Curvaton with double well potential

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context

- Introduction
- A model
    - Cosmological evolution
- Density perturbation
    - Formulation
    - Small and large initial amplitude
- Summary

§ Introduction

- Inflation

  is an elegant solution to

  horizon, flatness, monopole problem

  and

  provides the seed of density fluctuation of

  adiabatic,

  almost scale invariant,

  Gaussian and

  (if we are lucky) gravitation wave

Gaussian fluctuation?

- Inflaton in a single field inflation

  ≈ very weakly interacting (slow roll)

  ≈ nearly free field

  ≈ Gaussian quantum fluctuation

nonlinear parameters are of order of slow roll parameters; $f_{NL} = O(\varepsilon,\eta)$

Maldacena 2003

A large non-Gaussianity

- Curvaton is a promising mechanism

Lyth et al 2003

- What is a "curvaton"?

Lyth and Wands, Moroi and Takahashi, Enqvist and Sloth 2001~2

· Not inflaton!

· Scalar field with a flat potential, which condensates during inflation

- Field fluctuation is converted into density fluctuation

Curvaton models

- Different forms of Potential
- Quadratic [many many...]

$$V = \frac{1}{2} m_\sigma^2 \sigma^2$$

- +Quartic [Enqvist et al, Huang,...]
- Cosin type [Kawasaki et al, Huang]
- Double well [Choi and Seto]

$$V(\sigma) = \frac{\lambda}{4} \left(\sigma^2 - v^2 \right)^2$$

Why a large non-Gaussianity from curvaton?

- From inflaton
- $\zeta \approx \Phi/(1+w) \approx (1/\varepsilon)(1/H M_P^2)\, \dot{\phi}\, \delta\phi$

  $\approx H\, \delta\phi/\dot{\phi} + O(\delta\phi^2/M_P^2)$
- Second/(first)$^2 \approx O(\dot{\phi}^2/ H^2 M_P^2) = O(\varepsilon)$

- From curvaton
- $\zeta \approx \delta\rho/\rho = (1+\delta\sigma/\sigma)^2 - 1 = 2\, \delta\sigma/\sigma + (\delta\sigma/\sigma)^2$

Curvaton models

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- Quadratic [many many...]

$$V = \frac{1}{2} m_\sigma^2 \sigma^2$$

- +Quartic [Enqvist et al, Huang,...]
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$$V(\sigma) = \frac{\lambda}{4} \left(\sigma^2 - v^2\right)^2$$

§ A Model

- Double well potential with a large VEV [DW problem] and a small self-coupling [Flatness]

$$\mathcal{L} = -\frac{1}{2}(\partial\sigma)^2 - V(\sigma)$$

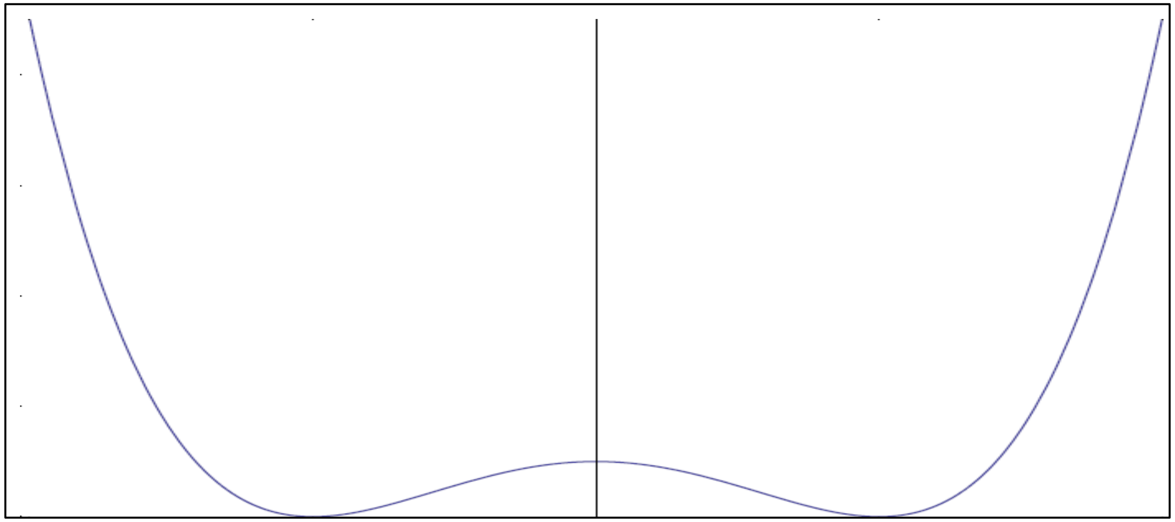
$$V(\sigma) = \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2$$

