Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model

(hep-th/0504217)

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Collaborated with S. Bal and J. Nishimura

Contents

1 Introduction

Large-N reduced models

$\Rightarrow$ **promising candidates for the constructive definition of superstring theory.**

The IIB matrix model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115

$$S = -\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right).$$

Relation with the type IIB superstring theory:

- **Matrix regularization of the Green-Schwarz action of type IIB superstring theory.**
- **D-brane interaction.**
- **Derivation of the string field theory.**

Matrix models on a homogeneous space

Motivations of fuzzy manifold studies:

- **Relation between the non-commutative field theory and the superstring.**
- **Novel regularization scheme alternative to lattice regularization.**
- **Prototype of the curved-space background in the large-N reduced models.**

Matrix models on a homogeneous space G/H:

G= (a Lie group), H = (a closed subgroup of G).

$\mathrm{S}^2 = \mathrm{SU}(2)/\mathrm{U}(1)$, $\mathrm{S}^2 \times \mathrm{S}^2$, $\mathrm{S}^4 = \mathrm{SO}(5)/\mathrm{U}(2)$, $\mathrm{CP}^2 = \mathrm{SU}(3)/\mathrm{U}(2)$, $\cdots$.

Fuzzy spheres are compact ⇒ realized by finite matrices.

The Yang-Mills-Chern-Simons (YMCS) model ⇒ fuzzy sphere background.

2 The model and its classical solution

3d massive Yang-Mills-Chern-Simons model

⇒ a toy model with fuzzy sphere solutions:

$$S[A] = N\alpha^4 \text{tr}\left(-\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{2i}{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho + \frac{\rho^2}{2}A_\mu^2\right).$$

- **Defined in the 3-dimensional Euclidean space ($\mu, \nu, \rho = 1, 2, 3$).**
- **Convergence of the path integral** P. Austing and J. F. Wheater, hep-th/0310170.
  (mass term just suppresses the path integral)
- **Classical equation of motion:**

$$-[A_\nu, [A_\mu, A_\nu]] + i\,\epsilon_{\mu\nu\lambda}[A_\nu, A_\lambda] + \rho^2 A_\mu = 0.$$

Fuzzy sphere classical solution:

$$A_\mu = X_\mu = \chi \begin{pmatrix} L_\mu^{(n_1)} \otimes 1_{k_1} & & \\ & \ddots & \\ & & L_\mu^{(n_s)} \otimes 1_{k_s} \end{pmatrix}, \text{ where } \begin{array}{l} \chi = \frac{1}{2}(1 + \sqrt{1 - 2\rho^2}), \\ \sum_{i=1}^{s} n_i k_i = N, \\ 0 < \rho < \frac{1}{\sqrt{2}}. \end{array}$$

- $L_\mu^{(n)}$: $n \times n$ representation of SU(2) Lie algebra:

$$[L_\mu^{(n)}, L_\nu^{(n)}] = i\epsilon_{\mu\nu\rho} L_\rho^{(n)}, \quad (L_\mu^{(n)})^2 = \frac{n^2 - 1}{4}.$$

- Collection of k_i coincident fuzzy spheres.
- Expansion around this solution $\Rightarrow$ $\mathrm{U}(k_1) \times \mathrm{U}(k_2) \times \cdots \times \mathrm{U}(k_s)$ gauge theory.
- $A_\mu = 0$ solution $\Rightarrow$ $s = 1$, $n_1 = 1$, $k_1 = N$.
- Classical free energy:

$$F_{\rm cl} = S_{\rm cl} = \frac{N\alpha^4}{24} f(\chi) \left(\sum_{i=1}^{s} k_i (n_i^3 - n_i) \right), \text{ where } f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2 \chi^2}{2} (\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}).$$

3 Review of the massless ($\rho = 0$) case

T. Azuma, S. Bal, K. Nagao and J. Nishimura, hep-th/0401038.

First-order phase transition

Monte Carlo simulation launched from fuzzy sphere classical solution:

Critical point at $\alpha_{\rm cr} \simeq \frac{2.1}{\sqrt{N}}$.

