Monte Carlo studies of the six-dimensional IKKT model

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1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)

$\Rightarrow$ **Promising candidate for the constructive definition of superstring theory.**
Ishibashi, Kawai, Kitazawa and Tsuchiya, hep-th/9612115.

$$S = N \left(-\frac{1}{4} \mathrm{tr}\, [A_\mu, A_\nu]^2 + \frac{1}{2} \mathrm{tr}\, \bar{\psi}_\alpha (\Gamma_\mu)_{\alpha\beta} [A_\mu, \psi_\beta] \right).$$

- **Dimensional reduction of 10-dim $\mathcal{N} = 1$ super Yang-Mills theory to 0 dimension.**
- **A_μ (10d vector) and ψ_α (10d Majorana-Weyl spinor) $\Rightarrow$ $N \times N$ hermitian matrices .**
- **A_μ's eigenvalues $\Rightarrow$ spacetime coordinate.**
- **Evidences for spontaneous breakdown of SO(10) symmetry to SO(4)**

  $\Rightarrow$ **emergence of four-dimensional spacetime.**
  Nishimura and Sugino, hep-th/0111102, Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.
- **Complex fermion determinant:**
  - **Crucial for rotational symmetry breaking.** Nishimura and Vernizzi, hep-th/0003223.
  - **Difficulty of Monte Carlo simulation.**

2 6d IKKT model

T .Aoyama, T. Azuma, M. Hanada and J. Nishimura

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

- A_μ (6d vector) and ψ (6d Weyl spinor) are $N \times N$ hermitian matrices .

  $\Gamma_1 = i\sigma_1 \otimes \sigma_2,\ \Gamma_2 = i\sigma_2 \otimes \sigma_2,\ \Gamma_3 = i\sigma_3 \otimes \sigma_2,\ \Gamma_4 = i1 \otimes \sigma_1,\ \Gamma_5 = i1 \otimes \sigma_3,\ \Gamma_6 = 1 \otimes 1.$

- SO(6) rotational symmetry and SU(N) gauge symmetry.

- Presence of $\mathcal{N} = 2$ supersymmetry.

- $Z = \int dA d\psi d\bar{\psi} e^{-S} = \int dA e^{-S_B} \underbrace{(\det \mathcal{M})}_{=\int d\psi d\bar{\psi} e^{-S_F}} = \int dA e^{-S_0} e^{i\Gamma}$. CPU cost is $O(N^6)$.

  4d $\rightarrow$ det $\mathcal{M}$ is real positive

  6d and 10d $\rightarrow$ det $\mathcal{M}$ is complex.

  Complex phase is important in SO(6) breakdown.

- **Previous works on this model:**

  * **Simulation of phase-quenched 6d and 10d IKKT model**

    ⇒ no symmetry breakdown of SO(6) (and SO(10)).

    J. Ambjorn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, hep-th/0005147

  * **Simulation of one-loop effective action (CPU cost is O(N^3)).**

    K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041.

  * **Gaussian expansion method ⇒ symmetry breakdown of SO(6) to SO(3).**

    T. Aoyama, J. Nishimura and T. Okubo

    Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

    λ_i $(i = 1, \cdots, 6)$ **: eigenvalues of** $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_6)$

    At large N, $\langle \lambda_{1,2,3} \rangle \gg \langle \lambda_{4,5,6} \rangle$

3 Monte Carlo simulation

Factorization method

An approach to the complex action problem in Monte Carlo simulation.

K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041,
J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, hep-lat/0208025.

Standard reweighting method:

$$\langle \lambda_i \rangle = \frac{\langle \lambda_i e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0} = \frac{\langle \lambda_i \cos\Gamma \rangle_0}{\langle \cos\Gamma \rangle_0},$$ where $\langle * \rangle_0 =$ (V.E.V. for the phase-quenched model Z_0).

Under parity transformation $A_i^P = A_i (i = 1, \cdots, 5)$, $A_6^P = -A_6 \Rightarrow$

- λ_i (eigenvalues of $T_{\mu\nu} = \frac{1}{N} \mathrm{tr}\, A_\mu A_\nu$) are invariant.
- $\det \mathcal{M}$ is complex conjugate.

