⑰ The effect of fermions on the emergence of (3+1)-dimensional space-time in the Lorentzian type IIB matrix model

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SETSUDAI

1. Introduction

Type IIB matrix model (a.k.a. IKKT model)

Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_{\rm b}} + \underbrace{\frac{-N}{2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_{\rm f}}$$

- Dimensional reduction of the D=10 SYM theory to 0 dim.
- A_μ,(μ=0,1,…,9), Ψ_α (α=1,2,…,16 after Weyl projection) ⇒N × N Hermitian traceless matrices.
- $\mathcal{N}$=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinates.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

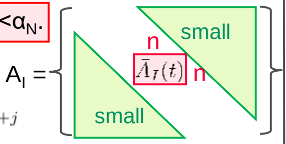
⇒indices are contracted by Lorentzian metric $\eta = \mathrm{diag}(-1, 1, 1, \cdots, 1)$

Time evolution: gauge fixing to diagonalize A_0

$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_N$.

Band-diagonal structure of A_I

(ν=1,2,…,N-n, and i,j=1,2,…,n)



n × n matrix for time $(\bar{A}_I)_{ij}(t) = (A_I)_{\nu+i,\nu+j}$

$$t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|, \text{ where } \bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i}$$

Sign problem of the Lorentzian version

$$Z = \int dA \left(\underbrace{e^{iS_{\rm b}}}_{\text{complex}} \underbrace{\int d\psi e^{iS_{\rm f}}}_{=\mathrm{Pf}\,\mathcal{M}} \right)$$

real

$\mathcal{M}$ is a $16(N^2-1) \times 16(N^2-1)$ sparse matrix

⇒We employ the Complex Langevin Method (CLM) .

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

Equivalence between the Lorentzian and Euclidean version

[Y. Asano, private communication]

Contour deformation $A_0 = e^{-3i\pi u/8}\tilde{A}_0, \ A_I = e^{i\pi u/8}\tilde{A}_I$

$$S_{\rm b} \to \tilde{S}_{\rm b} = \underbrace{e^{i\pi u/2}}_{\text{w.r.t. worldsheet}} \left\{ \frac{-N}{4}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 + \frac{N}{2}\underbrace{e^{-i\pi u}}_{\text{w.r.t. target space}}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2 \right\}$$

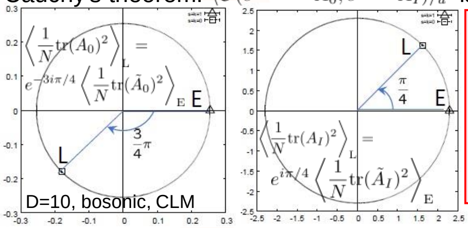
u=1: Euclidean model, with SSB SO(10)→SO(3) (3dim space*time*)

[K.N. Anagnostopoulos, T. Azuma, Y. Ito J. Nishimura, T. Okubo and S.K. Papadoudis arXiv:2002.07410]

$$e^{iS_{\rm b}(A)} = e^{-S(\tilde{A})}, \quad \text{where } S(\tilde{A}) = -\frac{N}{4}e^{-i\pi(1-u)/2}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 - \frac{N}{2}e^{i\pi(1-u)/2}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2$$

real part is positive (0<u≦1).

Cauchy's theorem: $\langle \mathcal{O}(e^{-3i\pi u/8}\tilde{A}_0, e^{i\pi u/8}\tilde{A}_I)\rangle_u$ is independent of u.



Equivalence of the Euclidean (u=1) and Lorentzian (u→+0) model

⇒The spacetime is Euclidean.

[K. Hatakeyama et. al. arXiv:2112.15368]

Adding the Lorentzian mass term

$$Z = \int dA d\psi e^{i(S+S_\gamma)}, \ S_\gamma = \frac{-N\gamma}{2}\mathrm{tr}(A^\mu A_\mu) = \frac{N\gamma}{2}\{\mathrm{tr}(A_0)^2 - \mathrm{tr}(A_I)^2\}$$

$$e^{iS_\gamma(A)} = e^{-S_\gamma(\tilde{A})}, \quad S_\gamma(\tilde{A}) = \frac{N\gamma}{2}\{e^{-i\pi(2+3u)/4}\mathrm{tr}(\tilde{A}_0)^2 + e^{i\pi(2+u)/4}\mathrm{tr}(\tilde{A}_I)^2\}$$

real part is negative (0<u≦1).

At γ>0, we cannot define the model by contour deformation

⇒Equivalence to the Euclidean model (u=1) is violated.

We consider the limit N→∞ ⇒γ→0.

3. Complex Langevin Method (CLM)

Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

- Introduce τ_a ⇒time order $\alpha_1 < \alpha_2 < \cdots < \alpha_N$ for complexified α_i .