- $\alpha < \alpha_{\rm cr}$: Yang-Mills phase

  Strong quantum effects.

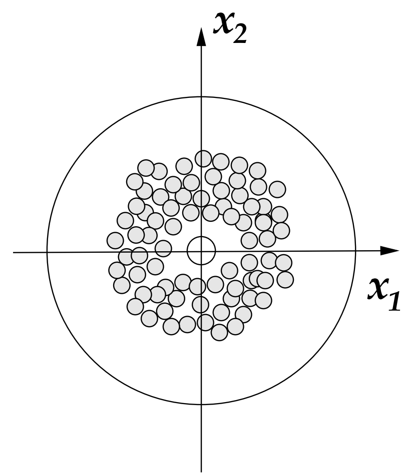
  behavior like the $\alpha = 0$ case.

  T. Hotta, J. Nishimura and A. Tsuchiya, hep-th/9811220,

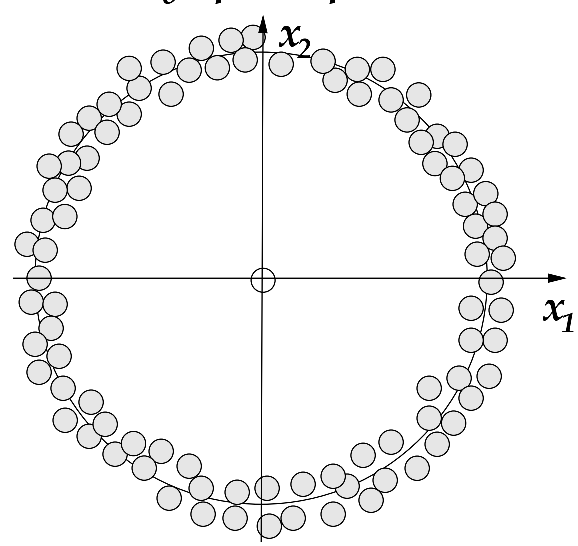
- $\alpha > \alpha_{\rm cr}$: fuzzy sphere phase.

  Fuzzy sphere configuration is stable.

Yang–Mills phase



Fuzzy sphere phase



Phase transition from the one-loop effective action

The effective action Γ is saturated at the one-loop level at large N.

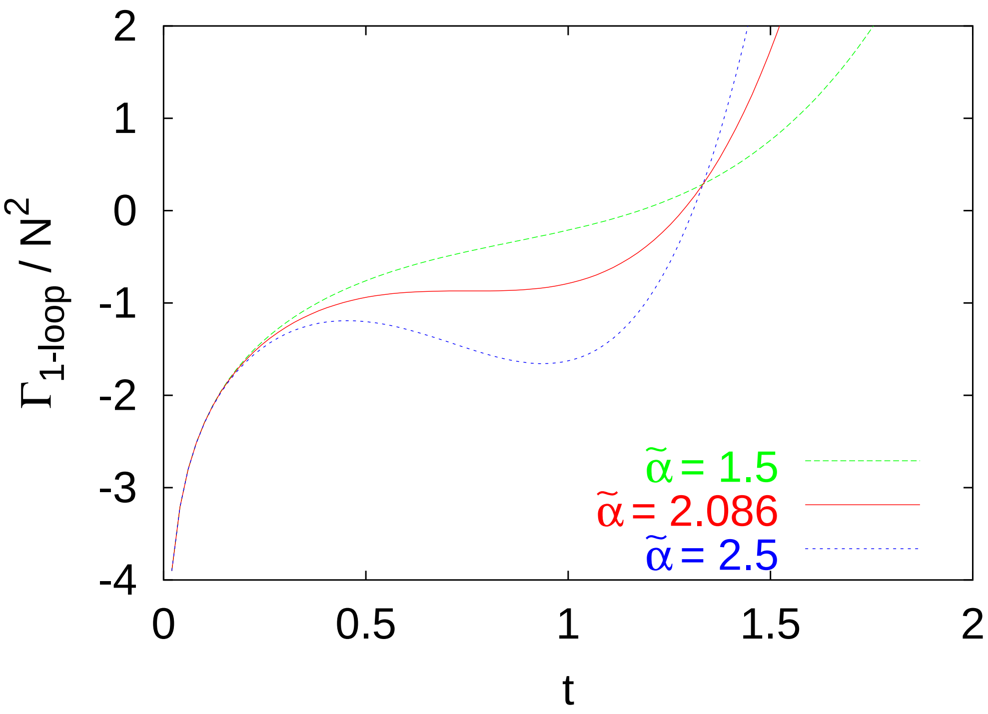
T. Imai, Y. Kitazawa, Y. Takayama and D. Tomino, hep-th/0307007.

