Monte Carlo Studies of the GWW Phase Transition in Large-N Gauge Theories

(arXiv:0710.5873)

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JPS meeting 2008 at Yamagata University, Sep. 20th 2008, 16:30 ∼ 16:45

Collaboration with Pallab Basu and Spenta R. Wadia

Contents

1 Introduction

AdS/CFT correspondence: J. M. Maldacena, hep-th/9711200

duality between type IIB superstring on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM theory.

- **Nonperturbative studies of superstring.**
- **Blackhole-blackstring transition**

  L. Alvarez-Gaume, C. Gomez, H. Liu and S. Wadia, hep-th/0502227,
  L. Alvarez-Gaume, P. Basu, M. Marino and S. R. Wadia, hep-th/0605041,
  S. R. Wadia, hep-th/0609052

Thermodynamic aspects of quantum gravity in AdS spacetime.

AdS spacetime allows two Schwarzschild blackhole solutions.

- **Small black hole (SBH): Unstable. Horizon radius smaller than AdS.**
- **Big black hole (BBH): Stable. Horizon radius comparable to AdS.**

Third-order phase transition of gauge theory and the blackhole's phase transition

L. Alvarez-Gaume, C. Gomez, H. Liu and S. Wadia, hep-th/0502227.

2 GWW Phase transition of the finite-temperature gauge theory

Zero-mode action of the bosonic sector of $\mathcal{N} = 4$ SYM on S^3 at finite temperature.

Phenomenological model dual to $AdS_5 \times S^5$ at finite temperature.

$$Z = \int dM_\mu dA e^{-S}, \quad \text{where}$$
$$S = N \int_0^\beta dt \left\{ \mathrm{tr} \sum_{\mu=1}^{D} (D_t M_\mu(t))^2 - \frac{\lambda}{2} \mathrm{tr} \sum_{\mu,\nu=1}^{D} [M_\mu(t), M_\nu(t)]^2 + m^2 \mathrm{tr} \sum_{\mu=1}^{D} M_\mu^2(t) \right\}.$$

- $D_t M_\mu(t) = \partial_t M_\mu(t) - i[A, M_\mu(t)]$

  (A = zero mode of the time component of the gauge field on S^3)

- $M_\mu(t)$: SO(6) scalar fields ($\mu, \nu, \cdots = 1, 2, \cdots, D$, here $D = 6$)

- $\frac{1}{\beta} = T$: temperature

  Periodic boundary condition : $A(t + \beta) = A(t)$, $M_\mu(t + \beta) = M_\mu(t)$.

- Static and diagonal gauge: $A = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$, where $\{\alpha_a\} \in [-\pi, \pi]$.

  Gauge fixing term : $S_{\mathrm{g.f.}} = -\sum_{a,b=1,a\neq b}^{N} \log \sin |(\alpha_a - \alpha_b)/2|$.

Effective action of the SYM theory on S^3 at finite temperature

→ **Described by Polyakov line $U = P \exp\left(i\int_0^\beta dt A(t)\right)$.**

Phase structure of the YM theory and blackhole states in supergravity.

