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Complex Langevin studies of the spacetime structure in the Lorentzian type IIB matrix model

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1. Introduction

Type IIB matrix model (a.k.a. IKKT model)
⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_\mathrm{b}} + \underbrace{\frac{-1}{2g^2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_\mathrm{f}}$$

- Dimensional reduction of the D=10 super-Yang-Mills theory to 0 dimension
- A_μ, Ψ_α ⇒N×N Hermitian traceless matrices.
- N=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinate.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

⇒contracted by the Lorentzian metric $\eta = \mathrm{diag}(-1, 1, 1, \cdots, 1)$

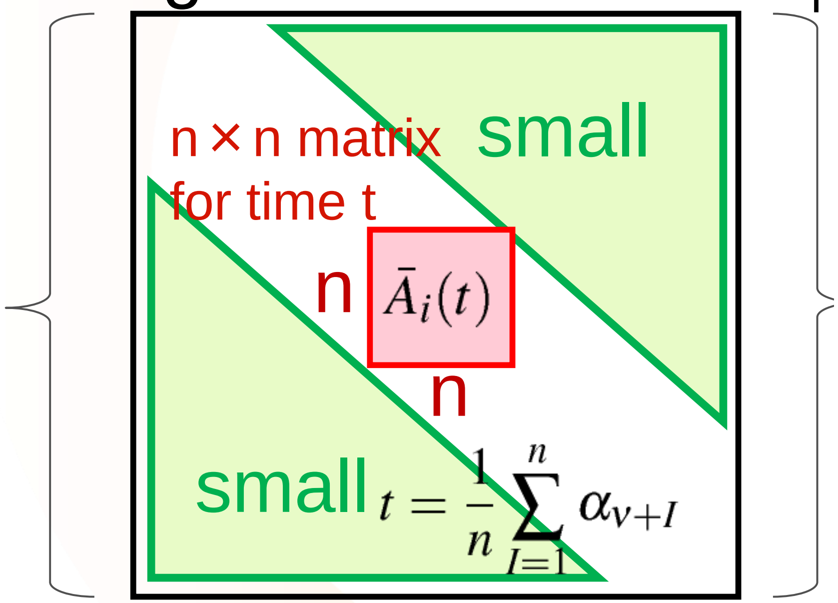
Time development: gauge fixing to diagonalize A_0

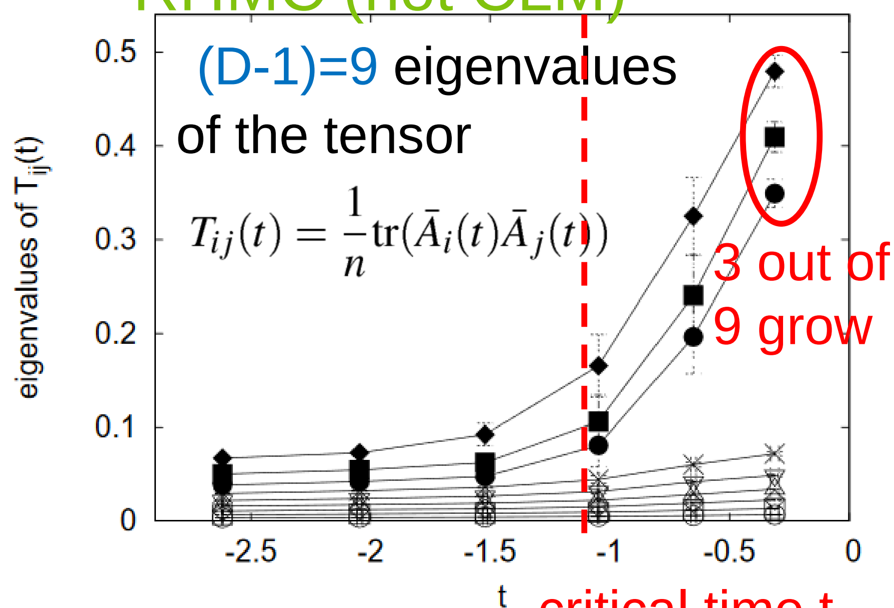
$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N), \text{ where } \alpha_1 < \alpha_2 < \cdots < \alpha_N.$$

Band-diagonal structure of A_i

D=10, N=16, n=4, RHMC (not CLM)

$A_i =$





Dynamical emergence of (3+1)-dim spacetime.

