大阪素粒子セミナー　文献紹介

Seiberg Duality in Chern-Simons Theory

A. Giveon and D. Kutasov　arXiv:0808.0360

とその周辺.

☆ 3D $\mathcal{N}=2$ supersymmetric Chern-Simons theory で strong-weak coupling の Seiberg-type duality を brane configuration を使って議論する.

1. Introduction
2. Review of brane configurations
   - ⅡA → 4D theories
   - ⅡB → 3D theories
   - Induced CS term
3. Electric theory
4. Magnetic theory and duality

1. Introduction

◦ Why CS ?

black 2-brane solution in M/IID SUGRA

$\downarrow$ near-horizon

M-theory on $AdS_4 \times S^7 \xleftrightarrow[\text{dual}]{} 3D\ \mathcal{N}=8$ SCFT.

• 3D YM

F^2 term は dim. 4 → coupling は dimensionful

→ IRで strongly coupled.

scale inv. であるには dim. 3 terms と主張 (Schwarz, 2004)

⇒ Chern-Simons term.

• 2008年の進展色々.

BLG (3D $\mathcal{N}=8$, 3-Lie alg.) → $SU(2)\times SU(2)$ CS

ABJM (3D $\mathcal{N}=6$, $U(N)\times U(N)$ CS)

• SUSYの低いCS.

3D $\mathcal{N}=2$ with matter chiral superfield.

~ nontrivial IR dynamics. e.g.) Gaiotto, Yin 0704.3740.

add superpot. interaction (matter に) → ~~conf. sym.~~ → nontrivial RG flows

$\mathcal{N}=3$ CS = $\mathcal{N}=2$ CS + superpot.

- Seiberg duality

4D $\mathcal{N}=1$ SUSY gauge theory を調べる important tool.

「SU(N_c) SQCD with $\underbrace{N_f\text{-flavors}}_{\text{quark } Q^i,\, \tilde{Q}_i \ (i=1\sim N_f)}$ の理論の IR limit は

SU(N_f-N_c) SQCD with $\underbrace{N_f\text{-flavors}}_{\text{dual quark } q_i,\, \tilde{q}^i}$ + $\underbrace{N_f^2 \text{ gauge singlet}}_{\text{meson } M^i_j = Q^i\tilde{Q}_j}$

で記述される」

2. Review of brane configurations

c.f.) Giveon-Kutasov のレビュー
hep-th/9802067

∘ IIA → 4D theories

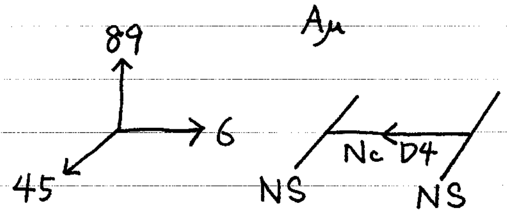
・ $\mathcal{N}=2$

	0	1	2	3	4	5	6	7	8	9
NS	○	○	○	○	○	○				
N_c D4	○	○	○	○			○			

→ $\frac{1}{4}$ SUSY = 8 supercharges

→ $U(N_c)$.

A_μ　　X_5, X_6 ~ adj. cpx. scalar　　Neumann



⇒ $\mathcal{N}=2$ vector multiplet

	0	1	2	3	4	5	6	7	8	9
D6	○	○	○	○				○	○	○
D4	○	○	○	○			○			

Neumann

A_6, X_7, X_8, X_9
~ 2 cpx. scalar

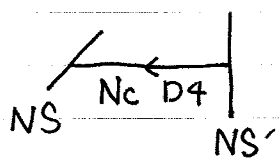
⇒ $\mathcal{N}=2$ hyper multiplet

・ $\mathcal{N}=1$

NSの1つを回す : $\mathcal{N}=2 \to \mathcal{N}=1$.

