ゲージ重力対応における「バブリング」の考え方

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Introduction

- Candidates for nonperturbative definition of superstring

  ~ matrix model, large-N gauge theory

- Proposed models

  ~reduction of 10D $\mathcal{N}=1$ SYM to lower dimensions

1.	0D	IIB matrix model	IKKT	type IIB superstring
2.	1D	Matrix theory	BFSS	M theory
3.	2D	Matrix string theory	DVV	type IIA superstring
4.	4D	$\mathcal{N}=4$ SYM	Maldacena	type IIB superstring on $AdS_5 \times S^5$

- In 1 and 2, space-time is dynamically generated through distribution of matrix eigenvalues `emergence of space-time'

  fluctuation of eingenvalue dist.~ particle

  talks by Kawai, Aoki, Umetsu , Nishimura and Sugino

- A similar interpretation for 4?
- $AdS_5 \times S^5$ *is directly derived from SYM?*
- *This partially succeeds in bubbling AdS* Lin-Lunin-Maldacena

Contents

Analogy with c=1 string

matrix quantum mechanics (open string picture) $\Longleftrightarrow$ 2D string with linear dilaton b.g. (closed string picture)

open-closed duality

$$S = \int dt \, \mathrm{Tr}\left(\frac{1}{2}\dot{\Phi}(t)^2 + \frac{1}{2}\Phi(t)^2\right)$$

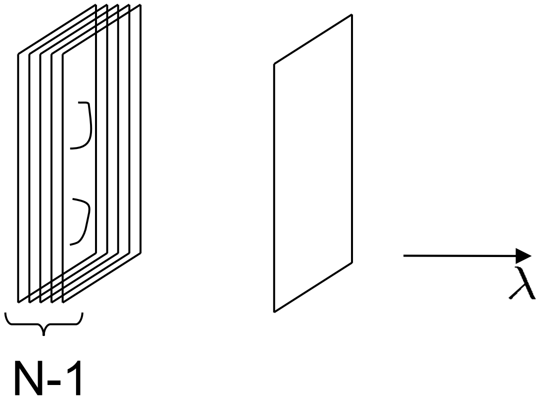
$$\Phi = U \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N) U^\dagger$$

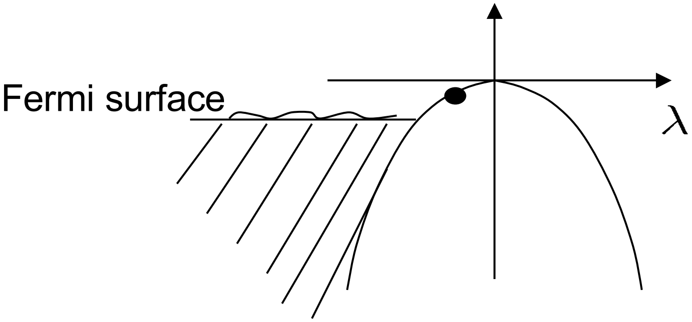
$(\lambda_1, \lambda_2, \cdots, \lambda_N)$: N free fermions

$$S_{2D} = \frac{1}{8\pi}\int d^2\sigma \sqrt{\hat{g}}(g^{ab}\eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\sqrt{2}\hat{R}\phi - \mu\phi e^{\sqrt{2}\phi}) + S_{ghost}$$

$$X^\mu = (\sqrt{2}t, \phi)$$

$$\lambda \sim -e^{-\phi/\sqrt{2}}$$





Das-Jevicki
Gross-Klebanov
Polchinski

Mcgreevy-Verlinde

Kuroki's talk

eigenvalue distribution ~ fermi surface ←——→ one dimension

fluctuation of eigenvalue distribution ~waves on fermi surface ←——→ closed string tacyon

isolated eigenvalue ←——→ ZZ brane

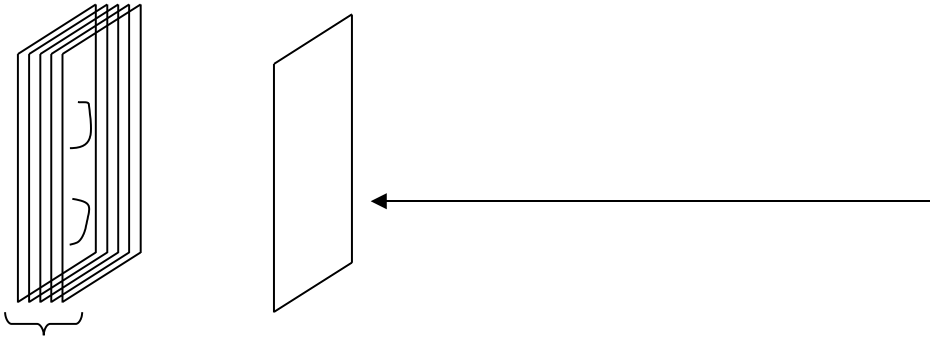
(open string picture) ⟺ (closed string picture)

N=4 SYM ~theory on D3-branes

open-closed duality

type IIB string on AdS$_5$xS^5

AdS$_5$xS^5 ~near horizon limit of D3-brane solution



← D3-brane in AdS$_5$xS^5

chiral primary states

⇨ matrix model ~ free fermions

eigenvalue distribution ↔ part of S^5

fluctuation of eigenvalue distribution ~waves ↔ KK graviton

on fermi surface

isolated eigenvalue ↔ (AdS) giant graviton (D3-brane)

Bubbling AdS

Landau problem

- charged particle in magnetic field in 2D

$$H_L = \frac{1}{2}(p_1 + x_2)^2 + \frac{1}{2}(p_2 - x_1)^2 = H_{HO} - L$$

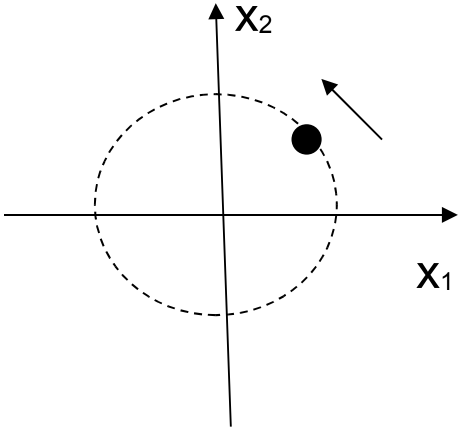
$$H_{HO} = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}(x_1^2 + x_2^2)$$ 2D harmonic oscillator

$$L = x_1 p_2 - x_2 p_1$$ angular momentum

- classical ground state (lowest Landau level)

$$H_L = 0 \Longleftrightarrow H_{HO} = L \Longleftrightarrow x_1 = p_2 = \dot{x}_2 \qquad x_2 = -p_1 = -\dot{x}_1$$

$$H_{HO} = L = x_1^2 + x_2^2$$



- complex coordinate $z = \frac{1}{\sqrt{2}}(x_1 + ix_2)$
- creation-annihilation operators

$$\hat{c}_1 = \frac{1}{\sqrt{2}}(z + \frac{\partial}{\partial z^*}) \quad \hat{c}_1^\dagger = \frac{1}{\sqrt{2}}(z^* - \frac{\partial}{\partial z})$$

$$\hat{c}_2 = \frac{1}{\sqrt{2}}(z^* + \frac{\partial}{\partial z}) \quad \hat{c}_2^\dagger = \frac{1}{\sqrt{2}}(z - \frac{\partial}{\partial z^*}) \qquad [\hat{c}_i, \hat{c}_j^\dagger] = \delta_{ij}$$

$$\Downarrow$$

$$H_L = 2\hat{c}_1^\dagger \hat{c}_1 + 1 \qquad H_{HO} = \hat{c}_1^\dagger \hat{c}_1 + \hat{c}_2^\dagger \hat{c}_2 + 1 \qquad L = \hat{c}_2^\dagger \hat{c}_2 - \hat{c}_1^\dagger \hat{c}_1$$

