

# Non-Gaussianities from isocurvature perturbations

David Langlois  
(APC, Paris)



Astroparticules  
et Cosmologie

# Outline

1. Introduction
2. Evolution of non-linear perturbations at some decay
3. Generalized bispectra
4. Generalized trispectra
5. CMB angular bispectrum

**Based on** DL, F. Vernizzi & D. Wands, JCAP 0812 (2008) 004 [arXiv:0809.4646]  
DL & A. Lepidi, JCAP 1101 (2011) 008 [arXiv:1007.5498]  
DL & T. Takahashi, JCAP 1102 (2011) 020 [arXiv:1012.4885]  
DL & B. van Tent, work in progress

## **See also**

Kawasaki, Nakayama, Sekiguchi, Suyama, Takahashi, JCAP 0811 (2008) 019 &  
JCAP 0901 (2009) 042

Hikage, Koyama, Matsubara, Takahashi, Yamaguchi, MNRAS 398, 2188 (2008)

Kawakami, Kawasaki, Nakayama, Takahashi, JCAP 0909 (2009) 002

# Isocurvature perturbations

- Several matter components in the Universe

$$S_X \equiv \frac{\delta n_X}{n_X} - \frac{\delta n_\gamma}{n_\gamma} = \frac{1}{1+w_X} \frac{\delta \rho_X}{\rho_X} - \frac{3}{4} \frac{\delta \rho_\gamma}{\rho_\gamma} = 3(\zeta_X - \zeta_\gamma)$$

- Observational constraints on  $\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} = \alpha \equiv \frac{a}{1-a}$

$$a_0 < 0.064 \quad (95\% \text{CL})$$

$$a_1 < 0.0037 \quad (95\% \text{CL})$$

depending on the correlation  $c \equiv \frac{\mathcal{P}_{S,\zeta}}{\sqrt{\mathcal{P}_S \mathcal{P}_\zeta}}$

[ WMAP7+BAO+SN ]

- Non-Gaussianities from isocurvature modes ?
  - If isocurvature modes exist, can they contribute to NG ?
  - What could be their amplitude ?

# Primordial non-Gaussianities

- **Adiabatic** non-Gaussianities of local type

$$\Phi(\mathbf{x}) = \hat{\Phi}(x) + f_{\text{NL}}(\hat{\Phi}^2(\mathbf{x}) - \langle \hat{\Phi}^2 \rangle)$$

or

$$\zeta(\mathbf{x}) = \hat{\zeta}(x) + \frac{3}{5}f_{\text{NL}}(\hat{\zeta}^2(\mathbf{x}) - \langle \hat{\zeta}^2 \rangle)$$

- Present observational constraints

$$-10 < f_{\text{NL}}^{\text{local}} < 74 \quad (95\% \text{ CL}) \quad [\text{WMAP7: Komatsu et al.}]$$

- Detection of significant local NG would imply
  - single field inflation ruled out
  - several degrees of freedom during inflation ?

# Non-linear perturbations

- In a multi-fluid system, one can define for each fluid

$$\zeta_A = \delta N + \frac{1}{3(1+w_A)} \ln \frac{\rho_A}{\bar{\rho}_A} \qquad \rho_A = \bar{\rho}_A e^{3(1+w_A)(\zeta_A - \delta N)}$$

- Non-linear isocurvature perturbation

$$S_{A,B} = 3(\zeta_A - \zeta_B)$$

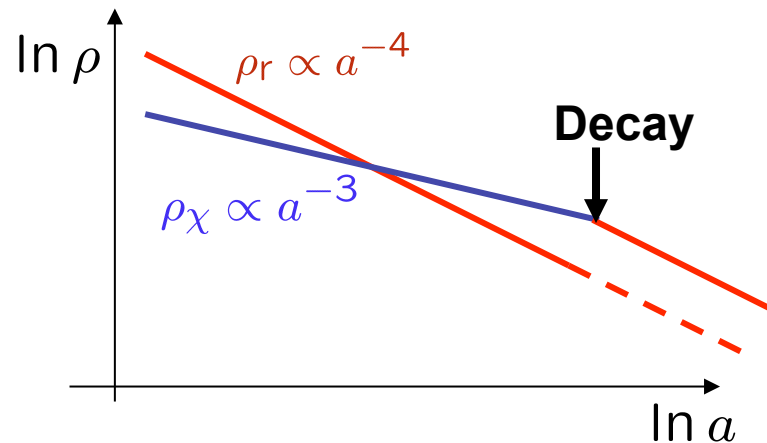
- Primordial entropy/isocurvature perturbation

$$S_m = 3(\zeta_m - \zeta_r) = \ln \left( \frac{\rho_m}{\bar{\rho}_m} \right) - \frac{3}{4} \ln \left( \frac{\rho_r}{\bar{\rho}_r} \right)$$

# The curvaton scenario

Mollerach (1990); Linde & Mukhanov (1997) ;  
Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

- Light scalar field during inflation (when  $H > m$ )  
which later oscillates (when  $H < m$ ), and finally decays.



- **Mixed curvaton-inflaton** scenario: both inflaton and curvaton fluctuations contribute to the observable perturbations.

