Smart and Human

常翔学園

摂南大学

Complex Langevin analysis of the spontaneous rotational symmetry breaking in the Euclidean type IIB matrix model (arXiv:1712.07562, arXiv:2001.XXXXX)

Takehiro Azuma (Setsunan Univ.)

NSM2019 IISER Bhopal, Dec 27th 2019, 15:15～15:45

with Konstantinos N. Anagnostopoulos (NTUA), Yuta Ito (KEK),
Jun Nishimura (KEK, SOKENDAI), Toshiyuki Okubo (Meijo Univ.)
and Stratos Kovalkov Papadoudis(NTUA)

1. Introduction

Difficulties in simulating complex partition functions.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

Sign problem:

The reweighting $\langle \mathscr{O} \rangle = \frac{\langle \mathscr{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. exp[O(N^2)]

$\langle * \rangle_0$ = (V.E.V. for the phase-quenched partition function Z_0)

Various methods to address the sign problem:
(Complex Langevin Method (CLM), factorization method, Lefschetz-thimble method…)

In the following, we discuss CLM.

2. Euclidean type IIB matrix model

SETSUDAI

type IIB matrix model model (a.k.a. IKKT model)
⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$Z = \int dA d\psi e^{-(S_b + S_f)}$$

$$S_b = -\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2, \quad S_f = N \mathrm{tr} \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]$$

Euclidean case after Wick rotation $A_0 \to iA_D, \Gamma^0 \to -i\Gamma_D$.
⇒Path integral is finite without cutoff.

[W. Krauth, H. Nicolai and M. Staudacher, hep-th/9803117, P. Austing and J.F. Wheater, hep-th/0103059]

- A_μ, Ψ_α ⇒N × N Hermitian traceless matrices.

$$\mu = 1,2,\cdots,D, \quad \alpha,\beta = \begin{cases} 1,2,3,4 & (D=6) \\ 1,2,\cdots,16 & (D=10) \end{cases}$$

- Originally defined in **D=10** (ψ: Majonara-Weyl)
  We consider the ***simplified D=6 case as well***
  (ψ: Weyl, not Majorana $d\psi \to d\psi d\bar{\psi}$)

2. Euclidean type IIB matrix model

SETSUDAI

- Result of Gaussian Expansion Method (GEM)

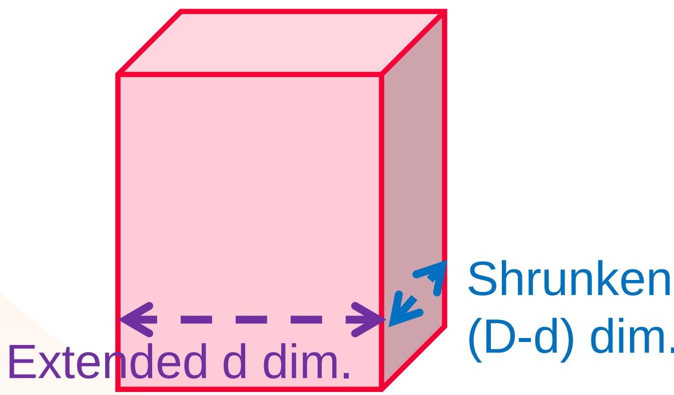
[T.Aoyama, J.Nishimura, and T.Okubo, arXiv:1007.0883、J.Nishimura, T.Okubo and F.Sugino, arXiv:1108.1293]

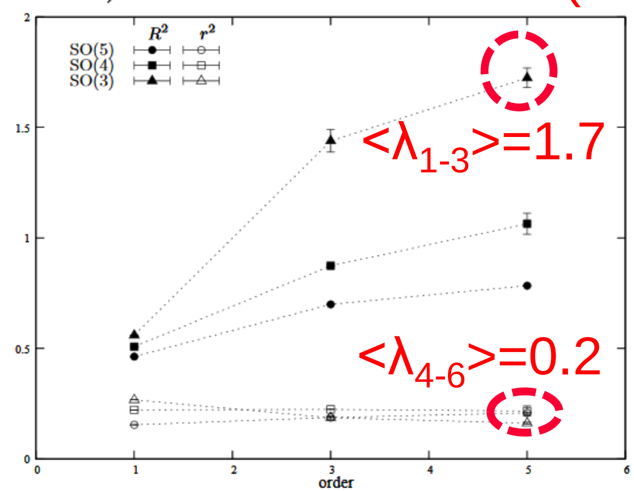
SSB **SO(6)→SO(3)** (In D=10, too, SO(10)→SO(3))
Dynamical compactification to 3-dim spacetime.

$$\lambda_n(\lambda_1 \geq \cdots \geq \lambda_D) : \text{eigenvalues of } T_{\mu\nu} = \frac{1}{N}\text{tr}(A_\mu A_\nu)$$

$$\rho_\mu = \frac{\langle\lambda_\mu\rangle}{\sum_{\nu=1}^{D}\langle\lambda_\nu\rangle} = \begin{cases} 0.30 & (\mu = 1,2,3) \\ 0.035 & (\mu = 4,5,6) \end{cases} \quad (D=6)$$

arXiv:1007.0883 (D=6)





2. Euclidean type IIB matrix model

$$Z = \int dA\, e^{-S_b} \underbrace{\left(\int d\psi\, e^{-S_f}\right)}_{=\det/\mathrm{Pf}\mathcal{M} = |\det/\mathrm{Pf}\mathcal{M}|e^{i\Gamma}} = \int dA \underbrace{e^{-S}}_{e^{-\{S_b - \log(\det/\mathrm{Pf}\mathcal{M})\}}}$$

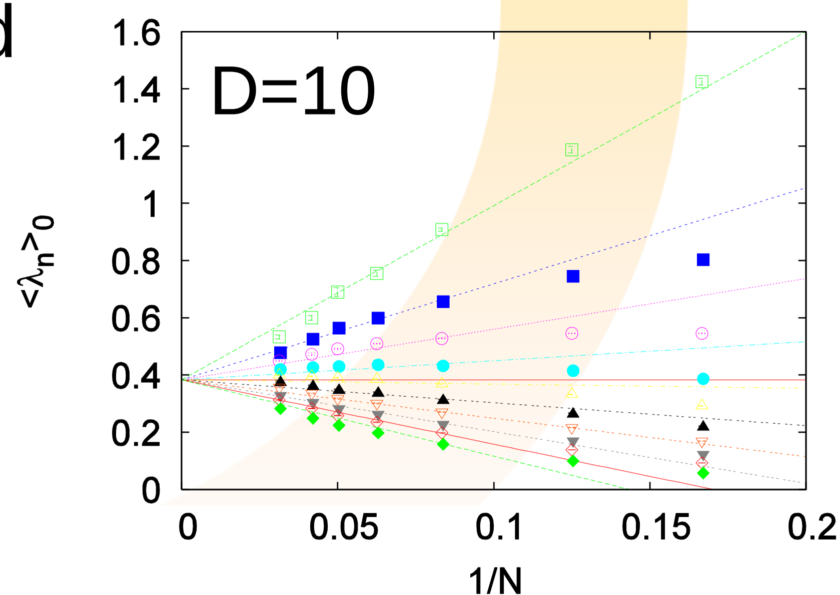
- Integrating out ψ yields $\det\mathcal{M}$ in D=6 ($\mathrm{Pf}\mathcal{M}$ in D=10)
- $\det/\mathrm{Pf}\,\mathcal{M}$'s *complex phase* contributes to the Spontaneous Symmetry Breaking (SSB) of SO(D).