- eigenstates

$$\Phi_{n_1,n_2}(z, z^*) = (\hat{c}_1^\dagger)^{n_1}(\hat{c}_2^\dagger)^{n_2}\Phi_{0,0} \qquad \Phi_{0,0}(z, z^*) = e^{-|z|^2}$$

- Lowest Landau level $H_L = 1 \Longleftrightarrow n_1 = 0$

$$\Phi_n(z, z^*) \equiv \Phi_{0,n}(z, z^*) = z^n e^{-|z|^2}$$

J=n

holomorphic except for exponential factor

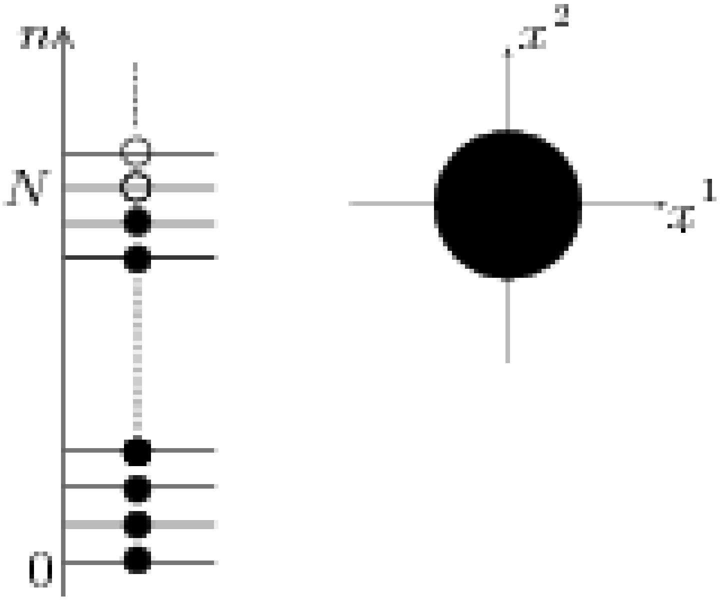
system of N fermions

subtract the minimum $\sum_{n=0}^{N-1} n = \frac{N(N-1)}{2}$ from the total angular momentum and rename it as J

$$N \to \infty, \quad \hbar \to 0 \text{ with } N\hbar = \text{fixed}$$

state with J=0

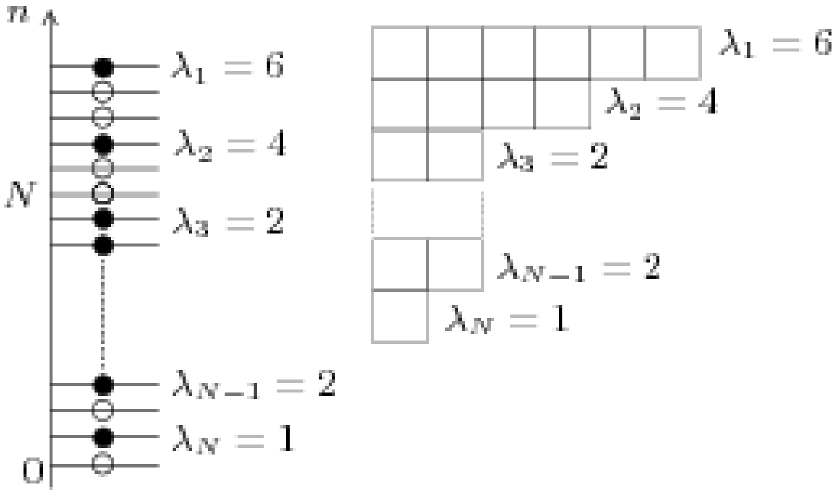
AdS5 x S5



classification of states with the total angular momentum J

$$\langle\lambda\rangle = (\lambda_1,\lambda_2,\cdots,\lambda_N) \quad \text{with} \quad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \geq 0 \quad \text{and} \quad \sum_{i=1}^{N} \lambda_i = J$$

$$\begin{aligned}\Phi_{\langle\lambda\rangle} &= \det_{i,j} \Phi_{N-i+\lambda_i}(z_j,\bar{z}_j)\\ &= \det_{i,j} z_j^{N-i+\lambda_i} \times e^{-\sum_{k=1}^{N}|z_k|^2}\end{aligned}$$



Young tableau specifies irreducible representations of GL(N,C) and S_J simultaneously

Schur polynomial

Kimura's talk

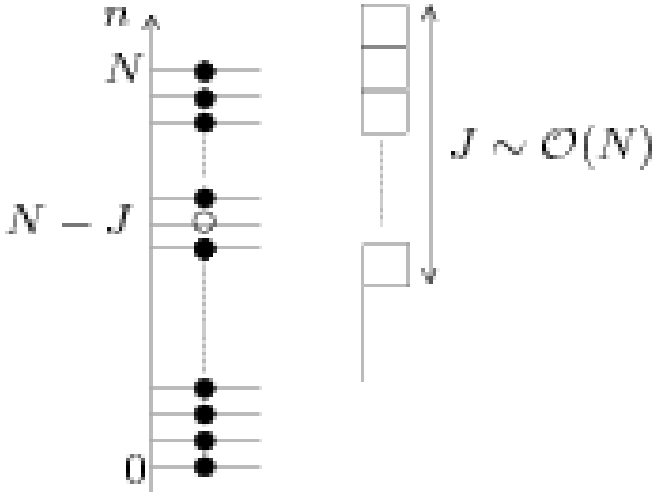
$$\chi_{\langle\lambda\rangle}(Z) = \frac{1}{J!}\sum_{\sigma\in S_J}\chi_{\langle\lambda\rangle}(\sigma)\sum_{i_1,i_2,\cdots,i_J=1}^{N} Z_{i_1 i_{\sigma(1)}} Z_{i_2 i_{\sigma(2)}}\cdots Z_{i_J i_{\sigma(J)}} = \text{l.c. of } \prod_{a=1}^{K}\mathrm{Tr}(Z^{J_a})$$