- Decay rate [saxion-like]

$$\Gamma_\sigma = C\frac{m_\sigma^3}{v^2} = C(2\lambda)^{3/2}v$$

§ § Cosmological evolution

- Rough history



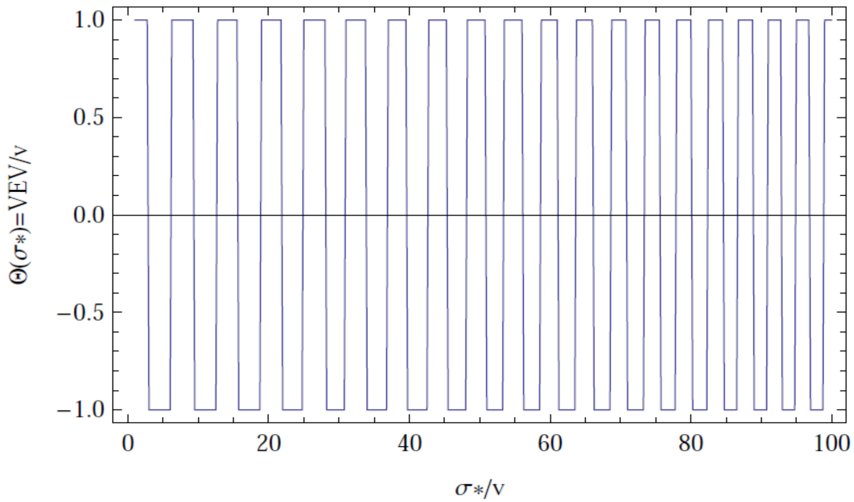
- Quartic to quadratic transition

At the time when the energy density becomes comparable with the height of potential hill

$$\rho_\sigma\big|_v = \frac{\lambda}{4}(\sigma_v^2 - v^2)^2$$

§ § Cosmological evolution 2

- +v or –v?



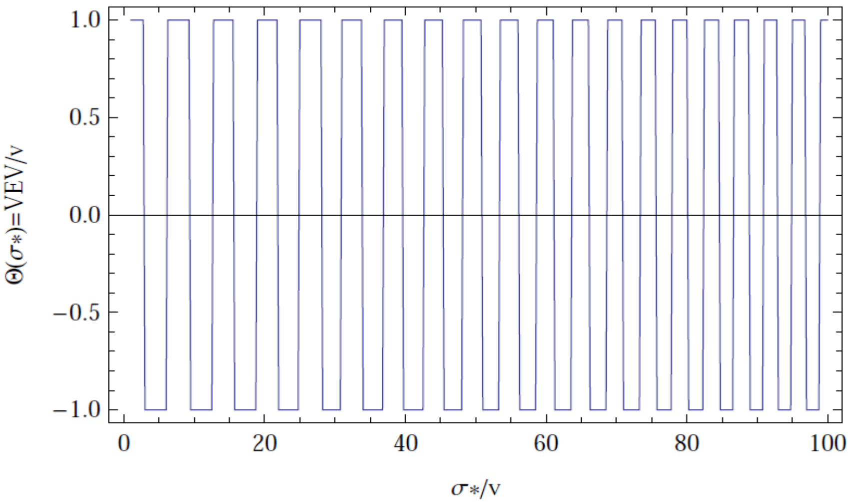
- Analytic solutions from the scaling law

$$\sigma(t) \simeq v\Theta(\sigma_{\rm os}) + \frac{\sigma_{2\rm os}}{(m_\sigma t)^{3/4}} \sin m_\sigma t$$