Effective action at one-loop around $A_\mu = tX_\mu$ (where $\tilde{\alpha} = \alpha\sqrt{N}$).

$$\frac{\Gamma_{1-\text{loop}}}{N^2} \simeq \tilde{\alpha}^4 \left(\frac{t^4}{8} - \frac{t^3}{6} \right) + \log t.$$

The local minimum disappears at $\tilde{\alpha} < \tilde{\alpha}_{\text{cr}} = (\frac{8}{3})^{\frac{3}{4}} \simeq 2.086 \cdots$.

Consistent with the Monte Carlo simulation.



Properties of the multi-fuzzy spheres

Expansion around k coincide fuzzy spheres $A_\mu = X_\mu + \tilde{A}_\mu$, where

$$X_\mu = L_\mu^{(n)} \otimes \mathbf{1}_k.$$

Quantum field theory with U(k) gauge group.

Realized by the finite $N = nk$ matrices.

Simulation from zero start $A_\mu^{(0)} = 0$ for $N = 16, \alpha = 2.0$.

Metastability of multi-fuzzy-sphere state.

$$\underbrace{A_\mu^{(0)} = 0}_{\text{initial state}} \to \cdots \to \underbrace{A_\mu = \begin{pmatrix} L_\mu^{(6\to5\to4\to3\to2\to1)} & 0 \\ 0 & L_\mu^{(10\to11\to12\to13\to14\to15)} \end{pmatrix}}_{\text{metastable vacuum}} \to \underbrace{A_\mu = L_\mu}_{\text{stable vacuum}}.$$

Calculation of the free energy

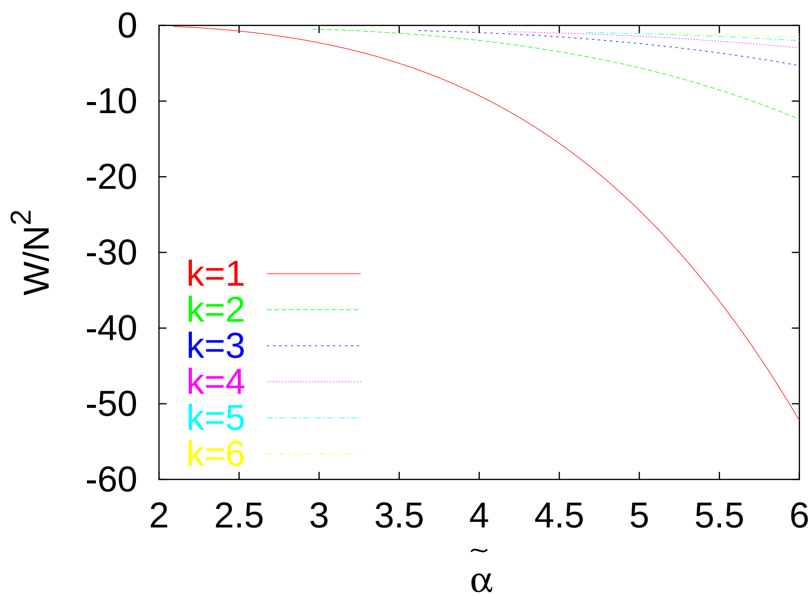
$$W = -\log \left(\int d\tilde{A} e^{-S} \right).$$