The phase Γ oscillates violently.

(Number of configurations required) $\simeq e^{O(N^2)}$. $\Rightarrow$ sign problem.

$\tilde{\lambda}_i \stackrel{\text{def}}{=} \lambda_i/\langle\lambda_i\rangle_0$: deviation from 1 $\Rightarrow$ effect of the phase.

Overlap problem: Discrepancy of a distribution function between the phase-quenched model Z_0 and the full model Z.

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle\delta(x-\tilde{\lambda}_i)\rangle = \frac{\langle\delta(x-\tilde{\lambda}_i)\cos\Gamma\rangle_0}{\langle\cos\Gamma\rangle_0} = \frac{\langle\delta(x-\tilde{\lambda}_i)\rangle_0\langle\cos\Gamma\rangle_{i,x}}{\langle\cos\Gamma\rangle_0} = \frac{1}{C}\rho_i^{(0)}(x)w_i(x),$$

where

$$C = \langle\cos\Gamma\rangle_0, \quad \rho_i^{(0)}(x) = \langle\delta(x-\tilde{\lambda}_i)\rangle_0, \quad w_i(x) = \langle\cos\Gamma\rangle_{i,x},$$

$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0}\delta(x-\tilde{\lambda}_i)].$$

Simulation of partition function $Z_{i,x} \Rightarrow x$ is trapped at $\tilde{\lambda}_i$.

The system visits the configurations important for full partition function Z.

Resolution of overlap problem.

Monte Carlo evaluation of $\langle\tilde{\lambda}_i\rangle$

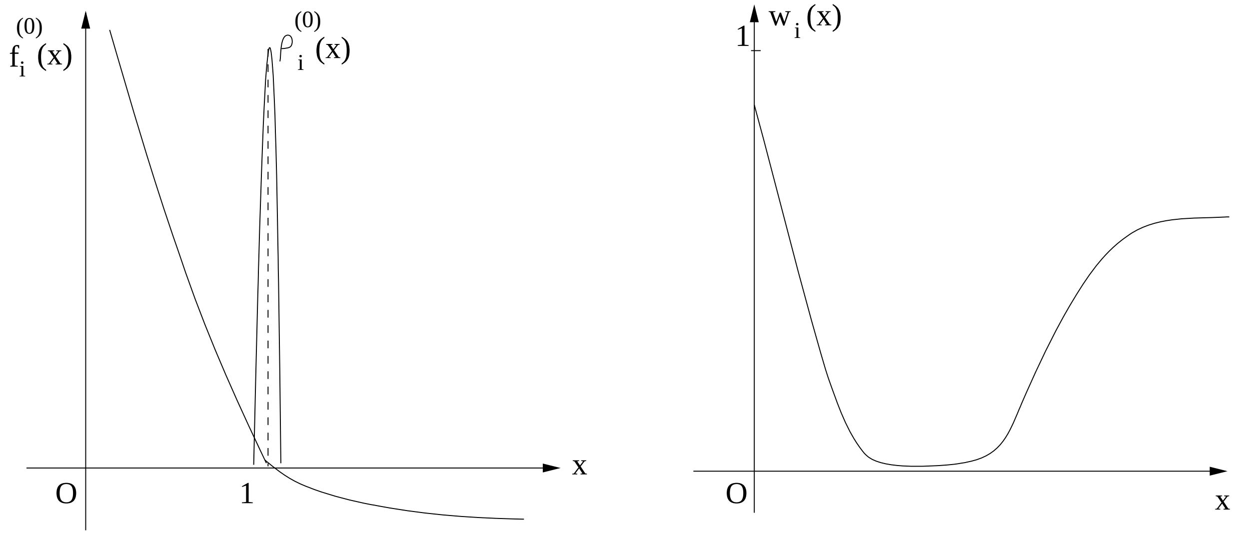
$w_i(x) > 0 \Rightarrow \langle\tilde{\lambda}_i\rangle$ **is the minimum of** $\mathcal{F}_i(x)$:

$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2}\log\rho_i(x).$$

We solve $\mathcal{F}'_i(x) = 0$, **namely** $\frac{1}{N^2}f_i^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_i(x)\right\}$, $\left(\text{where } f_i^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x}\log\rho_i^{(0)}(x)\right)$