L. Alvarez-Gaume, C. Gomez, H. Liu and S.R. Wadia hep-th/0502227

Gross-Witten-Wadia (GWW) third-order phase transition of the partition function

$$Z_\mu = \int dU \exp(2N\mu(\mathrm{tr}\, U + \mathrm{tr}\, U^\dagger)),$$

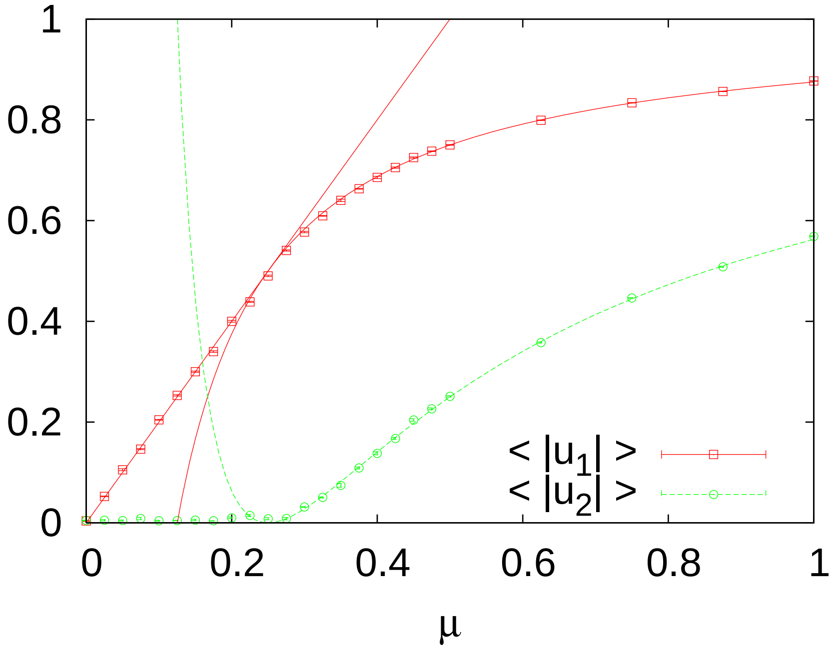
$$\langle |u_1| \rangle = \begin{cases} 2\mu & (\mu < 0.25), \\ 1 - \frac{1}{8\mu} & (\mu > 0.25). \end{cases}$$

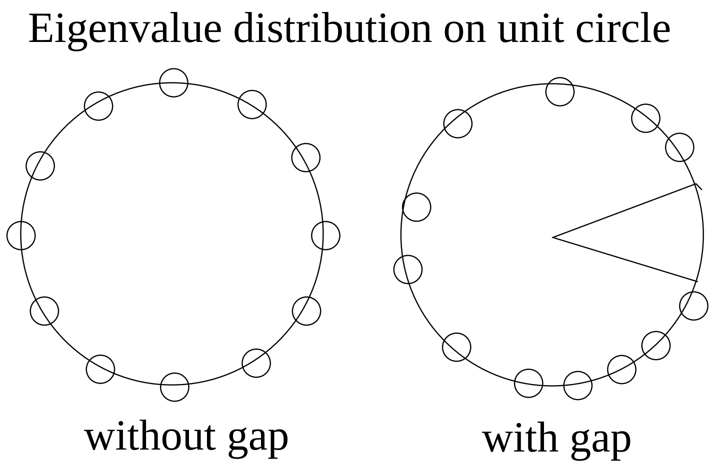
$$\langle |u_2| \rangle = \begin{cases} 0 & (\mu < 0.25), \\ 1 - \frac{1}{2\mu} + \frac{1}{16\mu^2} & (\mu > 0.25), \end{cases}$$

where $u_n = \frac{1}{N}\mathrm{tr}\, U^n$.

MC simulation for $N = 128$.

- $\mu < 0.25$: **no gap on unit circle.**
- $\mu > 0.25$: **a system has a gap.**





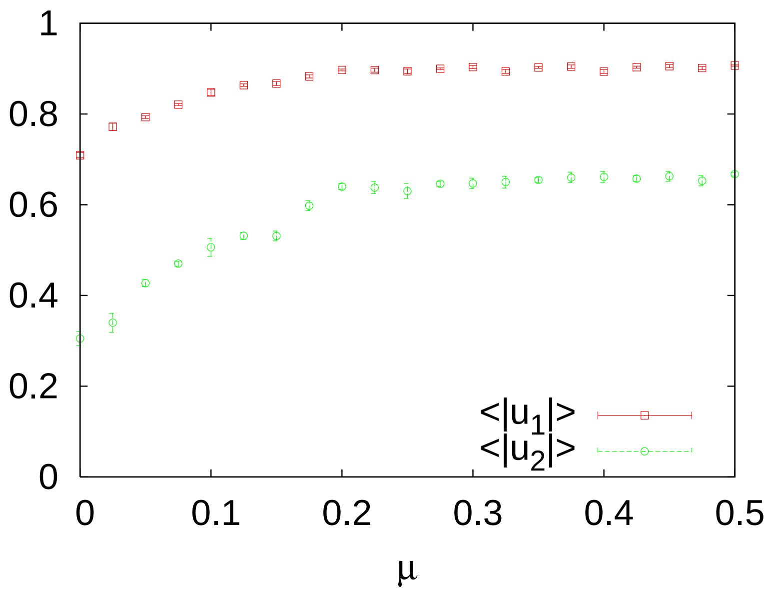
$$A = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_{\mathrm{N}})$$