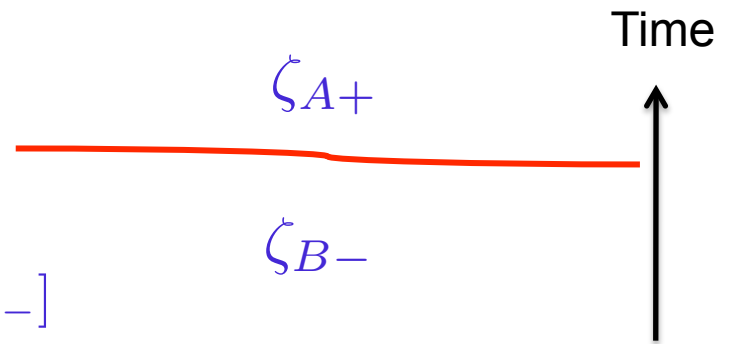
DL & Vernizzi '04; Ferrer, Rasanen & Valiviita '04 + many others

# Post-decay perturbations

- Decay hypersurface:  $H = \Gamma_\sigma$

$$\sum_B \rho_{B-} = \bar{\rho}_{\text{decay}} = \sum_A \rho_{A+}$$

$$\sum_B \Omega_B e^{3(1+w_B)(\zeta_{B-} - \zeta)} = 1 \implies \zeta = \zeta[\zeta_{B-}]$$



- For each species:  $\rho_{A+} = \rho_{A-} + \gamma_{A\sigma} \rho_\sigma$ ,  $\sum_A \gamma_{A\sigma} = 1$

$$e^{\beta_A(\zeta_{A+} - \zeta)} = (1 - f_A) e^{\beta_A(\zeta_{A-} - \zeta)} + f_A e^{\beta_\sigma(\zeta_{\sigma-} - \zeta)}$$

$$f_A \equiv \frac{\gamma_{A\sigma} \Omega_\sigma}{\Omega_A + \gamma_{A\sigma} \Omega_\sigma}$$

# Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_B T_A^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-} \zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-} \zeta_{C-} \zeta_{D-}$$

with the background-dependent coefficients

$$T_A^A = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_A + (1 - f_A),$$

$$T_A^\sigma = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_\sigma + f_A \frac{\beta_\sigma}{\beta_A},$$

$$T_A^C = f_A \left( 1 - \frac{\beta_\sigma}{\beta_A} \right) \lambda_C, \quad C \neq A, \sigma.$$

$$\beta_A \equiv 3(1 + w_A), \quad \lambda_A \equiv \frac{\beta_A \Omega_A}{\sum_B \beta_B \Omega_B}$$



# Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_B T_A^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-} \zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-} \zeta_{C-} \zeta_{D-}$$

with the background-dependent coefficients

$$U_A^{BC} = \frac{1}{2} \left[ \sum_E \beta_E T_A^E (\delta_{EB} - \lambda_B)(\delta_{EC} - \lambda_C) - \beta_A (T_{AB} - \lambda_B)(T_{AC} - \lambda_C) \right]$$

# Post-decay perturbations

- Up to third order, one can write

$$\zeta_{A+} = \sum_B T_A^B \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-} \zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-} \zeta_{C-} \zeta_{D-}$$

with the background-dependent coefficients

$$\begin{aligned} V_A^{BCD} &= -\frac{1}{2} \sum_{E,F} \beta_E T_{AE} (\delta_{EB} - \lambda_B) \lambda_F \beta_F (\delta_{FC} - \lambda_C) (\delta_{FD} - \lambda_D) \\ &+ \frac{1}{6} \sum_E \beta_E^2 T_{AE} (\delta_{EB} - \lambda_B) (\delta_{EC} - \lambda_C) (\delta_{ED} - \lambda_D) \\ &- \beta_A (T_{AB} - \lambda_B) \left[ U_A^{CD} - \frac{1}{2} \sum_E \beta_E \lambda_E (\delta_{EC} - \lambda_C) (\delta_{ED} - \lambda_D) \right] \\ &- \frac{1}{6} \beta_A^2 (T_{AB} - \lambda_B) (T_{AC} - \lambda_C) (T_{AD} - \lambda_D). \end{aligned}$$

# Mixed curvaton-inflaton scenario

Simple example: radiation + cdm + single curvaton

- **After the decay** (assuming  $\zeta_{c-} = \zeta_{r-} = \zeta_{\text{inf}}$  and  $\Omega_c \ll 1$ )

$$\zeta_{c+} = \zeta_c + \frac{1}{3}f_c(S_\sigma - S_c) + \frac{1}{6}f_c(1 - f_c)(S_\sigma - S_c)^2 + \frac{1}{18}f_c(1 - 3f_c + 2f_c^2)(S_\sigma - S_c)^3$$

$$\zeta_{r+} = \zeta_{r-} + \frac{r}{3}S_{\sigma-} + \frac{r}{18} \left[ 3 - 4r + \frac{2r}{\xi} - \frac{r^2}{\xi^2} \right] S_{\sigma-}^2 + \frac{r}{162} \left[ 9 + 18(1 - 2\xi)\frac{r}{\xi} + 4(8\xi^2 - 6\xi - 3)\frac{r^2}{\xi^2} + 2(6\xi - 1)\frac{r^3}{\xi^3} + 3\frac{r^4}{\xi^4} \right] S_{\sigma-}^3$$

- Parameters:  $f_c$ ,  $r \equiv \xi \tilde{r}$ ,  $\xi \equiv \frac{f_r}{\Omega_\sigma} = \frac{\gamma_{r\sigma}}{1 - (1 - \gamma_{r\sigma})\Omega_\sigma}$   
 $\tilde{r} = \frac{3\Omega_\sigma}{4 - \Omega_\sigma}$