	0	1	2	3	4	5	6	7	8	9
NS	○	○	○	○	○	○				
NS′	○	○	○	○					○	○
N_c D4	○	○	○	○			○			

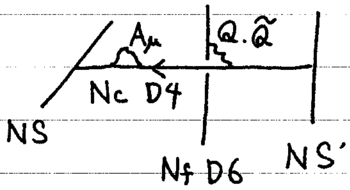
Neumann



⇒ $\mathcal{N}=1$ vector multiplet

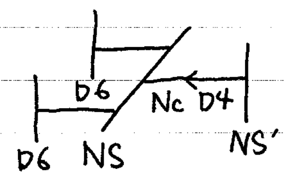
◦ 4D $\mathcal{N}=1$ SQCD, N_f flavors

	0	1	2	3	4	5	6	7	8	9
NS	◦	◦	◦	◦	◦	◦				
NS′	◦	◦	◦	◦					◦	◦
N_f D6	◦	◦	◦	◦				◦	◦	◦
N_c D4	◦	◦	◦	◦			◦			



D6 は SUSY を破らずに加えられる.
N_f chiral quarks : $Q^i, \tilde{Q}_i$

$\updownarrow$ Hanany-Witten transition



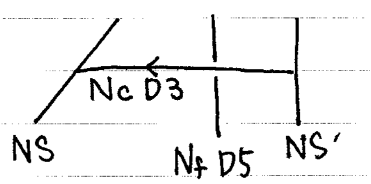
◦ IIB → 3D theories

IIA から x^3 方向を T-dualize

	0	1	2	3	4	5	6	7	8	9
NS	◦	◦	◦	◦	◦	◦				
NS′	◦	◦	◦	◦					◦	◦
N_f D5	◦	◦	◦					◦	◦	◦
N_c D3	◦	◦	◦				◦			

Neumann

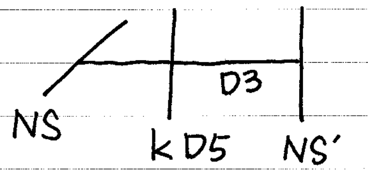
A_μ $(\mu=0,1,2)$, $\sigma = X_3$



∴ 3D $\mathcal{N}=2$ gauge theory on (012)

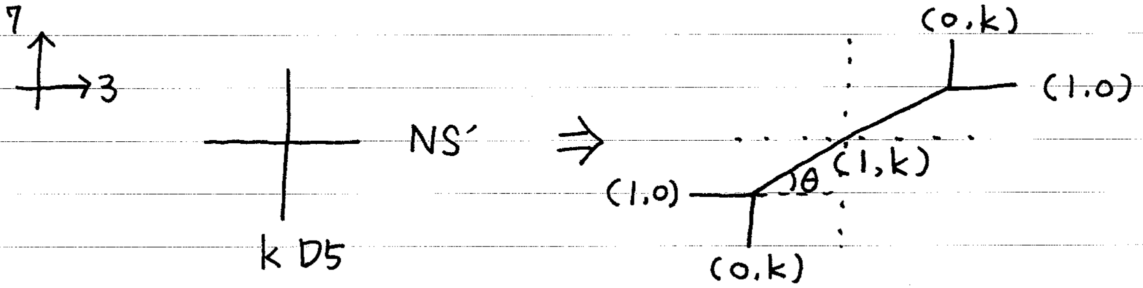
c.f.) Hanany-Witten hep-th/9611230

- Chern-Simons term. (Kitao, Ohta, Ohta hep-th/9808111)

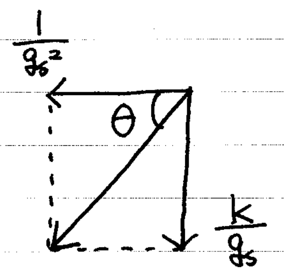


k D5 が NS′ に交わる.

⇒ (1, k) fivebrane.



$$\begin{cases} T_{NS} = \dfrac{1}{g_s^2 l_s^6} \\ T_{D5} = \dfrac{1}{g_s l_s^6} \end{cases}$$



$\tan\theta = g_s k$

- D3 上の eff. field theory.

x^6 方向は線分 : $0 \le x^6 \le L$.