No SSB with the phase-quenched partition function.

[J. Ambjorn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, hep-th/0003208,0005147, K.N. Anagnostopoulos, T. Azuma, J.Nishimura arXiv:1306.6135, 1509.05079]

$$Z_0 = \int dA\, e^{-S_0} = \int dA\, e^{-S_b} |\det/\mathrm{Pf}\mathcal{M}|$$

$\langle * \rangle_0$=V.E.V. for Z_0



3. Complex Langevin Method

Complex Langevin Method (CLM)

⇒Solve the complex version of the Langevin equation.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_\mu)_{ij}}{dt} = -\boxed{\frac{\partial S}{\partial (A_\mu)_{ji}}} + \eta_{\mu,ij}(t)$$

drift term

$$\boxed{\frac{\partial S}{\partial (A_\mu)_{ji}}} = \frac{\partial S_{\rm b}}{\partial (A_\mu)_{ji}} - c_d \mathrm{Tr}\left(\frac{\partial \mathcal{M}}{\partial (A_\mu)_{ji}} \mathcal{M}^{-1} \right) \qquad c_d = \begin{cases} 1 & (D=6 \to \det \mathcal{M}) \\ \frac{1}{2} & (D=10 \to \mathrm{Pf} \mathcal{M}) \end{cases}$$

- A_μ : Hermitian→general complex traceless matrices.
- η_μ: Hermitian-matrix white noise obeying the probability distribution $\exp\left(-\frac{1}{4}\int \mathrm{tr}\eta^2(t)dt\right)$

3. Complex Langevin Method

CLM does not work when it encounters these problems:

(1) Excursion problem: A_μ is too far from Hermitian

⇒Gauge Cooling minimizes the Hermitian norm

$$\mathcal{N} = \frac{-1}{DN} \sum_{\mu=1}^{D} \mathrm{tr}[(A_\mu - (A_\mu)^\dagger)^2]$$

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1604.07717]

A_μ : Hermitian→general complex traceless matrices.

⇒We make use of this extra symmetry:

After each step of discretized Langevin equation,

$$A_\mu \to g A_\mu g^{-1}, \;\; g = e^{\alpha H}, \;\; H = \frac{-1}{N} \sum_{\mu=1}^{D} [A_\mu, A_\mu^\dagger]$$

α: real parameter, such that $\mathcal{N}$ is minimized.

(2) Singular drift problem:

The drift term $dS/d(A_\mu)_{ji}$ diverges due to $\mathcal{M}$'s near-zero eigenvalues.

We trust CLM when the distribution p(u) of the drift norm

$$u = \sqrt{\frac{1}{DN^3} \sum_{\mu=1}^{D} \sum_{i,j=1}^{N} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2}$$

falls exponentially as $p(u) \propto e^{-au}$.

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

Look at the drift term ⇒ Get the drift of CLM!!

3. Complex Langevin Method

Mass deformation [Y. Ito and J. Nishimura, arXiv:1609.04501]

- SO(D) symmetry breaking term $\Delta S_{\mathrm{b}} = \frac{1}{2} N\varepsilon \sum_{\mu=1}^{D} m_{\mu} \mathrm{tr}(A_{\mu})^2$

Order parameters for SSB of SO(D): $\lambda_{\mu} = \mathrm{Re}\left\{ \frac{1}{N} \mathrm{tr}(A_{\mu})^2 \right\}$

- Fermionic mass term:

$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}\left(\bar{\psi}_{\alpha} \gamma_{\alpha\beta} \psi_{\beta} \right), \quad \gamma = \begin{cases} \Gamma_6 & (D=6) \\ i\Gamma_8 \Gamma_9^{\dagger} \Gamma_{10} & (D=10) \end{cases}$$

Avoids the singular eigenvalue distribution of $\mathscr{M}$.

This breaks SO(6)→SO(5) (SO(10)→SO(7))

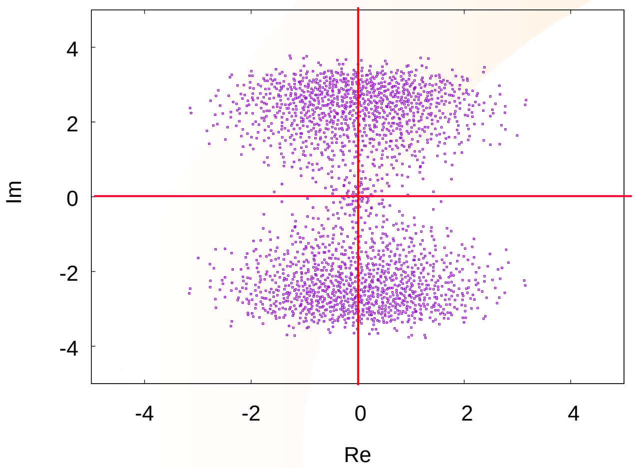
We study the SSB of the remaining symmetry.

Extrapolation (i) N→∞ ⇒(ii)ε→0 ⇒ (iii) m_{f}→0.

