Brane Tilings, CS Theories, M2 Branes

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Motivation

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- What is the world volume theory of a stack of M2 branes in M theory?
- Understand Chern Simons (CS) theories better

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- M2 probing CY4 = Cone over H^7

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- Finite renormalization - typically at 1 loop
- $\mathcal{N}$=2 supersymmetry (4 supercharges): no corrections
- Infinite family of SCFT's parametrized by CS terms

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- If put c conditions on CS levels G-c dimensional sub - lattice of SCFT's

Back to 3+1 dimensions

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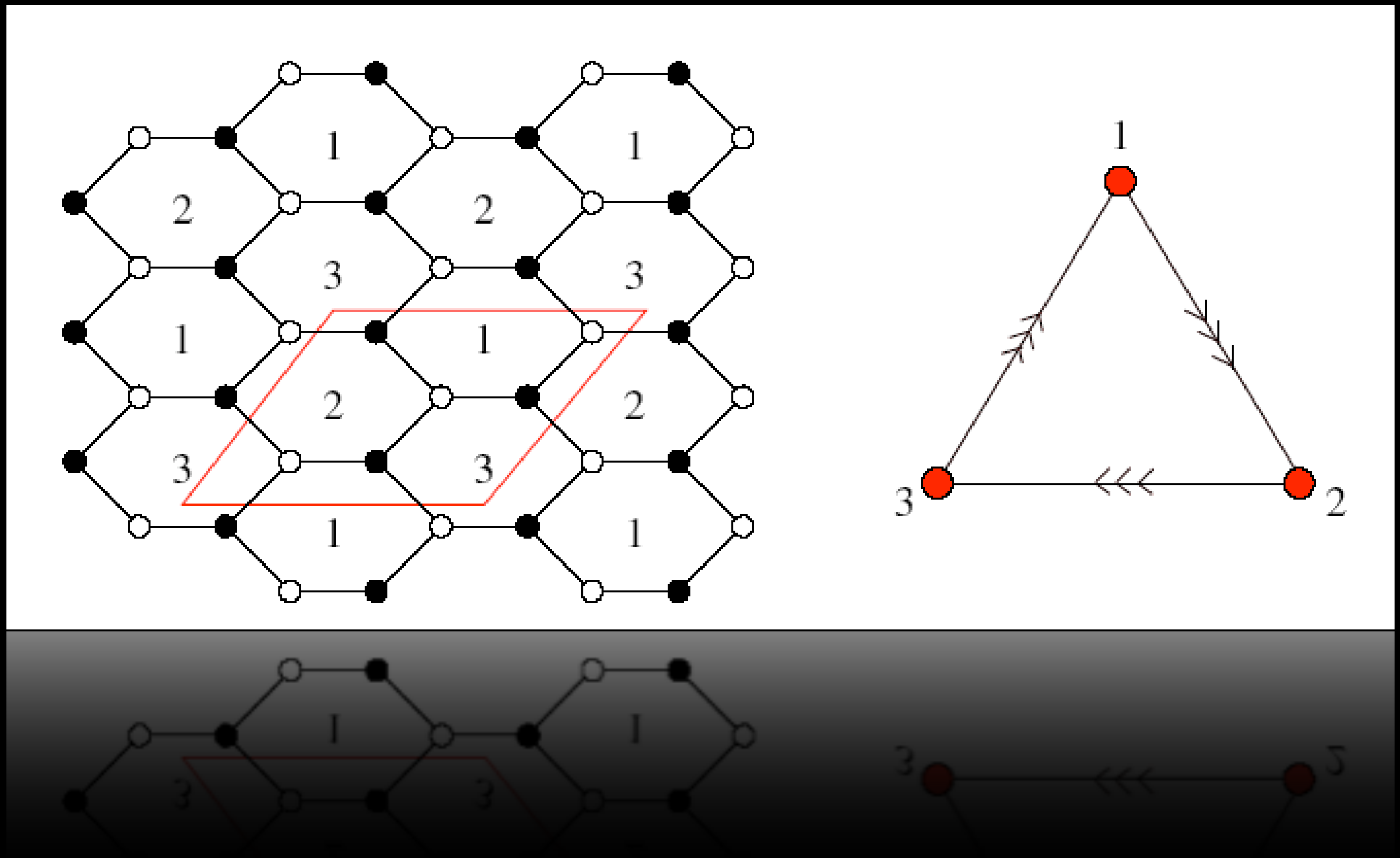
- Best description in terms of "Brane Tilings"



