Complex Langevin studies of the signature change of spacetime in the type IIB matrix model

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1. Introduction

Type IIB matrix model (a.k.a. IKKT model)
⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \frac{-1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2 + \frac{-1}{2g^2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]$$

$$\underbrace{\qquad\qquad\qquad}_{=S_{\mathrm{b}}} \quad \underbrace{\qquad\qquad\qquad\qquad}_{=S_{\mathrm{f}}}$$

・Dimensional reduction of the D=10 super-Yang-Mills theory to 0 dimension

・A_μ, Ψ_α ⇒N×N Hermitian traceless matrices.

・N=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinate.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model
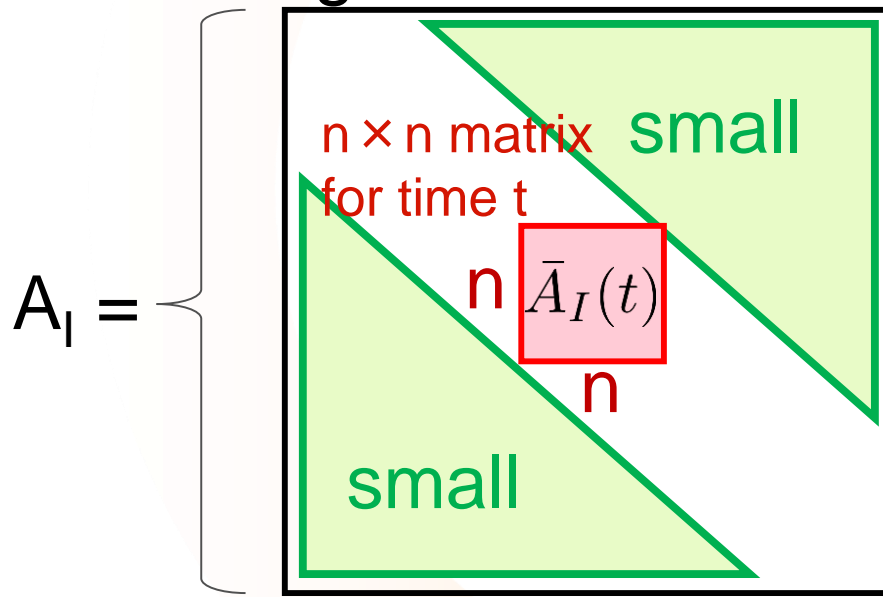
Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

⇒contracted by the Lorentzian metric $\eta = \text{diag}(-1,1,1,\cdots,1)$

Time development: gauge fixing to diagonalize A_0

$$A_0 = \text{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N), \text{ where } \alpha_1 < \alpha_2 < \cdots < \alpha_N.$$

Band-diagonal structure of A_I

$A_I =$

n × n matrix for time t

small

n $\bar{A}_I(t)$

n

small

$$t_1 = 0, \quad t_\nu = \sum_{k=1}^{\nu-1} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

where $\bar{\alpha}_k = \dfrac{1}{n} \sum_{i=1}^{n} \alpha_{i+k}$

2. Lorentzian type IIB matrix model Sign problem of the Lorentzian version

$$Z = \int dA \left(\boxed{e^{iS_{\mathrm{b}}}} \underbrace{\int d\psi\, e^{iS_{\mathrm{f}}}}_{= \mathrm{Pf}\,\mathcal{M}} \right)$$

complex   real

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

・We employ the Complex Langevin Method (CLM)

・We introduce parameters of Wick rotation:

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

・multiply overall $e^{is\pi/2}$ (w.r.t. world sheet)