# Curvaton perturbations

- During inflation: fluctuations  $\delta\sigma_* \sim \frac{H_*}{2\pi}$

- Oscillating phase:  $\rho_\sigma = m^2 \sigma^2$

$$\rho_\chi = \bar{\rho}_\chi e^{3(\zeta_\chi - \delta N)} \quad \left[ \zeta_m = \delta N + \frac{1}{3} \ln \left( \frac{\rho_m}{\bar{\rho}_m} \right) \right]$$

- On a constant energy density hypersurface (subdominant curvaton)

$$\delta N = \zeta_r$$

$$\rho_\sigma = \bar{\rho}_\sigma e^{3(\zeta_\sigma - \zeta_r)} = \bar{\rho}_\sigma e^{S_\sigma} \quad \Longrightarrow \quad e^{S_\sigma} = \left( 1 + \frac{\delta\sigma}{\bar{\sigma}} \right)^2$$

- Using the conservation of  $\delta\sigma/\bar{\sigma}$ , one gets

$$S_\sigma = \hat{S} - \frac{1}{4}\hat{S}^2 + \frac{1}{12}\hat{S}^3, \quad \hat{S} \equiv 2\frac{\delta\sigma_*}{\bar{\sigma}_*}$$

# “Primordial” perturbations

- **Curvature perturbation**  $\zeta_r = \zeta_{\text{inf}} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \frac{1}{6} z_3 \hat{S}^3$

$$z_1 = \frac{r}{3}, \quad z_2 = \frac{r}{18} \left( 3 - 8r + \frac{4r}{\xi} - 2 \frac{r^2}{\xi^2} \right),$$

$$z_3 = \frac{r^2}{54} \left( \frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36 \right)$$

- **Isocurvature perturbation**  $S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 + \frac{1}{6} s_3 \hat{S}^3$

$$s_1 = f_c - r, \quad s_2 = \frac{1}{6} \left( 3f_c(1 - 2f_c) + \frac{2r^3}{\xi^2} - \frac{4r^2}{\xi} + 8r^2 - 3r \right),$$

$$s_3 = -\frac{1}{2} f_c^2 (3 - 4f_c) - \frac{r^2}{18} \left( \frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36 \right)$$

# Power spectra

- Curvature:  $\mathcal{P}_{\zeta_r} = \mathcal{P}_{\zeta_{\text{inf}}} + \frac{r^2}{9} \mathcal{P}_{\hat{S}} \equiv \Xi^{-1} \frac{r^2}{9} \mathcal{P}_{\hat{S}}$
- Isocurvature:  $\mathcal{P}_{S_c} = (f_c - r)^2 \mathcal{P}_{\hat{S}}$
- Correlation:  $\mathcal{C} = \frac{\mathcal{P}_{S_c, \zeta_r}}{\sqrt{\mathcal{P}_{S_c} \mathcal{P}_{\zeta_r}}} = \varepsilon_f \sqrt{\Xi}, \quad \varepsilon_f \equiv \text{sgn}(f_c - r)$

The observational constraint on  $\alpha = \frac{\mathcal{P}_{S_c}}{\mathcal{P}_{\zeta_r}} = 9 \left(1 - \frac{f_c}{r}\right)^2 \Xi$  is satisfied if

$$\Xi \ll 1 \quad \text{or} \quad |f_c - r| \ll r \quad (\text{e.g. } f_c = 1, r \simeq 1)$$

# Bispectra

- Generalized bispectra:

$$\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) X^K(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta(\sum_i \mathbf{k}_i) B^{IJK}(k_1, k_2, k_3)$$

- One substitutes  $X^I = N_a^I \delta\phi^a + \frac{1}{2} N_{ab}^I \delta\phi^a \delta\phi^b + \dots$