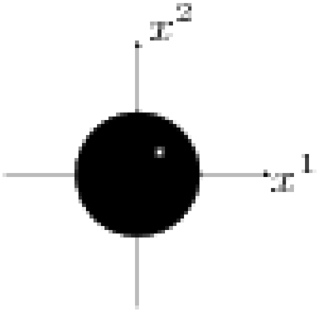
$$= \frac{\det_{i,j} z_j^{N-i+\lambda_i}}{\Delta(z)} \qquad \Delta(z) = \prod_{i<j}(z_i - z_j)$$

Corley-Jevicki-Ramgoolam

$$\Longrightarrow \quad \boxed{\Phi_{\langle\lambda\rangle} = \chi_{\langle\lambda\rangle}(Z)\Delta(z)e^{-\sum_{i=1}^{N}|z_i|^2}}$$

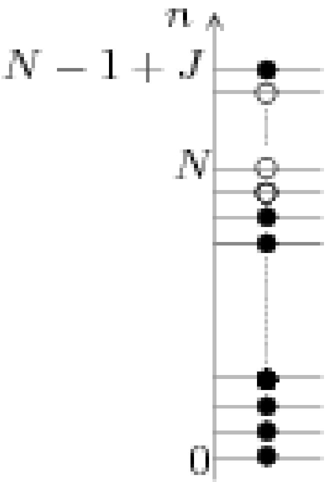
giant graviton

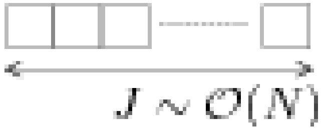


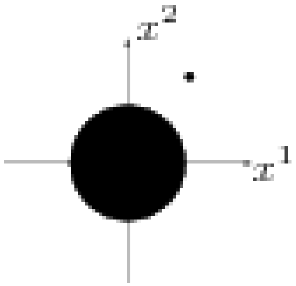


$$\chi_{\langle\lambda\rangle}(Z) = \det{}_J Z = \frac{1}{J!}\epsilon_{i_1 i_2 \cdots i_J k_1 k_2 \cdots k_{N-J}}\epsilon_{j_1 j_2 \cdots j_J k_1 k_2 \cdots k_{N-J}} Z_{i_1 j_1} Z_{i_2 j_2} \cdots Z_{i_J j_J}$$

AdS giant graviton





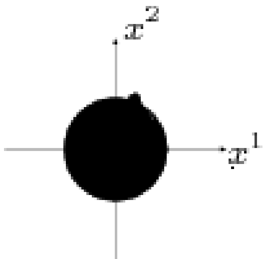


$$\frac{1}{J!}\sum_{\sigma\in S_J} Z_{i_1 i_{\sigma(1)}} Z_{i_2 i_{\sigma(2)}} \cdots Z_{i_J i_{\sigma(J)}}$$

KK graviton

$\mathrm{Tr}(Z^J)$

$J \sim \mathcal{O}(1)$



relation to phase space of 1D harmonic oscillator

Wigner phase space distribution for 1D harmonic oscillator

$$\hat{u}(p,q,t) = \int dx e^{ipx} \psi^\dagger(q+\frac{x}{2},t)\psi(q-\frac{x}{2},t)$$

relation between charge density and WPSD

Iso-Karabali-Sakita

$$\hat{U}(z,z^*,t) = \int \frac{d\Lambda d\Lambda^*}{4\pi^2} e^{-\Lambda^* z + \Lambda z^* - \frac{1}{4}\Lambda\Lambda^*} \int \frac{dpdq}{2\pi} e^{-\frac{\Lambda}{2}(q+ip)+\frac{\Lambda^*}{2}(q-ip)} \hat{u}(p,q,t)$$

in the classical limit

$$\hat{U}(z,z^*,t) = \frac{1}{\pi}\hat{u}(p,q,t) \qquad z = \frac{1}{2}(q-ip)$$

Chiral primary operators and giant gravitons

$\mathcal{N}=4$ SYM on R^4

- chiral primary operators of type

$$\prod_{b=1}^{K}\mathrm{Tr}(Z(x)^{J_b}) \qquad Z=\frac{1}{\sqrt{2}}(X^1+iX^2)$$

- half-BPS $\quad \Delta = J = \sum_{b=1}^{K} J_b$

- extremal correlator

$$\left\langle (\prod_{b=1}^{K}\mathrm{Tr}(Z(y)^{J_b}))^* \prod_{b_1=1}^{K_1}\mathrm{Tr}(Z(x_1)^{J_{b_1}^{(1)}})\cdots\prod_{b_n=1}^{K_n}\mathrm{Tr}(Z(x_n)^{J_{b_n}^{(n)}})\right\rangle$$

- non-renormalization theorem

extremal correlators is independent of g_{YM} it can be calculated using the free part

Eden-Howe-Sokatchev-West

propagator $\langle Z_{ij}(x)Z^*_{kl}(y)\rangle = \frac{\delta_{ik}\delta_{jl}}{(x-y)^2}$

- **KK graviton** with angular momentum J~O(1) ⟷ $\mathrm{Tr}(Z^J)$

- **giant graviton** = D3-brane wrapped on S^3 in S^5

McGreevy-Suskind-Toumbas
Balasubramanian-Berkooz-Naqvi-Strassler

size $r = \sqrt{\frac{J}{N}}R$

J~O(N)

R: radius of S^5 ⟶ $J \leq N$

$$\det{}_J Z = \frac{1}{J!}\epsilon_{i_1 i_2\cdots i_J k_1 k_2\cdots k_{N-J}}\epsilon_{j_1 j_2\cdots j_J k_1 k_2\cdots k_{N-J}} Z_{i_1 j_1} Z_{i_2 j_2}\cdots Z_{i_J j_J}$$

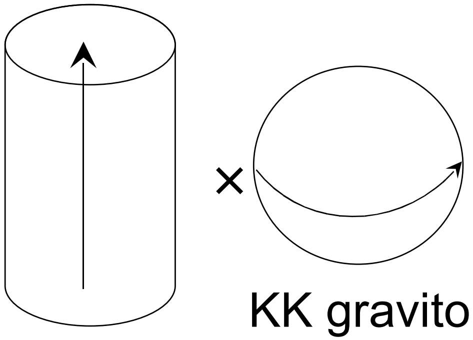
- **AdS giant graviton** = D3-brane wrapped on S^3 in AdS_5

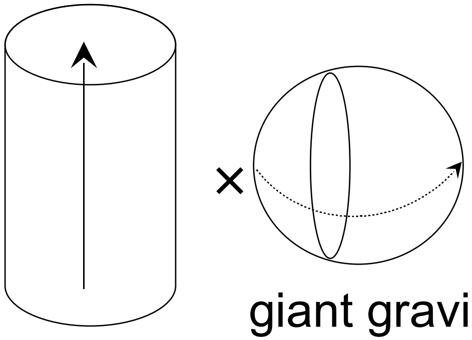
Hashimoto-Hirano-Itzhaki
Grisaru-Myers-Tafjord
Corley-Jevicki-Ramgoolam

