Complex Langevin studies of the continuum limit of the Lorentzian type IIB matrix model

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1. Introduction

Type IIB matrix model (a.k.a. IKKT model)

Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{-\frac{1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_b} + \underbrace{\frac{1}{g^2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu,\psi_\beta]}_{=S_f}$$

・Dimensional reduction of the D=10 super-Yang-Mills theory to 0 dimension

・A_μ, Ψ_α ⇒N×N Hermitian traceless matrices.

・N=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinate.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

⇒indices are contracted by Lorentzian metric $\eta = \mathrm{diag}(-1,1,1,\cdots,1)$

Time development: gauge fixing to diagonalize A_0

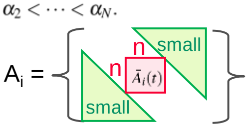
$A_0 = \mathrm{diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_N$.

Band-diagonal structure of A_i

(v=0,1,2,…,N-n, and I,J=1,2,…,n)

n×n matrix for time $t = \frac{1}{n}\sum_{I=1}^{n}\alpha_{v+I}$

$(\bar{A}_i)_{IJ}(t) = (A_i)_{v+I,v+J}$

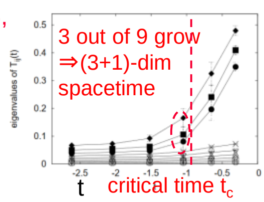


Original Lorentzian works on D=10, N=16, supersymmetric (no sign problem).

Order parameter of the breakdown of spatial symmetry:

(D-1) eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$

Dynamical emergence of (3+1)-dim spacetime.



Difficulties in putting the Lorentzian version on a computer:

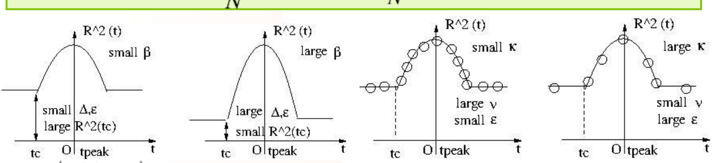
1. Sign problem $Z = \int dA \left(e^{iS_b}\int d\psi e^{iS_f}\right)$ (complex; real; $=\mathrm{Pf}\mathscr{M}$)

⇒We use Complex Langevin Method (CLM) [J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

2. The action is not bounded below

bosonic part: $S_b = \frac{N\beta}{4}\mathrm{tr}\left\{2\sum_{i=1}^{D-1}[A_0,A_i]^2 - \sum_{i,j=1}^{D-1}[A_i,A_j]^2\right\}$ $\left(\frac{1}{g^2 N} = \beta\right)$

⇒Infrared cutoff: $\frac{1}{N}\mathrm{tr}(A_0)^2 = \kappa$ $\quad \frac{1}{N}\mathrm{tr}(A_i)^2 = 1$



$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\sum_{i=1}^{D-1}(\bar{A}_i(t))^2\right\rangle$ $\quad \nu = (\text{number of } \alpha_i \text{ in } t_c < t \le t_{\mathrm{peak}})$

$\Delta = \frac{t_{\mathrm{peak}} - t_c}{R(t_c)}$: volume of temporal direction $\quad \varepsilon = \frac{\Delta}{\nu}$: lattice space

In the following, we study the D=6 bosonic model:

Similar emergence of the spacetime for D=6, 10, and bosonic/supersymmetric model. [Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795]

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1. Sign problem [J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

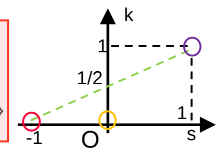
Parameters of Wick rotation: $Z = \int dA e^{-\tilde{S}_b}$ where

$$\tilde{S}_b = -iS_b = -\underbrace{i}_{=e^{i\pi/2}}\frac{N\beta}{4}\mathrm{tr}\left\{2\sum_{i=1}^{D-1}[A_0,A_i]^2 + \underbrace{(-1)}_{=e^{-i\pi}}\sum_{i,j=1}^{D-1}[A_i,A_j]^2\right\}$$

multiply $e^{is\pi/2}$ (world sheet Wick rotation)

$A_0 \to A_0 e^{-ik\pi/2}$ (target space Wick rotation)

$$\tilde{S}_b = -\frac{N\beta}{4}\mathrm{tr}\left\{2e^{i(1+s-2k)\pi/2}\sum_{i=1}^{D-1}[A_0,A_i]^2 + e^{i(s-1)\pi/2}\sum_{i,j=1}^{D-1}[A_i,A_j]^2\right\}$$



・(s,k)=(-1,0) : Original Lorentzian works $\int dA e^{iS_b} \to \int dA e^{\beta S_b}$

→no sign problem [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

・(s,k)=(1,1) : Euclidean version ($\mathrm{Pf}\mathscr{M}(e^{-ik\pi/2}A_0, A_i)$ has sign problem)

・k=(1+s)/2 : minimize noncommutativity $[A_0,A_i]$

Paulian-matrix structure of the space at (s,k)=(-1,0)

[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

$\bar{A}_i(t) \propto \sigma_i \oplus 0_{n-2} (i=1,2,3)$

2 of the n eigenvalues of $Q(t) = \sum_{i=1}^{D-1}(\bar{A}_i(t))^2$ grow.