Periodic bipartite tiling

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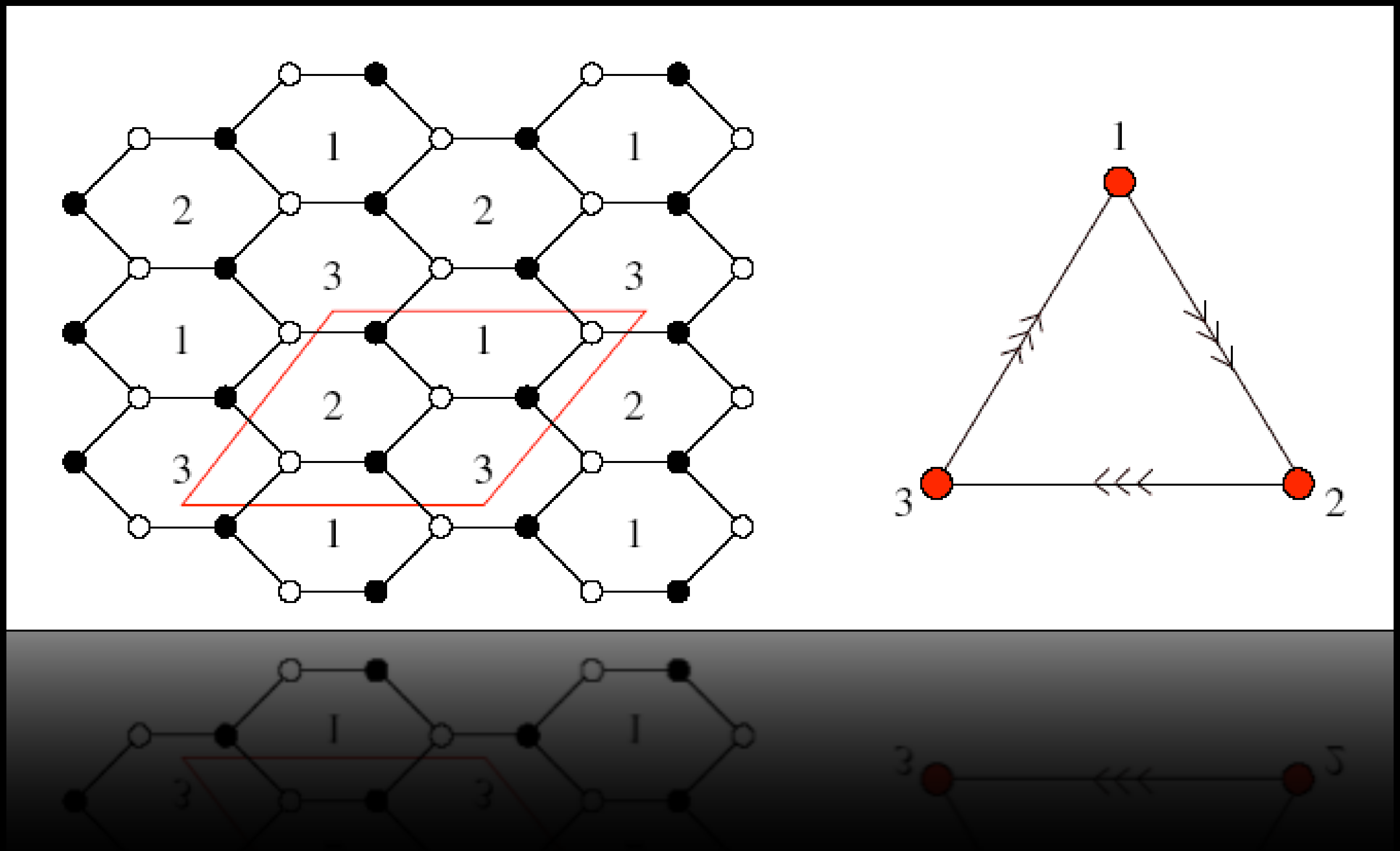
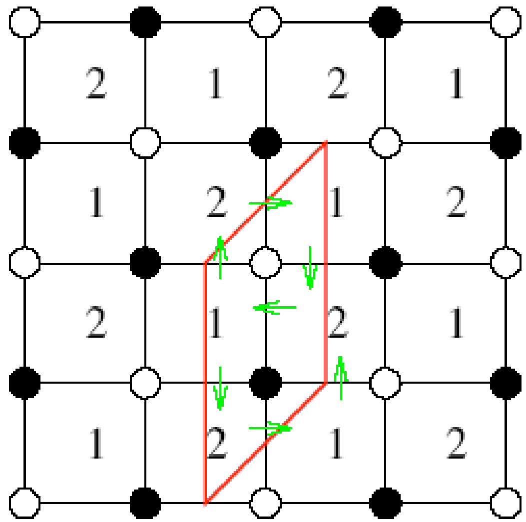
- k valent node - A k-th order interaction term in the superpotential