$\alpha_1 = 0, \ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i}$ (k=2,3,…,N), $A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$

- Mass term to avoid the near-zero modes of the Dirac operator:

$$\mathrm{Pf}\,\mathcal{M} = \int d\psi e^{iS_{m_{\rm f}}}, \ S_{m_{\rm f}} = \frac{-N}{2}\mathrm{tr}\left\{\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta] + im_{\rm f}\bar{\psi}_\alpha(\Gamma^7\Gamma^{8\dagger}\Gamma^9)_{\alpha\beta}\psi_\beta\right\}$$

(m_f→∞: bosonic model, m_f=0:SUSY at γ=0)

$$Z = \int dA e^{-S_{\rm eff}}, \ S_{\rm eff} = -i(S_{\rm b} + S_\gamma) - \log \mathrm{Pf}\,\mathcal{M} - \log \prod_{1 \le a < b \le N}(\alpha_a - \alpha_b)^2 - \sum_{a=1}^{N-1}\tau_a$$

- Gauge is already fixed ⇒No gauge cooling.

$$\frac{d(A_I)_{ab}(\sigma)}{d\sigma} = -\frac{dS_{\rm eff}}{d(A_I)_{ba}} + \eta_{I,ab}(\sigma), \quad \frac{d\tau_a(\sigma)}{d\sigma} = -\frac{dS_{\rm eff}}{d\tau_a} + \eta_a(\sigma)$$

fictitious Langevin time / drift term / Hermitian-matrix white noise / fictitious Langevin time / drift term / real-number white noise

- A_I : Hermitian→general complex traceless matrices.
- τ_a : Real number → complex number.

The condition to justify the CLM: [K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

The probability dist. of the drift norms falls exponentially or faster.

$$u_A = \sqrt{\frac{1}{9N^3}\sum_{I=1}^{9}\sum_{a,b=1}^{N}\left|\frac{dS_{\rm eff}}{d(A_I)_{ba}}\right|^2}, \ u_\alpha = \sqrt{\frac{1}{N}\sum_{a=1}^{N-1}\left|\frac{dS_{\rm eff}}{d\tau_a}\right|^2}$$

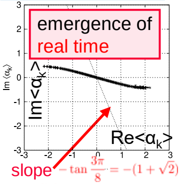
The following results satisfy this criterion.

Dynamical stabilization: [F. Attanasio and B. Jäger arXiv:1808.04400]

After each Langevin step, $A_I \to \frac{A_I + \eta A_I^\dagger}{1+\eta}$ (here,η=0.005)

4. Results

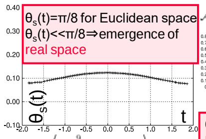
(1) bosonic model (without $\log \mathrm{Pf}\,\mathcal{M}$), N=96,n=12,γ=4



emergence of real time

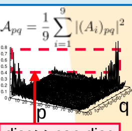
slope $-\tan\frac{3\pi}{8} = -(1+\sqrt{2})$

Euclidean time



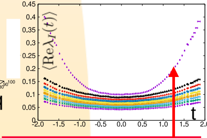
θs(t)=π/8 for Euclidean space
θs(t)<<π/8⇒emergence of real space

$$R^2(t) = \left\langle \frac{1}{n}\sum_{I=1}^{9}\mathrm{tr}(\bar{A}_I(t))^2 \right\rangle = e^{2i\theta_s(t)}|R^2(t)|$$



$$A_{pq} = \frac{1}{9}\sum_{i=1}^{9}|(A_i)_{pq}|^2$$

diag>>non-diag band-diagonal structure



1 out of 9 grows⇒ emergence of 1-dim space

Order parameter of the SSB of spatial SO(9) symmetry $T_{IJ}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_I(t)\bar{A}_J(t))$

$\lambda_I(t)$: eigenvalues of $T_{IJ}(t) (I, J = 1, 2, \cdots, 9)$

(2) fermionic model (with $\log \mathrm{Pf}\,\mathcal{M}$), N=96,n=12,γ=4, m_f=3.5

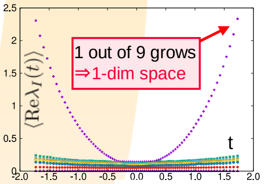
$\mathrm{Pf}\,\mathcal{M} = 0$ when $A_1, A_2 \neq O$, $A_3 = \ldots A_9 = O$ (m_f=0)

[W. Krauth, H. Nicolai and M.Staudacher, hep-th/9803117, J. Nishimura and G.Vernizzi hep-th/0003223]

At m_f→0, the SUSY effect may contribute to the expansion of 3-dim space.

In the following, the results share the emergence of real time and space, and the band-diagonal structure:

1-dim space emerges at m_f=3.5, because m_f weakens the SUSY effect.



1 out of 9 grows ⇒1-dim space

5. Deformation of Lorentzian mass term

Control bosonic quantum fluctuations to mimic SUSY cancellation.

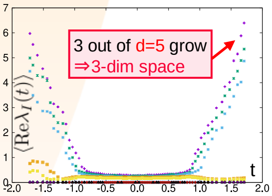
$$S_\gamma \to \frac{N\gamma}{2}\left\{\mathrm{tr}(A_0)^2 - \sum_{I=1}^{d}\mathrm{tr}(A_I)^2 - \xi\sum_{I=d+1}^{9}\mathrm{tr}(A_I)^2\right\}$$

This reduces to S_γ at ξ→1.

Space is restricted to at most d dim for ξ>>1.

N=96, n=12, γ=4, m_f=3.5, d=5, ξ=16 (with $\log \mathrm{Pf}\,\mathcal{M}$)

⇒emergence of 3-dim space.



3 out of d=5 grow ⇒3-dim space

6. Discussions

Balance between fermionic and bosonic fluctuations stabilizes the 3-dim space.

Large-scale N≧128 simulations are in progress.

At smaller m_f, can we obtain 3-dim space at ξ→1?

Study scaling at N→∞⇒γ→0.