Emergence of an expanding (3+1)-dimensional spacetime in the type IIB matrix model

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SETSUDAI

1. Introduction

Type IIB matrix model (a.k.a. IKKT model)

Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_b} + \underbrace{\frac{-N}{2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_f}$$

- Dimensional reduction of the D=10 SYM theory to 0dim.
- A_μ, Ψ_α ⇒ N × N Hermitian traceless matrices.
- N=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinate.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

⇒indices are contracted by Lorentzian metric $\eta = \mathrm{diag}(-1,1,1,\cdots,1)$

Time evolution: gauge fixing to diagonalize A_0

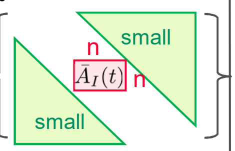
$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_N$.

Band-diagonal structure of A_I

(v=1,2,…,N-n, and p,q=1,2,…,n)

n × n matrix for time $(\bar{A}_I)_{pq}(t) = (A_I)_{\nu+p,\nu+q}$

$$t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|, \text{ where } \bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i}$$



Sign problem of the Lorentzian version

$\mathcal{M}$ is a 16(N²-1) × 16(N²-1) sparse matrix

$$Z = \int dA \left(e^{iS_b} \int d\psi e^{iS_f} \right), \quad \int d\psi e^{iS_f} = \mathrm{Pf}\,\mathcal{M}$$

complex / real

⇒We employ the Complex Langevin Method (CLM).

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

Results with the approximation to avoid the sign problem

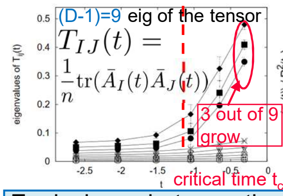
SUSY (include S_f), D=10, N=16, n=4, HMC (not CLM)

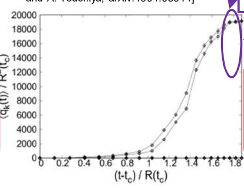
[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

Bosonic (omit S_f), D=10, N=256, n=18, HMC

[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

2 of the n eig of $Q(t) = \sum_{I=1}^{9} (\bar{A}_I(t))^2$ grow.





Dynamical emergence of (3+1)-dim spacetime, but with Pauli-matrix space structure.

Equivalence between the Lorentzian and Euclidean version

Contour deformation $A_0 = e^{-3i\pi u/8}\tilde{A}_0$, $A_I = e^{i\pi u/8}\tilde{A}_I$

$$S_b \to \tilde{S}_b = e^{i\pi u/2}\left\{ \frac{-N}{4}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 + \frac{N}{2} e^{-i\pi u}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2 \right\}$$

w.r.t. worldsheet; w.r.t. target space

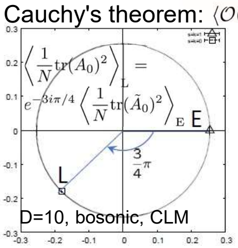
u=1: Euclidean model, with SSB SO(10)→SO(3) (3dim spacetime)

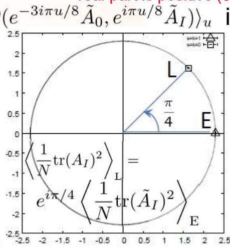
[K.N. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo and S.K. Papadoudis arXiv:2002.07410]

$$e^{i\tilde{S}_b(\tilde{A})} = e^{-S(\tilde{A})}, \quad \text{where } S(\tilde{A}) = -\frac{N}{4} e^{-i\pi(1-u)/2}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 - \frac{N}{2} e^{i\pi(1-u)/2}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2$$

real part is positive (0<u≦1).

Cauchy's theorem: $\langle \mathcal{O}(e^{-3i\pi u/8}\tilde{A}_0, e^{i\pi u/8}\tilde{A}_I)\rangle_u$ is independent of u.





Equivalence of the Euclidean (u=1) and Lorentzian (u→+0) model

⇒The spacetime is Euclidean.

[K. Hatakeyama et. al. arXiv:2112.15368]

Adding the Lorentzian mass term

$$Z = \int dA d\psi e^{i(S+S_\gamma)}, \quad S_\gamma = \frac{-N\gamma}{2}\mathrm{tr}(A^\mu A_\mu) = \frac{N\gamma}{2}\{\mathrm{tr}(A_0)^2 - \mathrm{tr}(A_I)^2\}$$

$$e^{iS_\gamma(A)} = e^{-S_\gamma(\tilde{A})}, \quad S_\gamma(\tilde{A}) = \frac{N\gamma}{2}\{e^{-i\pi(2+3u)/4}\mathrm{tr}(\tilde{A}_0)^2 + e^{i\pi(2+u)/4}\mathrm{tr}(\tilde{A}_I)^2\}$$

real part is negative (0<u≦1).

