Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model

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Contents

1 Introduction

Matrix models on a homogeneous space

Motivations of fuzzy manifold studies:

- **Relation between the non-commutative field theory and the superstring.**
- **Novel regularization scheme alternative to lattice regularization.**
- **Prototype of the curved-space background in the large-N reduced models.**

Matrix models on a homogeneous space G/H:

G= (a Lie group), H = (a closed subgroup of G).

$S^2 = SU(2)/U(1)$, $S^2 \times S^2$, $S^4 = SO(5)/U(2)$, $CP^2 = SU(3)/U(2)$, $\cdots$.

Fuzzy spheres are compact $\Rightarrow$ realized by finite matrices.

The Yang-Mills-Chern-Simons (YMCS) model $\Rightarrow$ fuzzy sphere background.

2 The model and its classical solution

3d massive Yang-Mills-Chern-Simons model

⇒ a toy model with fuzzy sphere solutions:

$$S[A] = N\alpha^4 \mathrm{tr}\left(-\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{2i}{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho + \frac{\rho^2}{2}A_\mu^2\right).$$

- **Defined in the 3-dimensional Euclidean space ($\mu, \nu, \rho = 1, 2, 3$).**
- **Convergence of the path integral** P. Austing and J. F. Wheater, hep-th/0310170.
  (mass term just suppresses the path integral)
- **Classical equation of motion:**

$$-[A_\nu, [A_\mu, A_\nu]] + i\,\epsilon_{\mu\nu\lambda}\,[A_\nu, A_\lambda] + \rho^2 A_\mu = 0.$$

Fuzzy sphere classical solution:

$$A_\mu = X_\mu = \chi \begin{pmatrix} L_\mu^{(n_1)} \otimes 1_{k_1} & & \\ & \ddots & \\ & & L_\mu^{(n_s)} \otimes 1_{k_s} \end{pmatrix}, \text{ where } \begin{array}{l} \chi = \frac{1}{2}(1 + \sqrt{1 - 2\rho^2}), \\ \sum_{i=1}^{s} n_i k_i = N, \\ 0 < \rho < \frac{1}{\sqrt{2}}. \end{array}$$

- $L_\mu^{(n)}$: $n \times n$ representation of SU(2) Lie algebra:

$$[L_\mu^{(n)}, L_\nu^{(n)}] = i\epsilon_{\mu\nu\rho} L_\rho^{(n)}, \quad (L_\mu^{(n)})^2 = \frac{n^2 - 1}{4}.$$

- Collection of k_i coincident fuzzy spheres.
- Expansion around this solution ⇒ $\mathrm{U}(k_1) \times \mathrm{U}(k_2) \times \cdots \times \mathrm{U}(k_s)$ gauge theory.
- $A_\mu = 0$ solution ⇒ $s = 1$, $n_1 = 1$, $k_1 = N$.
- Classical free energy:

$$F_{\rm cl} = S_{\rm cl} = \frac{N\alpha^4}{24} f(\chi) \left(\sum_{i=1}^{s} k_i (n_i^3 - n_i) \right), \text{ where } f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2 \chi^2}{2} \left(\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}\right).$$

3 Dynamical generation of gauge group

Evaluation of free energy at one-loop level.

$\rho < \frac{2}{3}$ regime

Single $(s = 1, n_1 = N, k_1 = 1)$ fuzzy sphere $A_\mu = \chi L_\mu^{(N)} \Rightarrow F_{\rm cl} = -{\rm O}(\alpha^4 N^4) < 0$.

$\Rightarrow$ Coincident fuzzy spheres cannot be the true vacuum.

Free energy for single fuzzy sphere:

$$\frac{F_{\rm FS}}{N^2} = \frac{N^2\alpha^4}{4} f(\chi) + \frac{5}{2}\log N + 4\log\alpha - \delta(\rho).$$

Free energy in the Yang-Mills phase (based on Gaussian expansion)

J. Nishimura, T. Okubo and F. Sugino, hep-th/0205253.

$$\frac{F_{\rm YM}}{N^2} = \frac{3}{2}\log N + 3\log\alpha + \underbrace{\gamma}_{\simeq -4.5}.$$

Critical point: $\alpha_{\rm cr} = \frac{1}{\sqrt{N}}\left(\frac{2\log N}{|f(\chi)|}\right)^{1/4}$

