Complex Langevin analysis of the spontaneous rotational symmetry breaking in the Euclidean type IIB matrix model

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1. Introduction

Difficulties in putting complex partition functions on computers.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

Sign problem:

The reweighting $\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. $\exp[O(N^2)]$

$\langle * \rangle_0$ = (V.E.V. for phase-quenched Z_0)

2. The type IIB matrix model

Candidate for nonperturbative string theory (a.k.a. "IKKT model")

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$Z = \int dA d\psi e^{-(S_b + S_f)}$$

$$S_b = -\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2, \quad S_f = N \mathrm{tr}\bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]$$

・***Euclidean*** case after Wick rotation $A_0 \to iA_D$, $\Gamma^0 \to -i\Gamma_D$.

⇒Path integral is finite without cutoff.

・A_μ, Ψ_α ⇒N×N Hermitian traceless matrices. $\mu = 1,2,\cdots,D, \ \alpha,\beta = \begin{cases} 1,2,3,4 & (D=6) \\ 1,2,\cdots,16 & (D=10) \end{cases}$

・Originally defined in **D=10**.

We consider the ***simplified D=6 case*** as well.

・Eigenvalues of A_μ : spacetime coordinate⇒$\mathcal{N}$=2 SUSY

・Integrating out ψ yields det$\mathcal{M}$ in D=6 (Pf $\mathcal{M}$ in D=10)

$$Z = \int dA e^{-S_b} \underbrace{\left(\int d\psi e^{-S_f} \right)}_{=\det/\mathrm{Pf}\mathcal{M} = |\det/\mathrm{Pf}\mathcal{M}| e^{i\Gamma} = e^{-S_{f,\mathrm{eff}}}} = \int dA \underbrace{e^{-S}}_{e^{-(S_b + S_{f,\mathrm{eff}})}}$$

・det/Pf$\mathcal{M}$'s *complex phase*

⇒Spontaneous Symmetry Breaking (SSB) of SO(D).

Result of Gaussian Expansion Method (GEM)

[T.Aoyama, J.Nishimura, and T.Okubo, arXiv:1007.0883, J.Nishimura, T.Okubo and F.Sugino, arXiv:1108.1293]

SSB **SO(6)→SO(3)** (In D=10, SO(10)→SO(3))

Dynamical compactification to 3-dim spacetime.

$\lambda_n (\lambda_1 \geq \cdots \geq \lambda_D)$: eigenvalues of $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}(A_\mu A_\nu)$ $\quad \rho_\mu = \frac{\langle \lambda_\mu \rangle}{\sum_{\mu=1}^{6} \langle \lambda_\mu \rangle} = \begin{cases} 0.30 & (\mu = 1,2,3) \\ 0.035 & (\mu = 4,5,6) \end{cases}$ (D=6)

3. Complex Langevin Method (CLM)

Solve the complex version of the Langevin equation.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_\mu)_{ij}}{dt} = -\frac{\partial S}{\partial (A_\mu)_{ji}} + \eta_{\mu,ij}(t)$$ ← drift term

$$\frac{\partial S}{\partial (A_\mu)_{ji}} = \frac{\partial S_b}{\partial (A_\mu)_{ji}} - c_d \mathrm{Tr}\left(\frac{\partial \mathcal{M}}{\partial (A_\mu)_{ji}} \mathcal{M}^{-1} \right) \quad c_d = \begin{cases} 1 & (D=6 \to \det \mathcal{M}) \\ \frac{1}{2} & (D=10 \to \mathrm{Pf}\mathcal{M}) \end{cases}$$

・A_μ : Hermitian→general complex traceless matrices.

・η_μ: Hermitian white noise obeying $\exp\left(-\frac{1}{4}\int \mathrm{tr}\eta^2(t) dt\right)$

CLM does not work when it encounters these problems:

(1) Excursion problem: A_μ is too far from Hermitian

⇒Gauge Cooling minimizes the Hermitian norm $\mathcal{N} = \frac{-1}{DN}\sum_{\mu=1}^{D} \mathrm{tr}[(A_\mu - (A_\mu)^\dagger)^2]$

(2) Singular drift problem:

The drift term $\partial S / \partial (A_\mu)_{ji}$ diverges due to $\mathcal{M}$'s near-zero eigenvalues.

We trust CLM when the distribution p(u) of the drift norm $u = \sqrt{\frac{1}{DN^3}\sum_{\mu=1}^{D}\sum_{i,j=1}^{N}\left|\frac{\partial S}{\partial (A_\mu)_{ji}}\right|^2}$ falls exponentially as $p(u) \propto e^{-au}$.

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

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Look at the drift term

⇒ Get the drift of CLM!!