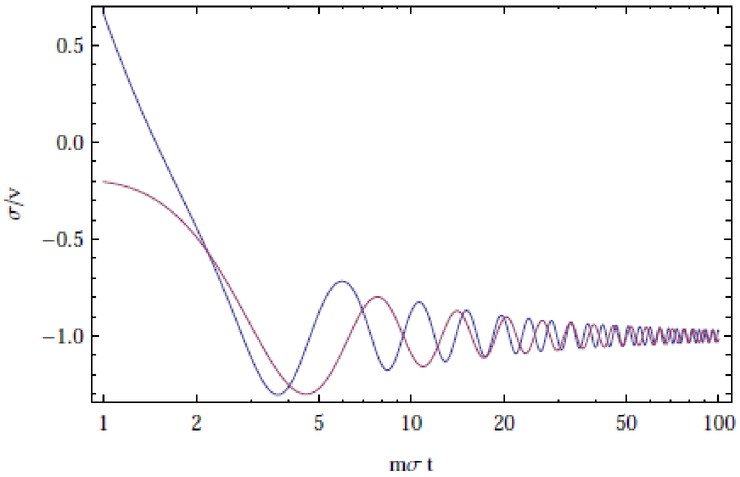
$$\sigma_{2\rm os} \simeq \begin{cases} (\sigma_v - v)\left(\frac{\rho_\sigma|_{\rm os}}{\rho_\sigma|_v}\right)^{3/8}\left(\frac{m_\sigma}{2\sqrt{3\lambda\sigma_{\rm os}^2}}\right)^{3/4}, \\ (\sigma_v - v)\left(\frac{\rho_\sigma|_{\rm os}}{\rho_\sigma|_v}\right)^{3/8}\left(\frac{H_R}{2\sqrt{3\lambda\sigma_{\rm os}^2}}\right)^{1/4}\left(\frac{m_\sigma}{2\sqrt{3\lambda\sigma_{\rm os}^2}}\right)^{3/4} \end{cases}$$

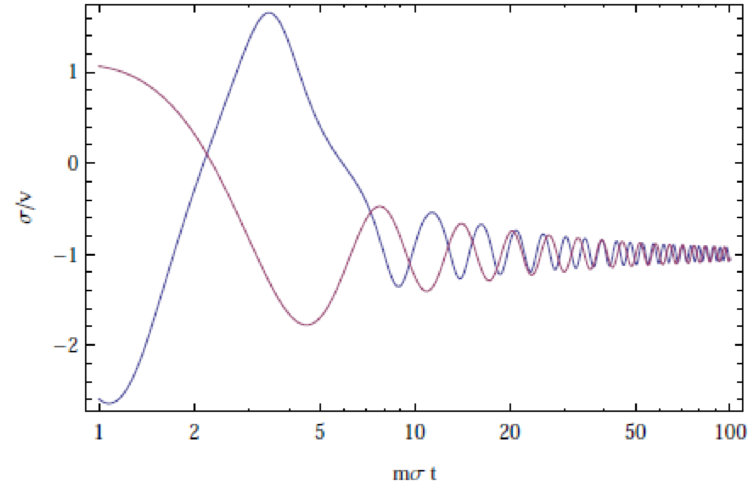
§ § Cosmological evolution 2

- +v or –v?



- Analytic solutions fit well!