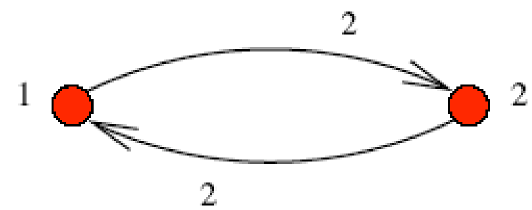
$\mathbb{Z}_3$ orbifold of $\mathbb{C}^3$

Example: Conifold

$$CY_6 = conifold$$



brane tiling



quiver

$$W = X_{12}^{(1)} X_{21}^{(1)} X_{12}^{(2)} X_{21}^{(2)} - X_{12}^{(1)} X_{21}^{(2)} X_{12}^{(2)} X_{21}^{(1)}$$

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- black (white) nodes connected to white (black) only

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- These rules define a unique Lagrangian in 3+1 dimensions

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- Hilbert Series - partition function to count the spectrum of the Chiral Ring

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- Largest known family of SCFT's in 2+1d!

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- Know for 2 terms in W and arbitrary G

The 2+1d Lagrangian

$$
\begin{aligned}
- & \int d^4\theta \sum_{X_{ab}} X_{ab}^{\dagger} e^{-V_a} X_{ab} e^{V_b} \\
+ & \; i \int d^4\theta \sum_{a=1}^{G} k_a \int_0^1 dt V_a \bar{D}^{\alpha} (e^{tV_a} D_{\alpha} e^{-tV_a}) \\
+ & \int d^2\theta W(X_{ab}) + \text{c.c.}
\end{aligned}
$$

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$$C = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ k_1 & k_2 & k_3 & \dots & k_G \end{pmatrix} .$$

Vacuum Equations

$$\begin{aligned} \partial_{X_{ab}} W &= 0 \\ \mu_a(X) := \sum_{b=1}^{G} X_{ab} X_{ab}^\dagger - \sum_{c=1}^{G} X_{ca}^\dagger X_{ca} + [X_{aa}, X_{aa}^\dagger] &= 4 k_a \sigma_a \\ \sigma_a X_{ab} - X_{ab} \sigma_b &= 0 \end{aligned}$$

Forward Algorithm

INPUT 1: Quiver $\rightarrow d_{G\times E} \rightarrow (Q_D)_{(G-2)\times c} = \ker(C)_{(G-2)\times G} \cdot \tilde{Q}_{G\times c}$;

with $d_{G\times E} := \tilde{Q}_{G\times c} \cdot (P^T)_{c\times E}$

INPUT 2: CS Levels $\rightarrow C_{2\times G} \nearrow$

$\nearrow$

INPUT 3: Superpotential $\rightarrow P_{E\times c} \rightarrow (Q_F)_{(c-G-2)\times c} = [\ker P]^t$;

$\downarrow$

$$(Q_t)_{(c-4)\times c} = \begin{pmatrix} (Q_D)_{(G-2)\times c} \\ (Q_F)_{(c-G-2)\times c} \end{pmatrix} \rightarrow$$