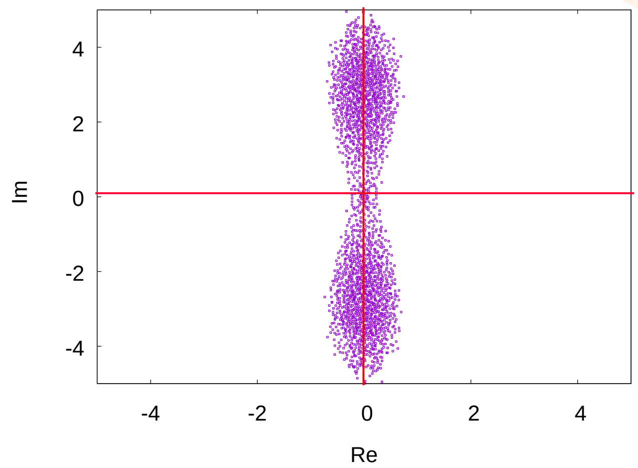
4. Result for D=6

The effect of adding these mass terms

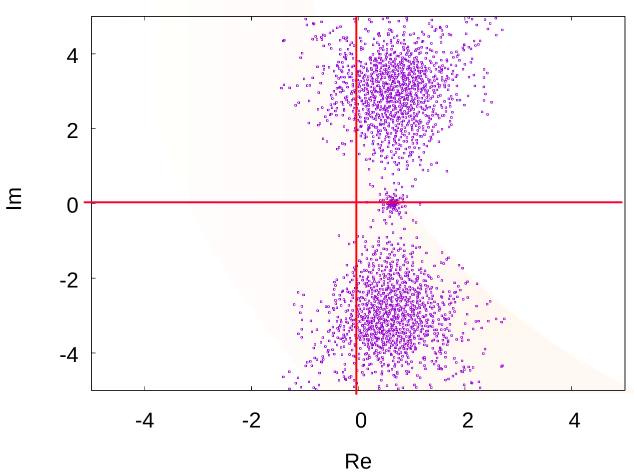
$(\varepsilon, m_f)=(0.00, 0.00)$



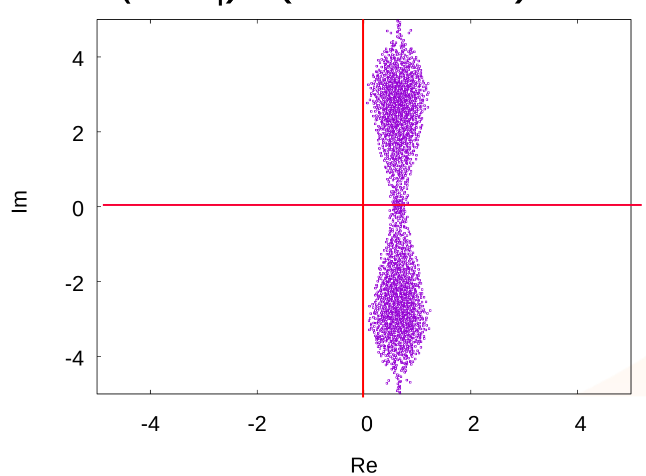
$(\varepsilon, m_f)=(0.25, 0.00)$



$(\varepsilon, m_f)=(0.00, 0.65)$



$(\varepsilon, m_f)=(0.25, 0.65)$



Scattering plots of the eigenvalues of the $4(N^2-1) \times 4(N^2-1)$ matrix $\mathcal{M}$ for D=6, N=24.

ΔS_b narrows the eigenvalue distribution.

ΔS_f shifts the eigenvalues, to evade the origin.

4. Result for D=6

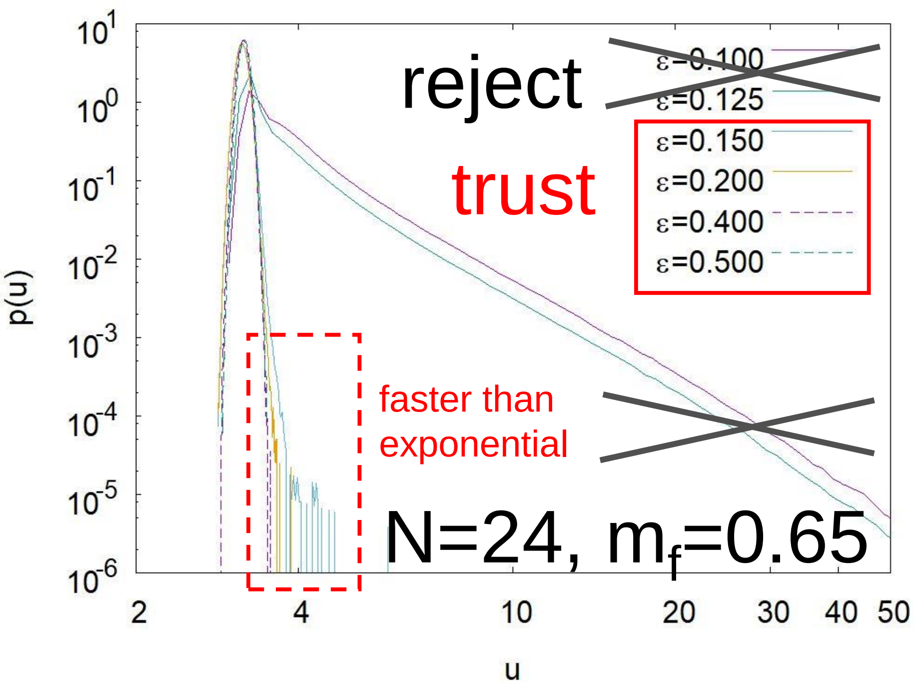
$$\Delta S_b = \frac{1}{2} N \varepsilon \sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2$$

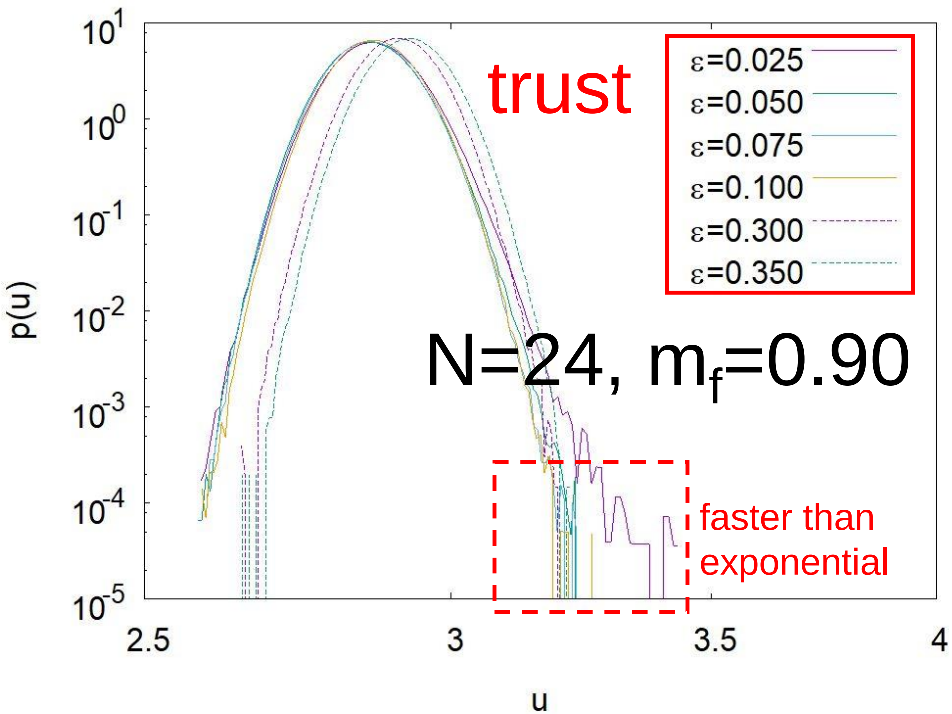
$$\Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_D)_{\alpha\beta} \psi_\beta) \quad (D=6)$$

m_μ=(0.5, 0.5, 1, 2, 4, 8)

$$u = \sqrt{\frac{1}{DN^3} \sum_{\mu=1}^{D} \sum_{i,j=1}^{N} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2}$$