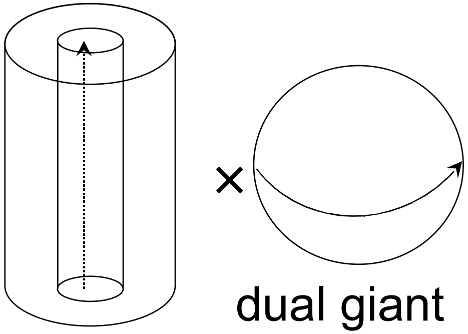
J~O(N)

$$\frac{1}{J!}\sum_{\sigma\in S_J} Z_{i_1 i_{\sigma(1)}} Z_{i_2 i_{\sigma(2)}}\cdots Z_{i_J i_{\sigma(J)}}$$

no restriction on J

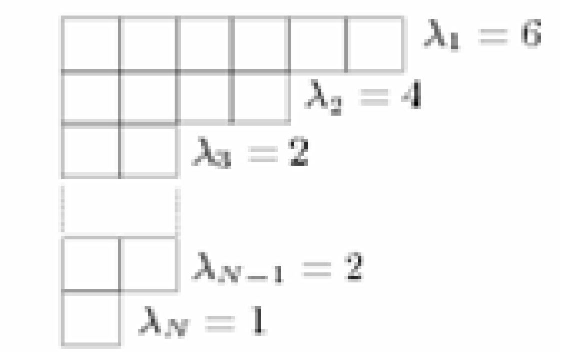

KK graviton


giant graviton


dual giant

Orhogonal basis

$$\langle \chi_{\langle\lambda\rangle}(x)\chi_{\langle\lambda'\rangle}(y)\rangle = \frac{c_{\langle\lambda\rangle}\delta_{\langle\lambda\rangle\langle\lambda'\rangle}}{|x-y|^{2J}}$$



$\mathcal{N}=4$ SYM on $R\times S^3$

- global coordinates of AdS$_5$

$$ds^2=R^2(-\cosh^2\rho dt^2+d\rho^2+\sinh^2\rho d\Omega_3^2)$$

boundary $(\rho\to\infty)\sim R\times S^3$

- relation between R^4 and $R\times S^3$

$$\begin{tikzcd}
R^4 \arrow[d, Leftrightarrow, "\text{conformally equiv.}"'] & ds^2=dr^2+r^2d\Omega_3^2 \arrow[r, "\ln r=\tau"'] & ds^2=e^{2\tau}(d\tau^2+d\Omega_3^2) \arrow[d, "ds^2=e^{2\tau}ds'^2"] \\
R\times S^3 & ds'^2=-dt^2+d\Omega_3^2 & ds'^2=d\tau^2+d\Omega_3^2 \arrow[l, "\tau=it"]
\end{tikzcd}$$

Δ on R^4 $\qquad$ H on $R\times S^3$

- operator on R^4 $\Longleftrightarrow$ state on $R\times S^3$

- chiral primary states $\lim_{t \to -\infty} \prod_{b=1}^{K} \mathrm{Tr}(Z^{J_b})|0\rangle$

$$\Delta = J = \sum_{b=1}^{K} J_b \longleftrightarrow$$

s-wave of KK decompositions
only t-dependence

- preserve 16 susy ×R ×SO(4)×SO(4)

$\Delta - J$

R-symmetry SO(6)

s-wave on S^3

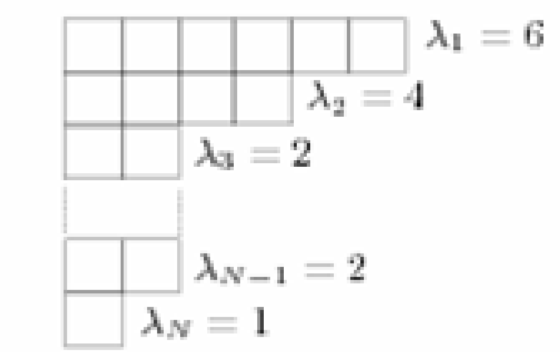
- free part of $Z(t)$~ complex matrix model

$$S = \int dt \mathrm{Tr}(\underbrace{\dot{Z}(t)\dot{Z}^{\dagger}(t)}_{\text{kinetic term}} \underbrace{- Z(t)Z^{\dagger}(t)}_{\text{coupling of conformal matter to curvature of } S^3})$$

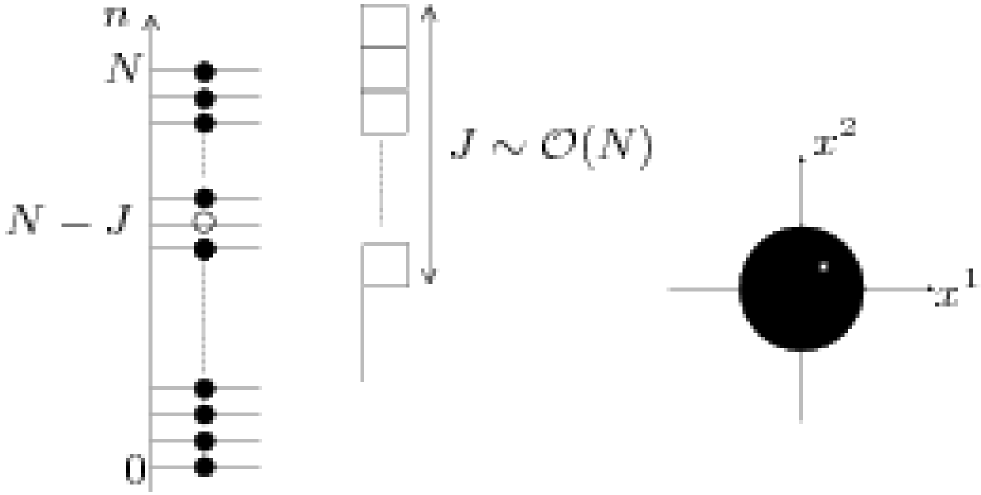
Hashimoto-Hirano-Itzhaki
Corley-Jevicki-Ramgoolam
Berenstein
Takayama-A.T.

operator-state correspondence

$$\chi_{\langle\lambda\rangle}(Z(x)) \quad \Longleftrightarrow \quad \begin{aligned} \Phi_{\langle\lambda\rangle} &= \chi_{\langle\lambda\rangle}(Z)\Delta(z)e^{-\sum_{i=1}^{N}|z_i|^2} \\ &= \det_{i,j} z_j^{N-i+\lambda_i} \times e^{-\sum_{k=1}^{N}|z_k|^2} \end{aligned}$$