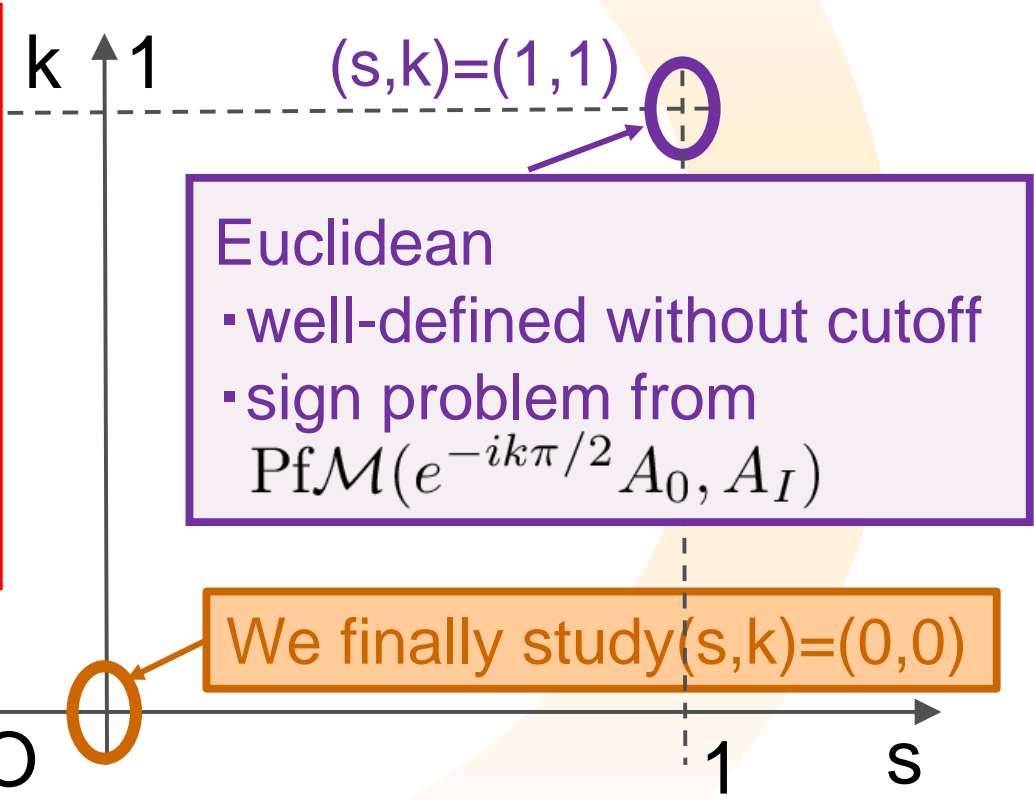・ $A_0 \rightarrow A_0 e^{-ik\pi/2}$ (w.r.t. target space)

- multiply overall $e^{is\pi/2}$ (w.r.t. world sheet)
- $A_0 \rightarrow A_0 e^{-ik\pi/2}$ (w.r.t. target space)

Original Lorentzian

[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

- approximation to avoid sign problem

$$\int dA e^{iS_b} \rightarrow \int dA e^{\beta S_b}$$

- IR cutoff $\frac{1}{N}\text{tr}(A_0)^2 = \kappa$ $\frac{1}{N}\text{tr}(A_I)^2 = 1$

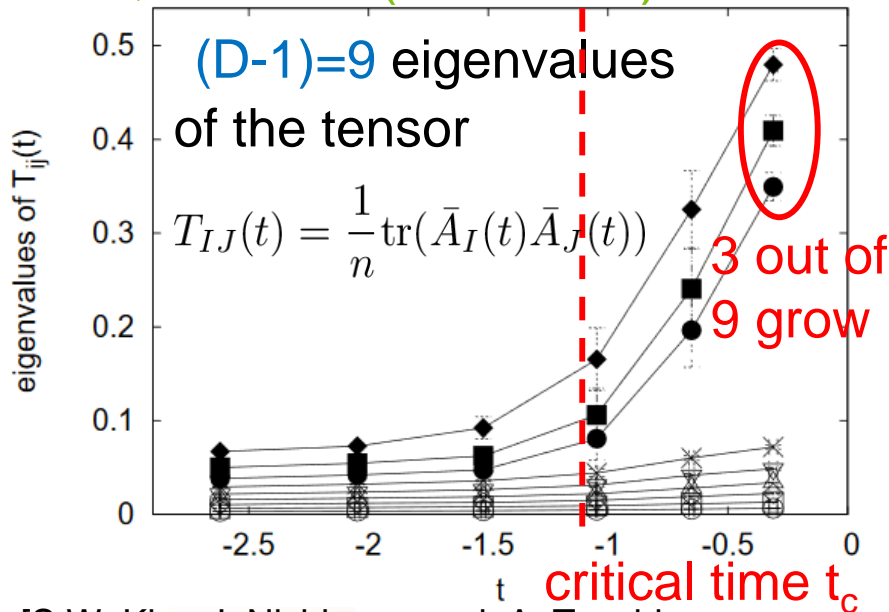$k \uparrow 1$ $(s,k)=(1,1)$

Euclidean
- well-defined without cutoff
- sign problem from $\text{Pf}\mathcal{M}(e^{-ik\pi/2}A_0, A_I)$