- $\alpha < \alpha_{\rm cr} \to F_{\rm YM} < F_{\rm FS}$: **Yang-Mills phase**
- $\alpha > \alpha_{\rm cr} \to F_{\rm YM} > F_{\rm FS}$: **single fuzzy sphere**

$\rho > \frac{2}{3}$ regime

$\frac{F_{\rm cl}}{N^2} = \frac{1}{4}\alpha^4 f(\chi)(\sum_{i=1}^{s} r_i(n_i^3 - n_i)) > 0$ ($r_i = \frac{k_i}{N}$, so that $\sum_{i=1}^{s} n_i r_i = 1$) and
$\frac{F_{1-\rm loop}}{N^2} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should have lower free energy than $F_{A=0} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should satisfy $n_i = {\rm O}(1)$, $k_i = r_i N = {\rm O}(N)$.

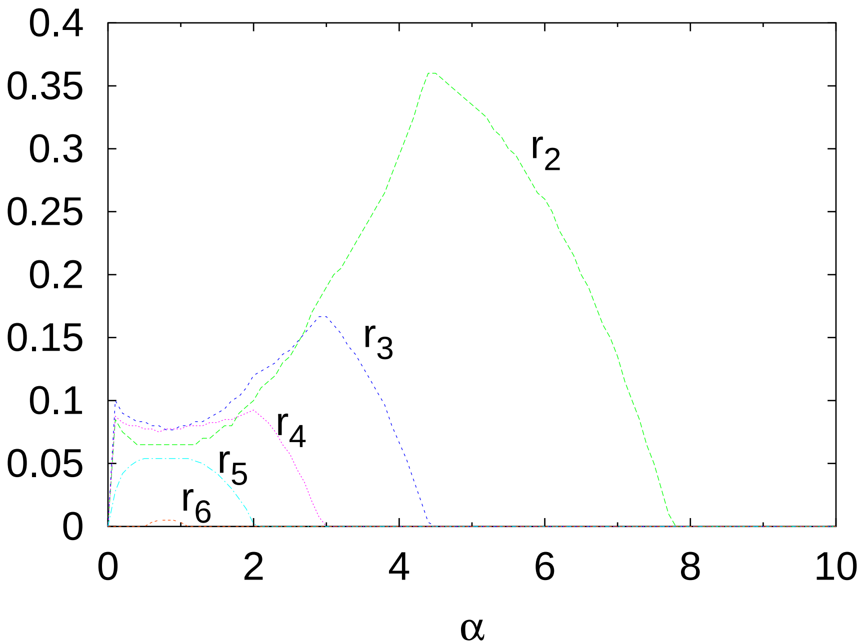
$U(k_i)$ gauge group ($k_i = {\rm O}(N)$) is dynamically generated.

Analogous to coincident transverse 5-branes in M-theory.

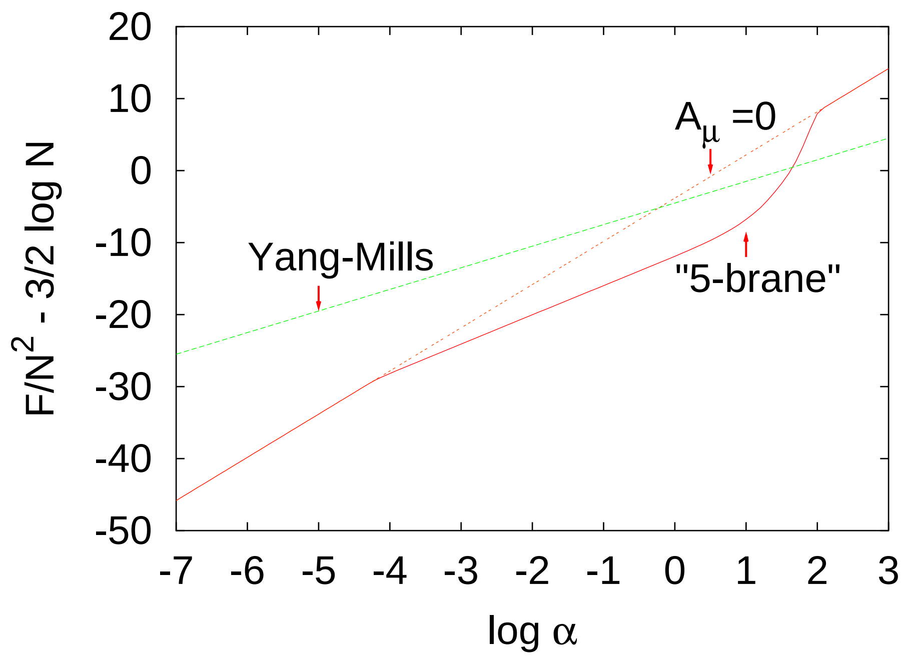
J. Maldacena, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0211139.