Saddle point of the gauge field for our model:

$$S' = S + N\mu\beta(\text{tr}\, U + \text{tr}\, U^{\dagger}).$$

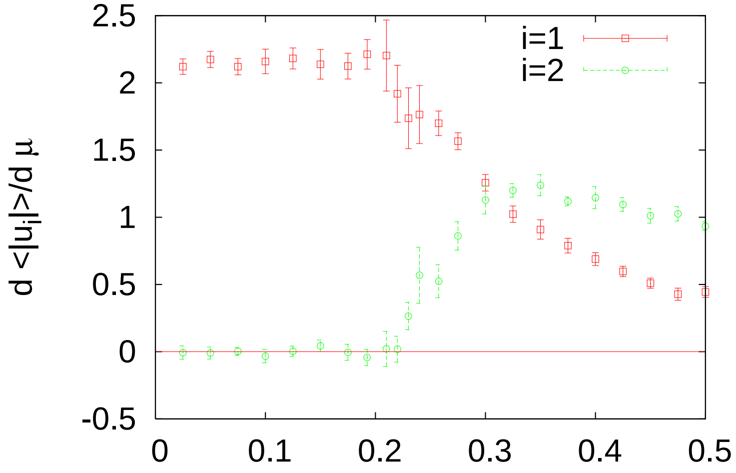
$D = 2$ case

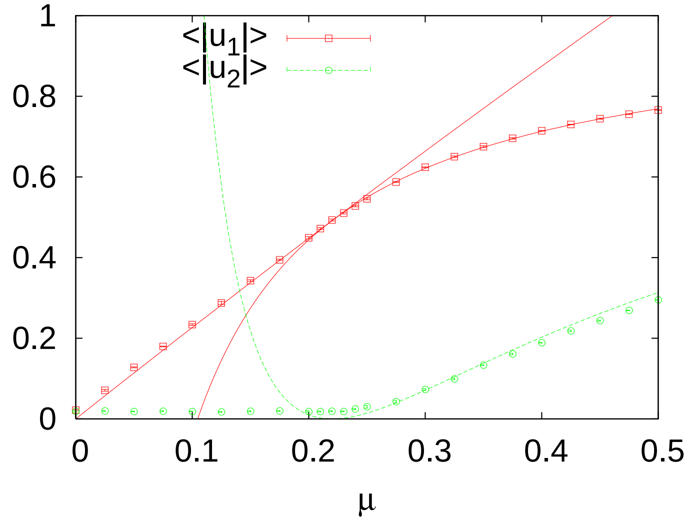
High-temperature ($\beta = 0.2 \Leftrightarrow T = 5.0$) case ($\lambda = m = 1.0$, $N = 48$).



No GWW-type phase transition.

Low-temperature $(\beta = 2.0 \Leftrightarrow T = 0.5)$ **case** $(\lambda = m = 1.0,\ N = 48)$.





Critical point is $\mu_c \simeq 0.22$.

- $\langle |u_1| \rangle_{|\mu=0} = 0,\ \langle |u_1| \rangle_{|\mu=\infty} = 1.$

- **At the critical point** $\mu = \mu_c$, $\langle |u_1| \rangle$ **and** $\frac{d\langle |u_1| \rangle}{d\mu}$ **are continuous.**

$$\langle |u_1| \rangle = \begin{cases} q_1 \frac{\mu}{\mu_c} + r_1 (\frac{\mu}{\mu_c})^2, & (\mu < \mu_c),\ \ r_1 = \frac{1}{2}(1 - \frac{3}{2} q_1 - \frac{1}{2} q_2), \\ 1 - q_2 (\frac{\mu}{\mu_c})^{-1} - r_2 (\frac{\mu}{\mu_c})^{-2}, & (\mu > \mu_c),\ \ r_2 = \frac{1}{2}(1 - \frac{1}{2} q_1 - \frac{3}{2} q_2), \end{cases}$$