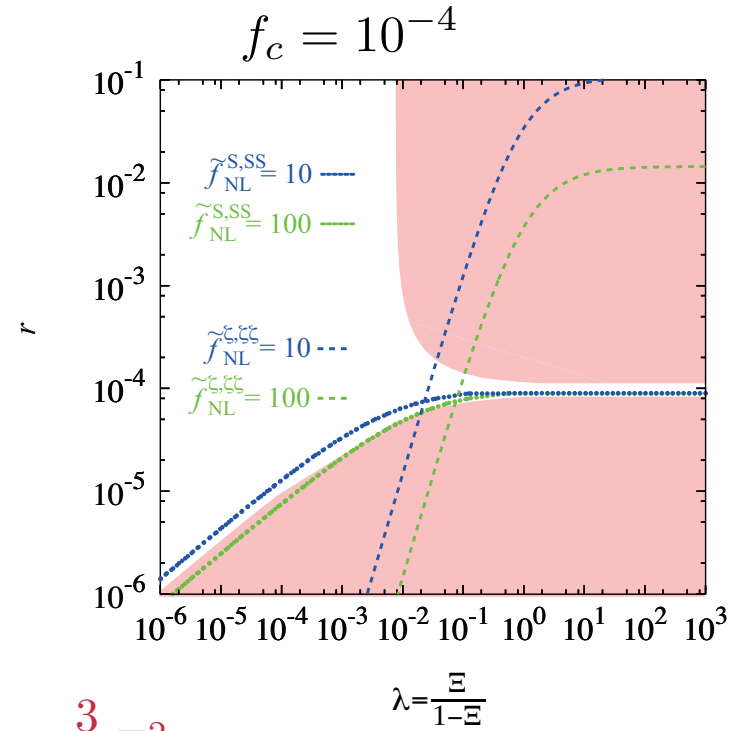
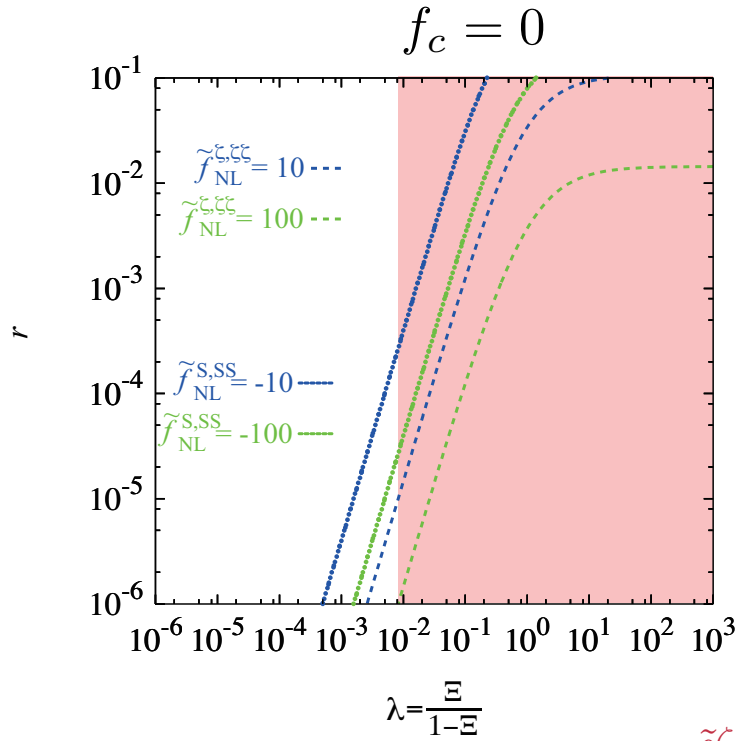
$$B^{IJK}(k_1, k_2, k_3) = \lambda^{I,JK} P(k_2) P(k_3) + \lambda^{J,KI} P(k_1) P(k_3) + \lambda^{K,IJ} P(k_1) P(k_2)$$

$$\lambda^{I,JK} \equiv N_{ab}^I N^{Ja} N^{Kb} \quad \tilde{f}_{NL}^{I,JK} \equiv \lambda^{I,JK} \left( \frac{P(k)}{P_\zeta(k)} \right)^2$$

- In our case  $X^I = \{\zeta, S\}$ , one finds **6 distinct coefficients**:

$$\begin{aligned} \tilde{f}_{NL}^{\zeta,\zeta\zeta} &= \frac{z_2}{z_1^2} \Xi^2, & \tilde{f}_{NL}^{\zeta,\zeta S} &= \frac{s_1 z_2}{z_1^3} \Xi^2, & \tilde{f}_{NL}^{S,\zeta\zeta} &= \frac{s_2}{z_1^2} \Xi^2, \\ \tilde{f}_{NL}^{\zeta,SS} &= \frac{s_1^2 z_2}{z_1^4} \Xi^2, & \tilde{f}_{NL}^{S,\zeta S} &= \frac{s_1 s_2}{z_1^3} \Xi^2, & \tilde{f}_{NL}^{S,SS} &= \frac{s_1^2 s_2}{z_1^4} \Xi^2 \end{aligned}$$

# Bispectra



$$\tilde{f}_{\text{NL}}^{\zeta, \zeta \zeta} \equiv \frac{6}{5} f_{\text{NL}} \simeq \frac{3}{2r} \Xi^2$$

$$f_c - r \simeq \varepsilon_f \frac{\sqrt{\alpha}}{3} r$$

$$\tilde{f}_{\text{NL}}^{S, SS} \simeq -27 \tilde{f}_{\text{NL}}^{\zeta, \zeta \zeta} \quad (f_c \ll r \ll 1)$$

$$\tilde{f}_{\text{NL}}^{S, SS} \simeq \left( \frac{3f_c}{r} \right)^3 \tilde{f}_{\text{NL}}^{\zeta, \zeta \zeta} \quad (r \ll f_c \ll 1)$$



# Bispectra

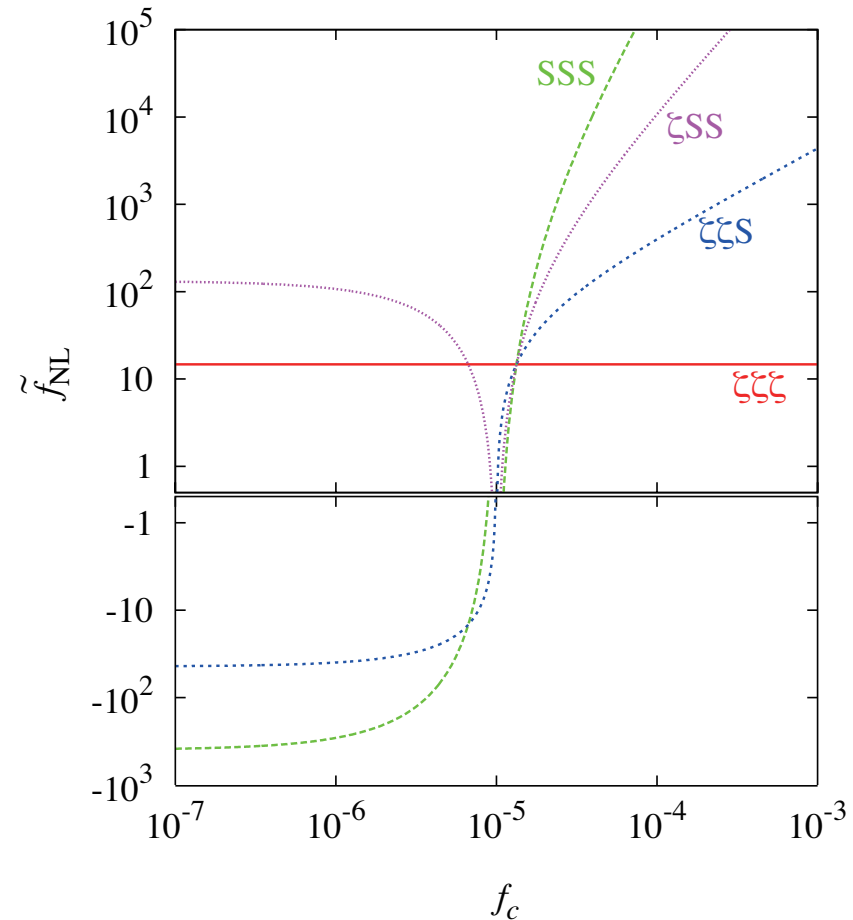
- Hierarchies between all the coefficients ?
- For  $f_c \ll r \ll 1$