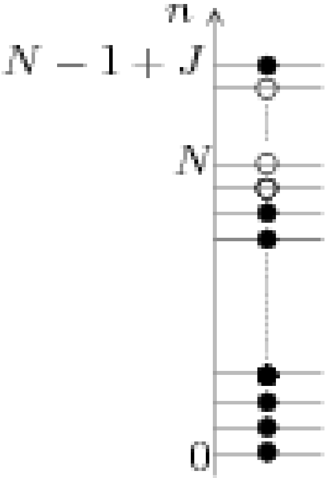


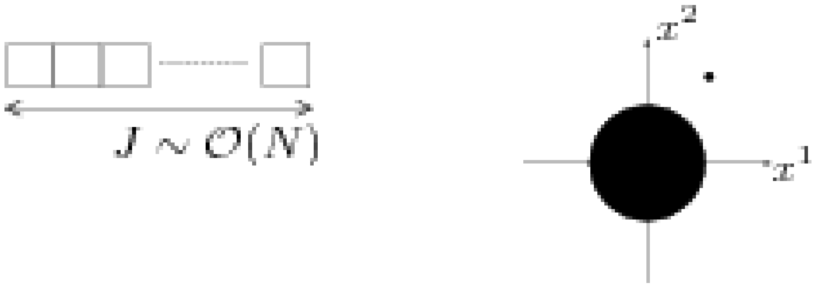
giant graviton



$$\chi_{\langle\lambda\rangle}(Z) = \mathrm{det}_J Z = \frac{1}{J!}\epsilon_{i_1 i_2 \cdots i_J k_1 k_2 \cdots k_{N-J}}\epsilon_{j_1 j_2 \cdots j_J k_1 k_2 \cdots k_{N-J}} Z_{i_1 j_1} Z_{i_2 j_2} \cdots Z_{i_J j_J}$$

AdS giant graviton



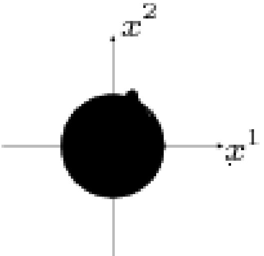


$$\frac{1}{J!}\sum_{\sigma \in S_J} Z_{i_1 i_{\sigma(1)}} Z_{i_2 i_{\sigma(2)}} \cdots Z_{i_J i_{\sigma(J)}}$$

KK graviton

$\mathrm{Tr}(Z^J)$

$J \sim \mathcal{O}(1)$



Bubbling AdS

Lin-Lunin-Maldacena

- general form of half-BPS sof type IIB sugra that preserve R ×SO(4)×SO(4) isometry

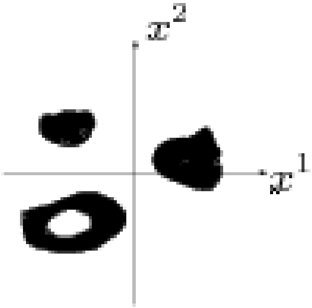
$$ds^2 = -h^{-2}\left[\underline{dt} + \sum_{i=1}^{2} V_i dx^i\right]^2 + h^2\left[dy^2 + \sum_{i=1}^{2} dx^i dx^i\right] + ye^G \underline{d\Omega_3^2} + ye^{-G}\underline{d\widetilde{\Omega}_3^2}\,,$$

$$h^{-2} = 2y\cosh G\,, \quad y\partial_y V_i = \epsilon_{ij}\partial_j z\,, \quad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z\,, \quad z = \frac{1}{2}\tanh G$$

characterized by a single function $z(x_1, x_2, y)$

- differential eq. satisfied by z

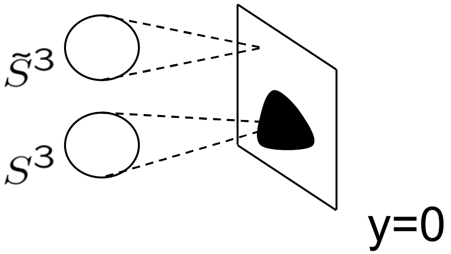
$$\partial_i\partial_i z + y\partial_y\left(\frac{\partial_y z}{y}\right)$$



- z must take ½ (white) or -½ (black) on the plane y=0 for the solution to be smooth

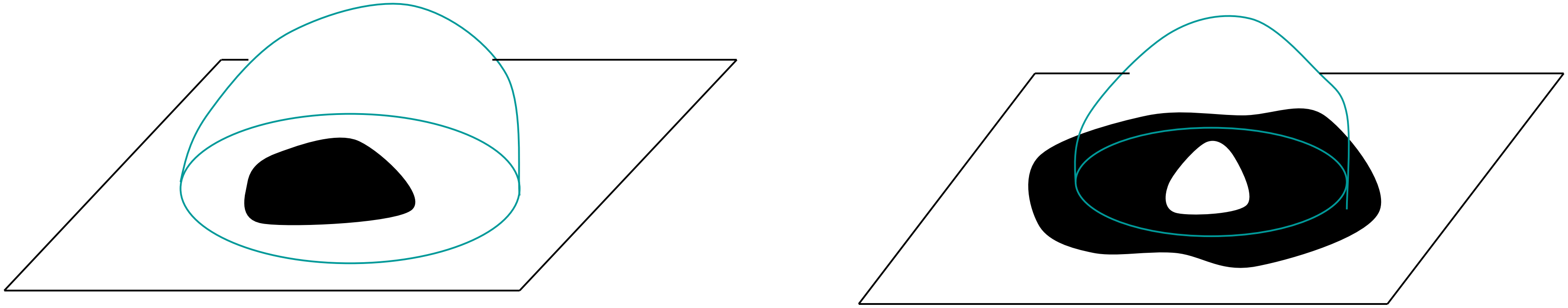
configuration of droplet ⟶ geometry

- shrinking S^3



topology of droplet (bubble) ⟺ topology of space-time

- flux



$$\tilde{N} = \frac{(\text{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4}$$

$$N = \frac{(\text{Area})_{z=\frac{1}{2}}}{4\pi^2 l_p^4}$$

Area is quantized

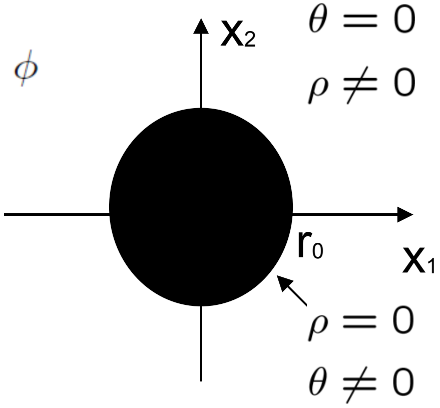
- example $AdS_5 \times S^5$ $\quad z(x_1, x_2, 0) = \frac{1}{2}\mathrm{sign}(r - r_0)$

$$z(r, y; r_0) = \frac{r^2 - r_0^2 + y^2}{2\sqrt{(r^2 + r_0^2 + y^2)^2 - 4r^2 r_0^2}}, \quad V_\phi = -\frac{1}{2}\left(\frac{r^2 + r_0^2 + y^2}{\sqrt{(r^2 + r_0^2 + y^2)^2 - 4r^2 r_0^2}} - 1\right), \quad V_r = 0$$

$$\Downarrow \quad y = r_0 \sinh\rho \sin\theta, \quad r = r_0 \cosh\rho \cos\theta, \quad \phi + t \to \phi$$

$$\boxed{ds^2 = r_0[-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2 + d\theta^2 + \cos^2\theta d\tilde{\phi}^2 + \sin^2\theta d\tilde{\Omega}_3^2]}$$