Parameters are fitted as

$$(r_1, r_2) = (-0.0121, -0.0293).$$

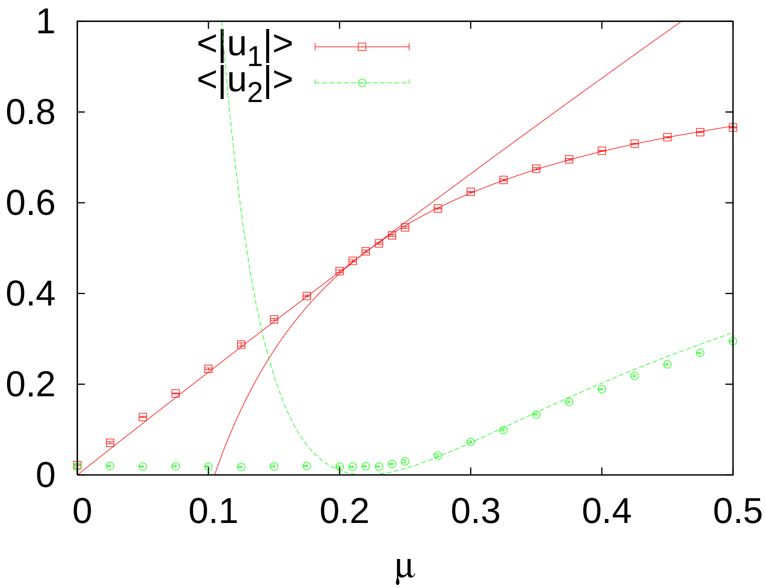
r_1, r_2**'s contribution is small. GWW-type third order phase transition.**

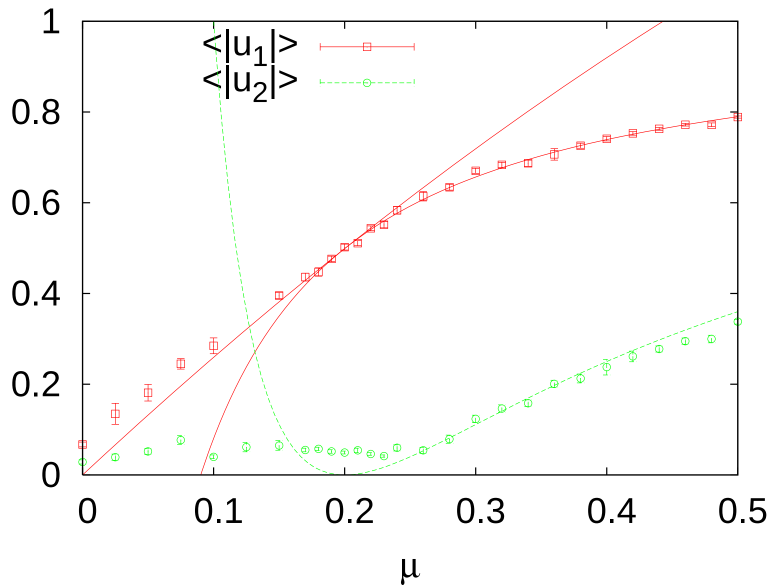
$$\langle |u_2| \rangle = 1 - \frac{2\mu_c}{\mu} + \frac{\mu_c^2}{\mu^2},\ \ (\mu > \mu_c).$$

$D = 6$ case

Low-temperature $(\beta = 2.0)$ case $(\lambda = m = 1.0)$

Left $D = 2, N = 48$, Right $D = 6, N = 16$.





Critical point is $\mu_c \simeq 0.20$.

$$\langle |u_1| \rangle = \begin{cases} q_1 \frac{\mu}{\mu_c} + r_1 (\frac{\mu}{\mu_c})^2, & (\mu < \mu_c), \;\; r_1 = \frac{1}{2}(1 - \frac{3}{2} q_1 - \frac{1}{2} q_2), \\ 1 - q_2 (\frac{\mu}{\mu_c})^{-1} - r_2 (\frac{\mu}{\mu_c})^{-2}, & (\mu > \mu_c), \;\; r_2 = \frac{1}{2}(1 - \frac{1}{2} q_1 - \frac{3}{2} q_2), \end{cases}$$

$$(r_1, r_2) = (-0.039, -0.041).$$

The results are insensitive to the dimensionality.

3 Conclusion

Zero mode effective action of the $\mathcal{N} = 4$ SYM theory on S^3.

- **Gross-Witten-Wadia (GWW) type third-order phase transition of the matrix model.**
- **Effect of fundamental matters (include vector fields)**

Further development

- **Extension to higher-dimensional system, such as $S^1 \times S^1 \times S^2$.**