepsilon_m}{2}\sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2. \quad \Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha(\Gamma_D)_{\alpha\beta}\psi_\beta) \quad (D=6)$$

(ii) $\varepsilon_m \to 0$ after $N \to \infty$

$$\rho_\mu(\varepsilon_m, m_f) = \frac{\langle \lambda_\mu \rangle_{\varepsilon_m, m_f}}{\sum_{\nu=1}^{D} \langle \lambda_\nu \rangle_{\varepsilon_m, m_f}}$$

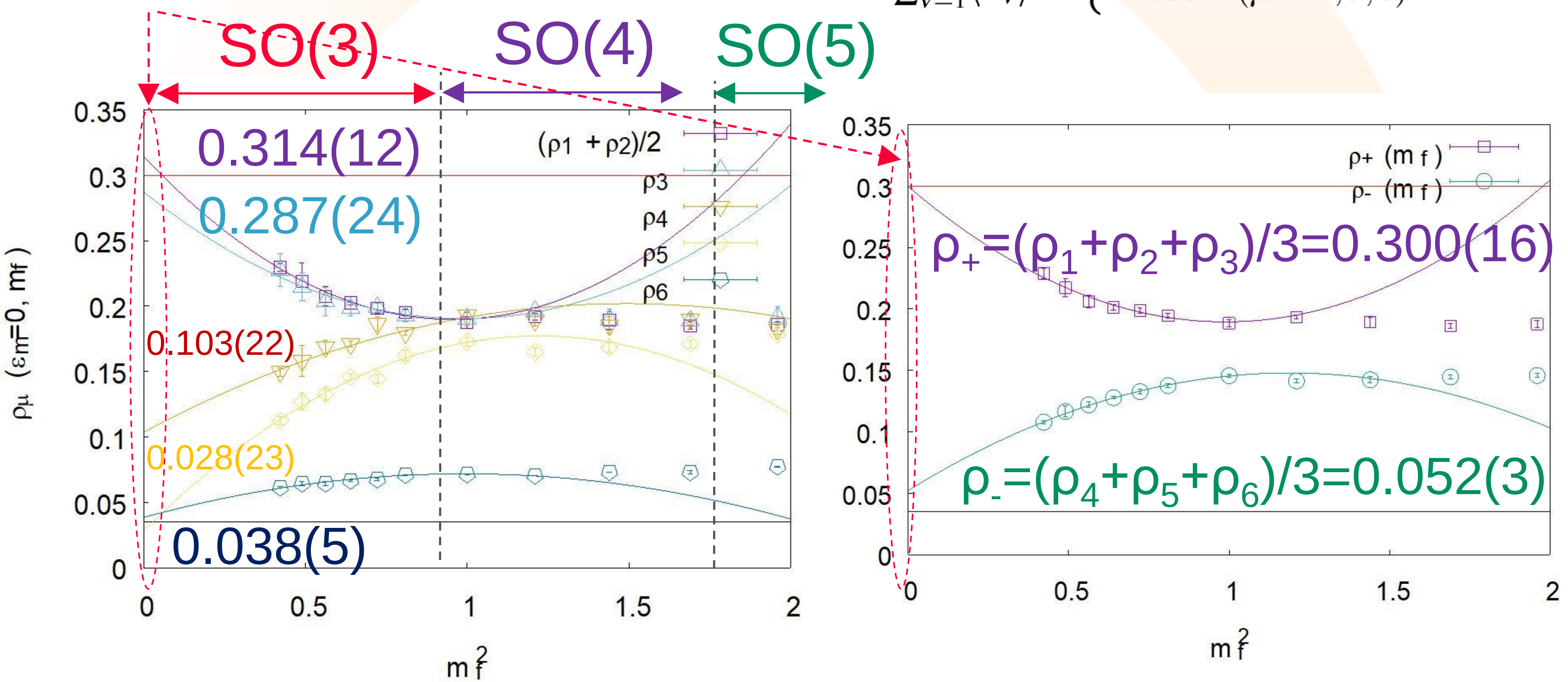


4. Result

$$\Delta S_b = N\frac{\varepsilon_m}{2}\sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2. \quad \Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_D)_{\alpha\beta}\psi_\beta) \ (D=6)$$

(iii) $m_f \to 0$ after $\varepsilon_m \to 0$

GEM $\rho_\mu = \frac{\langle\lambda_\mu\rangle}{\sum_{\nu=1}^{D}\langle\lambda_\nu\rangle} = \begin{cases} 0.30 & (\mu=1,2,3) \\ 0.035 & (\mu=4,5,6) \end{cases} \quad (D=6)$



SSB SO(6) → at most SO(3) Consistent with GEM.

5. Summary

Dynamical compactification of the spacetime in the simplified Euclidean D=6 IKKT model.

"Complex Langevin Method"
⇒trend of SSB SO(6)→SO(3).

Future works
Extension to D=10
Test various ideas
- Reweighting method [J. Bloch, arXiv:1701.00986]
- Other deformations than the mass deformation [Y. Ito, J. Nishimura, arXiv:1710.07929]