INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Status of Horava-Lifshitz gravity

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ref. Horava-Lifshitz Cosmology: A Review
arXiv: 1007.5199 [hep-th]
also arXiv: 110x.xxxx with K.Izumi
arXiv: 110x.xxxx with E.Gumrukcuoglu

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

$$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling
  $t \rightarrow b^z\, t$ ($E \rightarrow b^{-z} E$)
  $x \rightarrow b\, x$
  $\phi \rightarrow b^s\, \phi$
  $z+3-2z+2s = 0$
  $s = -(3-z)/2$
- $s = 0$ if $z = 3$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For $z = 3$, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \to t'(t), \quad x^i \to x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \to b^z\, t, \quad x^i \to b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}\,d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \Big[\quad D_i R_{jk} D^i R^{jk} \qquad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \qquad R R_i^j R_j^i \qquad R^3 \qquad \Big]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \Big(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{c_g^2 R - 2\Lambda}_{\text{IR potential}} \Big)$$

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Cosmological implications

- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The $z=3$ scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration “constant” (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (S.Maeda, Mukohyama, Shiromizu 2009).

Different versions of HL gravity

- There are at least four versions of the theory: w/wo detailed balance & w/wo projectability.
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable. [c.f. Henneaux, et.al. (2009)]
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- There is an attempt to extend the non-projectable theory by introducing $a_i = (\ln N)_{,i}$ [Blas, Pujolas and Sibiryakov 2009].

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f\dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f\dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f\dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

“Black holes” with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol
for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol
for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and H gets strongly coupled.

[NOTE: Strong coupling itself does not imply loss of predictability since all coefficients of infinite terms in perturbative expansion can be written in terms of the 11 parameters of the theory.]

- It is important to see if there is “Vainshtein effect”, i.e. decoupling of the strongly-coupled sector from the rest of the world.

Vainshtein effect in massive gravity

- Linearized analysis results in vDVZ discontinuity of the massless limit.
- However, perturbative expansion completely breaks down and cannot be trusted.
- Non-perturbative analysis shows continuity and GR is recovered in the massless limit.
- Continuity is not uniform as a function of distance. (e.g. 1/r expansion does not work.) However, Vainshtein radius can be pushed to infinity in the massless limit.

Strong coupling vs loss of predictability

- In low-E EFT like massive gravity (and DGP gravity), the strong coupling implies loss of predictability. Prediction requires knowledge of infinite number of terms, which we do not know.
- In HL gravity, if the theory is really renormalizable then all coefficients of infinite terms in perturbative expansion are written in terms of the 11 parameters of the theory. Therefore, the strong coupling itself does not imply loss of predictability.
- However, we need to see if the strongly coupled sector decouples from the other sector. This would be an **analogue of Vainshtein effect.**

Linear instability of scalar graviton

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = c_s^2 k^2 \times [1+ O(k^2/M^2)]$ with $c_s^2 = -(\lambda-1)/(3\lambda-1)<0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability of "DM as integration const" if
  $t_J \sim (G_N\rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/M$.
- Thus, the linear instability does not show up if
  $|c_s| = |(\lambda-1)/(3\lambda-1)|^{1/2} < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$. $(\Phi \sim -G_N\rho L^2)$
  for $L > \mathrm{Max}[0.01\mathrm{mm}, 1/M]$
  (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

Analogue of Vainshtein effect

- Breakdown of perturbation in the limit λ → 1

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = e^{2\zeta}\left[e^h\right]_{ij}$$

$$B = \frac{3\lambda - 1}{\lambda - 1}\frac{\dot\zeta}{\partial^2}, \quad n_i = 0$$

momentum constraint

$$\begin{aligned} I_{kin} &= M_{Pl}^2 \int dt d^3\vec{x} \left\{ (1+3\zeta)\left[\frac{3\lambda-1}{\lambda-1}\dot\zeta^2 + \frac{1}{8}\dot h^{ij}\dot h_{ij}\right] \right. \\ &\quad + \frac{1}{2}\zeta\partial^i(\partial_i B\partial^2 B + 3\partial^j B\partial_i\partial_j B) + \frac{1}{2}(\partial^k h_{ij}\partial_k B - 3\dot h_{ij}\zeta)\partial^i\partial^j B \\ &\quad \left. - \frac{1}{4}(\dot h^{ij}\partial_k h_{ij})\partial^k B \right\} + O(\epsilon^4), \end{aligned}$$

- No negative power of (λ-1) in potential part
- Non-perturbative analysis is needed for scalar graviton sector!