OUTPUT: $(G_t)_{4\times c} = [\mathrm{Ker}(Q_t)]^t$

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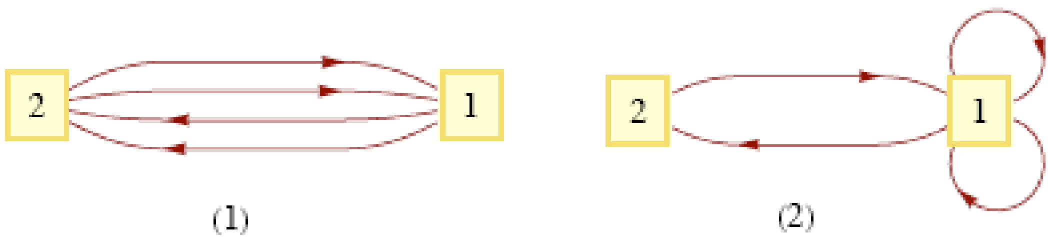
- Number of gauge groups G
- Number of fields in the quiver E

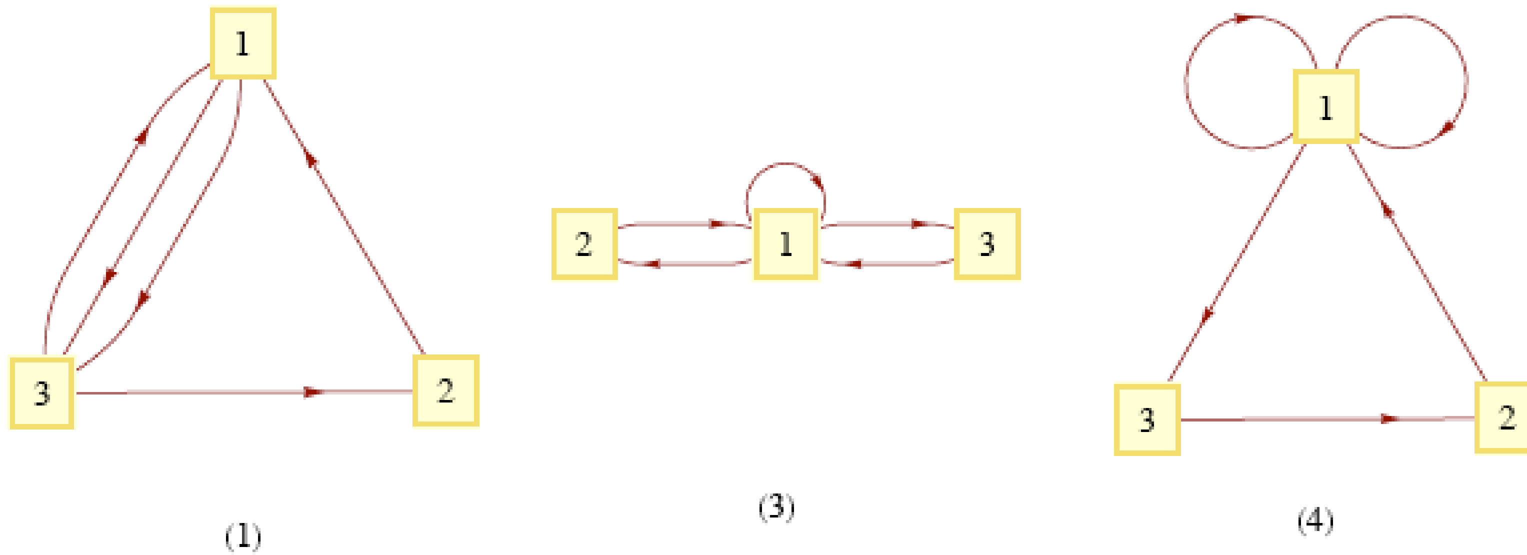
4 fields in the quiver



$$W_{(1)} = \mathrm{Tr}(X^1_{12}X^1_{21}X^2_{12}X^2_{21} - X^1_{12}X^2_{21}X^2_{12}X^1_{21}); \qquad W_{(2)} = \mathrm{Tr}(X_{12}[\phi^1_2, \phi^2_2]X_{21})$$