$AdS_5 \times S^5$ geometry

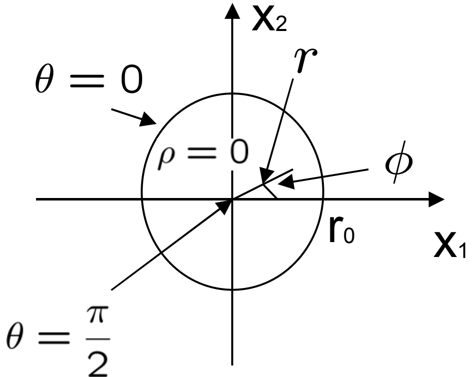


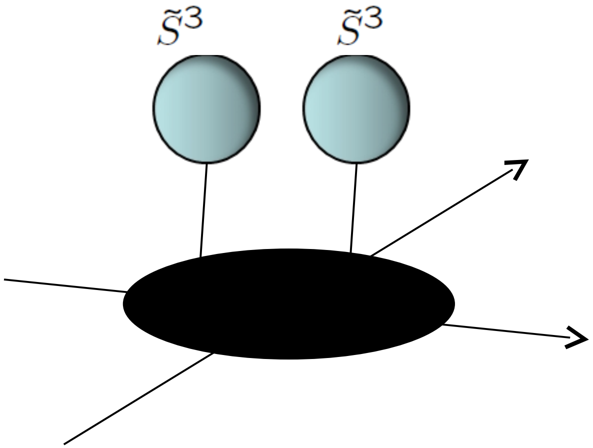
AdS/CFT duality $\longrightarrow$ $r_0^2 = R_{AdS_5}^4 = 4\pi\alpha'^2 g_s N$

area of droplet $= \pi r_0^2 = 4\pi^2\alpha'^2 g_s N \to \hbar = 2\pi\alpha'^2 g_s$ quantized !

$g_s N$:fixed $N \to \infty \implies \hbar \to 0$

droplet and S⁵



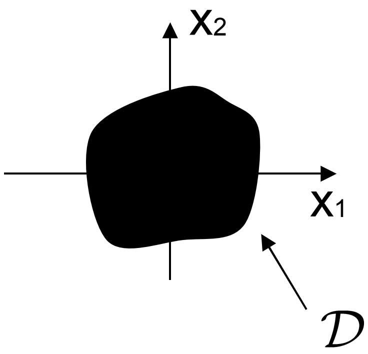


Identification with Fermi droplets

- quantization of area

area of droplet~ F_5 flux $\Longrightarrow$ $\boxed{\text{area of droplet} = 4\pi^2 l_p^4 N}$

- enegy and angular momentum



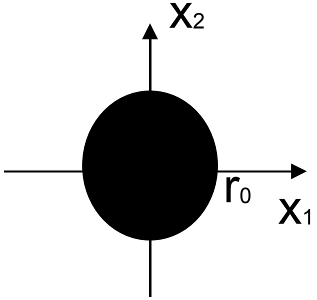
$$g_{t\phi} \longrightarrow \hbar\Delta = \hbar J = \underline{\int_{\mathcal{D}} \frac{d^2x}{2\pi\hbar}\frac{1}{2}(x_1^2+x_2^2)} - \underline{\frac{\hbar}{2}\left(\int_{\mathcal{D}} \frac{d^2x}{2\pi\hbar}\right)^2}$$

$\Downarrow$ total energy of N fermions

$\Downarrow$ ground state energy

- ground state

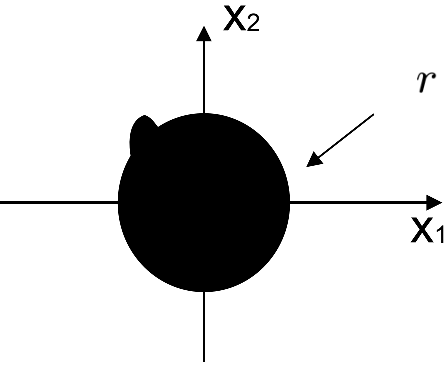
$AdS_5 \times S^5$ $\longleftrightarrow$



$\longleftrightarrow$ ground state of fermions

KK graviton and (AdS) giant graviton

- KK graviton with angular momentum J



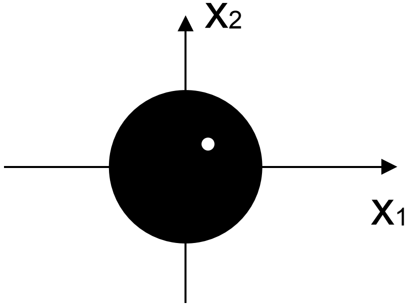
$$r = \bar{r}(\phi, t) = r_0 + \sum_J a_J e^{in\phi}, \quad a_{-J} = a_J^*$$

$$\hookrightarrow a_J$$

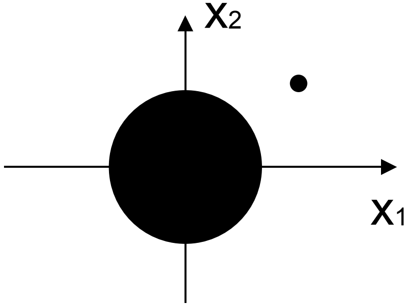
~KK graviton with angular momentum J

Grant-Maoz-Marsano-Papadodimas-Rychkov

- giant graviton with J～O(N)



- AdS giant graviton with J～O（N）



Further developments

- General half-BPS operators Yoneya

$c_{a_1 a_2 \cdots a_n} \mathrm{Tr}(X^{a_1} X^{a_2} \cdots X^{a_n})$ Cuntz algebra+free fermions

- Bubbling Wilson loop Yamaguchi, Kuroki's talk
- IIA bubbling~ SU(2|4) symmetric solution of IIA sugra

Lin-Maldacena

Electrostatics in an axially symmetric system in 3D

gravity duals of Vacua in SU(2|4) symmetric theories

⇨ Relations between SU(2|4) symmetric theories

Ishiki-Shimasaki-Takayma-Tsuchiya, Shimasaki's talk

⋮

Superstar

extremal limit of black hole with one charge in 5D N=2 gauged SUGRA → lifted to ten dimensions

Behrndt-Chamseddine-Sabra
Behrndt-Cvetic-Sabra
Cvetic et. al.