2. Lorentzian type IIB matrix model

In the following, we study the D=10 bosonic model:

Similar emergence of the spacetime for the bosonic/supersymmetric model. [Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795]

Difficulties in putting the Lorentzian version on a computer:

1. The action is not bounded below

bosonic part: $S_\mathrm{b} = \frac{N\beta}{4}\mathrm{tr}\left\{2\sum_{i=1}^{D-1}[A_0,A_i]^2 - \sum_{i,j=1}^{D-1}[A_i,A_j]^2\right\} \quad \left(\frac{1}{g^2N} = \beta\right)$

⇒infrared cutoff: $\frac{1}{N}\mathrm{tr}(A_0)^2 = \kappa \quad \frac{1}{N}\mathrm{tr}(A_i)^2 = 1$

2. Sign problem

$$Z = \int dA \left(\underbrace{e^{iS_\mathrm{b}}}_{\text{complex}} \underbrace{\int d\psi e^{iS_\mathrm{f}}}_{=\mathrm{Pf}\mathcal{M}\ \text{real}} \right)$$

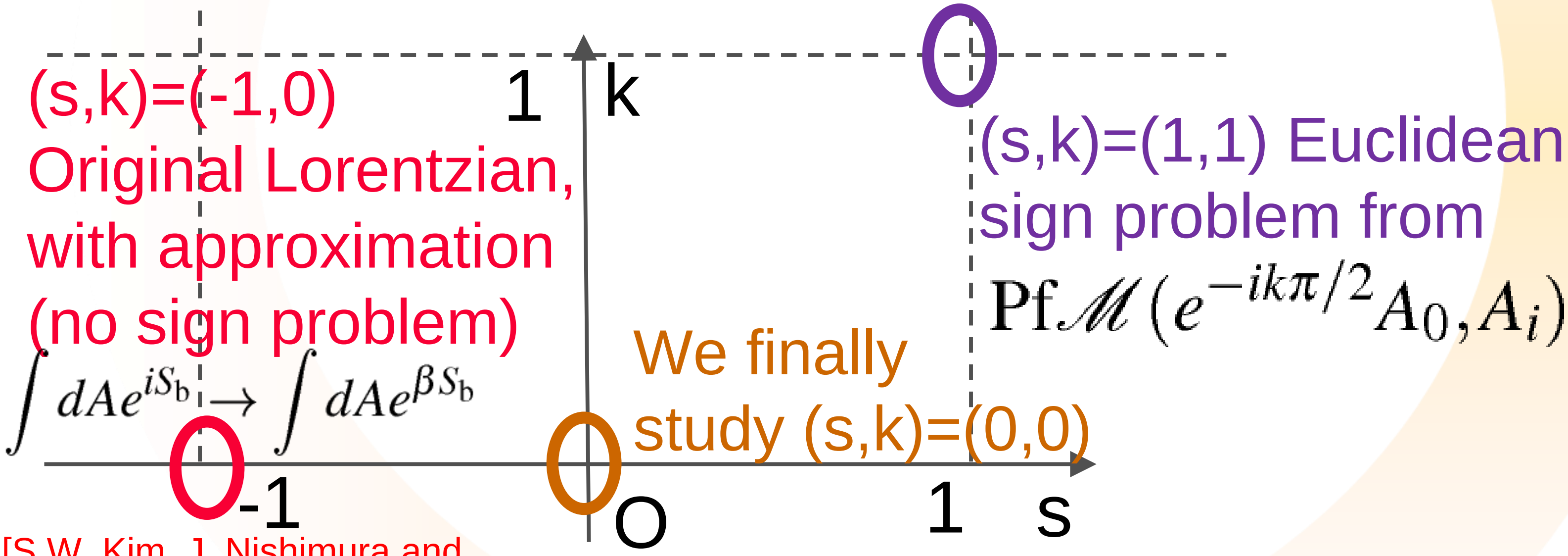
[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

⇒We employ the Complex Langevin Method (CLM)

2. Sign problem

Parameters of Wick rotation: [J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

- multiply overall $e^{is\pi/2}$ (w.r.t. world sheet)
- $A_0 \rightarrow A_0 e^{-ik\pi/2}$ (w.r.t. target space)



[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

2. Lorentzian type IIB matrix model

SETSUDAI

Paulian-matrix structure of space at (s,k)=(-1,0)