's distribution p(u) (log-log)





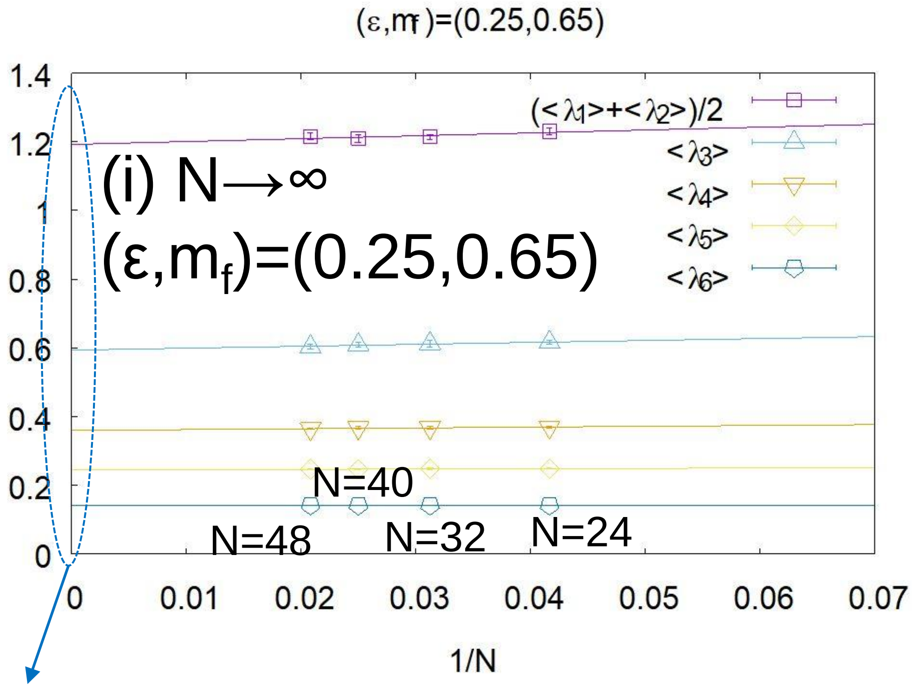
4. Result for D=6

SETSUDAI

$$\Delta S_b = \frac{1}{2} N \varepsilon \sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2 \qquad \Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_D)_{\alpha\beta} \psi_\beta) \ (D=6)$$

m_μ=(0.5, 0.5, 1, 2, 4, 8)

(i) N→∞ limit for fixed (ε,m_f)



$\langle \lambda_\mu \rangle_{\varepsilon, m_f}$ at large N

(ε,m_f)→(0,0) extrapolation for finite N

⇒We cannot observe SSB of SO(D).

4. Result for D=6

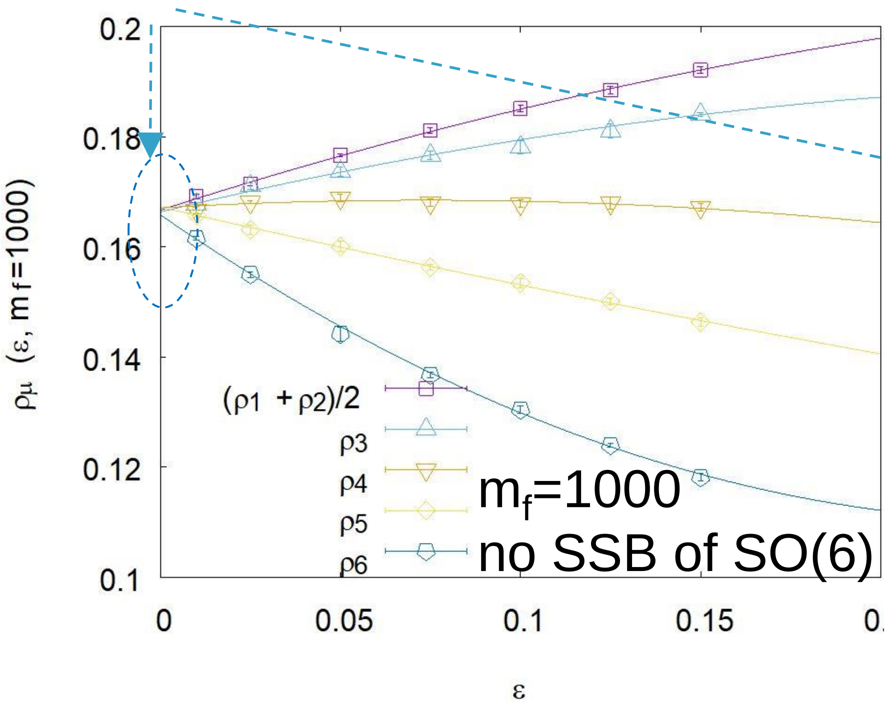
$$\Delta S_{\mathrm{b}} = \frac{1}{2} N \varepsilon \sum_{\mu=1}^{D} m_{\mu} \mathrm{tr}(A_{\mu})^2$$

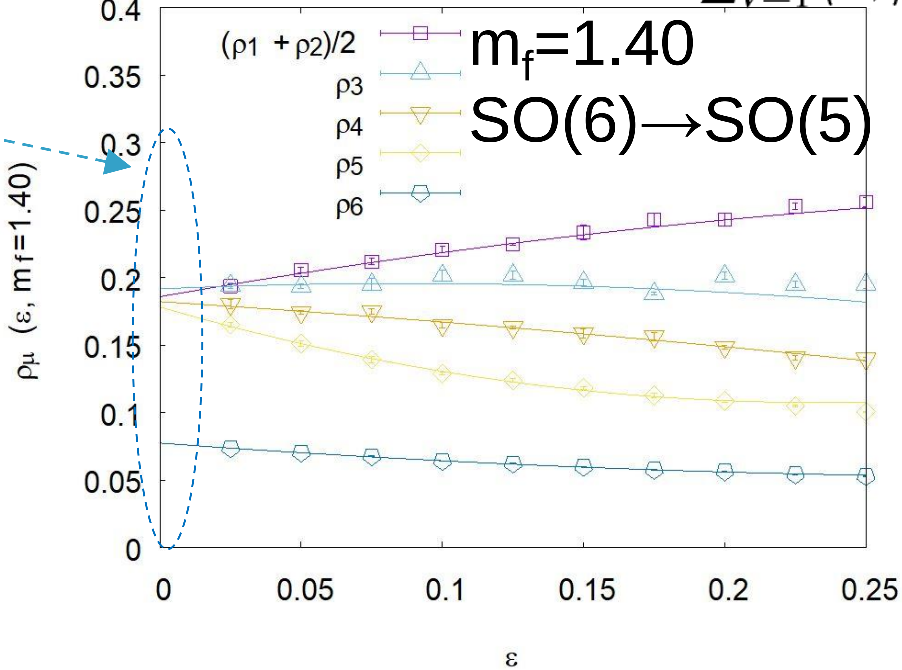
$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}(\bar{\psi}_{\alpha} (\Gamma_D)_{\alpha\beta} \psi_{\beta}) \ (D=6)$$