Superstar (ten-dimensional form)

$$
\begin{aligned}
ds^2 \; = \; & -\frac{1}{\sqrt{\Delta}}(\cos^2\theta + \frac{\rho^2}{L^2}\Delta)dt^2 + \frac{L^2 H}{\sqrt{\Delta}}\sin^2\theta d\phi^2 + \frac{2L}{\sqrt{\Delta}}\sin^2\theta dt d\phi \\
& +\sqrt{\Delta}(\frac{d\rho^2}{f} + \rho^2 d\Omega_3^2) + L^2\sqrt{\Delta}d\theta^2 + \frac{L^2}{\sqrt{\Delta}}\cos^2\theta d\tilde{\Omega}_3^2
\end{aligned}
$$

$$
H = 1 + \frac{Q}{\rho^2}, \quad f = 1 + \frac{Hr^2}{L^2}, \quad \Delta = \sin^2\theta + H\cos^2\theta
$$

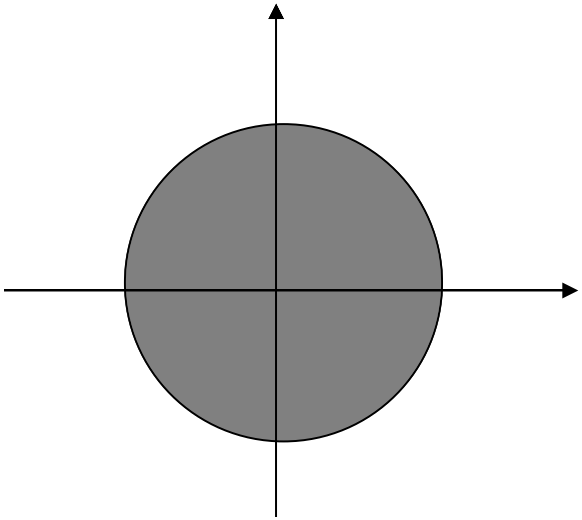
$Q = 0 \quad \Longrightarrow \quad AdS_5 \times S^5$

$Q > 0 \quad \Longrightarrow$ naked singularity at $\rho = 0$

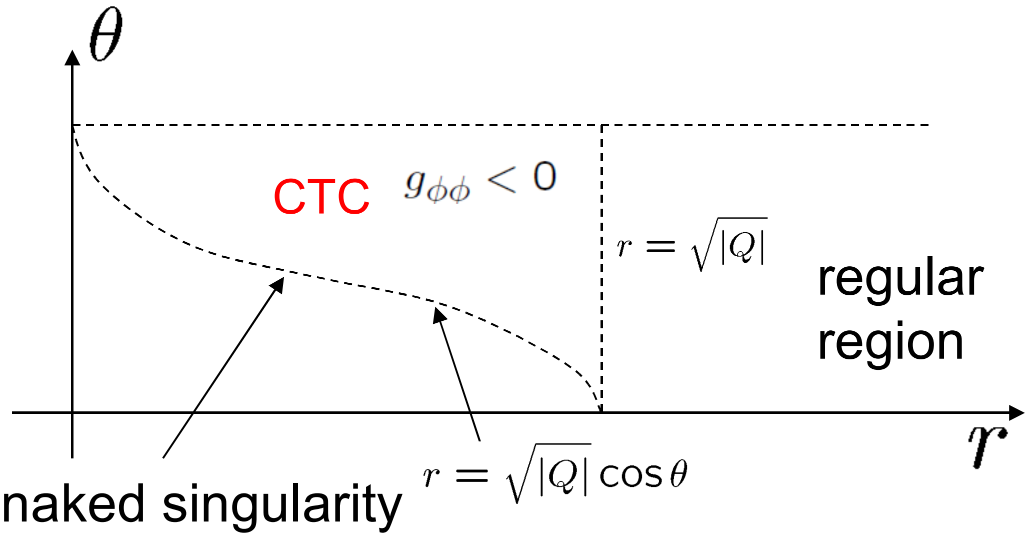
$$y = L\rho\cos\theta, \quad r = L^2\sqrt{f}\sin\theta, \quad t \to Lt \quad \Longrightarrow$$ LLM geometry

$$z = \frac{1}{2(1+Q/L^2)}\left[\frac{y^2+r^2-r_0^2}{\sqrt{(y^2+r^2+r_0^2)^2-4r^2r_o^2}} + \frac{Q}{L^2}\right] \qquad r_0^2 = L^2(L^2+Q)$$

$$u(r) = \begin{cases} \frac{1}{1+Q/L^2} & r < r_0 \\ 0 & r > r_0 \end{cases}$$



$$-L^2 < Q < 0 \quad \Longrightarrow \quad u > 1 \qquad \theta = \frac{\pi}{2}$$



Caldarelli-Klemm-Silva
Milanesi-O'Loughlin

chronology protection $\Longleftrightarrow$ Pauli exclusion principle smooth geometry ⟺ $u = 0, 1$ ⟺ pure state

superstar,
hyperstar ⟺ $0 < u < 1$ ⟺ mixed states

Ensemble of Young diagrams
~ limit shape

entropy ⟺ entropy

?

Balasubramanian-de Boer-Jejjala-Simon
Buchel, Suryanarayana, ,,,,,,,,,

Berenstein’s model

Berenstein, Berenstein-Correa-Vazquez, Berenstein-Vazquez, Berenstein-Cotta, Berenstein-Cotta-Leonardi

truncation

keep only s-wave on S3 of six scalars X^a and A_0

supersymmetric states excite only these modes in the free limit and supersymmetry should account for cancelations of quantum corrections

integration over A_0 ⟹ Gauss law constraint

hamiltonian

$$H = \mathrm{Tr}\left(\frac{1}{2}(P_a^2) + \frac{1}{2}(X_a^2) + \frac{g_{YM}^2}{8\pi^2}[X^a, X^b]^2\right)$$

large ‘t Hooft coupling $\lambda = g_{YM}^2 N$

$$[X^a, X^b] = 0$$

$$(\vec{x}_i)^a = X_{ii}^a$$

hamiltonian

$$H = \sum_i \left(-\frac{1}{2\mu^2} \vec{\nabla}_i \mu^2 \vec{\nabla}_i + \frac{1}{2} |\vec{x}_i|^2 \right) \qquad \mu^2 = \prod_{i<j} |\vec{x}_i - \vec{x}_j|^2$$

inner product

$$\langle \Psi | \Psi' \rangle = \int d^{dN}x \, \mu^2 \, \Psi^* \Psi'$$

wave function for ground state

$$\Psi_0 = \exp \left(-\frac{1}{2} \sum_i |\vec{x}_i|^2 \right)$$

$$H\Psi_0 = \left(\frac{N(N-1)}{2} + \frac{dN}{2} \right) \Psi_0 \qquad E_0 = \frac{N(N-1)}{2} + \frac{dN}{2}$$

redefinition of wave function

$$\psi = \mu \Psi \quad \Longrightarrow \quad \langle \psi | \psi' \rangle = \int d^{dN}x \, \psi^* \psi'$$

probability distribution for N particles

cf.) d=2

$$|\psi_0|^2 = \mu^2 \exp \left(-\sum_i |\vec{x}_i|^2 \right) = \exp \left(-\sum_i |\vec{x}_i|^2 + \sum_{i \neq j} \log |\vec{x}_i - \vec{x}_j| \right)$$