$$\tilde{f}_{\text{NL}}^{I,JK} \simeq (-3)^{\mathcal{I}_S} \tilde{f}_{\text{NL}}^{\zeta,\zeta\zeta}$$

where  $\mathcal{I}_S$  is the number of S indices

- For  $r \ll f_c \ll 1$

$$\tilde{f}_{\text{NL}}^{I,JK} \simeq \left( \frac{3f_c}{r} \right)^{\mathcal{I}_S} \tilde{f}_{\text{NL}}^{\zeta,\zeta\zeta}$$



# Adiabatic trispectrum

- Adiabatic local trispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv (2\pi)^3 \delta(\sum_i \mathbf{k}_i) T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

- Assuming  $\zeta = N_a \delta\phi^a + \frac{1}{2} N_{ab} \delta\phi^a \delta\phi^b + \frac{1}{6} N_{abc} \delta\phi^a \delta\phi^b \delta\phi^c + \dots$

one gets

$$T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \tau_{\text{NL}} [P(k_{13})P(k_3)P(k_4) + 11 \text{ perms}] + \frac{54}{25} g_{\text{NL}} [P(k_2)P(k_3)P(k_4) + 3 \text{ perms}]$$

with

$$\tau_{\text{NL}} = \frac{N_{ab} N^{ac} N^b N_c}{(N_d N^d)^3}, \quad g_{\text{NL}} = \frac{25}{54} \frac{N_{abc} N^a N^b N^c}{(N_d N^d)^3}$$

- Constraints

$$-3.2 < \tau_{\text{NL}}/10^5 < 3.3 \quad (95\% \text{ CL})$$

$$-3.80 < g_{\text{NL}}/10^6 < 3.88 \quad (95\% \text{ CL}) \quad [\text{Smidt et al. '10}]$$

# Trispectra

- Generalized trispectra:

$$\langle X_{\mathbf{k}_1}^I X_{\mathbf{k}_2}^J X_{\mathbf{k}_3}^K X_{\mathbf{k}_4}^L \rangle_c \equiv (2\pi)^3 \delta(\sum_i \mathbf{k}_i) T^{IJKL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

- Substitute  $X^I = N_a^I \delta\phi^a + \frac{1}{2} N_{ab}^I \delta\phi^a \delta\phi^b + \frac{1}{6} N_{abc}^I \delta\phi^a \delta\phi^b \delta\phi^c + \dots$