$(s,k)=(-1,0)$
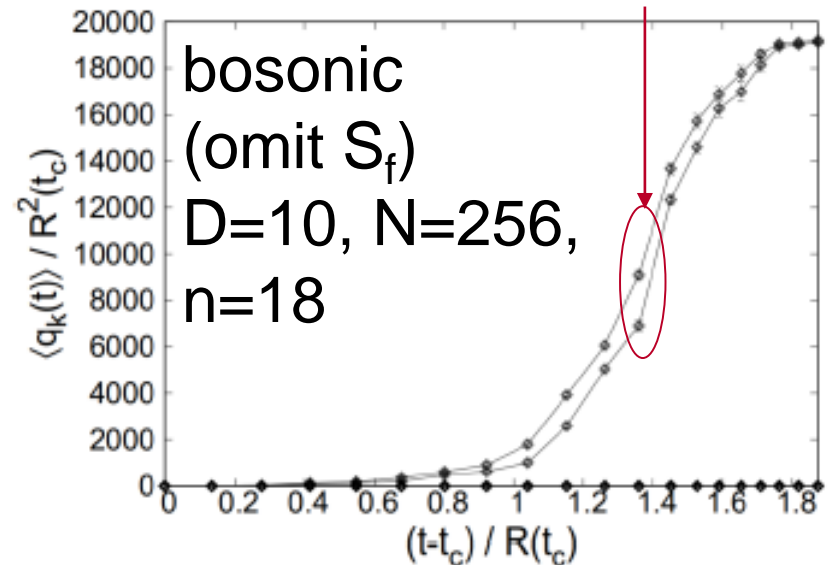
We finally study $(s,k)=(0,0)$

-1

O 1 s

Results at (s,k)=(-1,0), with the approximation to avoid the sign problem:

SUSY (include S_f) D=10, N=16, n=4, RHMC (not CLM)

2 of the n eigenvalues of

$$Q(t) = \sum_{I=1}^{D-1} (\bar{A}_I(t))^2 \quad \text{grow.}$$

(D-1)=9 eigenvalues of the tensor

$$T_{IJ}(t) = \frac{1}{n} \text{tr}(\bar{A}_I(t)\bar{A}_J(t))$$

3 out of 9 grow

critical time t_c

[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

bosonic (omit S_f) D=10, N=256, n=18

[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

Dynamical emergence of (3+1)-dim spacetime.

$$\bar{A}_I(t) \propto \sigma_I \oplus 0_{n-2} \quad (I = 1, 2, 3)$$

Pauli-matrix space structure ⇒hollow 3-dim sphere

Complex Langevin Method (CLM)

⇒Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

drift term Hermitian-matrix white noise drift term real-number white noise

$$\frac{d(A_i)_{ab}(t_l)}{dt_l} = -\frac{dS_{\text{eff}}}{d(A_i)_{ba}} + \eta_{i,ab}(t_l) \qquad \frac{d\tau_a(t_l)}{dt_l} = -\frac{dS_{\text{eff}}}{d\tau_a} + \eta_a(t_l)$$

fictitious Langevin time   fictitious Langevin time

· A_i : Hermitian→general complex traceless matrices.

· τ_a : Real number → complex number.

Introducing time order $\alpha_1 < \alpha_2 < \dots < \alpha_N$ for complexified α_i

$$\alpha_1 = 0, \ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i} \ (k=2,3,\cdots,N) \quad t_1 = 0, \ t_\nu = \sum_{k=1}^{\nu-1} |\bar\alpha_{k+1} - \bar\alpha_k| \quad \bar\alpha_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{i+k}$$
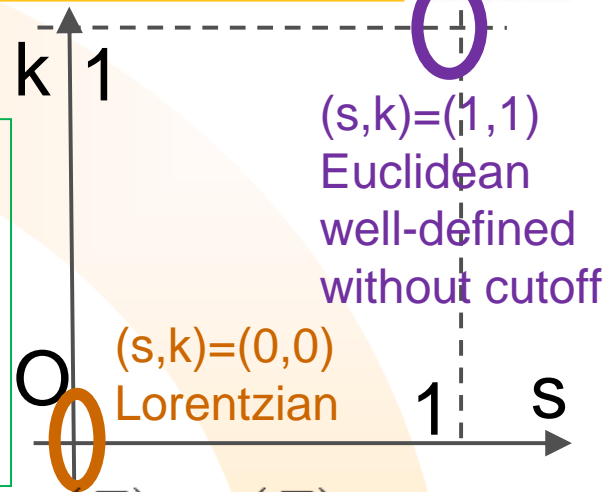
Bosonic action after the Wick rotation