Analogue of Vainshtein effect

- Spherically symmetric, static ansatz

$$N=1,\quad N_i dx^i=\beta(x)dx,\quad g_{ij}dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$$\Downarrow\quad R\equiv \beta^{(\lambda-1)/(2\lambda)}r$$ without HD terms

$$R''+\frac{\lambda-1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2R}{4\lambda^2\beta^2}+\frac{(\lambda-1)\beta'R'}{\lambda\beta}-\frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta}-\frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2+\frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda}+[(2\lambda-1)\beta^2-1](R')^2}{(3\lambda-1)\beta^2+(\lambda-1)} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1\pm\sqrt{1+4AB}}{2A},$$

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'},\quad B\equiv\frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda}+[(2\lambda-1)\beta^2-1](R')^2}{(3\lambda-1)\beta^2+(\lambda-1)}$$

- "-" branch recovers GR in the $\lambda \to 1$ limit

Analogue of Vainshtein effect

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$ choose the “-” branch

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'} \frac{\boxed{\beta^{(\lambda-1)/\lambda}} + [(2\lambda-1)\beta^2-1](R')^2}{\boxed{(3\lambda-1)\beta^2+(\lambda-1)}}$$

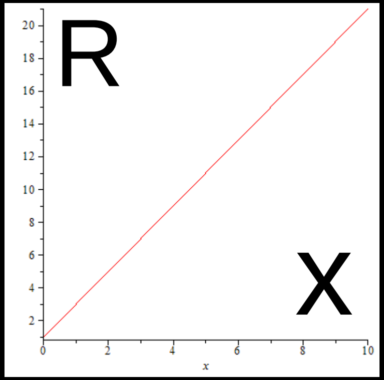
- $(3\lambda-1)\beta^2 << (\lambda-1)$
  perturbative regime, 1/r expansion
- $(3\lambda-1)\beta^2 >> (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius???

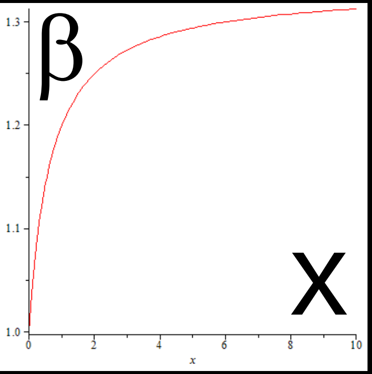
Izumi & Mukohyama 2009
“Steller center is dynamical”

Analogue of Vainshtein effect

- Numerical integration in the "-" branch with $\beta(x=0)=1$, $r(x=0)=1$, $r'(x=0)$ given

for
$\lambda-1=10^{-6}$
$r'(x=0)=2$



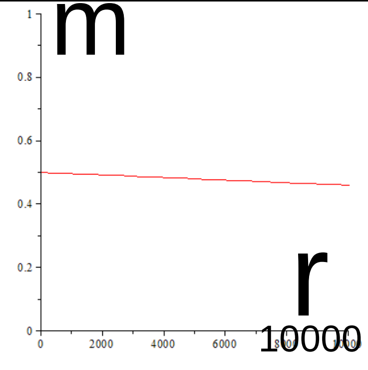


- Misner-Sharp energy

$$m \equiv \frac{r}{2}\left[1-\left(1-\beta^2\right)\left(r'\right)^2\right]$$

almost constant

GR is recovered!



Nonlinear cosmological perturbation and $\lambda \rightarrow 1$

(with K.Izumi, to appear soon)

- HL gravity @ IR → GR + DM [Mukohyama 2009]
- $^{\exists}$Subtleties with $\lambda \rightarrow 1$
- Nonlinear cosmological perturbation in vacuum HL gravity
- Gradient expansion
- No problem with $\lambda \rightarrow 1$ in any order

Caustic avoidance

JCAP 0909:005,2009

- In GR, congruence of geodesics forms caustics because gravity is attractive.
- HL gravity is repulsive at short distances, due to higher curvature (HC) terms. (c.f. bouncing FRW universe)
- With codimension 2 and 3, HC terms can bounce would-be caustics.
- With codimension 1, deviation of λ from 1 is also needed to bounce would-be caustics.