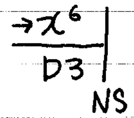
$$S_{D3} = -\frac{1}{4g_{4D}^2}\int d^3x\, dx^6\, F_{MN}F^{MN} \quad (M.N : 0.1.2.6)$$

$$= -\frac{1}{4g_{4D}^2}\int d^3x\, dx^6 \Big[F_{\mu\nu}F^{\mu\nu} + 2(\partial_6 A_\mu)^2 \quad (\mu\nu : 0.1.2)$$
$$+ 2(\partial_\mu A_6)^2 - 4\partial_\mu A_6\, \partial_6 A^\mu \Big]$$

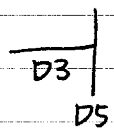
$$\delta S_{D3} = \cdots + \frac{1}{g_{4D}^2}\int d^3x\, dx^6\, \partial_6 \left(\delta A_\mu (\partial_6 A^\mu - \partial^\mu A_6) \right)$$

$$\therefore\ \delta S_{D3}\Big|_{\text{boundary}} = \left[\frac{1}{g_{4D}^2}\int d^3x\, \delta A_\mu (\partial_6 A^\mu - \partial^\mu A_6) \right]_{x^6=0}^{x^6=L}.$$

・boundary conditions

NS 

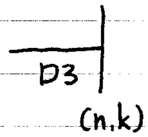
: $F_{\mu 6} = \partial_\mu A_6 - \partial_6 A_\mu = 0$

D5 

: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = 0$

or $\epsilon_{\mu\nu\lambda} F^{\nu\lambda} = 0$

$\downarrow$

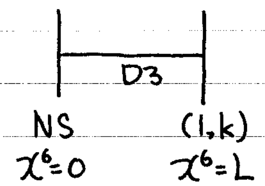
(n,k) 

: $\partial_\mu A_6 - \partial_6 A_\mu - a\frac{k}{n}\epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda = 0$

n : # NS

k : # D5

for $^\exists a$ $\left(a = \frac{g_{4D}^2}{2\pi}\right)$

・

$x^6=0$ は boundary term は消える

$x^6=L$ は消えない. $\to$ additional contrib.

・$S_{CS} = \frac{k}{4\pi}\int d^3x\, \epsilon_{\mu\nu\lambda} A^\mu \partial^\nu A^\lambda$

$\delta S_{CS} = \frac{k}{2\pi}\int d^3x\, \delta A_\mu \epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda$

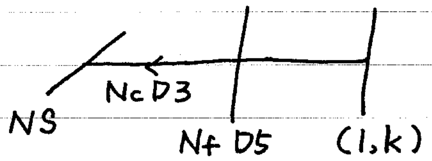
$\Rightarrow \delta S_{D3}\big|_{\text{boundary}} + \delta S_{CS} = 0$

※このScsは boundary cond. の式を $x^6=L$ で課して解くと.

S_{D3} の $\partial_\mu A_6 \partial_6 A^\mu$ の所から出てくる.

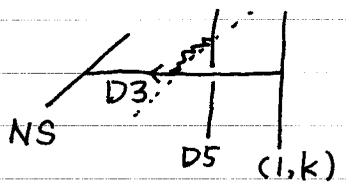
3. Electric theory.

- $\mathcal{N}=2$ $U(N_c)$ CS with N_f fundamentals $Q^i, \tilde{Q}_i$.



- D5を動かす ~ moduli.

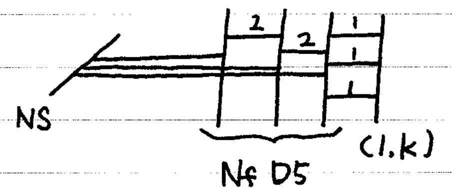
45方向



cpx. mass for $Q^i, \tilde{Q}_i$

superpot. $W = m_i Q^i \tilde{Q}_i$

N_f D5を6方向にバラす.



complex moduli

$$\dim \mathcal{M} = \begin{cases} N_f^2 & (N_f < N_c) \\ 2N_cN_f - N_c^2 & (N_f \geq N_c) \end{cases}.$$

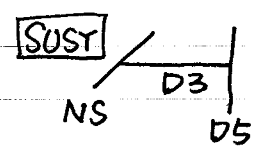
-

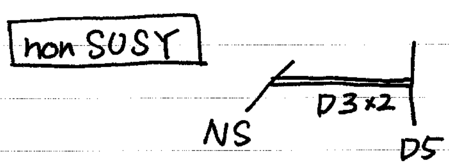
	0	1	2	3	4	5	6	7	8	9
(1,k)	o	o	o	★				★	o	o
D5	o	o	o					o	o	o
D3	o	o	o				o			

D5 − (1,k)
1 cpx. dof.