⇒sphere whose inside is empty.

2. Infrared cutoff $Z = \int dA e^{-\tilde{S}_b(A)}\delta\left(\frac{1}{N}\mathrm{tr}(A_0)^2 - \kappa\right)\delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right)$

Change variables $A_0 \to \sqrt{\frac{\kappa}{u}}A_0,\ A_i \to \frac{1}{\sqrt{v}}A_i$ ⇒integrate out u,v

$Z = \int dA e^{-S_{\mathrm{eff}}}$ where $S_{\mathrm{eff}} = \tilde{S}_b(Y_0,Y_i) + \frac{N}{2}\{\mathrm{tr}(A_0)^2 + \mathrm{tr}(A_i)^2\}$ $\quad Y_0 = A_0\sqrt{\frac{\kappa N}{\mathrm{tr}(A_0)^2}}$ $\quad Y_i = A_i\sqrt{\frac{N}{\mathrm{tr}(A_i)^2}}$

3. Complex Langevin Method (CLM)

Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

Introducing time order $\alpha_1<\alpha_2<\ldots<\alpha_N$ for complexified α_i

$A_0 = \mathrm{diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)$ $\quad \alpha_1 = 0,\ \alpha_k = \sum_{i=1}^{k-1}e^{\tau_i}\ (k=2,3,\cdots,N)$

Render A_0 traceless $A_0 \to \tilde{A}_0 = A_0 - \mathbf{1}\left(\frac{1}{N}\mathrm{tr}(A_0)\right)$

Add gauge fixing term $S_{\mathrm{f}} = \left(-\log\prod_{1\le b<a\le N}(\alpha_a-\alpha_b)^2 - \sum_{a=1}^{N-1}\tau_a\right)$ to S_{eff}.

Gauge is already fixed ⇒No gauge cooling

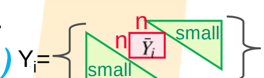
$$\frac{d(A_i)_{ab}(t_l)}{dt_l} = -\frac{dS_{\mathrm{eff}}}{d(A_i)_{ba}} + \eta_{i,ab}(t_l) \qquad \frac{d\tau_a(t_l)}{dt_l} = -\frac{dS_{\mathrm{eff}}}{d\tau_a} + \eta_a(t_l)$$

(drift term; Hermitian-matrix white noise; fictitious Langevin time; real-number white noise)

・A_i : Hermitian→general complex traceless matrices.

・τ_a : Real number → complex number.

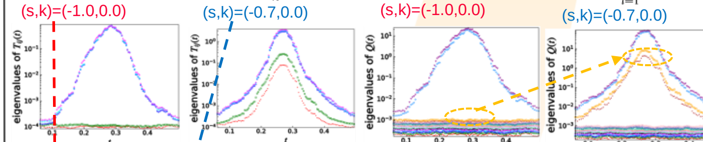
4. Results (preliminary)

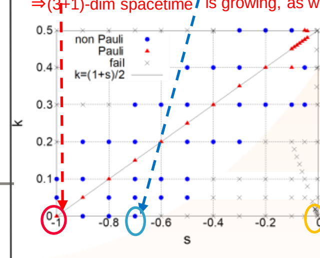


Y_i are not Hermitian → n×n Hermitian matrices $\bar{H}_i(t) = \frac{1}{2}(\bar{Y}_i(t) + \bar{Y}_i^\dagger(t))$

D=6, N=128, bosonic, n=18, (β,κ)=(8.0, 0.02), (s,k)=(-1.0,0.0), (-0.7, 0.0)

(D-1) eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{H}_i(t)\bar{H}_j(t))$ $\quad$ n eigenvalues of $Q(t) = \sum_{i=1}^{D-1}(\bar{H}_i(t))^2$





5. Summary

CLM of the Lorentzian type IIB matrix model

Departure from Paulian-matrix structure

Eventually we take (s,k)→(0,0), N→∞.

・approach (s,k)→(0,0) at larger N.

・study smaller κ (covering late time)