Do both $\frac{1}{N^2}\log w_i(x)$ **and** $\frac{1}{N^2}f_i^{(0)}(x)$ **scale at large** N **as**

$$\frac{1}{N^2}\log w_i(x) \to \Phi_i(x), \quad \frac{1}{N^2}f_i^{(0)}(x) \to F_i(x)?$$



Behavior of $\Phi_i(x)$

Asymptotic behavior of $\Phi_i(x) = \frac{1}{N^2}\log w_i(x)$ **at** $x \ll 1$ **and** $x \gg 1$.
When we fix the i**-th largest eigenvalue** $\rightarrow$

- $x \ll 1$ $(i = 2, \cdots, 6)$ $\Rightarrow$ $(i-1)$**-dimensional configuration**
- $x \gg 1$ $(i = 1, \cdots, 5)$ $\Rightarrow$ i**-dimensional configuration**

Fermion determinant $\det \mathcal{M}$ **is complex conjugate under**

$$A_i^P = A_i (i = 1, \cdots, 5), \quad A_6^P = -A_6$$

$$\Omega_d = \{\{A_\mu\}; n_\mu^{(i)} A_\mu = 0 \text{ for } \exists n_\mu^{(i)} (i = 1, \cdots 6 - d)\}$$

5-dimensional configuration $\Omega_5 \Rightarrow$ **Fermion determinant is real.**

J. Nishimura and G. Vernizzi, hep-th/0003223.

For d-dimensional configuration Ω_d,

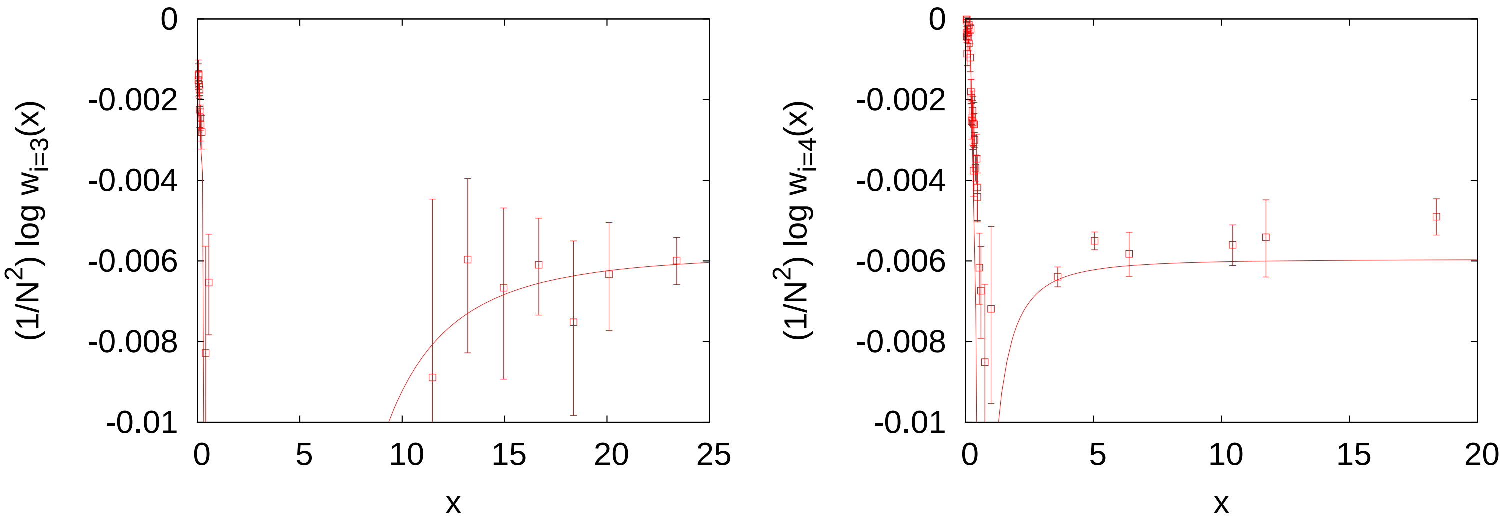
$$\frac{\partial^n \Gamma}{\partial A_{\mu_1}^{a_1} \cdots \partial A_{\mu_n}^{a_n}} = 0 \text{ for } n = 1, \cdots 5-d \Rightarrow \Gamma = \mathrm{O}(A^{6-d})$$