- In our case

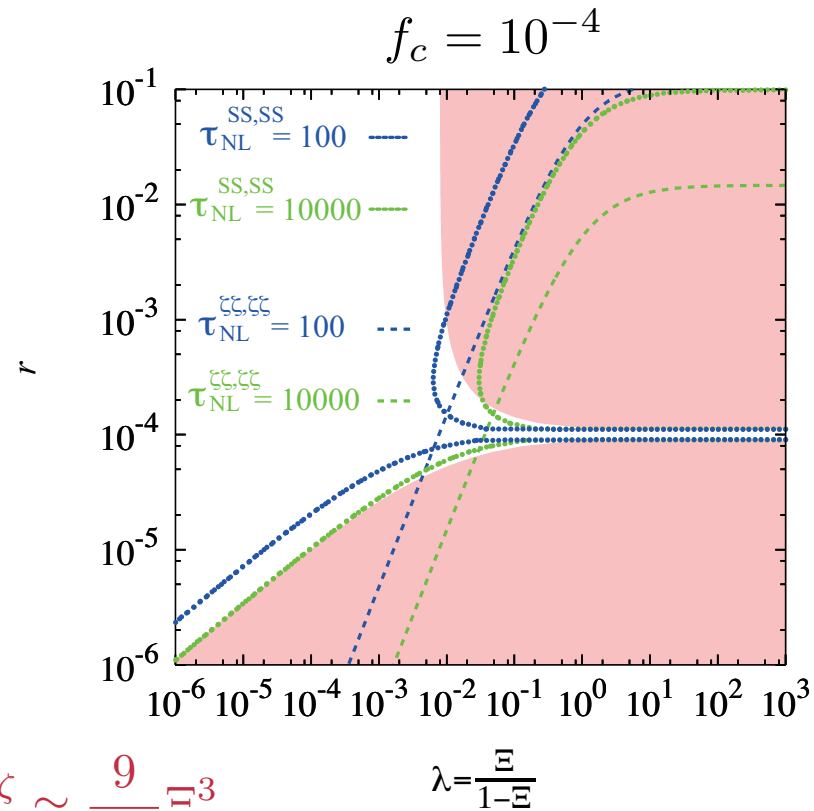
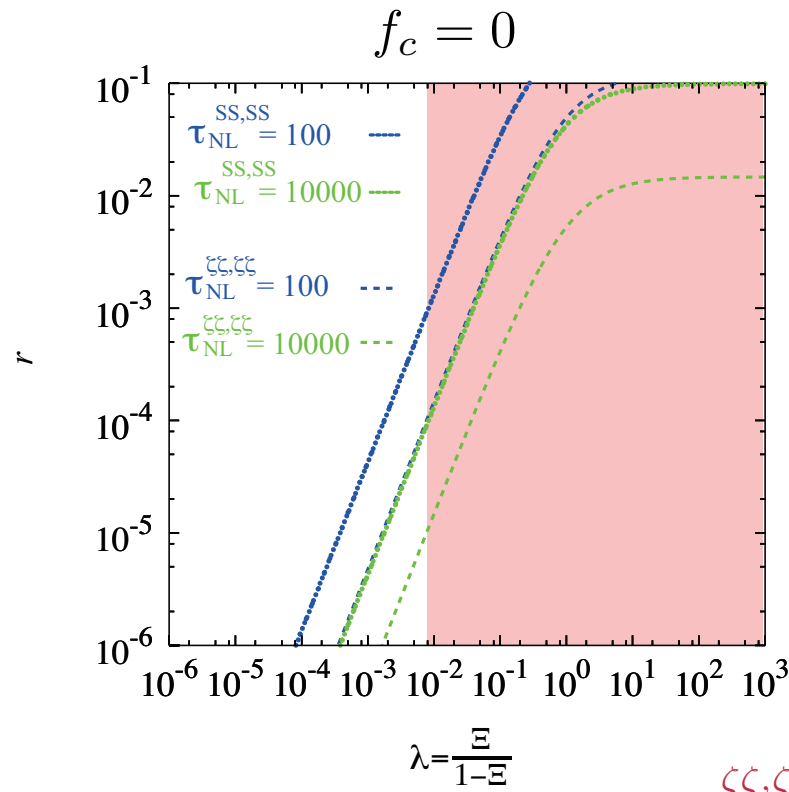
$$T^{IJKL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = t_{NL}^{I,JKL} P_{\hat{S}}(k_2) P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) + 3 \text{ perms}$$

$$+ \hat{t}_{NL}^{IJ,KL} [P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) P_{\hat{S}}(k_{13}) + P_{\hat{S}}(k_3) P_{\hat{S}}(k_4) P_{\hat{S}}(k_{14})] + 5 \text{ perms}$$

$$\tilde{g}_{NL}^{I,JKL} \equiv \frac{54}{25} g_{NL}^{I,JKL} \equiv \frac{N_{(3)}^I N_{(1)}^J N_{(1)}^K N_{(1)}^L}{z_1^6} \Xi^3 \quad \tau_{NL}^{IJ,KL} \equiv \frac{N_{(2)}^I N_{(2)}^J N_{(1)}^K N_{(1)}^L}{z_1^6} \Xi^3$$

**8 coefficients**  $\tilde{g}_{NL}^{I,JKL}$ , **9 coefficients**  $\tau_{NL}^{IJ,KL}$

# Trispectra: $\tau_{\text{NL}}^{IJ,KL}$



$$\tau_{\text{NL}}^{\zeta\zeta,\zeta\zeta} \simeq \frac{9}{4r^2} \xi^3$$

$$\tau_{\text{NL}}^{IJ,KL} \simeq (-3)^{\mathcal{I}_S} \tau_{\text{NL}}^{\zeta\zeta,\zeta\zeta}$$