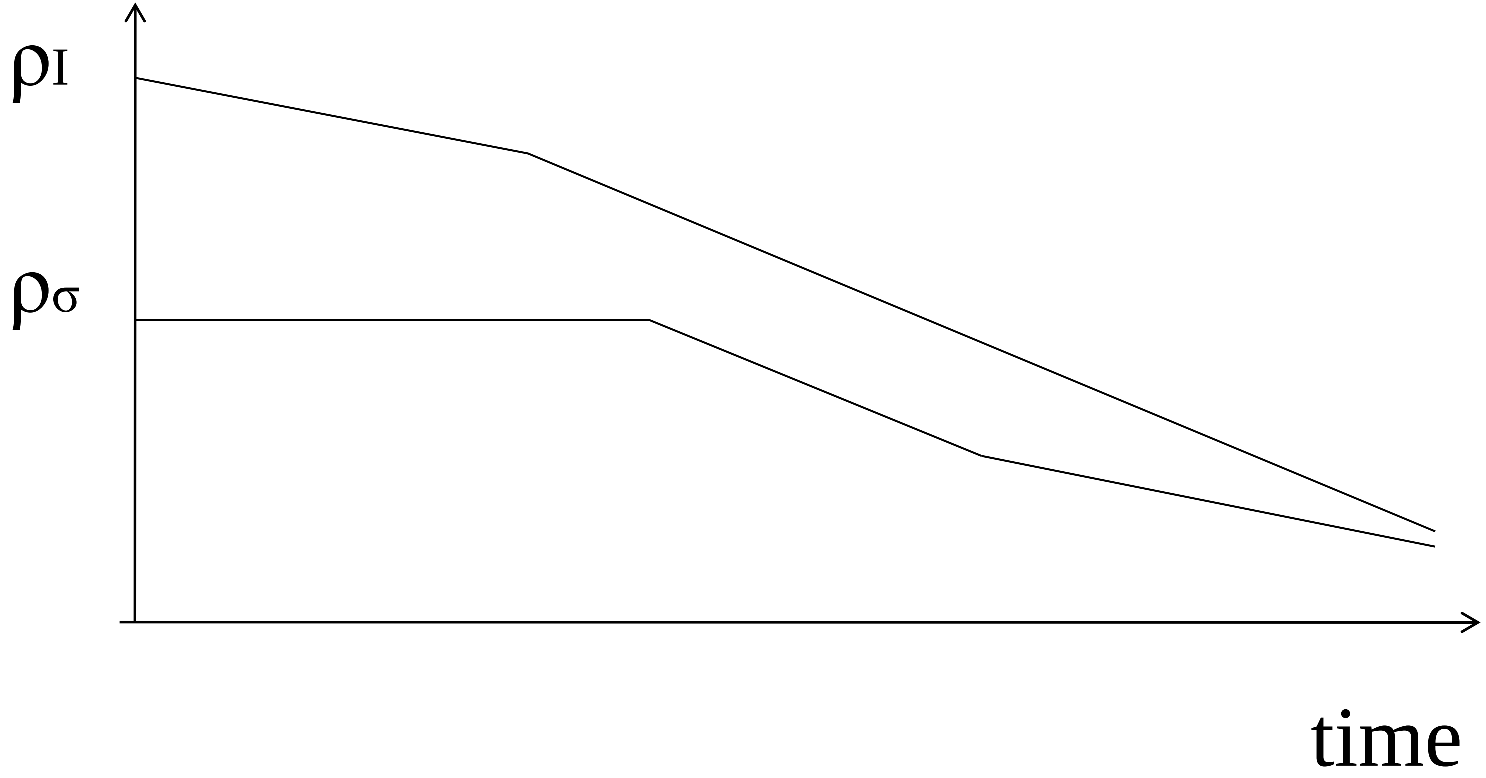




§ § Cosmological evolution 3.1

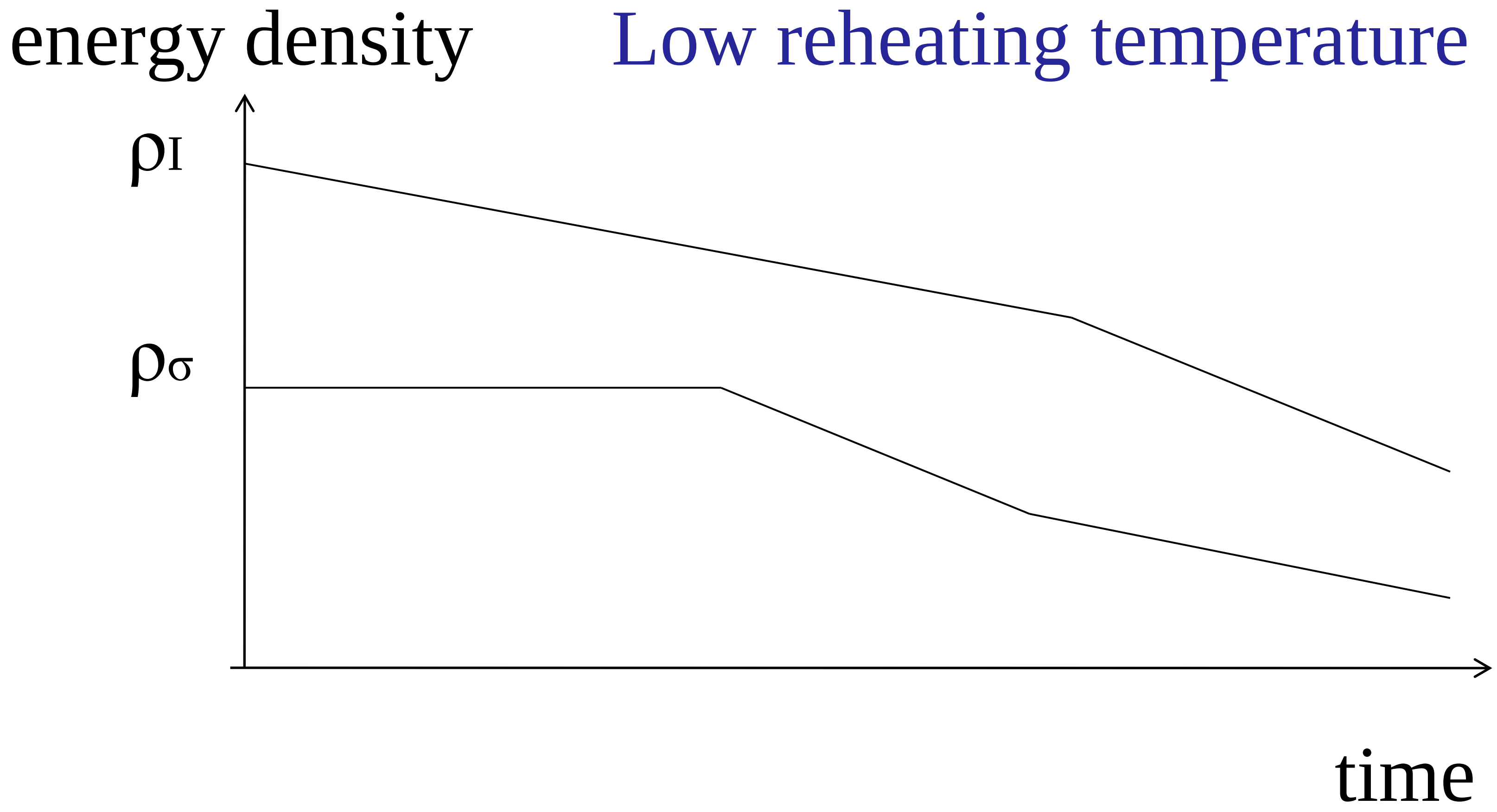
- Reheating temperature T_R dependence

energy density High reheating temperature



§ § Cosmological evolution 3.2

- Reheating temperature T_R dependence



§ Density perturbation

- We estimate power spectrum and non-Gaussianity in terms of nonlinear parameters f_{NL} and g_{NL}

- We impose the constraints from the null detection of tensor mode fluctuation

§ § Formulation 1

- We employ δN formalism for perturbation
  Lyth et al, Sasaki et al

$$\zeta_\sigma = \delta N + \frac{1}{3}\int_{\rho_0(t)}^{\rho(t,\mathbf{x})} \frac{d\tilde{\rho}}{\tilde{\rho}+\tilde{p}}$$

  on the uniform curvaton density surface.

- Expand this with the late time energy density expression $\rho_\sigma(t,x) \simeq \dfrac{m_\sigma^2 \sigma_{2\mathrm{os}}^2(t,x)}{2(mt)^{3/2}}$

§ § Formulation 2

- After the expansion

