# Monte Carlo studies of the six-dimensional IKKT model

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## Contents

# 1 Introduction

**Matrix models as a constructive definition of superstring theory**

**IKKT model (IIB matrix model)**

⇒ **Promising candidate for the constructive definition of superstring theory.**
Ishibashi, Kawai, Kitazawa and Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right).$$

- **Dimensional reduction of 10-dim $\mathcal{N} = 1$ super Yang-Mills theory to 0 dimension.**
- $A_\mu$ **(10d vector) and** $\psi_\alpha$ **(10d Majorana-Weyl spinor) ⇒** $N \times N$ **hermitian matrices .**
- $A_\mu$**'s eigenvalues ⇒ spacetime coordinate.**
- **Evidences for spontaneous breakdown of SO(10) symmetry to SO(4)**

  **⇒emergence of four-dimensional spacetime.**
  Nishimura and Sugino, hep-th/0111102, Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.
- **Complex fermion determinant:**
  - **Crucial for rotational symmetry breaking.** Nishimura and Vernizzi, hep-th/0003223.
  - **Difficulty of Monte Carlo simulation.**

## 2 6d IKKT model

T .Aoyama, T. Azuma, M. Hanada and J. Nishimura

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

- $A_\mu$ (6d vector) and $\psi$ (6d Weyl spinor) are $N \times N$ hermitian matrices .

  $\Gamma_1 = i\sigma_1 \otimes \sigma_2,\ \Gamma_2 = i\sigma_2 \otimes \sigma_2,\ \Gamma_3 = i\sigma_3 \otimes \sigma_2,\ \Gamma_4 = i1 \otimes \sigma_1,\ \Gamma_5 = i1 \otimes \sigma_3,\ \Gamma_6 = 1 \otimes 1.$

- SO(6) rotational symmetry and SU($N$) gauge symmetry.

- Presence of $\mathcal{N} = 2$ supersymmetry.

- $Z = \int dA d\psi d\bar{\psi} e^{-S} = \int dA e^{-S_B} \underbrace{(\det \mathcal{M})}_{=\int d\psi d\bar{\psi} e^{-S_F}} = \int dA e^{-S_0} e^{i\Gamma}$. CPU cost is O($N^6$).

  4d → det $\mathcal{M}$ is real positive

  6d and 10d → det $\mathcal{M}$ is complex.

  Complex phase is important in SO(6) breakdown.

- **Previous works on this model:**

  * **Simulation of phase-quenched 6d and 10d IKKT model**

    **⇒ no symmetry breakdown of SO(6) (and SO(10)).**

    J. Ambjorn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, hep-th/0005147

  * **Simulation of one-loop effective action (CPU cost is $O(N^3)$).**

    K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041.

  * **Gaussian expansion method ⇒ symmetry breakdown of SO(6) to SO(3).**

    T. Aoyama, J. Nishimura and T. Okubo

    **Observable for probing dimensionality :** $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

    $\lambda_i$ $(i = 1, \cdots, 6)$ **: eigenvalues of** $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_6)$

    **At large** $N$, $\langle \lambda_{1,2,3} \rangle \gg \langle \lambda_{4,5,6} \rangle$

## 3 Monte Carlo simulation

### Factorization method

An approach to the complex action problem in Monte Carlo simulation.

K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041,
J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, hep-lat/0208025.

$\tilde{\lambda}_i \stackrel{\text{def}}{=} \lambda_i / \langle \lambda_i \rangle_0$: deviation from 1 $\Rightarrow$ effect of the phase.

Overlap problem: Discrepancy of a distribution function between the phase-quenched model $Z_0$ and the full model $Z$.

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle = \frac{\langle \delta(x - \tilde{\lambda}_i) \cos \Gamma \rangle_0}{\langle \cos \Gamma \rangle_0} = \frac{\langle \delta(x - \tilde{\lambda}_i) \rangle_0 \langle \cos \Gamma \rangle_{i,x}}{\langle \cos \Gamma \rangle_0} = \frac{1}{C} \rho_i^{(0)}(x) w_i(x),$$

where

$$\langle * \rangle_0 = (\text{ V.E.V. for the phase-quenched model } Z_0$$
$$C = \langle \cos \Gamma \rangle_0, \quad \rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0, \quad w_i(x) = \langle \cos \Gamma \rangle_{i,x},$$
$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)].$$

Simulation of partition function $Z_{i,x} \Rightarrow x$ is trapped at $\tilde{\lambda}_i$.