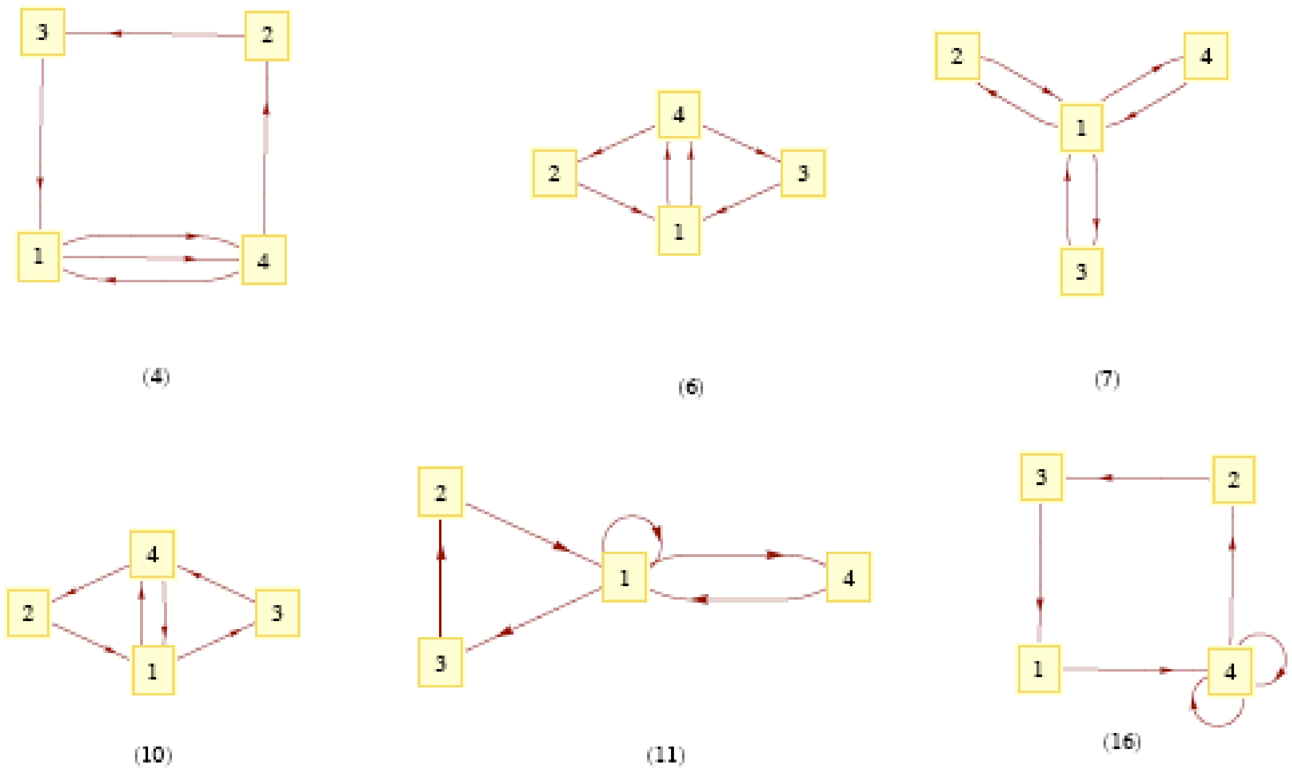
Figure 1: The quivers with 4 fields and 2 nodes. There are 2 solutions and the 2-term superpotentials are also given. The moduli space in both cases is just the trivial CY 4-fold $\mathbb{C}^4$.

5 fields in the Quiver



$$W_{(1)} = \mathrm{Tr}(X_{21}X_{13}^{1}X_{31}X_{13}^{2}X_{32} - X_{21}X_{13}^{2}X_{31}X_{13}^{1}X_{32}) \; ;$$
$$W_{(3)} = \mathrm{Tr}(X_{21}\phi_1 X_{13}X_{31}X_{12} - X_{21}X_{13}X_{31}\phi_1 X_{12}) \; ;$$
$$W_{(4)} = \mathrm{Tr}(X_{21}[\phi_1^1, \phi_1^2]X_{13}X_{32}).$$