[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

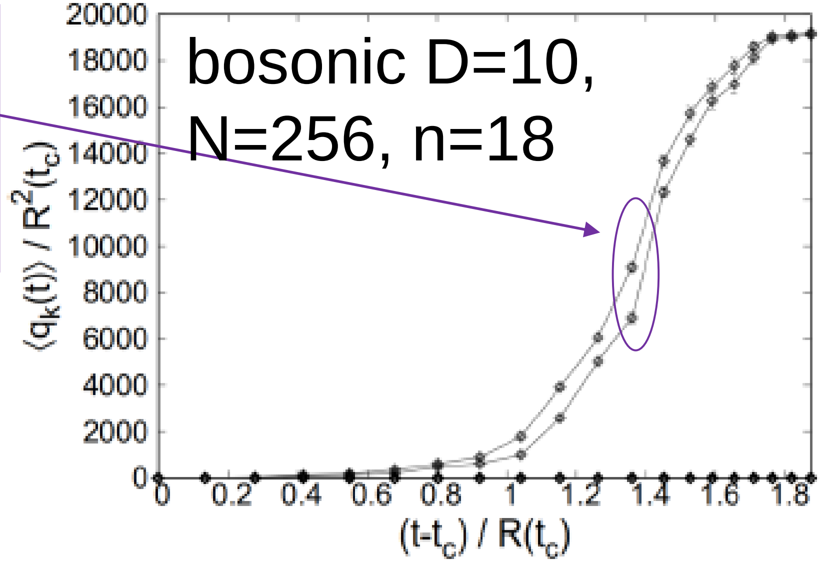
$$\bar{A}_i(t) \propto \sigma_i \oplus 0_{n-2} (i = 1, 2, 3)$$

2 of the n eigenvalues of

$$Q(t) = \sum_{i=1}^{D-1} (\bar{A}_i(t))^2$$

grow.

⇒sphere whose inside is empty.



3. Complex Langevin Method

Complex Langevin Method (CLM)

⇒Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_i)_{ab}(t_l)}{dt_l} = -\frac{dS_{\mathrm{eff}}}{d(A_i)_{ba}} + \eta_{i,ab}(t_l) \qquad \frac{d\tau_a(t_l)}{dt_l} = -\frac{dS_{\mathrm{eff}}}{d\tau_a} + \eta_a(t_l)$$

drift term; Hermitian-matrix white noise; drift term; real-number white noise; t_l: fictitious Langevin time

- A_i : Hermitian→general complex traceless matrices.
- τ_a : Real number → complex number.

Introducing time order $\alpha_1<\alpha_2<\ldots<\alpha_N$ for complexified α_i

$$\alpha_1 = 0, \ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i} \ (k = 2, 3, \cdots, N)$$

4. Result

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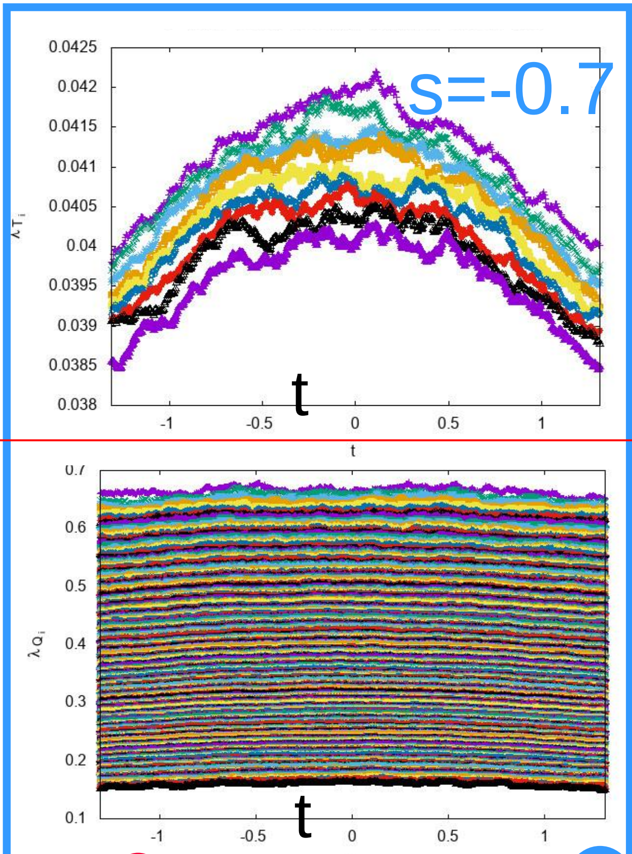
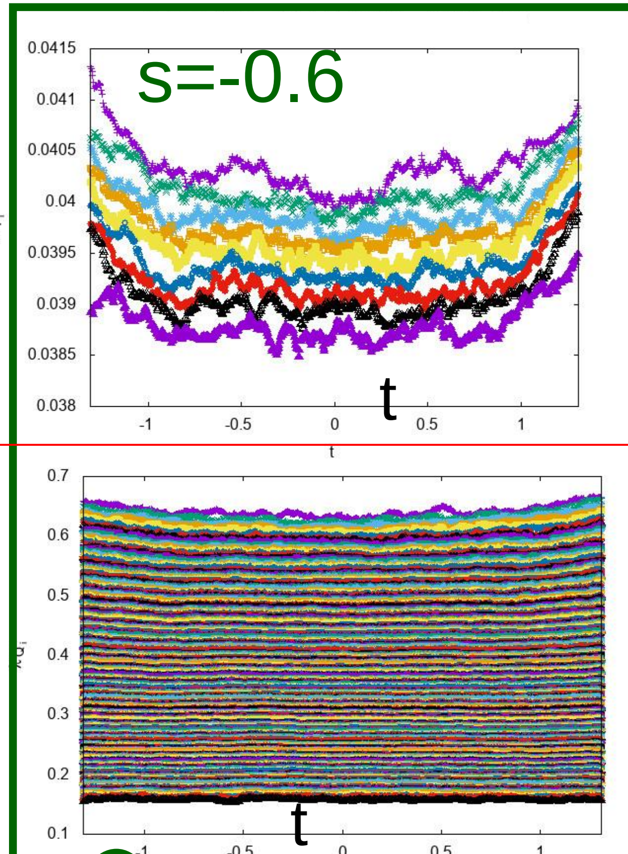
New phase of continuous space at $-1<s\leqq 0$.