k 1

$$A_0 \to A_0 e^{-ik\pi/2}$$ multiply overall $e^{is\pi/2}$

$\Downarrow$

$$Z = \int dA e^{-\tilde{S}_{b(s,k)}(A_0, A_I)}$$, where

$$\tilde{S}_b = N\beta \mathrm{tr}\left(\frac{-1}{2} e^{i(1+s-2k)\pi/2} \sum_{I=1}^{D-1}[A_0, A_I]^2 - \frac{1}{4} e^{i(s-1)\pi/2} \sum_{I,J=1}^{D-1}[A_I, A_J]^2 \right)$$
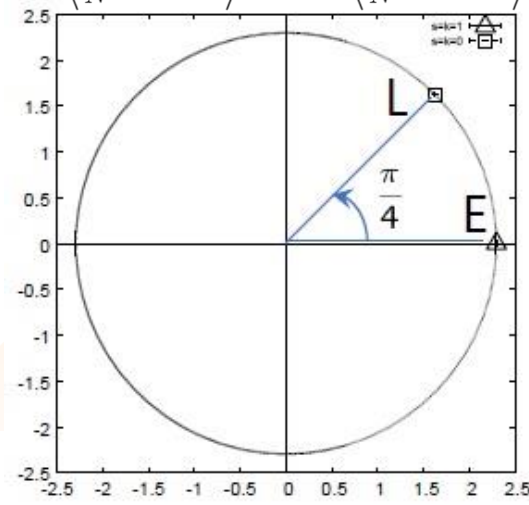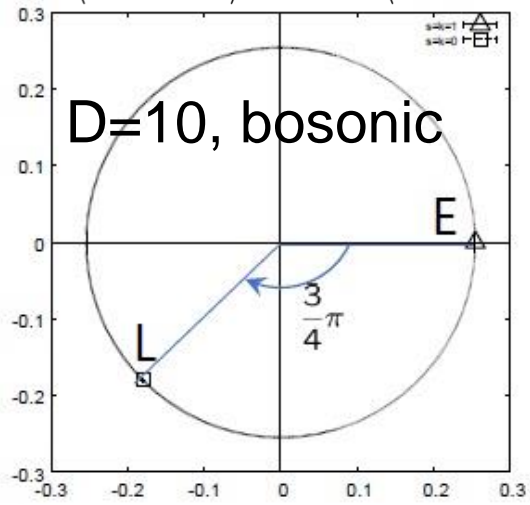
(s,k)=(1,1)
Euclidean
well-defined
without cutoff

(s,k)=(0,0)
Lorentzian

O   1   s

$$\tilde{S}_{b\,(0,0)}\big(\underbrace{A_0^{(E)} e^{-3i\pi/8}}_{=A_0^{(L)}}, \underbrace{A_I^{(E)} e^{i\pi/8}}_{=A_I^{(L)}}\big) = \tilde{S}_{b\,(1,1)}\big(A_0^{(E)}, A_I^{(E)}\big)$$

Lorentzian   Euclidean

$$\left\langle \frac{1}{N}\mathrm{tr}(A_0^{(L)})^2 \right\rangle = e^{-3i\pi/4}\left\langle \frac{1}{N}\mathrm{tr}(A_0^{(E)})^2 \right\rangle \qquad \left\langle \frac{1}{N}\mathrm{tr}(A_I^{(L)})^2 \right\rangle = e^{i\pi/4}\left\langle \frac{1}{N}\mathrm{tr}(A_I^{(E)})^2 \right\rangle$$