(Up to $(5-d)$-order perturbation $\Rightarrow$ configuration $\in \Omega_5$)

Expected power behaviors:

$$\Phi_i(x) \propto \begin{cases} c_{i,0} x^{7-i} + \cdots & (x \ll 1, i = 2, \cdots, 6) \\ \frac{d_{i,0}}{x^{6-i}} + \cdots & (x \gg 1, i = 1, \cdots, 5) \end{cases}$$

(*) x has the order of the eigenvalues of $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.



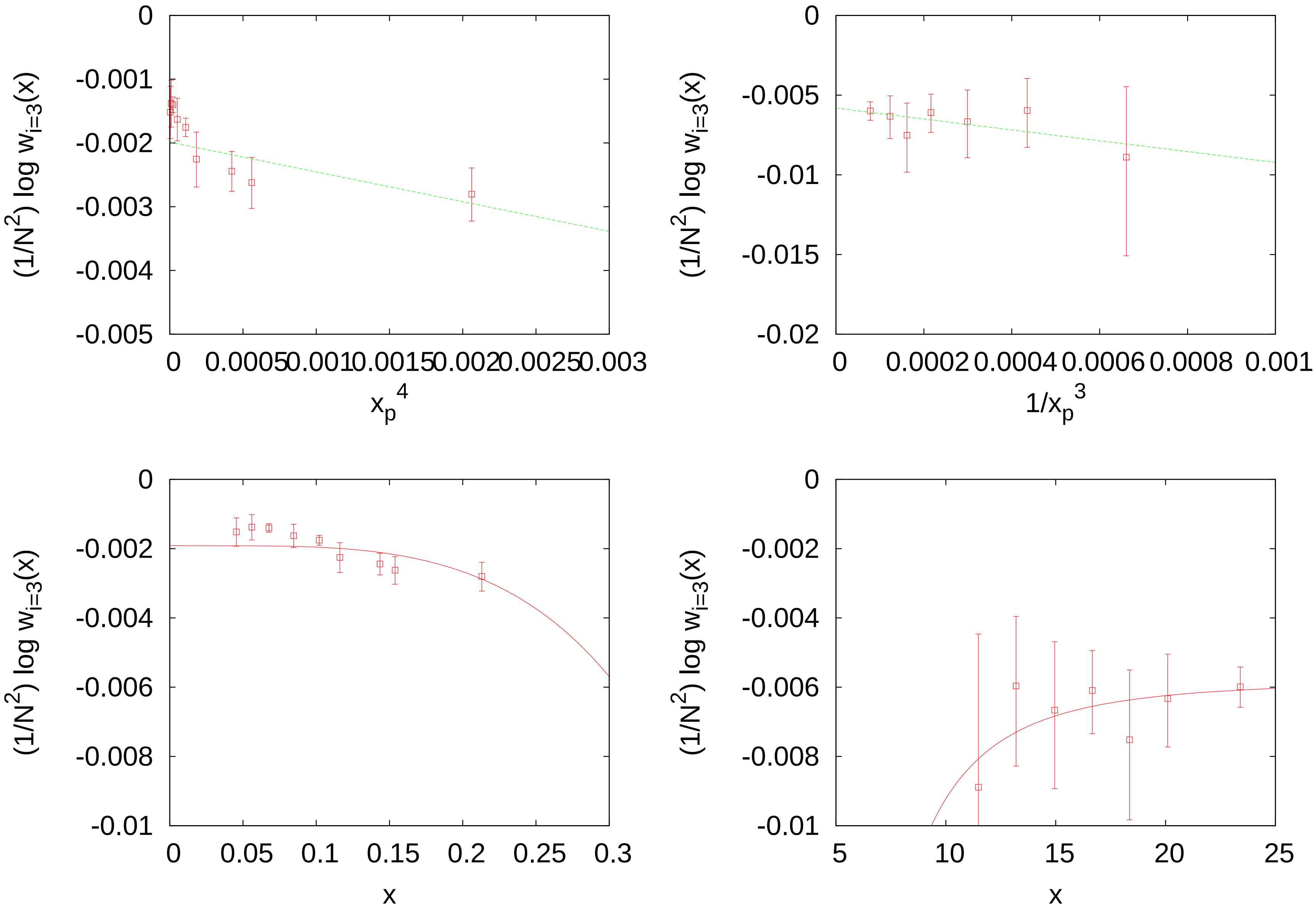


Figure 1: $\frac{1}{N}\log w_{i=3}(x)$ for $N = 8$