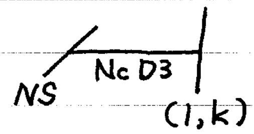
D5 − D5
2 cpx. dof.

- s-rule : 1本のD5に対し、NSから延びたD3は1本しか端をもてない. (SUSYを破る).

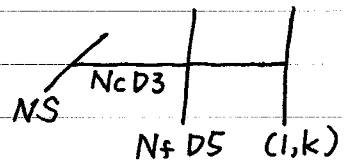




・ SUSYを破らず、NSと(1,k)にのびるD3の本数.



のとき、 $N_c \le k$ (Bergman, Hanany, Karch, Kol / Ohta 9908210.



のとき、 N_f本は NS − D5 − (1,k) にのびる.

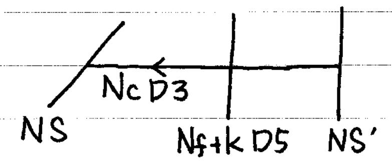
$\Rightarrow \quad N_c - N_f \le k$

$N_f + k - N_c \ge 0$.

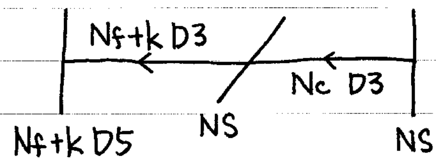
c.f.) Witten 9903005.

4. Magnetic theory and duality

- electric config. → magnetic one

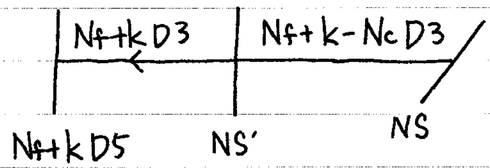


⇓



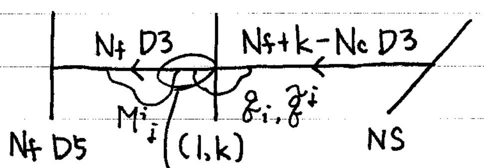
((p,q) fivebrane が (r,s) fivebrane と交差する
⇓
間に $|ps-qr|$ D3 ができる.)

⇓



⇓

magnetic $\mathcal{N}=2$ $U(N_f+k-N_c)$ CS with N_f matters



- N_f fundamentals : $q_i, \tilde{q}^i$
- N_f^2 singlet : $M^i{}_j$
- coupling : superpot. $W = M^i_j q_i \tilde{q}^j$

moduli は electric のときと同じ.

- proposal of duality

electric $U(N_c)$ CS $\longleftrightarrow$ magnetic $U(N_f+k-N_c)$ CS

identification : $M^i{}_j = Q^i \tilde{Q}_j$

※ ele の SUSY vac. $N_f+k-N_c \geq 0 \leftrightarrow$ gauge grp. の rank ≥ 0.

◦ coupling : weak ↔ strong.

・'t Hooft coupling

$$\begin{cases} \lambda_{ele} = \dfrac{N_c}{k} \\ \lambda_{mag} = \dfrac{N_f + k - N_c}{k} \end{cases}$$

イ) ele. が weak : N_f, N_c fixed で $k \to \infty$

このとき. $\lambda_{mag} \sim O(1)$.

ロ) mag. が weak : N_f, $N_f + k - N_c$ fixed で $k \to \infty$

このとき. $k \sim N_c$ でないといけない $\Rightarrow$ $\lambda_{ele} \sim O(1)$.
$(k = N_c)$　$(\lambda_{ele} = 1)$.

$\Rightarrow$ duality は strong ↔ weak である. と主張.