The system visits the configurations important for full partition function $Z$.

Resolution of overlap problem.

## Monte Carlo evaluation of $\langle \tilde{\lambda}_i \rangle$

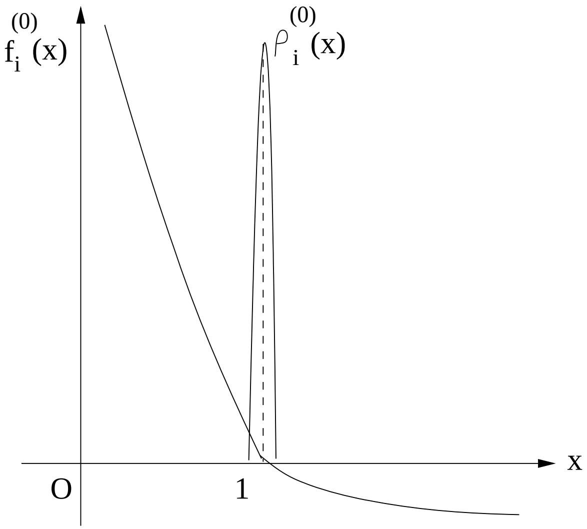
$w_i(x) > 0 \Rightarrow \langle \tilde{\lambda}_i \rangle$ is the minimum of $\mathcal{F}_i(x)$:

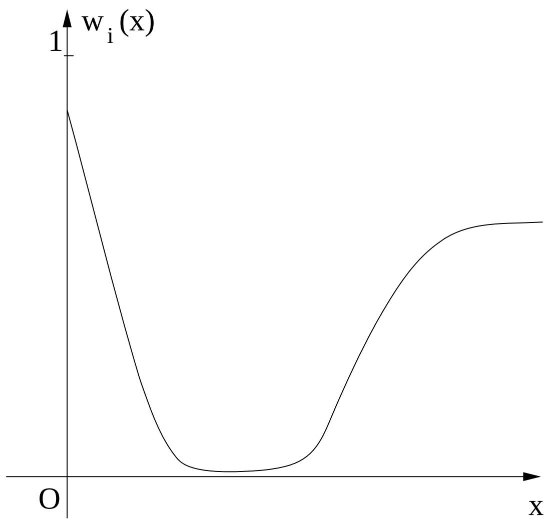
$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2} \log \rho_i(x).$$

We solve $\mathcal{F}'_i(x) = 0$, namely $\frac{1}{N^2} f_i^{(0)}(x) = -\frac{d}{dx}\left\{ \frac{1}{N^2} \log w_i(x) \right\}$, (where $f_i^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \log \rho_i^{(0)}(x)$)

Do both $\frac{1}{N^2} \log w_i(x)$ and $\frac{1}{N^2} f_i^{(0)}(x)$ scale at large $N$ as

$$\frac{1}{N^2} \log w_i(x) \to \Phi_i(x), \quad \frac{1}{N^2} f_i^{(0)}(x) \to F_i(x)?$$