$n_1 = 1, n_2 = 2, \cdots, n_s = s$, **for brevity.**

r_i**'s that minimize the free energy at** $\rho = 0.7$.

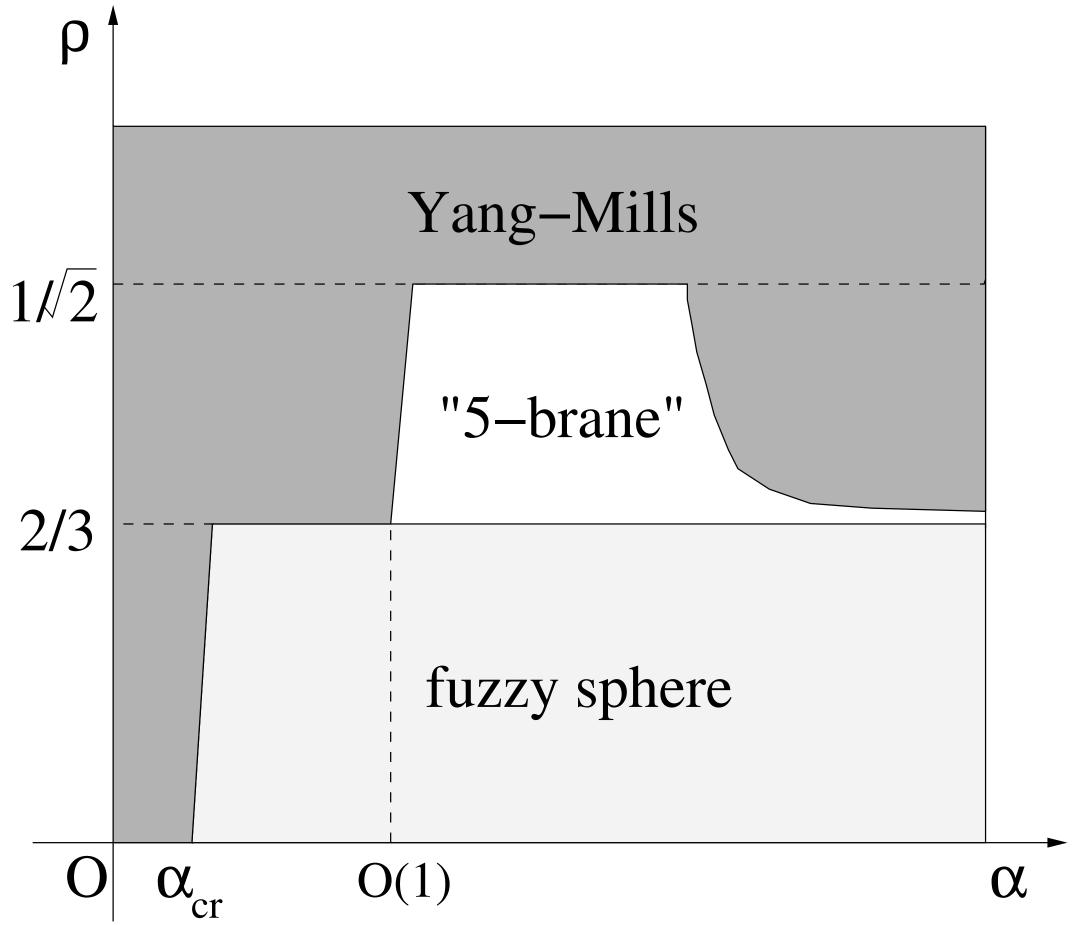


Comparison of free energy with Yang-Mills phase at $\rho = 0.7$.



The Yang-Mills phase takes over at $\alpha \simeq \mathrm{O}(1)$.

Phase diagram



4 Conclusion

Massive Yang-Mills-Chern-Simons model:

Many local minima with different gauge groups.

A nice laboratory for testing ideas and methods to study superstring theory by matrix models.

In this work, we have found

Dynamical generation of nontrivial gauge group.

The road to deriving the SU(3) $\times$ SU(2) $\times$ U(1) gauge group from the large-N reduced model?