$m_{\mu}=(0.5, 0.5, 1, 2, 4, 8)$

(ii) $\varepsilon \to 0$ after $N \to \infty$

$$\rho_{\mu}(\varepsilon, m_{\mathrm{f}}) = \frac{\langle \lambda_{\mu} \rangle_{\varepsilon, m_{\mathrm{f}}}}{\sum_{\nu=1}^{D} \langle \lambda_{\nu} \rangle_{\varepsilon, m_{\mathrm{f}}}}$$





- $m_f \to \infty$: Ψ decouples from A_μ and reduces to the bosonic IKKT.
- The bosonic IKKT S_b does not break SO(D).

[T. Hotta, J. Nishimura and A. Tsuchiya, hep-th/9811220]

- The SSB of SO(D) is not an artifact of $\varepsilon \to 0$ but a physical effect.

4. Result for D=6

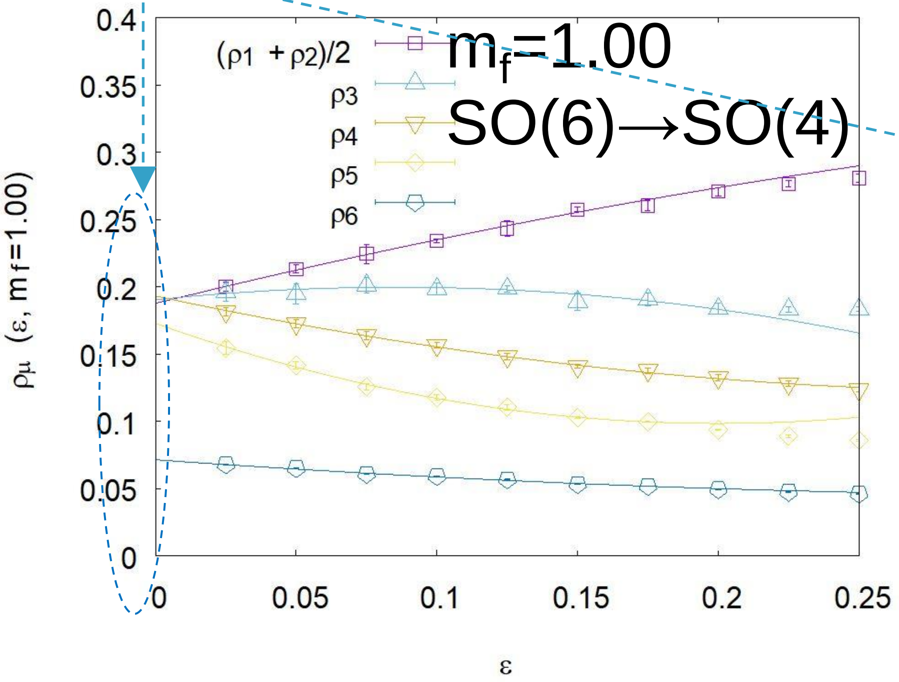
$$\Delta S_{\mathrm{b}} = \frac{1}{2} N \varepsilon \sum_{\mu=1}^{D} m_{\mu} \mathrm{tr}(A_{\mu})^2$$

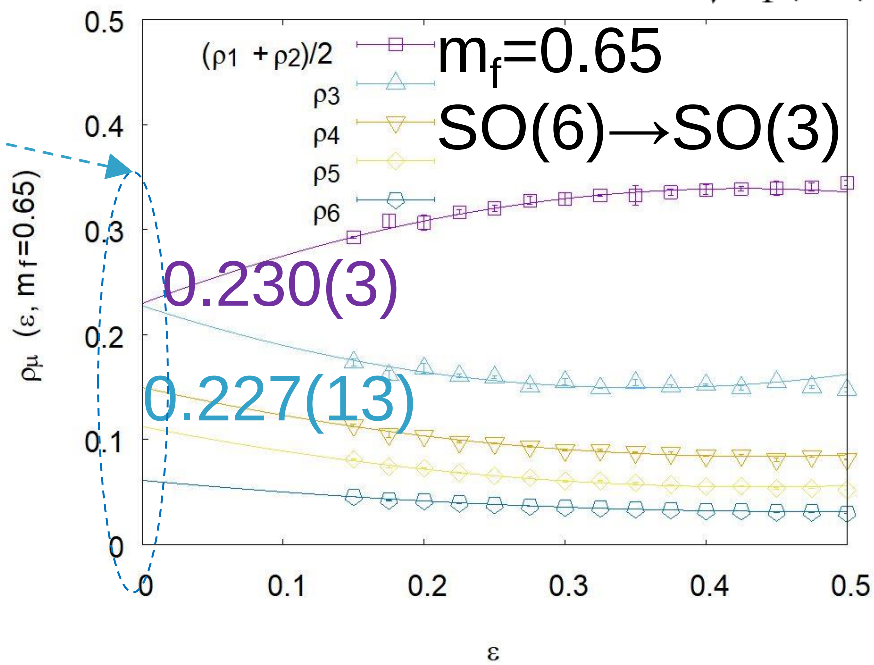
$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}(\bar{\psi}_{\alpha} (\Gamma_D)_{\alpha\beta} \psi_{\beta}) \quad (D=6)$$

m_{μ}=(0.5, 0.5, 1, 2, 4, 8)

$$\rho_{\mu}(\varepsilon, m_{\mathrm{f}}) = \frac{\langle \lambda_{\mu} \rangle_{\varepsilon, m_{\mathrm{f}}}}{\sum_{\nu=1}^{D} \langle \lambda_{\nu} \rangle_{\varepsilon, m_{\mathrm{f}}}}$$