6 fields in the Quiver



$$W_{(4)} = \mathrm{Tr}(X_{31}X^1_{14}X_{41}X^2_{14}X_{42}X_{23} - X_{31}X^2_{14}X_{41}X^1_{14}X_{42}X_{23})\ ;$$

$$W_{(6)} = \mathrm{Tr}(X_{42}X_{21}(X^1_{14}X_{43}X_{31}X^2_{14} - X^2_{14}X_{43}X_{31}X^1_{14}))\ ;$$

$$W_{(7)} = \mathrm{Tr}(X_{12}X_{21}(X_{14}X_{41}X_{13}X_{31} - X_{13}X_{31}X_{14}X_{41}))\ ;$$

$$W_{(10)} = \mathrm{Tr}(X_{42}X_{21}X_{14}X_{41}X_{13}X_{34} - X_{42}X_{21}X_{13}X_{34}X_{41}X_{14})\ ;$$

$$W_{(11)} = \mathrm{Tr}(X_{32}X_{21}\phi_1X_{14}X_{41}X_{13} - X_{32}X_{21}X_{14}X_{41}\phi_1X_{13})\ ;$$

$$W_{(16)} = \mathrm{Tr}(X_{42}X_{23}X_{31}X_{14}[\phi^1_4, \phi^2_4])$$

G=2, E=4, Model I

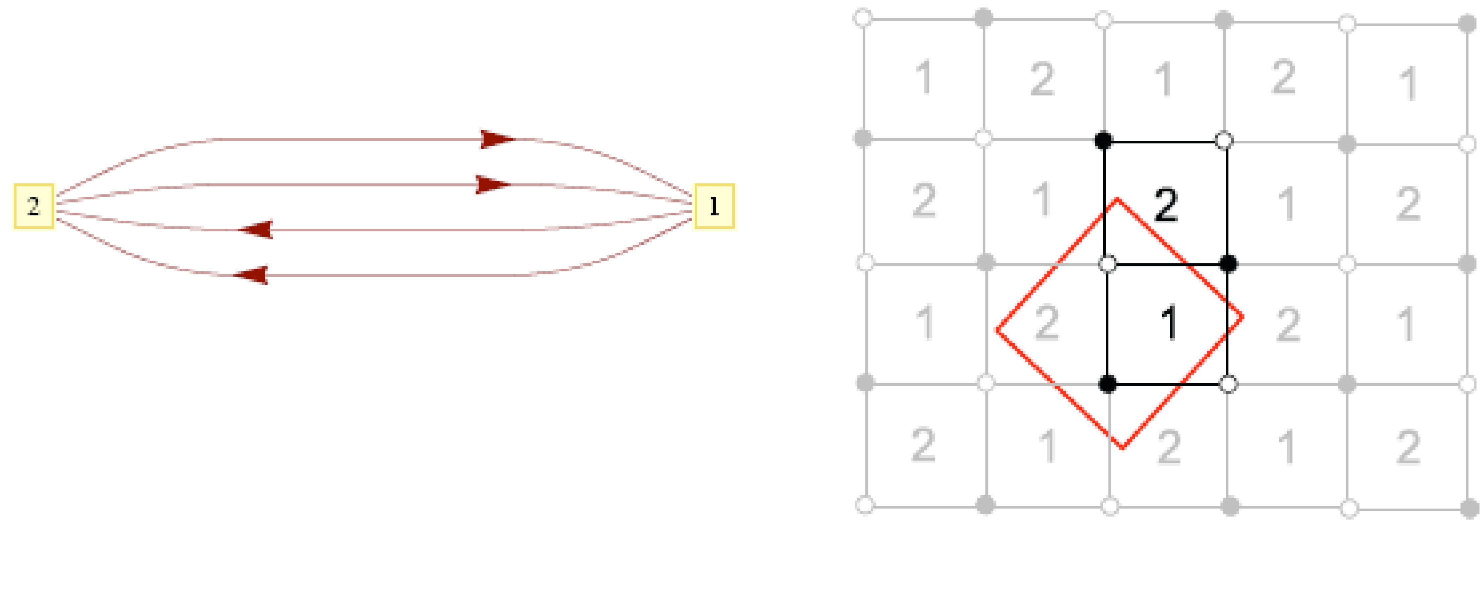


Figure 1: (i) Quiver diagram for the ABJM theory. (ii) Tiling for the ABJM theory.