$$\zeta_\sigma = \zeta_{\sigma 1} + \frac{1}{2}\zeta_{\sigma 2} + \frac{1}{6}\cdots$$

$$\begin{aligned}
\zeta_{\sigma 1} &= \frac{2\sigma'_{2\mathrm{os}}}{3\sigma_{2\mathrm{os}}}\delta\sigma_*, \\
\zeta_{\sigma 2} &= -\frac{3}{2}\left(1 - \frac{\sigma_{2\mathrm{os}}\sigma''_{2\mathrm{os}}}{\sigma'^2_{2\mathrm{os}}}\right)\zeta^2_{\sigma 1} \equiv A_2\zeta^2_{\sigma 1}, \\
\zeta_{\sigma 3} &= \frac{9}{4}\left(2 - 3\frac{\sigma_{2\mathrm{os}}\sigma''_{2\mathrm{os}}}{\sigma'^2_{2\mathrm{os}}} + \frac{\sigma^2_{2\mathrm{os}}\sigma'''_{2\mathrm{os}}}{\sigma'^3_{2\mathrm{os}}}\right)\zeta^3_{\sigma 1} \equiv A_3\zeta^3_{\sigma 1}.
\end{aligned}$$

- Total ζ

$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \frac{1}{6}\zeta_3 + \cdots$$

$$R \equiv \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}$$

$$\zeta_1 = (1 - R)\zeta_{r1} + R\zeta_{\sigma 1},$$

$$\zeta_2 = (1 - R)\zeta_{r2} + R\zeta_{\sigma 2} + R(1 - R)(3 + R)\left(\zeta_{r1} - \zeta_{\sigma 1}\right)^2,$$