(ii) ε→0 after N→∞





4. Result for D=6

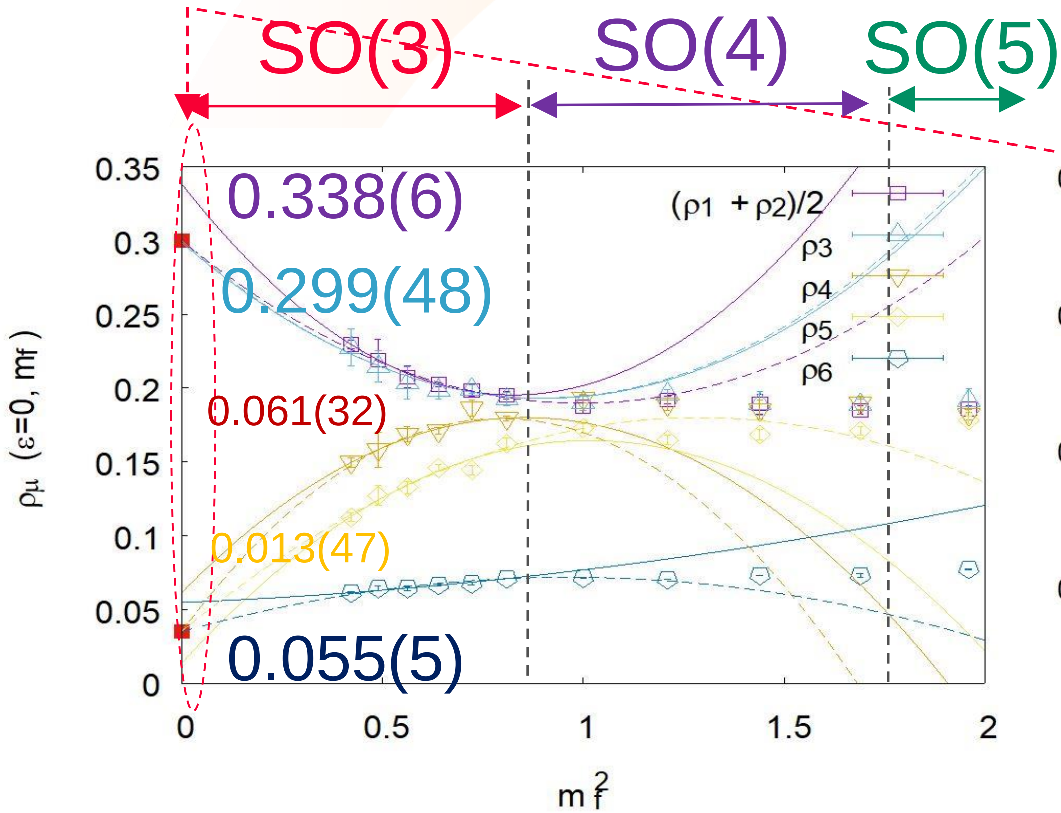
$$\Delta S_{\mathrm{b}} = \frac{1}{2} N \varepsilon \sum_{\mu=1}^{D} m_{\mu} \mathrm{tr}(A_{\mu})^2$$

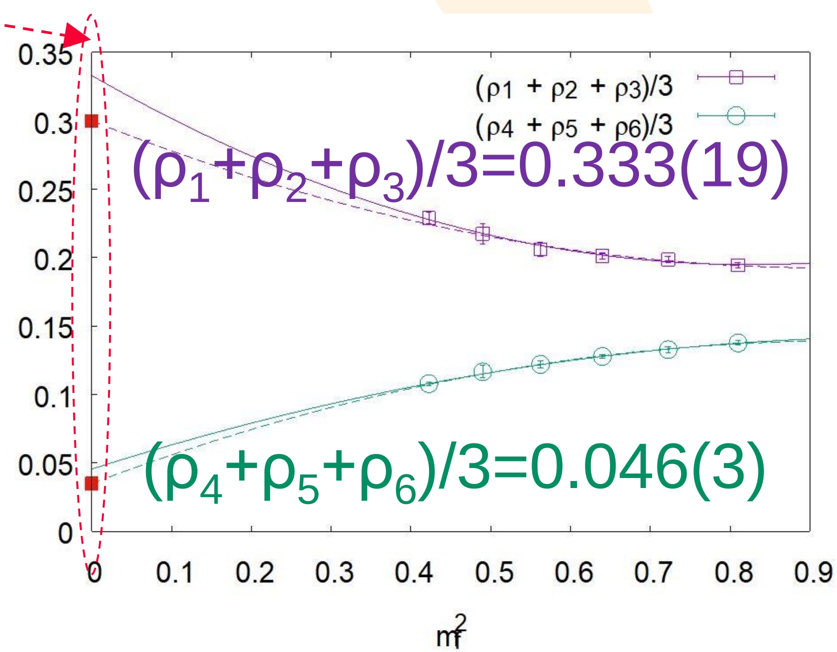
$$\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}(\bar{\psi}_{\alpha} (\Gamma_D)_{\alpha\beta} \psi_{\beta}) \quad (D=6)$$

m_μ=(0.5, 0.5, 1, 2, 4, 8)

(iii) $m_f \to 0$ after $\varepsilon \to 0$

$$\rho_\mu = \frac{\langle \lambda_\mu \rangle}{\sum_{\nu=1}^{D} \langle \lambda_\nu \rangle} = \begin{cases} 0.30 & (\mu = 1,2,3) \\ 0.035 & (\mu = 4,5,6) \end{cases} \quad (D=6)$$





(dotted line: $m_f \to 0$ limit fixed to GEM results)

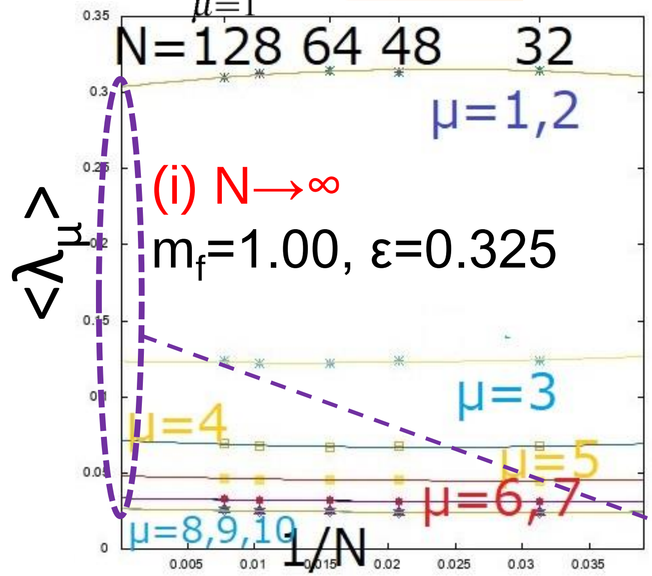
SSB SO(6) → at most SO(3)   Consistent with GEM.