Equivalence of the
Euclidean and Lorentzian
model without a cutoff
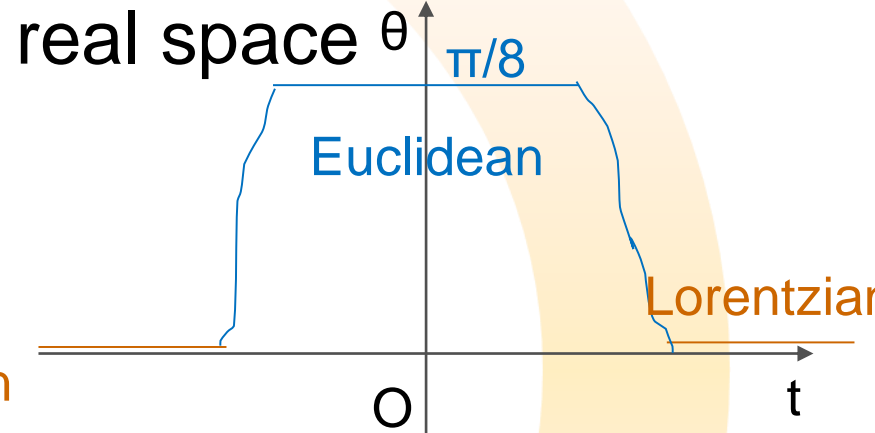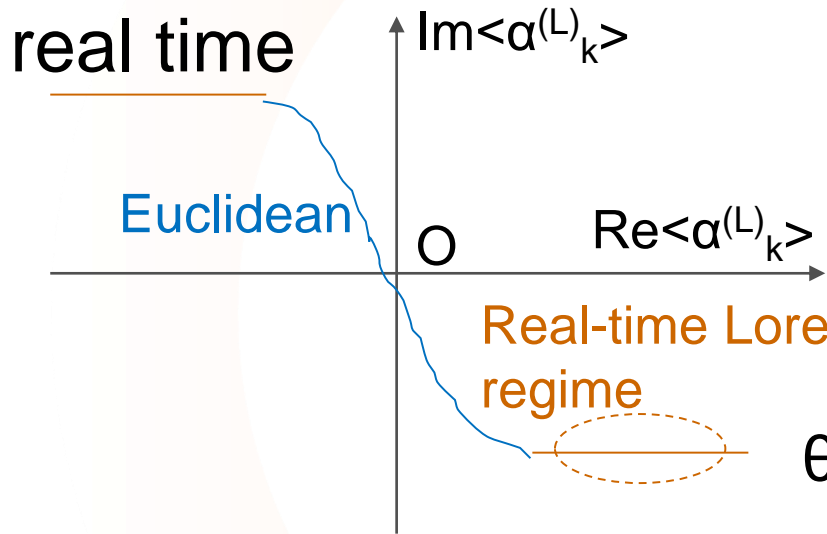
D=10, bosonic

⇒The emergent spacetime
is Euclidean.

How do we realize the real spacetime?

$$= -\mathrm{tr}(A_\mu)^2 \text{ at } \varepsilon = 0$$

Add a Lorentzian mass term $\quad S_{\mathrm{m}} = \dfrac{N\gamma_{\mathrm{L}}}{2}\{e^{i\varepsilon}\mathrm{tr}(A_0)^2 - e^{-i\varepsilon}\mathrm{tr}(A_I)^2\}$

Our expectation:

real time

Im$\langle\alpha^{(L)}{}_k\rangle$

Euclidean

O   Re$\langle\alpha^{(L)}{}_k\rangle$

Real-time Lorentzian regime

real space θ

π/8

Euclidean

Lorentzian

O   t

$\theta = \arg R(t) = \begin{cases} \pi/8 : \text{Euclidean regime} \\ 0 \quad : \text{Lorentzian regime} \end{cases}$
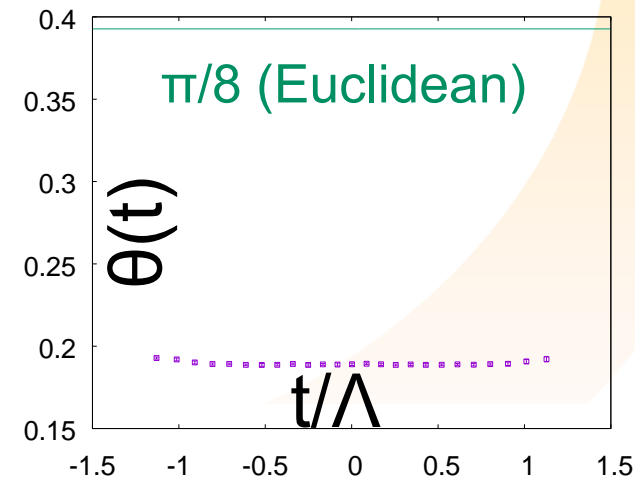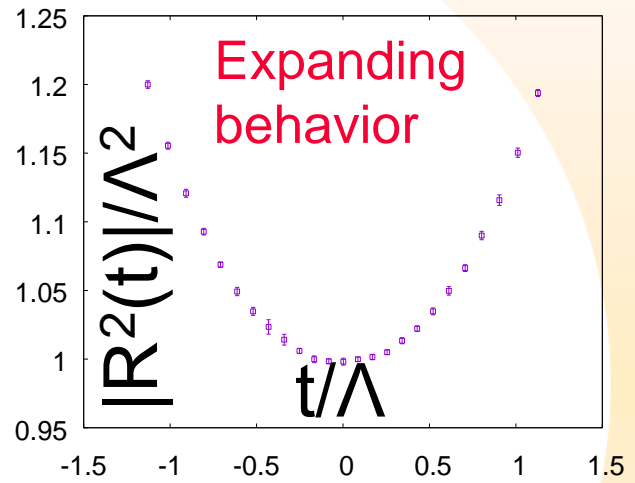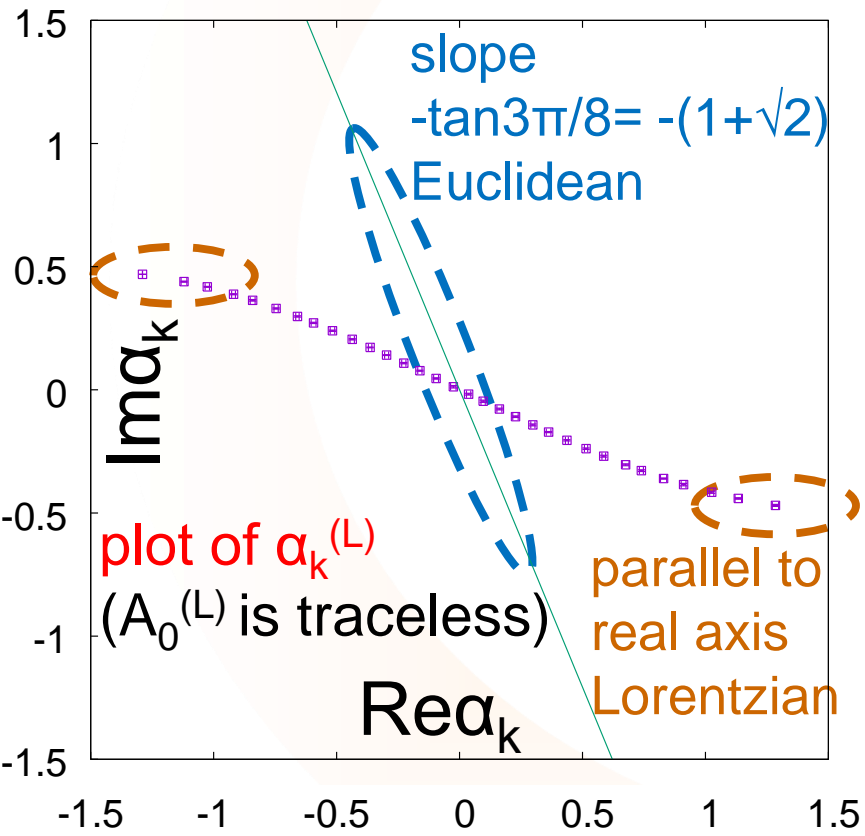
$$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\sum_{I=1}^{D-1}(\bar{A}_I^{(L)}(t))^2 \right\rangle = e^{2i\theta}|R^2(t)|$$

4. Result

Numerical result for bosonic model (omit S_f)
D=10, N=32, β=1.0, γ_L=3.0, ε=0.1π (preliminary)

$$R^2(t) = \left\langle \frac{1}{n}\text{tr}\sum_{I=1}^{D-1}(\bar{A}_I^{(L)}(t))^2 \right\rangle = e^{2i\theta}|R^2(t)|$$

$$\Lambda = |R(0)|$$

slope
-tan3π/8= -(1+√2)
Euclidean

Imα$_k$

plot of α$_k^{(L)}$
(A$_0^{(L)}$ is traceless)

parallel to
real axis
Lorentzian

Reα$_k$

$|R^2(t)|/\Lambda^2$

Expanding
behavior

t/Λ

θ(t)

π/8 (Euclidean)

t/Λ

5. Conclusion

Complex Langevin Method (CLM) for the type IIB matrix model.

Equivalence of the Euclidean and Lorentzian model without a cutoff.
⇒The emergent spacetime is interpreted as Euclidean.

Introduce a Lorentzian mass term
⇒・Signature change of spacetime
　・Expanding behavior of $|R^2(t)|$
　・Phase θ below π/8 (Euclidean) .

Future directions:
⇒impact of SUSY, larger-N simulations