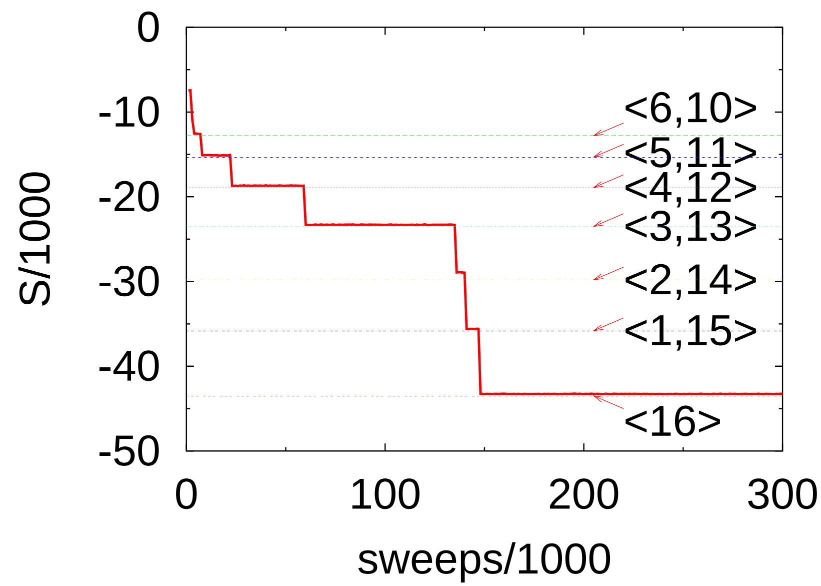
Eigenvalues of Casimir operator

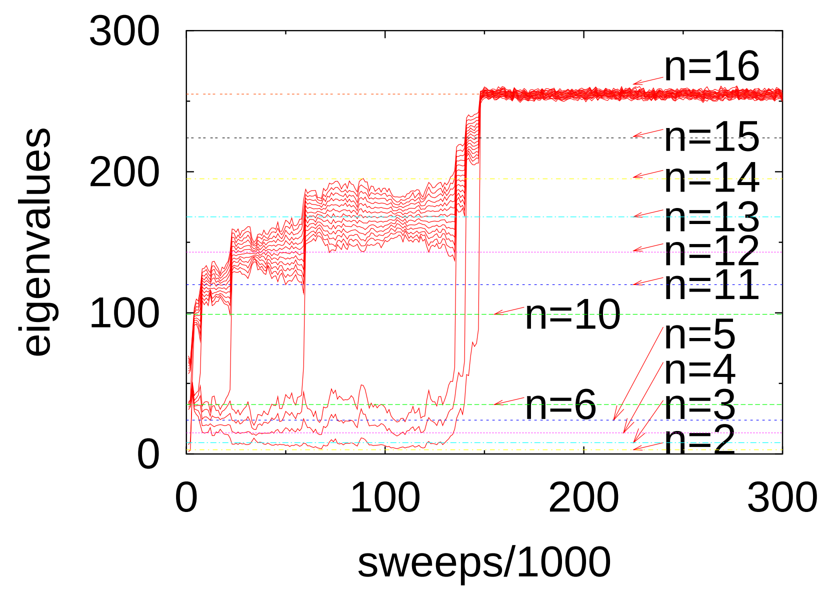
$$Q = \alpha^2 (A_1^2 + A_2^2 + A_3^2).$$

$k = 1$ has the lowest free energy to all order of perturbation.

T. Azuma, K. Nagao and J. Nishimura, hep-th/0410263.







Attempts for other models

Extension to the models with other fuzzy sphere solution:

- **Fuzzy S^4 sphere (`hep-th/0405096`) No fuzzy S^4 phase.**
- **Fuzzy CP^2 manifold (`hep-th/0405277`).**
- **Fuzzy $S^2 \times S^2$ sphere.**
- **Supersymmetric 4d YMCS model with fuzzy S^2 solution.**

In these cases (except S^4), U(1) gauge group is favored.