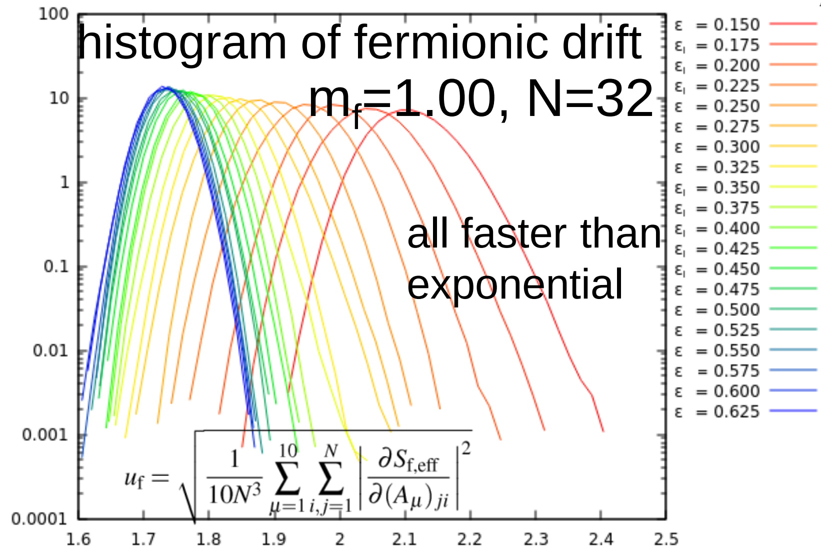
5. Result for D=10

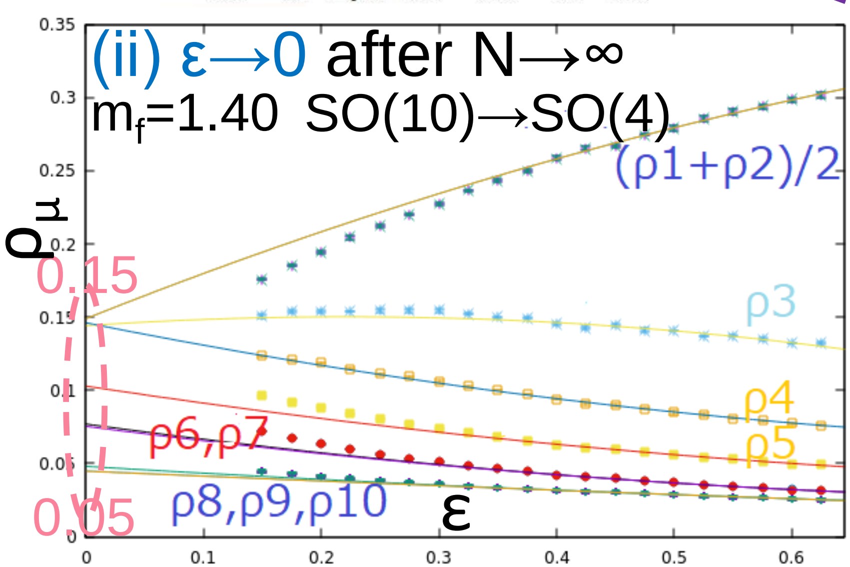
m_μ=(0.5,0.5,1,2,4,8,8,8,8,8)

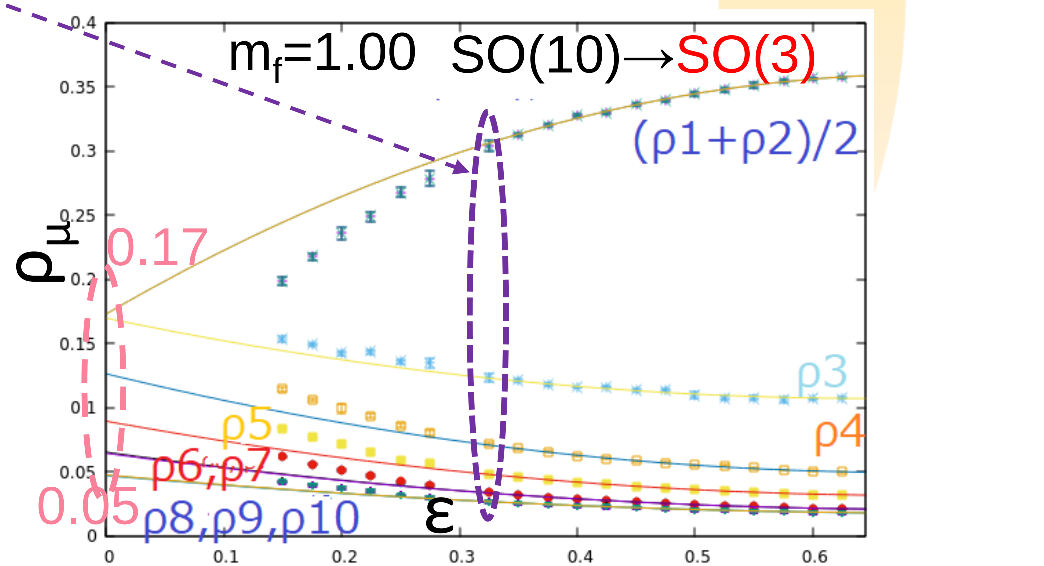
SETSUDAI

$$\Delta S_{\rm b} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu {\rm tr}(A_\mu)^2 \qquad \Delta S_{\rm f} = N m_{\rm f} {\rm tr}\left(\bar{\psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\psi_\beta\right) \qquad \rho_\mu(\varepsilon, m_{\rm f}) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\rm f}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\rm f}}}$$









5. Result for D=10

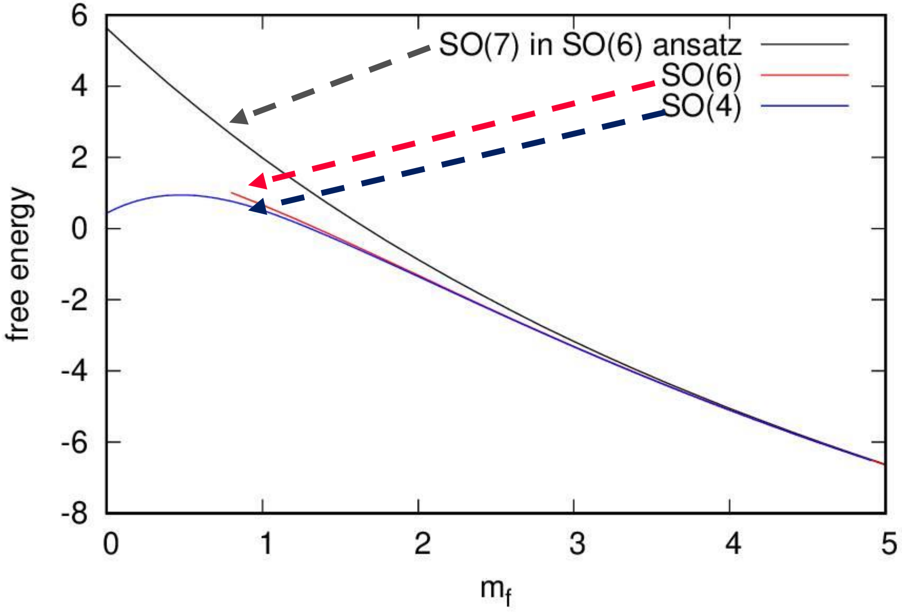
$m_\mu=(0.5,0.5,1,2,4,8,8,8,8,8)$

$$\Delta S_{\rm f} = N m_{\rm f} {\rm tr}\left(\bar{\psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\psi_\beta\right)$$