N →∞ limit

$$|\psi_0|^2 = \exp\left(-\int d^dx\,\rho(x)|\vec{x}|^2 + \int d^dx d^dy\,\rho(x)\rho(y)\log|\vec{x}-\vec{y}|\right)$$

constraint $\int d^dx\,\rho(x) = N$

saddle point approximation

$$\vec{x}^2+\lambda = 2\int d^dy\,\rho_0(y)\log|\vec{x}-\vec{y}|$$

$$(\nabla^2)^{\frac{d}{2}}\log|\vec{x}-\vec{y}| \propto \delta^d(x-y) \Longrightarrow \left[\begin{array}{lll} \rho_0 = 0 & \text{for } d>2 & \text{contradiction}\\ \rho_0 = const. & \text{for } d=2 & \text{OK}\end{array}\right.$$

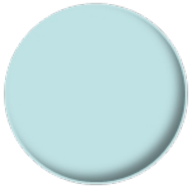
$$\boxed{\rho_0 = N\frac{\delta(|\vec{x}|-r_0)}{r_0^{d-1}\mathrm{Vol}(S^{d-1})}}$$

exponent $= -Nr_0^2 + N^2\log r_0 + N^2 c$

$$\boxed{r_0=\sqrt{\frac{N}{2}}}$$

Confirmed by numerical simulation

S^{d-1}



hamiltonian for off-diagonal part

$$H_{od} = \sum_{i\neq j}\left(\frac{1}{2}(\Pi_a)_{ij}(\Pi_a)_{ji} + \frac{1}{2}\omega_{ij}^2(X^a)_{ij}(X^a)_{ji}\right)$$

$$\omega_{ij}^2 = 1 + \frac{g_{YM}^2}{2\pi^2}|\vec{x}_i - \vec{x}_j|^2$$

BMN-type state

$$|\psi_k\rangle = \sum_{l=0}^{J} \exp(ikl/J) \sum_{j,j'=1}^{l} z_j^l Y_{jj'} z_{j'}^{J-l} X_{j'j}\psi_0|0\rangle_{od}$$

energy

$$E^{tot} = J + \langle E^{osc}\rangle \qquad E^{osc}_{jj'} = \sqrt{1 + \frac{g_{YM}^2}{2\pi^2}|\vec{x}_j - \vec{x}_{j'}|^2}$$

$$\langle E^{osc}\rangle$$

$$= T\frac{\int d^{dN}x\,|\psi_0|^2\sum_{jj'}|\sum_l e^{ikl/J} z_j^l z'^{J-l}_{j'}|^2 2E^{osc}_{jj'}}{\int d^{dN}x\,|\psi_0|^2\sum_{jj'}|\sum_l e^{ikl/J} z_j^l z'^{J-l}_{j'}|^2}$$

$$= \frac{\int d\Omega_5 d\Omega'_5|\sum_l e^{ikl/J} z^l z'^{J-l}|^2 2\sqrt{1 + \frac{g_{YM}^2}{2\pi^2}|\vec{x} - \vec{x}'|^2}}{\int d\Omega_5 d\Omega'_5|\sum_l e^{ikl/J} z^l z'^{J-l}|^2}$$

$$\left|\sum_{l=0}^{J} e^{ikl/J} z^l z'^{J-l}\right|^2 = r_0^{2J} \sum_{l,l'=0}^{J} (\cos\theta)^{l+l'} (\cos\theta')^{2J-l-l'} e^{i(l-l')(\frac{k}{J}+\phi'-\phi)}$$

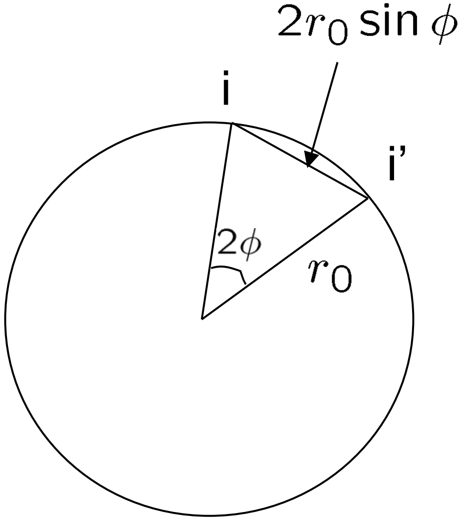
peak at $\theta = 0,\ \phi - \phi' = \frac{k}{J}$ $\Longrightarrow$ $|\vec{x} - \vec{x}'|^2 = 4 r_0^2 \sin^2 \frac{\phi - \phi'}{2}$

$$\boxed{\begin{aligned} \langle E^{osc} \rangle &= 2\sqrt{1 + \frac{2 g_{YM}^2 r_0^2}{\pi^2} \sin^2 \frac{k}{2J}} \\ &= 2\sqrt{1 + g_{YM}^2 N \left(\frac{n}{J}\right)^2} \end{aligned}}$$

Giant magnon

$$|\psi_k\rangle = \sum_{l} \mathrm{Tr}(\cdots ZYZ \cdots) \exp(ikl) \psi_0 |0\rangle_{od}$$

$$E = \sqrt{1 + \frac{2 g_{YM}^2 r_0^2}{\pi^2} \sin^2 \frac{k}{2}} = \sqrt{1 + \frac{g_{YM}^2 N}{\pi^2} \sin^2 \frac{k}{2}}$$



Summary

- Emergent geometry in AdS/CFT
- Analogy with c=1 string
- Bubbling AdS ~ chiral primary states

  emergence of part of S5
- chronology protection, mixed states
- Attempt for emergence of full S5

outlook

- emergence of AdS5xS5
- thermodynamics
- 1/N correction (string correction)
- Implication for IIB matrix model

- 例 1　　pp wave geometry　$z(x_1, x_2, 0) = \frac{1}{2}\text{sign}x_2$

$$\Longrightarrow \quad z(x_1, x_2, y) = \frac{1}{2}\frac{x_2}{\sqrt{x_2^2+y^2}}, \quad V_1 = \frac{1}{2}\frac{1}{\sqrt{x_2^2+y^2}}, \quad V_2 = 0$$

$$\xRightarrow[y = r_1 r_2, \quad x_2 = \frac{1}{2}(r_1^2 - r_2^2)]{} \quad \boxed{ds^2 = -2dtdx_1 - (r_1^2 + r_2^2)dt^2 + d\vec{r}_1^2 + d\vec{r}_2^2}$$