4 Dynamical generation of gauge group

One-loop free energy:

$$F = F_{\rm cl} \underbrace{- \log(\mathrm{vol}(H)\mathcal{N}\Delta(X_\mu)\mathcal{Z})}_{=F_{\rm 1-loop}}.$$

Perturbation around the fuzzy sphere solution $A_\mu = X_\mu + \tilde{A}_\mu$

⇒ **flat directions corresponding to the transformation:**

$$A_\mu \to A^g_\mu = g A_\mu g^\dagger, \text{ where } g \in H = \mathrm{U}(N)/\prod_{i=1}^{s} \mathrm{U}(k_i).$$

- **vol(H): gauge volume for the coset space H.**

$$\text{vol}(H) = \frac{\text{vol}(\text{U}(N))}{(\prod_{i=1}^{s} \text{vol}(\text{U}(k_i)))}, \text{ where vol}(n) = \frac{(2\pi)^{n(n+1)/2}}{(n-1)! \cdots 1! 0!}.$$

- $\mathcal{N} = (\frac{N\alpha^4}{2\pi})^{\frac{1}{2}\{N^2 - \sum_{i=1}^{s}(k_i)^2\}}$.

- **$\Delta(X_\mu)$: Faddeev-Popov determinant for the gauge fixing.**

$$\Delta(X_\mu) = \prod_{i,j=1}^{s} {\prod}'^{(n_i+n_j)/2-1}_{l=|n_i-n_j|/2} [\chi^2 l(l+1)]^{k_i k_j (2l+1)}.$$

($\prod' \to l = 0$ is excluded).

- **$\mathcal{Z}$: integration over the fluctuation $\tilde{A}_\mu$.**

Gauge fixing term: $S_{\text{g.f.}} = -\frac{1}{2} N\alpha^4 \text{tr}[X_\mu, A_\mu]^2$.

$$S_{\text{tot}} = S + S_{\text{g.f.}} = S_{\text{cl}} + \frac{1}{2} N\alpha^4 \text{tr}(\tilde{A}_\mu \mathcal{Q}_{\mu\nu} \tilde{A}_\nu), \text{ where}$$
$$\mathcal{Q}_{\mu\nu} = \{\chi^2 (\mathcal{L}_\lambda)^2 + \rho^2\}\delta_{\mu\nu} - i\rho\epsilon_{\mu\nu\lambda}\mathcal{L}_\lambda, \quad \mathcal{L}_\lambda M = [L_\lambda, M].$$

Eigenvalue problem $\mathcal{Q}_{\mu\nu}\tilde{A}_\nu = \lambda \tilde{A}_\mu$.

K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0205185.

For $n_i \times n_j$ blocks, $(\tilde{A}_\mu)_{n_i \times n_j} = \sum_{l=|n_i-n_j|/2}^{(n_i+n_j)/2-1} \sum_{m=-l}^{l} \tilde{A}_\mu^{(l,m)} |l, m\rangle$.

$$(\mathcal{L}_1 \pm i\mathcal{L}_2)|l,m\rangle = \underbrace{\sqrt{l(l+1) - m(m\pm 1)}}_{=b_\pm} |l, m\pm 1\rangle.$$

Eigenvalue equation for each block:

$$\{\Lambda - \rho^2(1-m)\}\tilde{A}_+^{(l,m)} = \rho^2 b_- \tilde{A}_3^{(l,m-1)} ,$$
$$\{\Lambda - \rho^2(1+m)\}\tilde{A}_-^{(l,m)} = -\rho^2 b_+ \tilde{A}_3^{(l,m+1)} ,$$
$$\{\Lambda - \rho^2\}\tilde{A}_3^{(l,m)} = \frac{1}{2}\rho^2 (b_+ \tilde{A}_+^{(l,m+1)} - b_- \tilde{A}_-^{(l,m-1)}) , \text{ where}$$
$$\tilde{A}_\pm^{(l,m)} = \tilde{A}_1^{(l,m)} \pm i\tilde{A}_2^{(l,m)}, \quad \Lambda = \lambda - \chi^2 l(l+1).$$

Three eigenvalues for $l \geq \frac{1}{2}$ ($l = 0 \to \lambda = \rho^2$ with 3-fold degeneracy):

$$\underbrace{\lambda_1 = \chi^2 l(l+1)}_{2l+1 \text{ degeneracy}}, \quad \underbrace{\lambda_2 = \chi^2 l(l+1) - \rho^2 l}_{2l-1 \text{ degeneracy}}, \quad \underbrace{\lambda_3 = \chi^2 l(l+1) + \rho^2(l+1)}_{2l+3 \text{ degeneracy}}.$$