bosonic, D=10, N=1024, n=256, k=0, (β,κ)=(2.5,1).

$$\bar{H}_i(t)=\frac{1}{2}(\bar{A}_i(t)+\bar{A}_i^\dagger(t))$$: Hermitian part





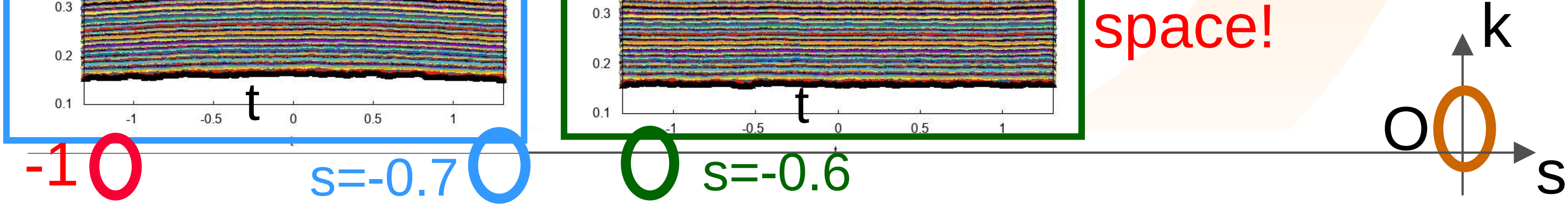
(D-1) eig. of

$$T_{ij}(t)=\frac{1}{n}\mathrm{tr}(\bar{H}_i(t)\bar{H}_j(t))$$

No spontaneous symmetry breaking (SSB) of SO(9) so far.

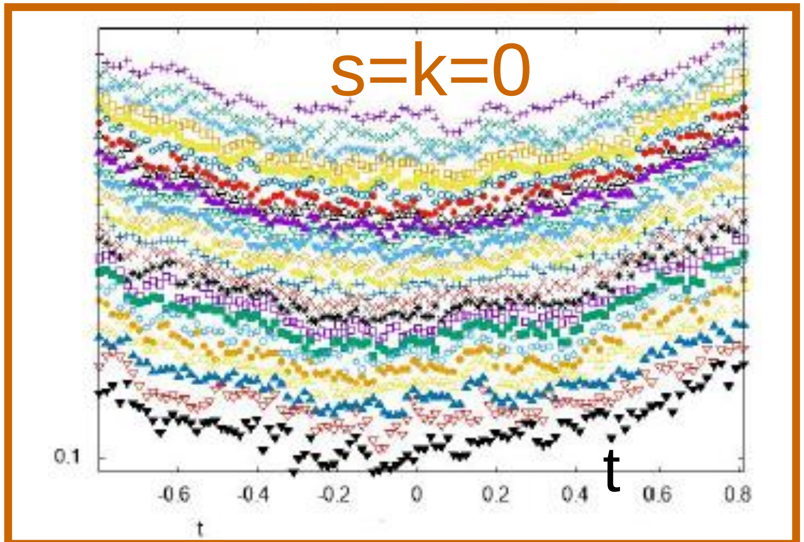
n eig. of $Q(t)=\sum_{i=1}^{D-1}(\bar{H}_i(t))^2$