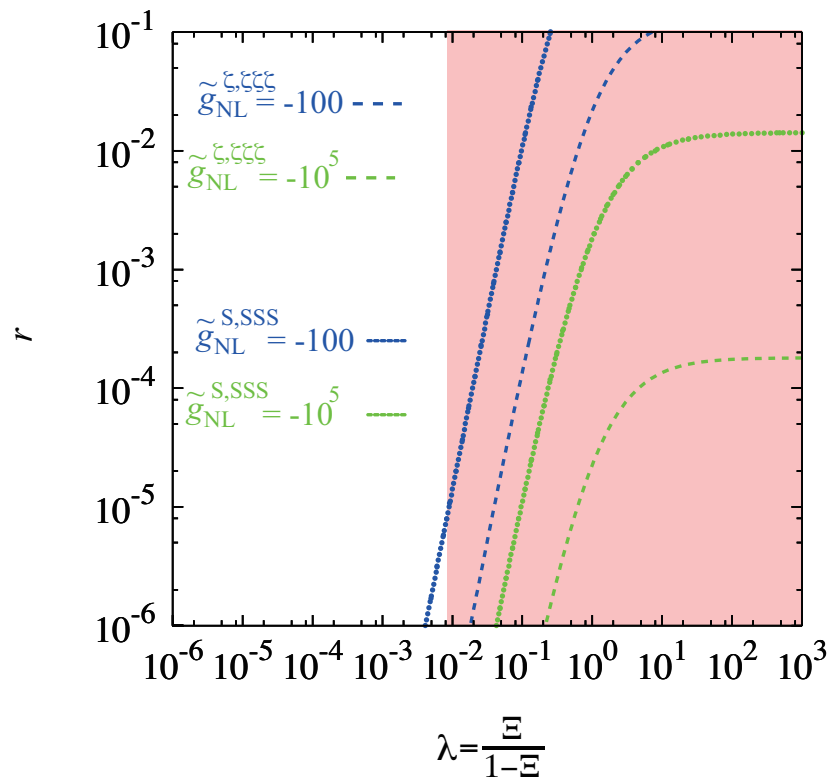
$(f_c \ll r \ll 1)$

$$\tau_{\text{NL}}^{IJ,KL} \simeq \left(\frac{3f_c}{r}\right)^{\mathcal{I}_S} \tau_{\text{NL}}^{\zeta\zeta,\zeta\zeta}$$

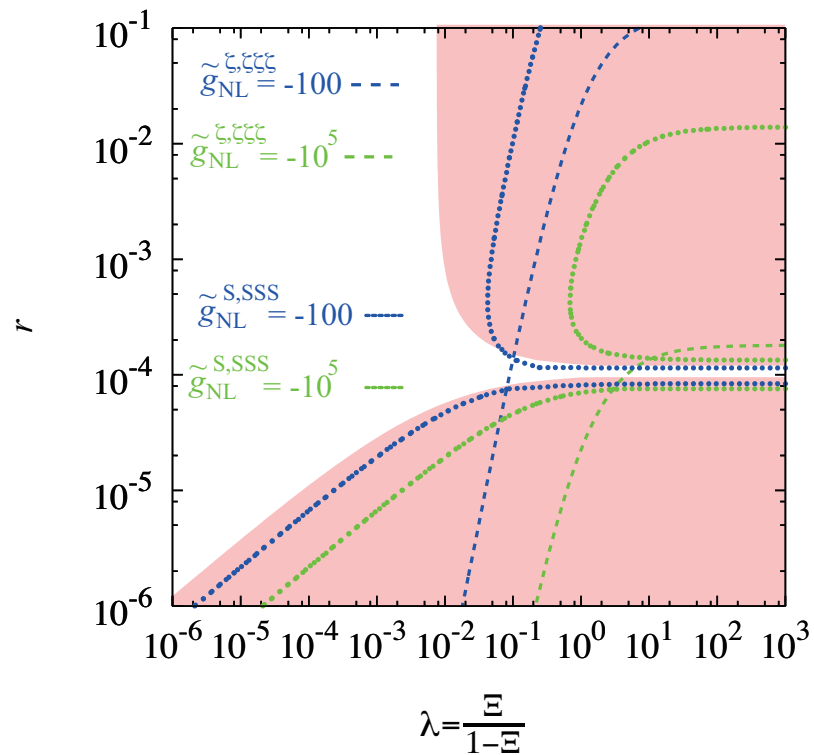
$(r \ll f_c \ll 1)$

# Trispectra: $\tilde{g}_{NL}^{I,JKL}$

$f_c = 0$



$f_c = 10^{-4}$



$$\tilde{g}_{NL}^{\zeta, \zeta \zeta \zeta} \simeq -\frac{9}{r} \mathbb{E}^3$$

# Trispectra: $\tilde{g}_{NL}^{I,JKL}$

- Hierarchy between the coefficients ?

