Non-perturbative stability of the fuzzy sphere in a matrix model with the Chern-Simons term

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1 Introduction

Curved-space classical solution of the matrix model

The curved-space background of a matrix model is an important issue, if a matrix model is to be an eligible framework to describe the gravitational interaction.

The IIB matrix model has only a flat background, and we want to build a matrix model which describes the curved-space background more manifestly.

We realize such an action by the addition of the Chern-Simons term to the IIB matrix model.

In this talk, we focus on the following bosonic action:

$$S = Tr\left(-\frac{N}{4}[A_i, A_j]^2 + \frac{2i\alpha N}{3}\epsilon_{ijk}A_iA_jA_k\right).$$

- A_i is the 3-dimensional bosonic vector.
  Each component is an $N \times N$ hermitian matrix.
- This model is defined in the 3-dimensional Euclidean space.
  This model incorporates $SO(3)$ Lorentz symmetry and $SU(N)$ gauge symmetry.

The classical equation of motion

$$[A_j, [A_i, A_j]] + i\alpha\epsilon_{jkl}[A_k, A_l] = 0$$

incorporates an S^2 fuzzy-sphere classical solution.

$$A_i = \alpha L_i,$$

where L_i is the N-dimensional irreducible representation of the $SU(2)$ Lie algebra:

$$[L_i, L_j] = i\epsilon_{ijk}L_k.$$

The radius of the fuzzy-sphere solution is given by the Casimir of the $SU(2)$ Lie algebra:

$$A_1^2 + A_2^2 + A_3^2 = \alpha\frac{N^2 - 1}{4}1_{N\times N}.$$

The quantum stability has been investigated perturbatively through the one(multi)-loop computation:

S. Iso, Y. Kimura, K. Tanaka and K. Wakatsuki hep-th/0101102.
T. Imai, Y. Kitazawa, Y. Takayama and D. Tomino hep-th/0303120, 0307007.

In this work, we investigate the quantum stability of the fuzzy-sphere solution non-perturbatively through the Monte-Carlo simulation.

2 Numerical simulation of the matrix model with the Chern-Simons term

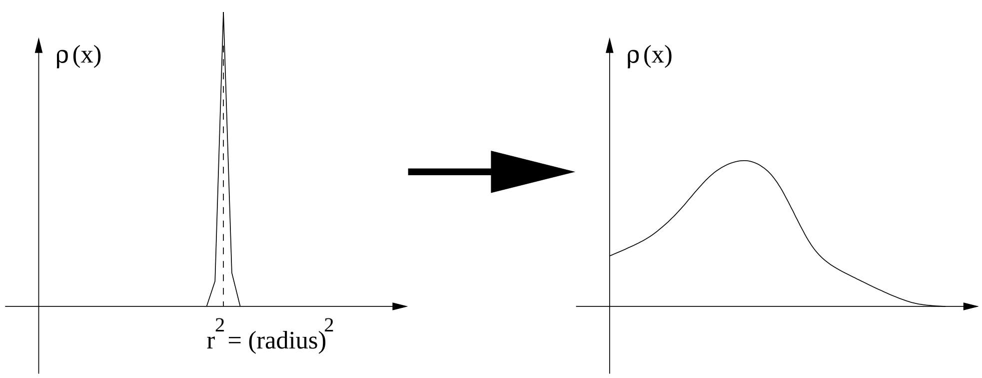
The S^2 fuzzy-sphere solution is stable under the quantum effect.

We analyzed the following observables in verifying the stability of the fuzzy sphere.

- $\frac{1}{N}\langle Tr A^2 \rangle$: **The spacetime extent.**
- **The histogram of the eigenvalues of the Casimir**

$$A_1^2 + A_2^2 + A_3^2.$$

When we start the Monte-Carlo simulation from the fuzzy-sphere solution $A_i = \alpha L_i$, the eigenvalues are peaked at the radius-square at the outset:

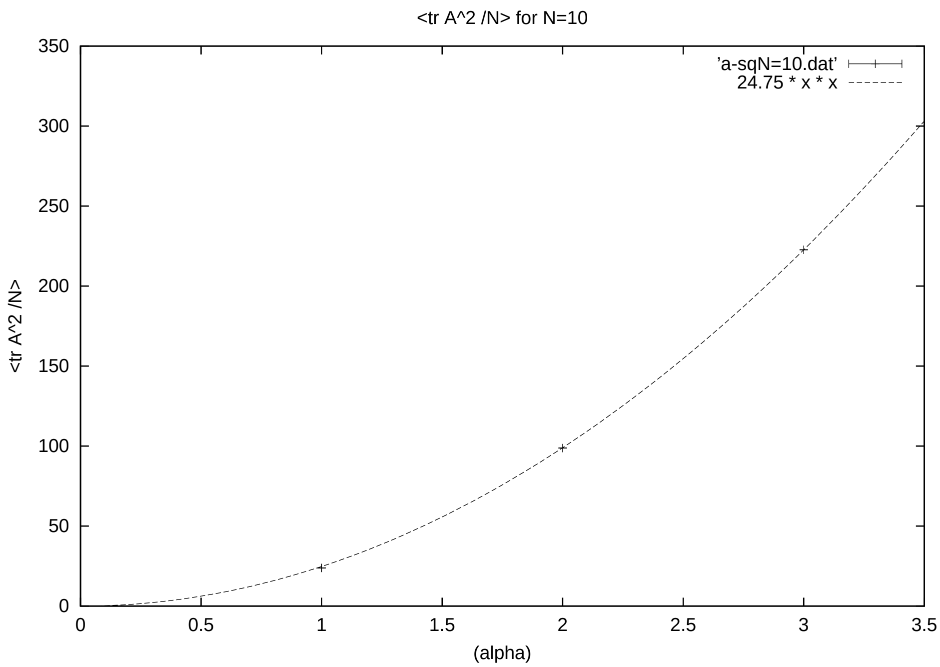


The behavior of the eigenvalue histogram indicates the stability of the fuzzy sphere solution.

In the following we perform a simulation for $N = 10$ and $\alpha = 1.0, 2.0, 3.0$.

The behavior of the spacetime extent

The spacetime extent $\frac{1}{N}\langle Tr A^2\rangle$ stays near the analytical value of the radius.



The analytical value of the fuzzy-sphere radius is (for $N = 10$) given by

$$A_1^2 + A_2^2 + A_3^2 = \frac{N^2 - 1}{4}\alpha^2 1_{N\times N} = \frac{99}{4}\alpha^2 1_{N\times N}.$$

$\Downarrow$

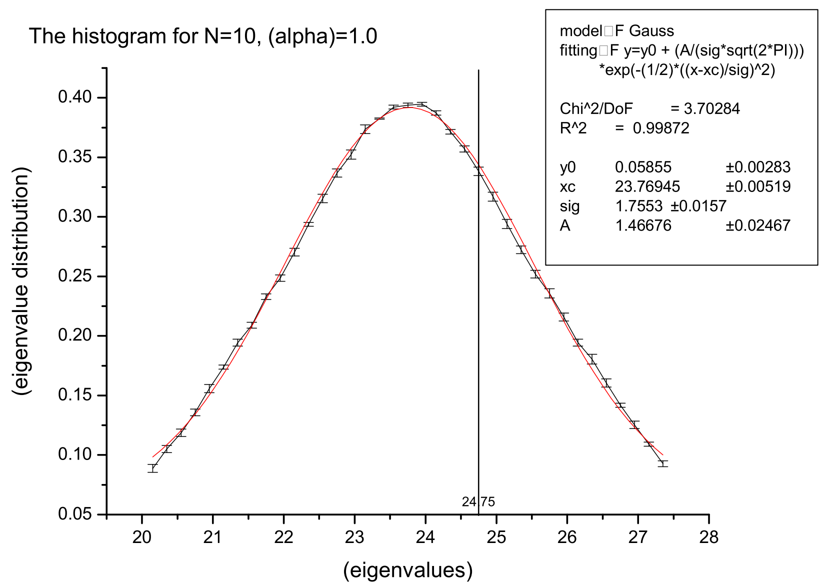
The eigenvalues of the Casimir concentrates in the vicinity of the original sphere.