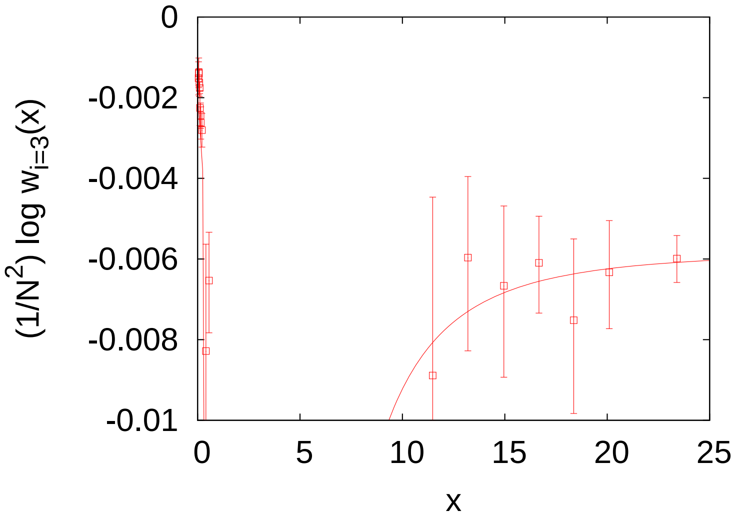


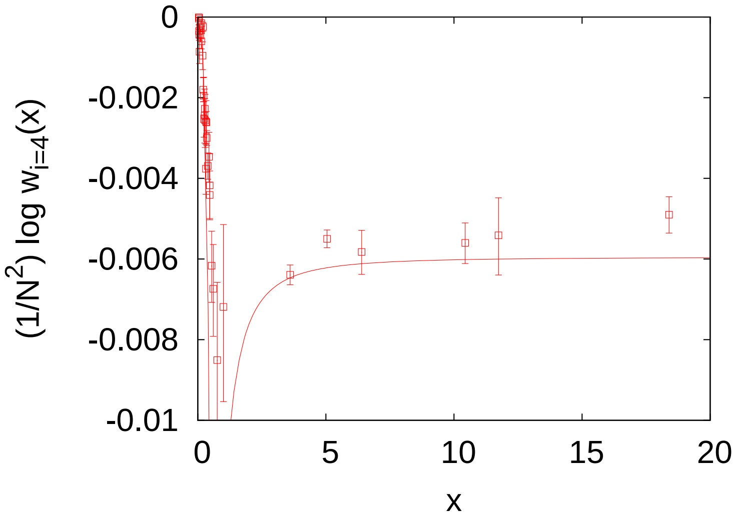



## Behavior of $\Phi_i(x)$

**Asymptotic behavior of $\Phi_i(x) = \frac{1}{N^2} \log w_i(x)$ at $x \ll 1$ and $x \gg 1$.**

$$\Phi_i(x) \propto \begin{cases} c_{i,0} x^{7-i} + \cdots & (x \ll 1, i = 2, \cdots, 6) \\ \frac{d_{i,0}}{x^{6-i}} + \cdots & (x \gg 1, i = 1, \cdots, 5) \end{cases}$$

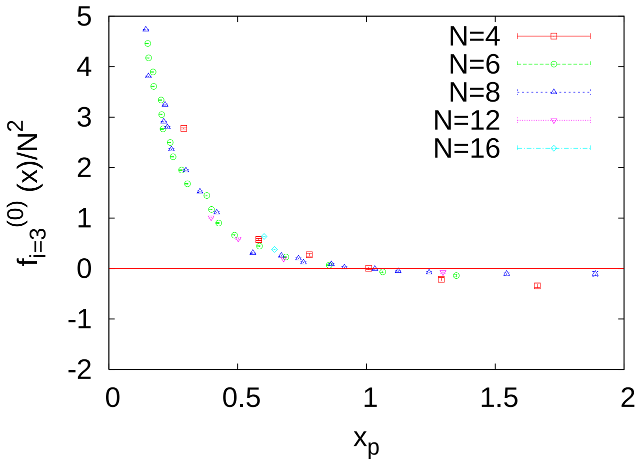


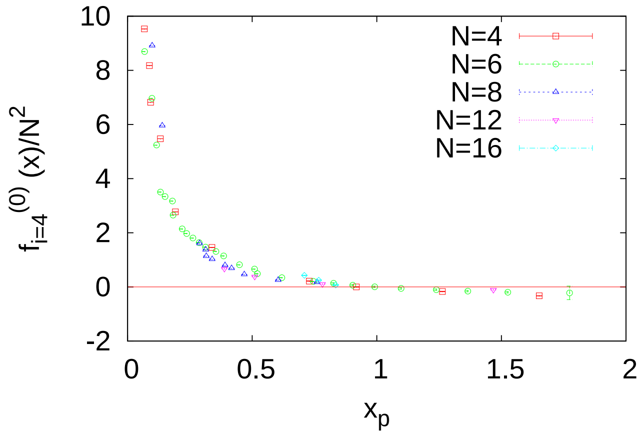



## Behavior of $\frac{1}{N^2} f_i^{(0)}(x)$

**Ansatz for all $x$:** $$\frac{1}{N^2} f_i^{(0)}(x) = \begin{cases} \frac{5}{x} \exp(-b_{i=1} x) & i = 1 \\ \frac{7-i}{2x} \exp(-b_i x) & i = 2, \cdots, 6 \end{cases}$$

**For $N = 8$ numerical data, we have $b_{i=3,4} \simeq 5$.**




**Symmetry breakdown SO(6) → SO(3) → Large-$N$ behavior is important.**

## 4 Conclusion

**Monte Carlo simulation of 6d IKKT model ⇒ spontaneous breakdown of SO(6) symmetry.**

**Can we understand the emergence of the spacetime?**

**Future works**

- **Simulation of larger $N$ ⇒ study the finite-$N$ effect.**
- **Ultimately, 10d IKKT model ⇒ 4d spacetime.**