G=2, E=4, Model II

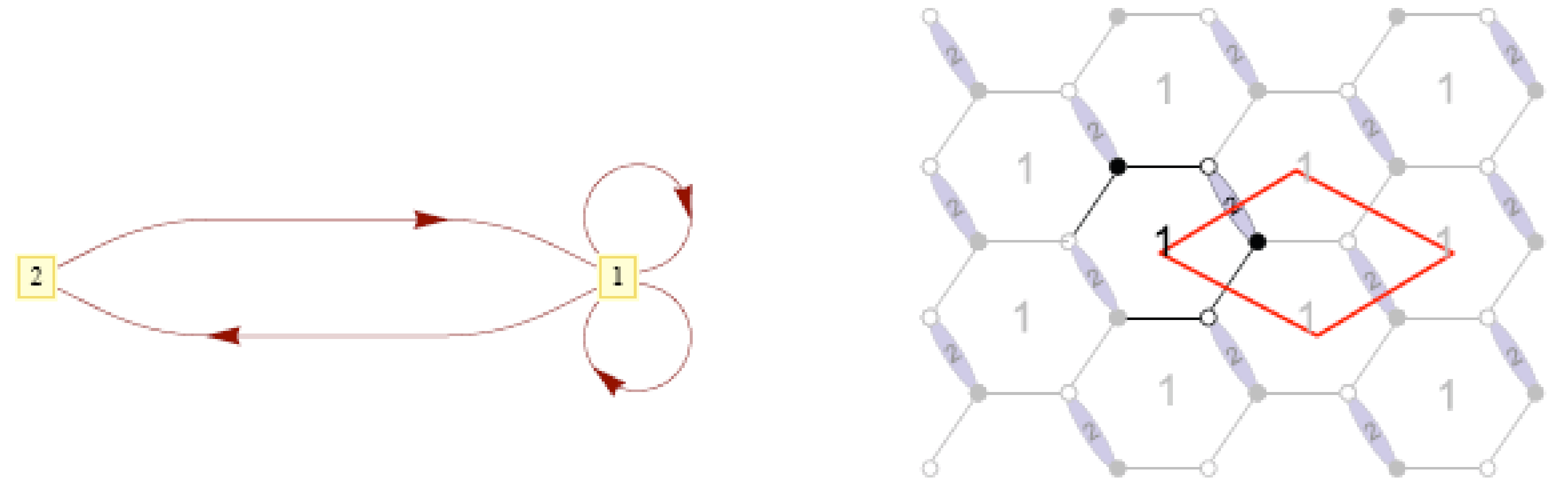


Figure 3: (i) Quiver diagram for phase 2 of the $\mathbb{C}^4$ theory. (ii) Tiling for phase 2 of the $\mathbb{C}^4$ theory.