Final result:

$$\mathcal{Z} = \left(\frac{2\pi}{N\alpha^4}\right)^{\frac{3}{2}N^2} \times \prod_{i=1}^{s} \rho^{-3(k_i)^2} \times \prod_{i,j=1}^{s} (q_{n_i n_j})^{k_i k_j}, \text{ where}$$

$$q_{nm} = \prod_{l=|n-m|/2}^{'\;(n+m)/2-1} \left[\lambda_1^{2l+1} \lambda_2^{2l-1} \lambda_3^{2l+3}\right]^{-\frac{1}{2}}.$$

$\rho < \frac{2}{3}$ regime

Single $(s = 1, n_1 = N, k_1 = 1)$ fuzzy sphere $A_\mu = \chi L_\mu^{(N)}$ $\Rightarrow F_{\rm cl} = -{\rm O}(\alpha^4 N^4) < 0$.

$\Rightarrow$ Coincident fuzzy spheres cannot be the true vacuum.

Free energy for single fuzzy sphere:

$$\frac{F_{\rm FS}}{N^2} = \frac{N^2\alpha^4}{4} f(\chi) + \frac{5}{2}\log N + 4\log\alpha - \delta(\rho), \text{ where}$$
$$f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2\chi^2}{2} (\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}).$$

Free energy in the Yang-Mills phase (based on Gaussian expansion)

J. Nishimura, T. Okubo and F. Sugino, hep-th/0205253.

$$\frac{F_{\rm YM}}{N^2} = \frac{3}{2}\log N + 3\log\alpha + \underbrace{\gamma}_{\simeq -4.5}.$$

Critical point: $\alpha_{\rm cr} = \frac{1}{\sqrt{N}} \left(\frac{2\log N}{|f(\chi)|} \right)^{1/4}$

- $\alpha < \alpha_{\rm cr} \to F_{\rm YM} < F_{\rm FS}$: Yang-Mills phase
- $\alpha > \alpha_{\rm cr} \to F_{\rm YM} > F_{\rm FS}$: single fuzzy sphere

$\rho > \frac{2}{3}$ regime

$\frac{F_{\rm cl}}{N^2} = \frac{1}{4}\alpha^4 f(\chi)(\sum_{i=1}^{s} r_i(n_i^3 - n_i)) > 0$ ($r_i = \frac{k_i}{N}$, so that $\sum_{i=1}^{s} n_i r_i = 1$) and

$\frac{F_{1-\rm loop}}{N^2} = \frac{3}{2}\log N + O(1)$.

The stable fuzzy sphere should have lower free energy than $F_{A=0} = \frac{3}{2}\log N + O(1)$.

The stable fuzzy sphere should satisfy $n_i = O(1)$, $k_i = r_i N = O(N)$.

$U(k_i)$ gauge group ($k_i = O(N)$) is dynamically generated.

Analogous to coincident transverse 5-branes in M-theory.

J. Maldacena, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0211139.

Rough estimate at large α

$$\frac{F}{N^2} \simeq \underbrace{\frac{1}{4}\alpha^4 f(\chi)}_{=A} \sum_{i=1}^{s} r_i (n_i^3 - n_i) + \underbrace{2\log\alpha}_{=B} \sum_{i=1}^{s} (r_i)^2.$$

Minimize this free energy under $\sum_{i=1}^{s} n_i r_i = 1$.

$$r_i = \frac{1}{2B}\left[\lambda n_i - A(n_i^3 - n_i)\right], \text{ where}$$

$$\lambda = \frac{1}{\sum_{i=1}^{s} n_i^2}\left[2B + A\sum_{i=1}^{s}(n_i^4 - n_i^2)\right].$$