4. Mass deformation

[Y. Ito and J. Nishimura, arXiv:1609.04501]

SO(D) breaking term $\Delta S_b = \frac{1}{2} N\varepsilon \sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2$

Order parameters for SO(D)'s SSB $\lambda_\mu = \mathrm{Re}\left\{\frac{1}{N}\mathrm{tr}(A_\mu)^2\right\}$

Fermionic mass term: $\Delta S_f = N m_f \mathrm{tr}(\bar{\psi}_\alpha \gamma_{\alpha\beta} \psi_\beta)$, $\gamma = \begin{cases} \Gamma_6 & (D=6) \\ i\Gamma_8 \Gamma_9^\dagger \Gamma_{10} & (D=10) \end{cases}$

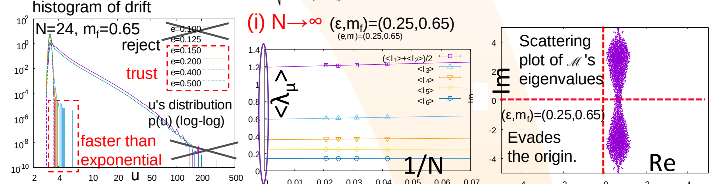
Avoids $\mathcal{M}$'s singular eigenvalue distribution

Extrapolation (i) N→∞ ⇒(ii)ε→0 ⇒ (iii) m_f→0

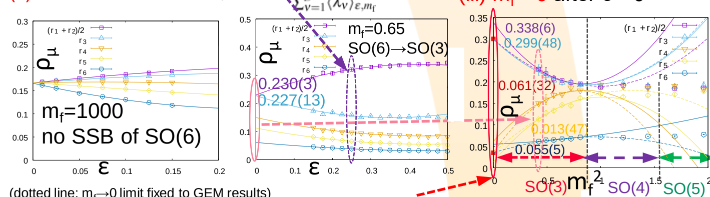
5. Result of D=6

[K.N. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura and S.K. Papadoudis, arXiv:1712.07562]

m_μ=(0.5,0.5,1,2,4,8) $\quad u = \sqrt{\frac{1}{DN^3}\sum_{\mu=1}^{D}\sum_{i,j=1}^{N}\left|\frac{\partial S}{\partial (A_\mu)_{ji}}\right|^2}$



(ii) ε→0 after N→∞ $\quad \rho_\mu(\varepsilon, m_f) = \frac{\langle \lambda_\mu \rangle_{\varepsilon, m_f}}{\sum_{\nu=1}^{D}\langle \lambda_\nu \rangle_{\varepsilon, m_f}}$ $\quad$ (iii) m_f→0 after ε→0



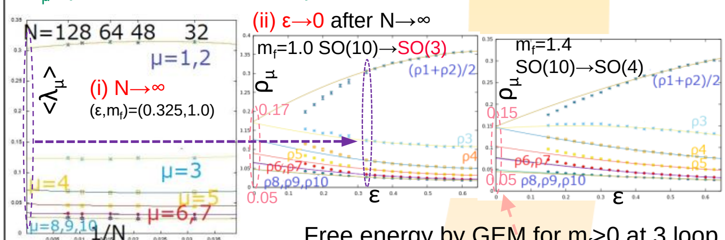
(dotted line: m_f→0 limit fixed to GEM results)

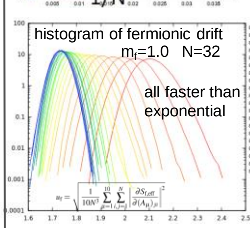
SSB SO(6)→SO(3) Consistent with GEM.

6. Result of D=10

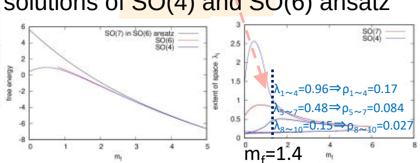
[K.N. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo and S.K. Papadoudis, work in progress]

m_μ=(0.5,0.5,1,2,4,8,8,8,8,8)





Free energy by GEM for m_f>0 at 3 loop (no $\Delta S_b = \frac{1}{2} N\varepsilon \sum_{\mu=1}^{D} m_\mu \mathrm{tr}(A_\mu)^2$ term) solutions of SO(4) and SO(6) ansatz



Trend of SSB SO(10)→SO(3).

(iii) m_f→0 after ε→0: hand in hand with GEM ???

7. Future works

・Reweighting method [J. Bloch, arXiv:1701.00986]

・Other deformations than the mass deformation

(z=1:original Euclidean, pure imaginary z: fermion det/Pf is real) [Y. Ito and J. Nishimura, arXiv:1710.07929]

$$N\mathrm{tr}\left(\bar{\psi}(z\Gamma_D)[A_D, \psi] + \sum_{k=1}^{D-1} \bar{\psi}\Gamma_k [A_k, \psi] \right)$$