G=3, E=5, Model I

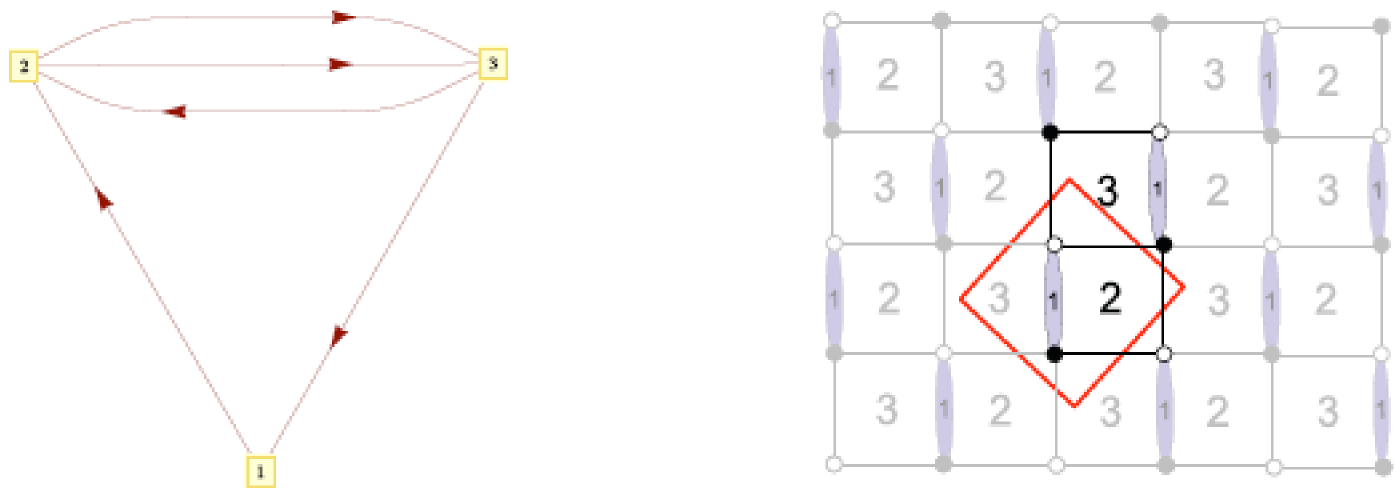


Figure 7: (i) Quiver of phase 2 of the $\widetilde{\mathcal{C}} \times \mathbb{C}$ theory. (ii) Tiling of phase 2 of the $\widetilde{\mathcal{C}} \times \mathbb{C}$ theory.

G=4, E=6, Model IV

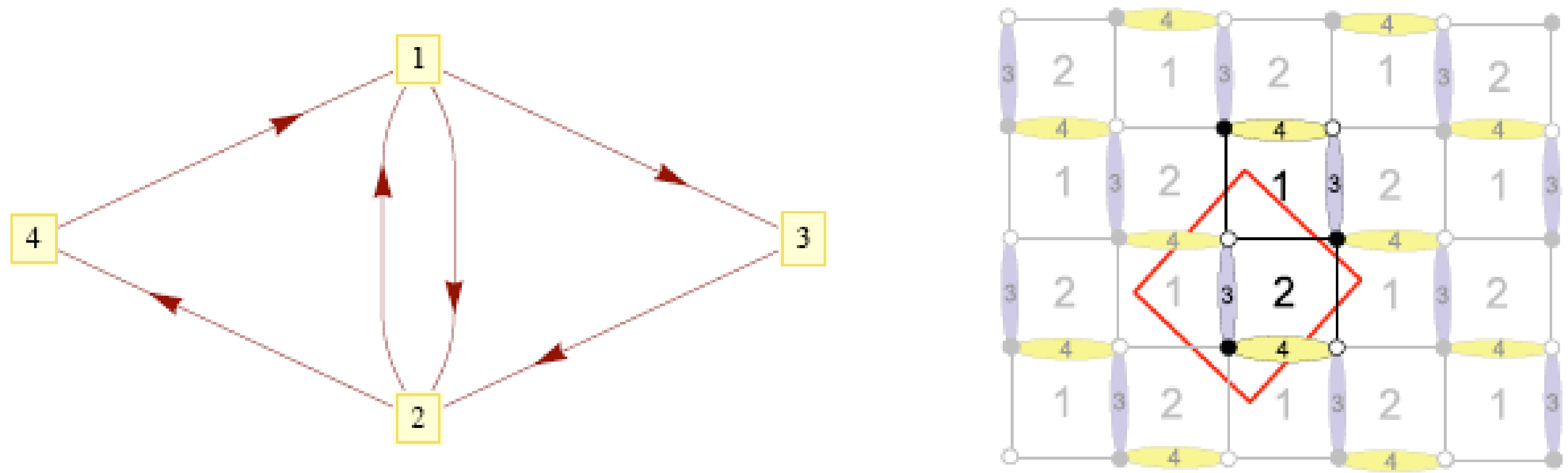


Figure 11: (i) Quiver diagram for phase 2 of the D_3 theory. (ii) Tiling for phase 2 of the D_3 theory.

Counting Quivers
1 Hexagon

$$
\begin{aligned}
f_1(t) &= \frac{1}{(1-t)(1-t^2)(1-t^3)} \\
&= 1 + t + 2t^2 + 3t^3 + \ldots
\end{aligned}
$$

Counting Quivers
Chessboard Tiling

$$\begin{aligned} f_2(t) &= \frac{1-t^6}{(1-t)(1-t^2)^2(1-t^3)(1-t^4)} \\ &= 1+t+3t^2+4t^3+8t^4+\dots \end{aligned}$$

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