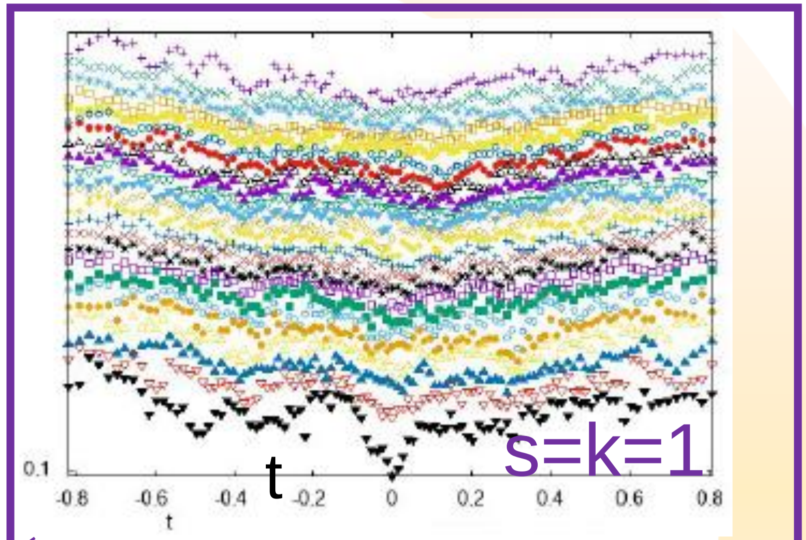
continuous space!

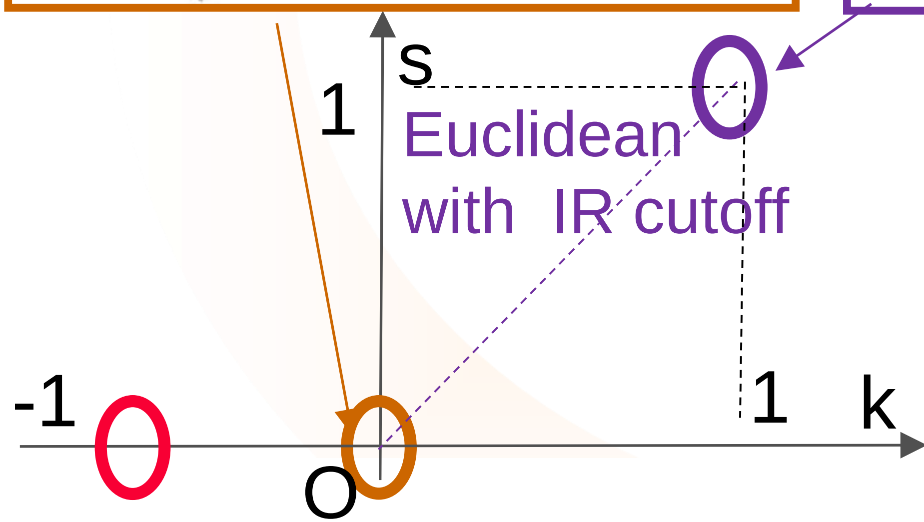


4. Result

The new phase is smoothly connected
from Lorentzian s=k=0 to Euclidean s=k=1.







n eig. of $Q(t) = \sum_{i=1}^{D-1} (\bar{H}_i(t))^2$

bosonic, D=10, N=128,
n=24, (β,κ)=(5.29, 0.32).

5. Conclusion

Complex Langevin Method (CLM) for the Lorentzian type IIB matrix model.

⇒We discovered a new phase of the continuous space.

- no SSB of SO(9) so far.
- smoothly connected to the Euclidean version.

Search for the continuous space with SSB SO(9)→SO(3)

- appropriate parameters (β,κ)
- simulation at larger N
- effect of the fermion

$$S_b = \frac{N\beta}{4}\mathrm{tr}\left\{2\sum_{i=1}^{D-1}[A_0,A_i]^2 - \sum_{i,j=1}^{D-1}[A_i,A_j]^2\right\} \quad \left(\frac{1}{g^2N}=\beta\right)$$

$$\frac{1}{N}\mathrm{tr}(A_0)^2 = \kappa \qquad \frac{1}{N}\mathrm{tr}(A_i)^2 = 1$$