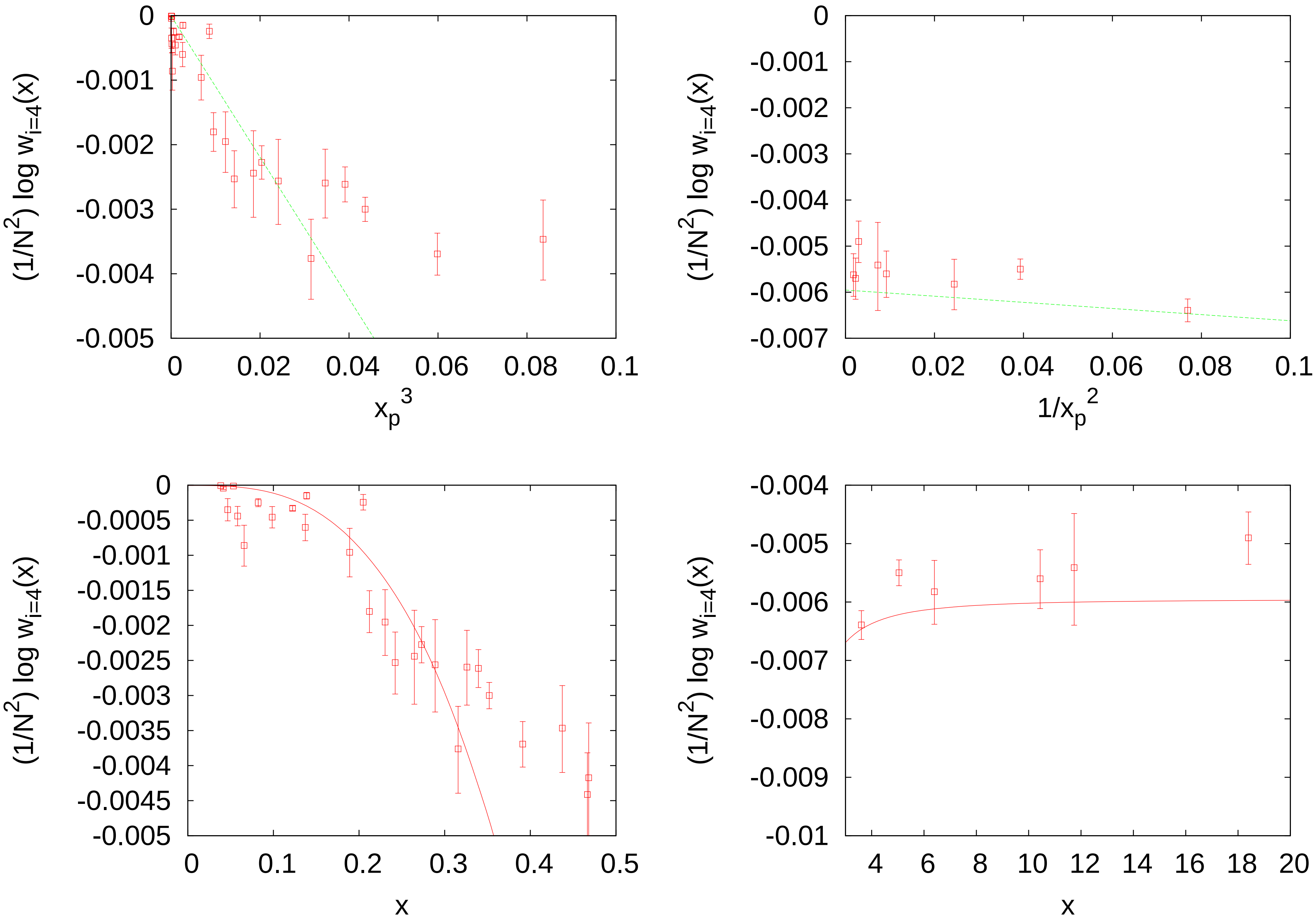


Figure 2: $\frac{1}{N}\log w_{i=4}(x)$ for $N = 8$

Behavior of $\frac{1}{N^2} f_i^{(0)}(x)$

Leading behavior at small x ($x \ll 1$) → $(7 - i)$ directions are shrunk.

- $i = 2, \cdots, 6$: $\rho_i^{(0)}(x) \simeq (\sqrt{x})^{N^2(7-i)} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \frac{7-i}{2x}$
- $i = 1$: **Eigenvalues of A_μ are collapsed to zero.**

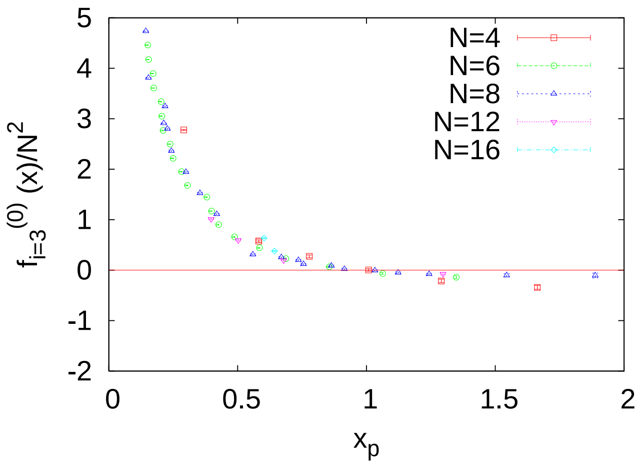
  $\Rightarrow$ **Add the effect of fermionic determinant (polynomial of A_μ with degree $4N^2$).**