The histogram of the eigenvalues of the Casimir

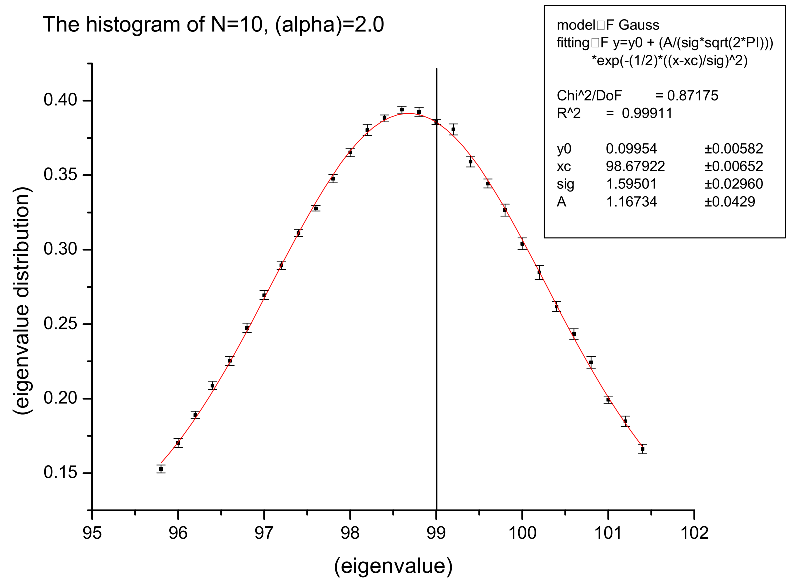
The following histograms indicate that the eigenvalues constitute the sphere-form shell.

The eigenvalues are distributed Gaussian-like around the analytical radius-square.

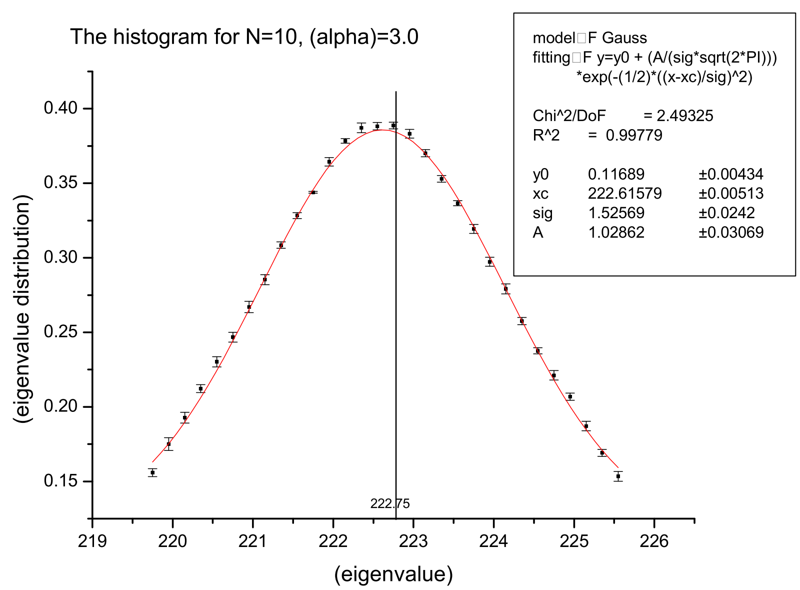
$\alpha = 1.0$



$\alpha = 2.0$



$\alpha = 3.0$



3 Conclusion

In this work, we have investigated the quantum stability of the S^2 fuzzy-sphere solution of the matrix model (with only the bosonic part).

We have found that the S^2 fuzzy-sphere solution is stable under the quantum effect.

- The spacetime extent $\frac{1}{N}\langle TrA^2\rangle$ stays near the analytical radius-square of the fuzzy sphere.
- The eigenvalues constitute the sphere-form shell.

Future works

- Analysis of the supersymmetric matrix model.
- Extension to the higher-dimensional fuzzy-sphere solution.

  Y. Kimura hep-th/0204256, 0301055, T. Azuma, M. Bagnoud hep-th/0209057.

  The matrix model with the higher-dimensional Chern-Simons term

$$S = -\frac{N}{4}Tr[A_\mu, A_\nu]^2 - gN\epsilon^{\mu_1\cdots\mu_{2k+1}}TrA_{\mu_1}\cdots A_{\mu_{2k+1}}$$

  incorporates the solution of the higher-dimensional fuzzy-sphere solution.

- The investigation of $\alpha < \mathcal{O}(\frac{1}{\sqrt{N}})$ region, in which the classical picture is conjectured to break down.