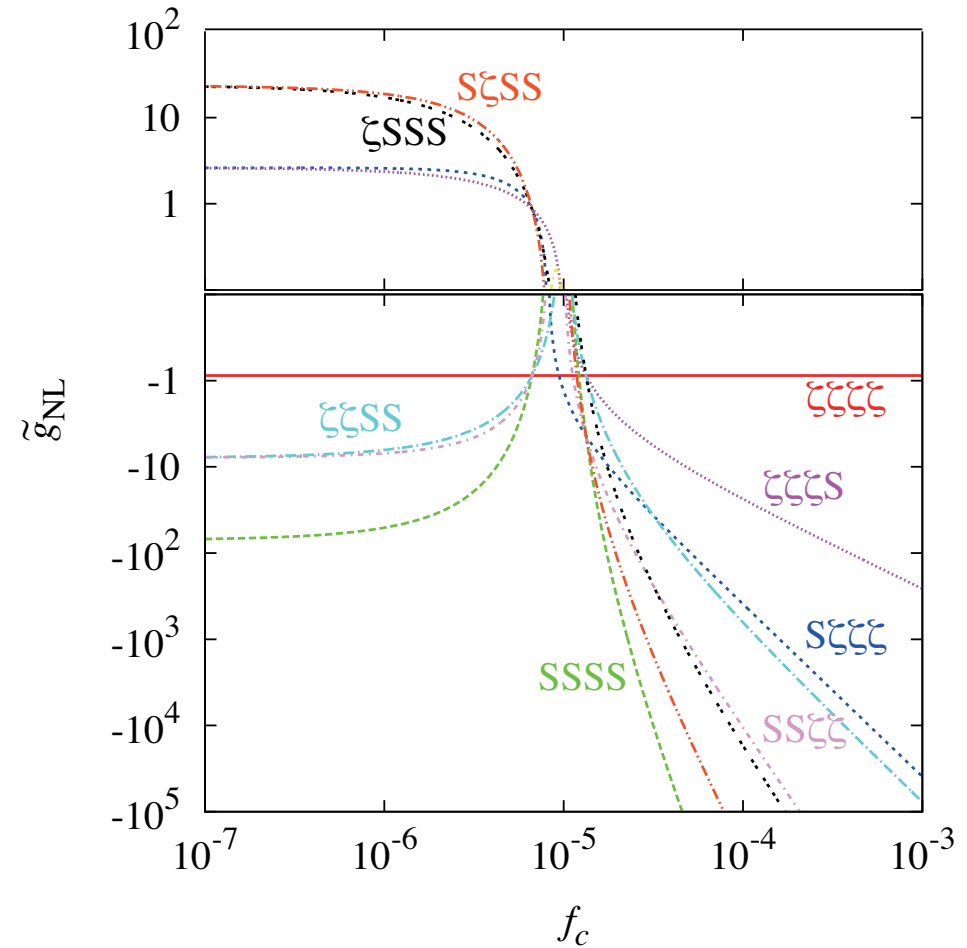
- For  $f_c \ll r \ll 1$

$$\tilde{g}_{NL}^{I,JKL} \simeq (-3)^{I_S} \tilde{g}_{NL}^{\zeta,\zeta\zeta\zeta}$$

- For  $r \ll f_c \ll 1$

$$\tilde{g}_{NL}^{S,JKL} \simeq \frac{9f_c^2}{2r^2} \tilde{g}_{NL}^{\zeta,JKL}$$

$$\tilde{g}_{NL}^{I,JKL} \simeq \left(\frac{3f_c}{r}\right)^{I_S^{\hat{3}}} \tilde{g}_{NL}^{I,\zeta\zeta\zeta}$$



# CMB constraints

- Temperature anisotropies

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm}, \quad a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left( \sum_I X^I(\mathbf{k}) g_l^I(k) \right) Y_{lm}^*(\hat{\mathbf{k}})$$

- Angular bispectrum

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3} b_{l_1 l_2 l_3}$$

$$\mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \int d^2 \hat{\mathbf{n}} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_2 m_2}(\hat{\mathbf{n}}) Y_{l_3 m_3}(\hat{\mathbf{n}})$$

$$b_{l_1 l_2 l_3} = \sum_{I,J,K} \left( \frac{2}{\pi} \right)^3 \int k_1^2 dk_1 \int k_2^2 dk_2 \int k_3^2 dk_3 g_{l_1}^I(k_1) g_{l_2}^J(k_2) g_{l_3}^K(k_3) \\ B_{IJK}(k_1, k_2, k_3) \int_0^\infty r^2 dr j_{l_1}(k_1 r) j_{l_2}(k_2 r) j_{l_3}(k_3 r)$$

# CMB constraints

- « Reduced » angular bispectrum

$$b_{l_1 l_2 l_3} = \sum_{I,J,K} \tilde{f}_{\text{NL}}^{I,JK} b_{l_1 l_2 l_3}^{I,JK}$$

- Six different shapes

$$b_{l_1 l_2 l_3}^{I,JK} = 3 \int_0^\infty r^2 dr \alpha_{(l_1)}^I(r) \beta_{l_2}^J(r) \beta_{l_3}^K(r) \quad \alpha_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g_l^I(k)$$

$$\beta_l^I(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g_l^I(k) P_\zeta(k)$$

- Total bispectrum  $B_{l_1 l_2 l_3} = \sum_i \tilde{f}_{\text{NL}}^{(i)} B_{l_1 l_2 l_3}^{(i)}$

with  $i = \{(\zeta, \zeta\zeta), (\zeta, \zeta S), (\zeta, SS), (S, \zeta\zeta), (S, \zeta S), (S, SS)\}$



# CMB constraints

- Minimization of

$$\chi^2 = \langle B^{obs} - \sum_i \tilde{f}_{NL}^{(i)} B^{(i)}, B^{obs} - \sum_i \tilde{f}_{NL}^{(i)} B^{(i)} \rangle$$

$$\begin{aligned} \langle B, B' \rangle &\equiv \sum_{l_i} \frac{B_{l_1 l_2 l_3} B'_{l_1 l_2 l_3}}{\sigma_{l_1 l_2 l_3}^2} \\ &= \sum_{l_i} \frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}^2 \frac{b_{l_1 l_2 l_3} b'_{l_1 l_2 l_3}}{\sigma_{l_1 l_2 l_3}^2} \end{aligned}$$

$$\sigma_{l_1 l_2 l_3}^2 \equiv \langle B_{l_1 l_2 l_3}^2 \rangle - \langle B_{l_1 l_2 l_3} \rangle^2 \approx \Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}$$

# CMB constraints

- 6x6 Fisher matrix  $F_{ij} \equiv \langle B^{(i)}, B^{(j)} \rangle$
- Error on the parameters  $\Delta \tilde{f}^i = \sqrt{(F^{-1})_{ii}}$
- To be compared with a single-parameter analysis  $\Delta \tilde{f}^i = 1/\sqrt{F_{ii}}$
- Correlations:  $r_{ij} = \frac{(F^{-1})_{ij}}{\sqrt{(F^{-1})_{ii}(F^{-1})_{jj}}}$

# Conclusions

- Allowing for an isocurvature mode in addition to the adiabatic mode leads to additional local bispectra:
  - one purely isocurvature shape
  - four shapes from adiabatic-isocurvature correlations
- For the trispectrum, one finds in total nine  $\tau_{\text{NL}}$  - like coefficients and eight  $g_{\text{NL}}$ - like coefficients.
- Models with interesting hierarchies between the various coefficients.
- It would be interesting to try to constrain or measure these new shape coefficients by using the CMB data.