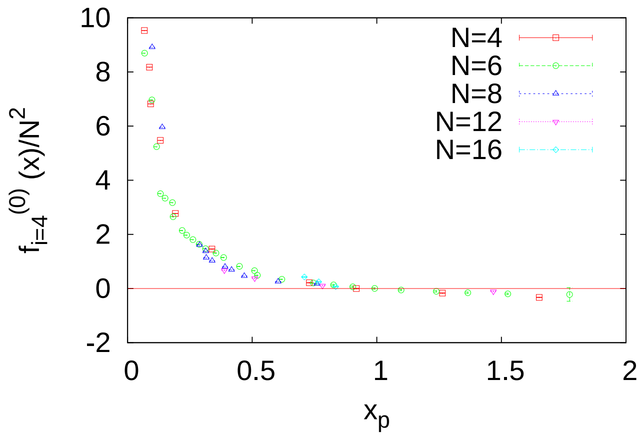
  $\Rightarrow \rho_{i=1}^{(0)}(x) \simeq (\sqrt{x})^{10N^2} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \frac{5}{x}$

At large x: $\frac{1}{N^2} f_i^{(0)}(x) \to 0.$

Ansatz for all x: $\frac{1}{N^2} f_i^{(0)}(x) = \begin{cases} \frac{5}{x} \exp(-b_{i=1} x) & i = 1 \\ \frac{7-i}{2x} \exp(-b_i x) & i = 2, \cdots, 6 \end{cases}$

For $N = 8$ numerical data, we have $b_{i=3,4} \simeq 5$.