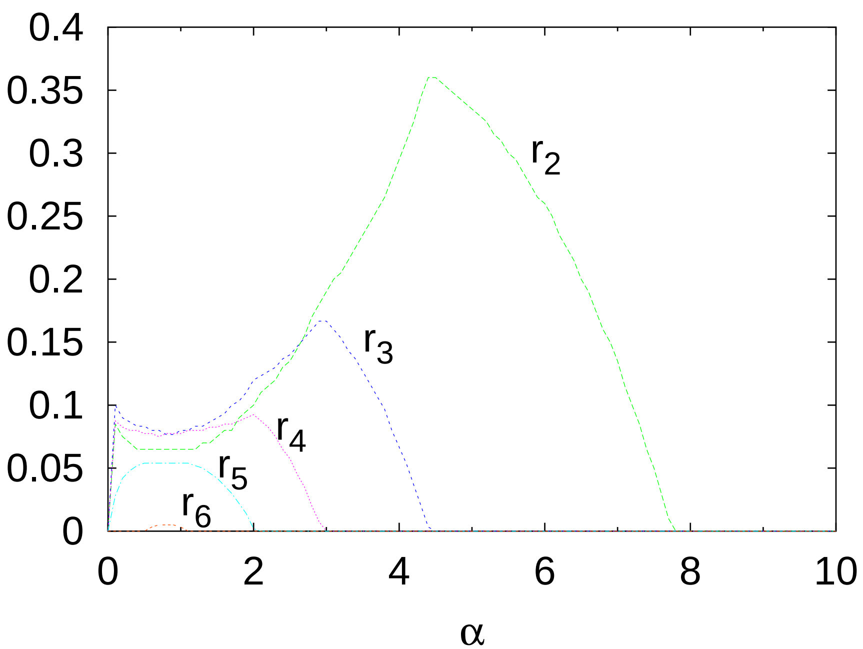
For brevity, $n_1 = 1, n_2 = 2, n_3 = 3, \cdots, n_s = s$.

Condition $r_i > 0 \Rightarrow s(s^2-1)(4s^2-1) < \frac{60B}{A} \left(= \frac{480\log\alpha}{\alpha^4 f(\chi)}\right)$.

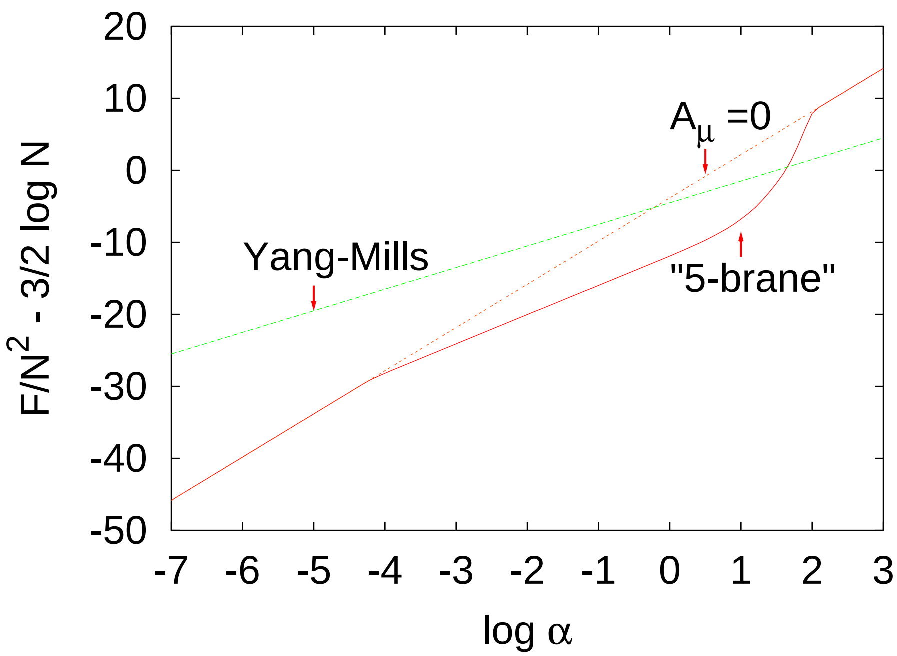
- **Large $\alpha \Rightarrow$ Only $s = 1$ is possible $\Rightarrow$ $A_\mu = 0$ is favored.**
- **$s = 2$ (2×2 spheres) appears at $\frac{A}{B} = \frac{2}{3}$.**

General analysis at moderate α

r_i's that minimize the free energy at $\rho = 0.7$.