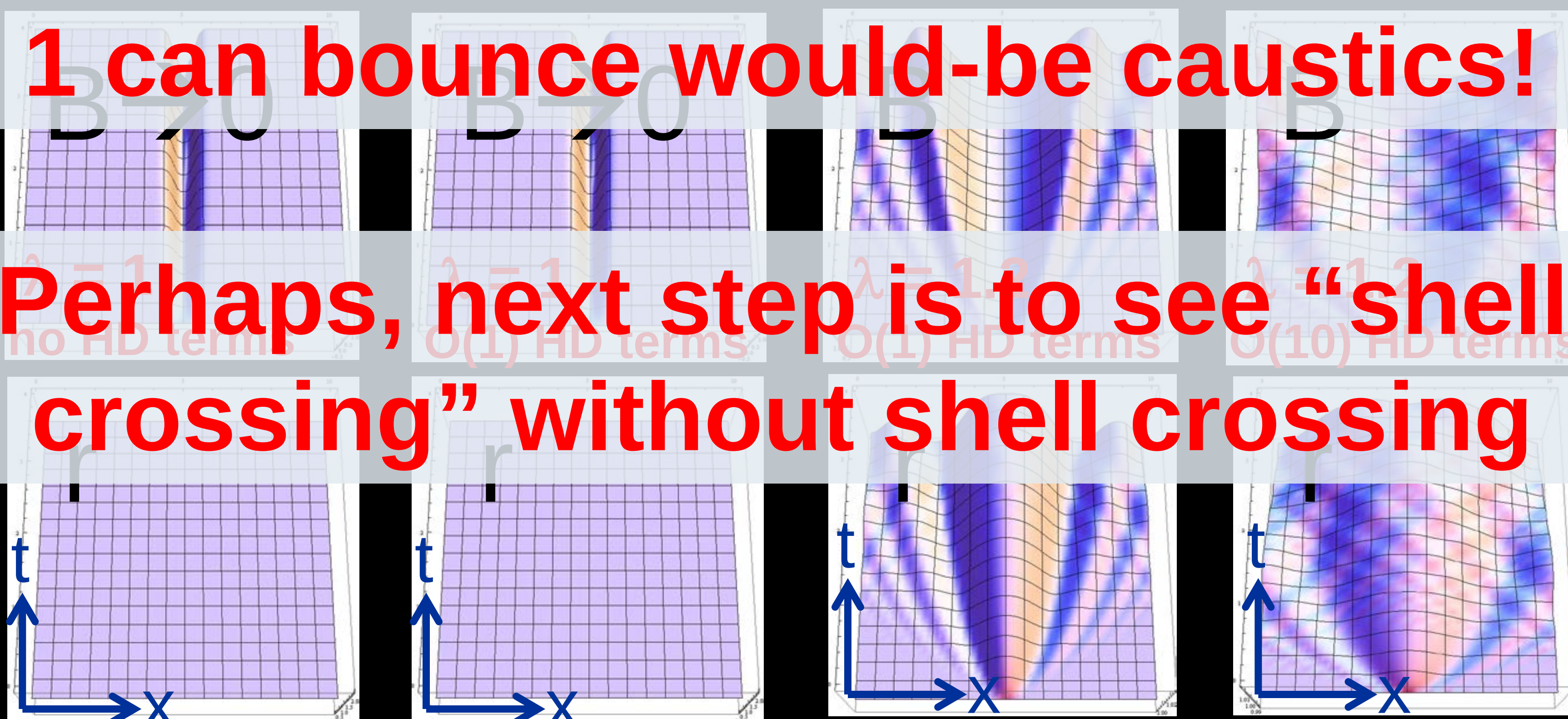
Caustic avoidance

(preliminary)

$$N = 1 \qquad N_i = 0$$

$$ds^2 = B(t,x)dx^2 + r^2(t,x)(dy^2 + dz^2)$$

HC terms and deviation of λ from 1 can bounce would-be caustics!



Perhaps, next step is to see "shell crossing" without shell crossing

What happens in the UV?

(with E.Gumrukcuoglu, to appear soon)

- $1/3 < \lambda < 1$ is forbidden because of ghost
- Recovery of GR requires $\lambda \to 1+0$ in the IR
- A natural candidate for the UV fixed point would be $\lambda \to \infty$
- Regular and simpler dynamics with $\lambda \to \infty$

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
  e.g. scale-invariant perturbations without inflation
  dark matter as an integral constant
- For spherically-symmetric, static, vacuum configurations, GR is recovered in the limit $\lambda \to 1$ non-perturbatively.
  $\to$ analogue of Vainshtein effect
- For superhorizon cosmological perturbations, GR + DM is recovered in the limit l $\to$ 1 non-perturbatively.
- Caustics avoidance requires higher curvature terms and deviation of λ from 1 in the UV. Next step is to see if bounce of shells can mimic shell crossing.
- A natural candidate for the UV fixed point would be $\lambda \to \infty$, with regular and simpler dynamics.

Future works

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ? Does it satisfy the stability condition for the scalar graviton?
  ($|c_s| < \mathrm{Max}\,[|\Phi|^{1/2},HL]$ for $L>\mathrm{Max}[M^{-1},0.01\mathrm{mm}]$)
- Can we get a common "limit of speed" ?
  (i) $M_{z=3} \ll M_{pl}$, (ii) supersymmetry, (iii) other ideas?
- How generic is 'Vainshtein effect'?
- How generic is caustic avoidance?
- Micro & macro behavior of "CDM"
- Adiabatic initial condition for "CDM" from the z=3 scaling
- Spectral tilt from anomalous dimension
- Extensions of the original theory: Blas, et.al; Horava & Melby-Thompson …

Backup slides

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No "local" Hamiltonian constraint

  E.o.m. of matter

  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq

  can be integrated to give

  Friedmann eq with "dark matter as integration constant"

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}\rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$