Solutions of the equation $\frac{1}{N^2} f_i^{(0)}(x) = -\frac{d}{dx}\left\{ \frac{1}{N^2} \log w_i(x) \right\}$ **:**

Double-peak structure for $i = 3, 4 \rightarrow$ **two solutions** x_s **and** x_l $(x_s < x_l = +\infty)$

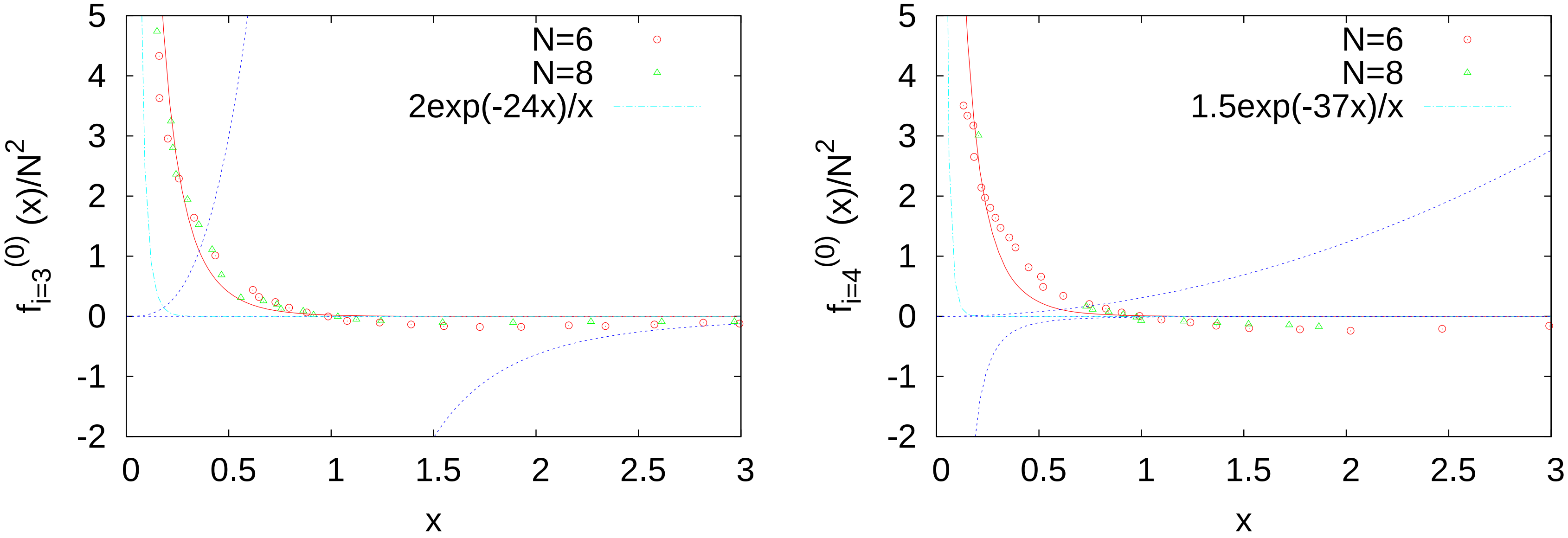
Result of Gaussian Expansion Method (GEM) **T. Aoyama, J. Nishimura and T. Okubo**

- **Symmetry breakdown SO(6) → SO(3):**

  x_l **should dominate for** $i = 3$**.** x_s **should dominate for** $i = 4$**.**

- **For collapsed directions,** $x_s = 0.18$**.**

  To have this solution, the coeffcient b_i **should be** $b_{i=3} \simeq 24, \quad b_{i=4} \simeq 37$**.**



- **Strong finite-N effect $\rightarrow$ For $i = 4$ we expect $x_s \simeq 0.18$ ($b_{i=4} \simeq 37$) at large N.**
- **For $b_{i=3} \simeq 24, b_{i=4} \simeq 37$ (expected behavior at large N) $\rightarrow$ x_l dominates for $i = 3$ while x_s dominates for $i = 4$.**

Evidence for symmetry breakdown SO(6) $\rightarrow$ SO(3)

4 Conclusion

Monte Carlo simulation of 6d IKKT model $\Rightarrow$ spontaneous breakdown of SO(6) symmetry.

Can we understand the emergence of the spacetime?

- **Gaussian Expansion Method: SO(6) $\rightarrow$ SO(3).**
- **Numerical evidence for symmetry breakdown SO(6) $\rightarrow$ SO(3).**

$\boxed{\textbf{Future works}}$

- **Simulation of larger N $\Rightarrow$ study the finite-N effect.**
- **Ultimately, 10d IKKT model $\Rightarrow$4 spacetime.**