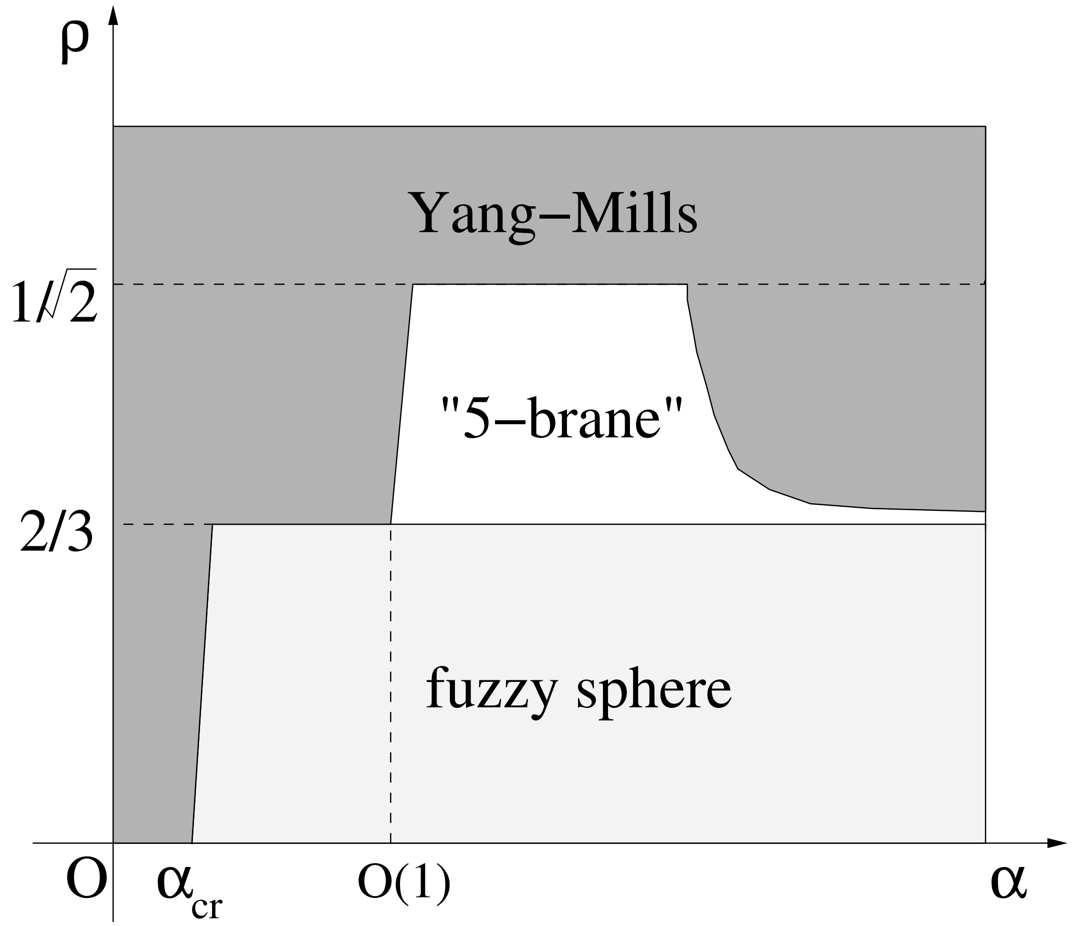


Comparison of free energy with Yang-Mills phase at $\rho = 0.7$.



The Yang-Mills phase takes over at $\alpha \simeq O(1)$.

Phase diagram



5 Conclusion

Massive Yang-Mills-Chern-Simons model:

Many local minima with different gauge groups.

A nice laboratory for testing ideas and methods to study superstring theory by matrix models.

In this work, we have found

Dynamical generation of nontrivial gauge group.

The road to deriving the SU(3) × SU(2) × U(1) gauge group from the large-N reduced model?