(no $\Delta S_{\rm b} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu {\rm tr}(A_\mu)^2$ term)

$$\rho_\mu(\varepsilon, m_{\rm f}) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\rm f}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\rm f}}}$$

GEM result for $m_f>0$ at 3 loop solutions of SO(4) and SO(6) ansatz



Favors lower symmetry at smaller m_f.

6. Summary

Dynamical compactification of the spacetime in the Euclidean type IIB matrix model.

"Complex Langevin Method" ⇒trend of SO(D)→SO(3).

Future works

Application of CLM to other cases

Lorentzian version of the type IIB matrix model

generalization to Gross-Witten-Wadia model

$$S_{\mathrm{g}} = N(a\mathrm{tr}U + b\mathrm{tr}U^{\dagger})$$

[P. Basu, K. Jaswin and A. Joseph, arXiv:1802.10381]

supersymmetric quantum mechanics

[A. Joseph and A. Kumar, arXiv:1908.04153]

backup: example of CLM

Example [G. Aarts, arXiv:1512.05145]

$$S(x) = \frac{1}{2}\underbrace{(a+ib)}_{=\sigma}x^2, \ (a, b \in \mathbf{R}, \ a > 0)$$

S(x) is complex for real x.
Complexify to z=x+iy.

$$S(z) = \frac{1}{2}\sigma z^2 = \frac{1}{2}(a+ib)\overbrace{(x+iy)^2}^{=z^2} = \frac{a(x^2-y^2)}{2} + ibxy, \quad \frac{\partial S}{\partial z} = \sigma z = (a+ib)(x+iy)$$

Complex Langevin equation for this action

$$\dot{x}(t) = -\mathrm{Re}\left(\frac{\partial S}{\partial z}\right) + \eta(t) = (-ax+by) + \eta(t) \qquad \dot{y}(t) = -\mathrm{Im}\left(\frac{\partial S}{\partial z}\right) = (-ay-bx)$$

The real white noise satisfies

$$\langle \eta(t_1)\eta(t_2)\rangle = 2\delta(t_1 - t_2) \qquad \langle \cdots \rangle = \frac{\int \mathscr{D}\eta \cdots \exp\left(-\frac{1}{4}\int \eta^2(t)dt\right)}{\int \mathscr{D}\eta \exp\left(-\frac{1}{4}\int \eta^2(t)dt\right)}$$

backup: example of CLM

Solution of the Langevin equation

$$x(t) = e^{-at}\underbrace{[x(0)\cos bt + y(0)\sin bt]}_{=A(t)} + \int_0^t \eta(s) e^{-a(t-s)}\cos[b(t-s)]ds$$

$$y(t) = e^{-at}[y(0)\cos bt - x(0)\sin bt] - \int_0^t \eta(s) e^{-a(t-s)}\sin[b(t-s)]ds$$

$$\begin{aligned}\langle x^2\rangle &= \lim_{t\to+\infty}\langle x^2(t)\rangle = \lim_{t\to+\infty}\left\{\underbrace{e^{-2at}A(t)^2}_{\to 0} + 2e^{-at}A(t)\int_0^t \underbrace{\langle\eta(s)\rangle}_{=0} e^{-a(t-s)}\cos[b(t-s)]ds\right.\\ &\left. + \int_0^t\int_0^t \underbrace{\langle\eta(s)\eta(s')\rangle}_{=2\delta(s-s')} e^{-a(2t-s-s')}\cos[b(t-s)]\cos[b(t-s')]dsds'\right\}\\ &= \lim_{t\to+\infty}\left\{2\int_0^t e^{-2a(t-s)}\cos^2[b(t-s)]\right\}ds = \frac{2a^2+b^2}{2a(a^2+b^2)}\end{aligned}$$

Similarly, $\langle y^2\rangle = \frac{b^2}{2a(a^2+b^2)},\ \langle xy\rangle = \frac{-b}{2(a^2+b^2)}$

This replicates $\langle z^2\rangle = \langle x^2\rangle - \langle y^2\rangle + 2i\langle xy\rangle = \frac{a-ib}{a^2+b^2} = \frac{1}{\sigma}$

backup: example of CLM

Fokker-Planck equation

$$\frac{\partial P}{\partial t} = L^\top P \quad \text{where} \quad L^\top = \frac{\partial}{\partial x}\left\{\underbrace{\mathrm{Re}\left(\frac{\partial S}{\partial z}\right)}_{=ax-by} + \frac{\partial}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\underbrace{\mathrm{Im}\left(\frac{\partial S}{\partial z}\right)}_{=ay+bx}\right\}$$

Ansatz for its static solution:

$$P(x,y) = N\exp\left(-\alpha x^2 - \beta y^2 - 2\gamma xy\right) = N\exp\left(-\beta\left(y+\frac{\gamma x}{\beta}\right)^2 - \overbrace{\left(\alpha - \frac{\gamma^2}{\beta}\right)}^{=a(a^2+b^2)/(2a^2+b^2)} x^2\right)$$

$$0 = \partial_t P = L^\top P = [\underbrace{(2a-2\alpha)}_{=0\to a=\alpha} + x^2\underbrace{(4\alpha^2 - 2a\alpha - 2b\gamma)}_{=0\to\gamma=a^2/b} + y^2\underbrace{(4\gamma^2 + 2b\gamma - 2a\beta)}_{=0\to\beta=a(1+2a^2/b^2)} + xy\underbrace{(4(2\alpha-a)\gamma + 2b(\alpha-\beta))}_{=0}]P$$

Using $\frac{\int_{-\infty}^{+\infty} t^2 e^{-At^2}\,dt}{\int_{-\infty}^{+\infty} e^{-At^2}\,dt} = \frac{1}{2A} \quad (A>0)$ we have

$$\langle x^2\rangle = \frac{\iint x^2 P(x,y)\,dxdy}{\iint P(x,y)\,dxdy} = \frac{1}{2} \div \frac{a(a^2+b^2)}{2a^2+b^2} = \frac{2a^2+b^2}{2a(a^2+b^2)}$$