At γ>0, we cannot define the model by contour deformation

⇒Equivalence to the Euclidean model (u=1) is violated.

We consider the limit N→∞ ⇒γ→0.

3. Complex Langevin Method (CLM)

Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

- Introduce τ_a ⇒time order $\alpha_1 < \alpha_2 < \cdots < \alpha_N$ for complexified α_i.

$\alpha_1 = 0, \ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i}$ (k=2,3,⋯,N), $A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$

- Mass term to avoid the near-zero modes of the Dirac operator:

$$\mathrm{Pf}\,\mathcal{M}_{m_f} = \int d\psi e^{iS_{m_f}}, \ S_{m_f} = \frac{-N}{2}\mathrm{tr}\{\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta] + m_f\bar{\psi}_\alpha(\Gamma^7\Gamma^{8\dagger}\Gamma^9)_{\alpha\beta}\psi_\beta\}$$

$$Z = \int dA e^{-S_{\mathrm{eff}}}, \ S_{\mathrm{eff}} = -i(S_b + S_\gamma) - \log \mathrm{Pf}\,\mathcal{M}_{m_f} - \log \prod_{1\le a<b\le N} (\alpha_a - \alpha_b)^2 - \sum_{a=1}^{N-1}\tau_a$$

- Gauge is already fixed ⇒No gauge cooling.

$$\frac{d(A_I)_{ab}(\sigma)}{d\sigma} = -\frac{dS_{\mathrm{eff}}}{d(A_I)_{ba}} + \eta_{I,ab}(\sigma), \quad \frac{d\tau_a(\sigma)}{d\sigma} = -\frac{dS_{\mathrm{eff}}}{d\tau_a} + \eta_a(\sigma)$$

fictitious Langevin time; drift term; Hermitian-matrix white noise; fictitious Langevin time; drift term; real-number white noise

- A_I : Hermitian→general complex traceless matrices.
- τ_a : Real number → complex number.

The drift term involves $\frac{d}{d(A_I)_{ba}}\{-\log \mathrm{Pf}\,\mathcal{M}_{m_f}\} = -\frac{1}{2}\mathrm{Tr}\left(\frac{d\mathcal{M}_{m_f}}{d(A_I)_{ba}}\mathcal{M}_{m_f}^{-1}\right)$

- $\mathcal{M}_{m_f}$'s near-zero modes cause the singular drift problem.
- We use conjugate gradient (CG) method and noisy estimator.

⇒Large-scale numerical simulation using supercomputers.

The condition to justify the CLM: [K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

The probability dist. of the drift norms falls exponentially or faster.

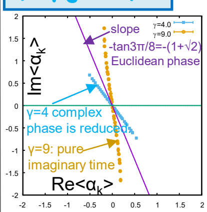
$$u_A = \sqrt{\frac{1}{9N^3}\sum_{I=1}^{9}\sum_{a,b=1}^{N}\left|\frac{dS_{\mathrm{eff}}}{d(A_I)_{ba}}\right|^2}, \quad u_\alpha = \sqrt{\frac{1}{N}\sum_{a=1}^{N-1}\left|\frac{dS_{\mathrm{eff}}}{d\tau_a}\right|^2}$$

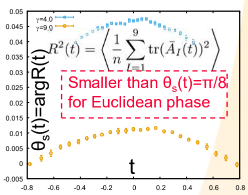
Dynamical stabilization: [F. Attanasio and B. Jäger arXiv:1808.04400]

After each Langevin step, $A_I \to \frac{A_I + \eta A_I^\dagger}{1+\eta}$ (here, η=0.01)

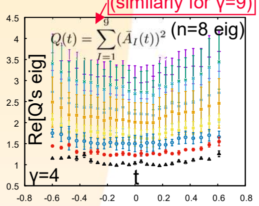
4. Results

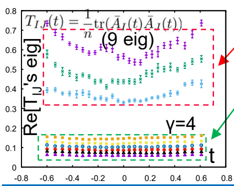
N=32, n=8, u=0, m_f=3.5 (preliminary)





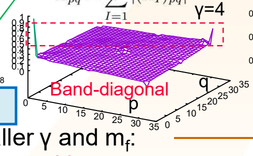
departure from Paulian structure (similarly for γ=9)

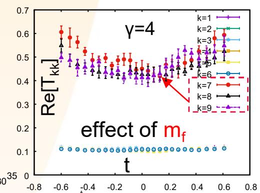




expanding 3dim space (similarly for γ=9)

shrunken 6dim space





5. Discussions

At larger N, and smaller γ and m_f:

Can we observe the transition from Euclidean to Lorentzian geometry?

Large-scale numerical simulation at larger N is important.