§ § Formulation 3

- The power spectrum

$$\mathcal{P}_\zeta = (1-R)^2\mathcal{P}_{\zeta_r} + R^2\mathcal{P}_{\zeta_\sigma}$$

$$\mathcal{P}_{\zeta_r} = \left(\frac{H_*^2}{2\pi|\dot{\phi}|}\right)^2 = \frac{H_*^2}{8\pi^2\epsilon M_P^2},$$

$$\mathcal{P}_{\zeta_\sigma} = \frac{H_*^2}{4\pi^2}\left(\frac{2\sigma'_{2\mathrm{os}}}{3\sigma_{2\mathrm{os}}}\right)^2$$

$$R \equiv \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}$$

$$\tilde{r} \equiv \frac{R^2\mathcal{P}_{\zeta_\sigma}}{(1-R)^2\mathcal{P}_{\zeta_r}}$$

- Non-linear parameters

$$f_{NL} = \frac{5}{6}\frac{\tilde{r}^2}{(1+\tilde{r})^2}\left[\frac{3+A_2}{R} - 2 - R\right],$$

$$g_{NL} = \frac{25}{54}\frac{\tilde{r}^3}{(1+\tilde{r})^3}\left[\frac{9+9A_2+A_3}{R^2} - \frac{18+6A_2}{R} - 4 - 3A_2 + 10R + 3R^2\right].$$

§ § Formulation 3

- The power spectrum

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$$\mathcal{P}_{\zeta_\sigma} = \frac{H_*^2}{4\pi^2}\left(\frac{2\sigma'_{2\mathrm{os}}}{3\sigma_{2\mathrm{os}}}\right)^2$$

$$R \equiv \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}$$

$$\tilde{r} \equiv \frac{R^2\mathcal{P}_{\zeta_\sigma}}{(1-R)^2\mathcal{P}_{\zeta_r}}$$

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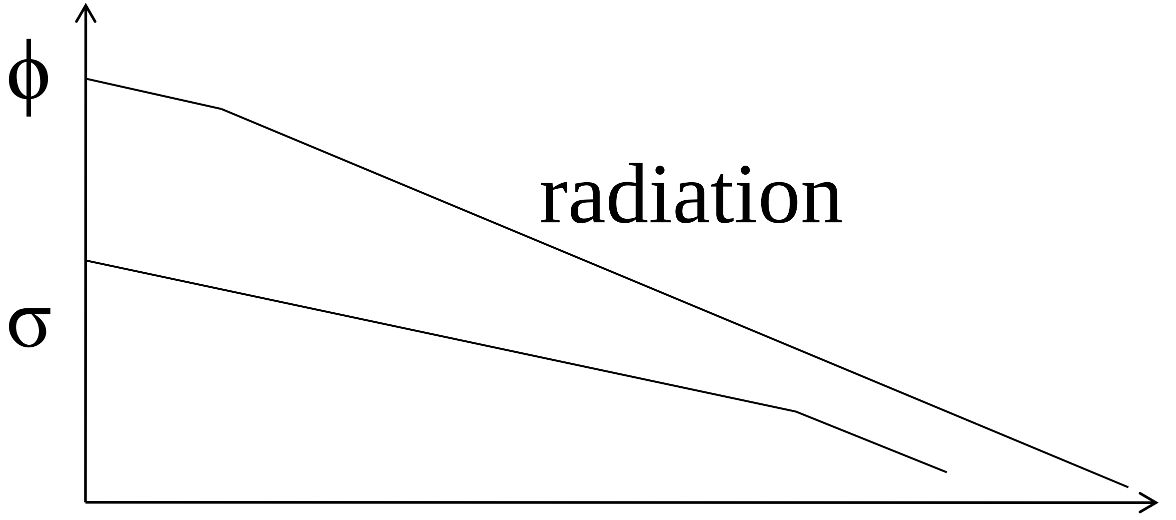
Intuitive understanding of 1/R

- If mainly subdominat curvaton generate $\delta T/T$



- ζ_σ is diluted, $\zeta_\sigma \rightarrow \zeta = R\,\zeta_\sigma$
- Second/(first)$^2 \approx O(R/R^2) = O(1/R)$

“subdominat curvaton predict a large f_{NL}”

Wands et al, Langlois et al, Enqvist et al,...

§ § Small initial expectation

- Quantities appears in ζs.

$$\sigma'_{2os} = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{\rm os}^2 - v^2)^{3/8}} \left(1 - \frac{9}{4}\frac{\sigma_{\rm os}(\sigma_{\rm os} - v)}{(3\sigma_{\rm os}^2 - v^2)}\right)$$

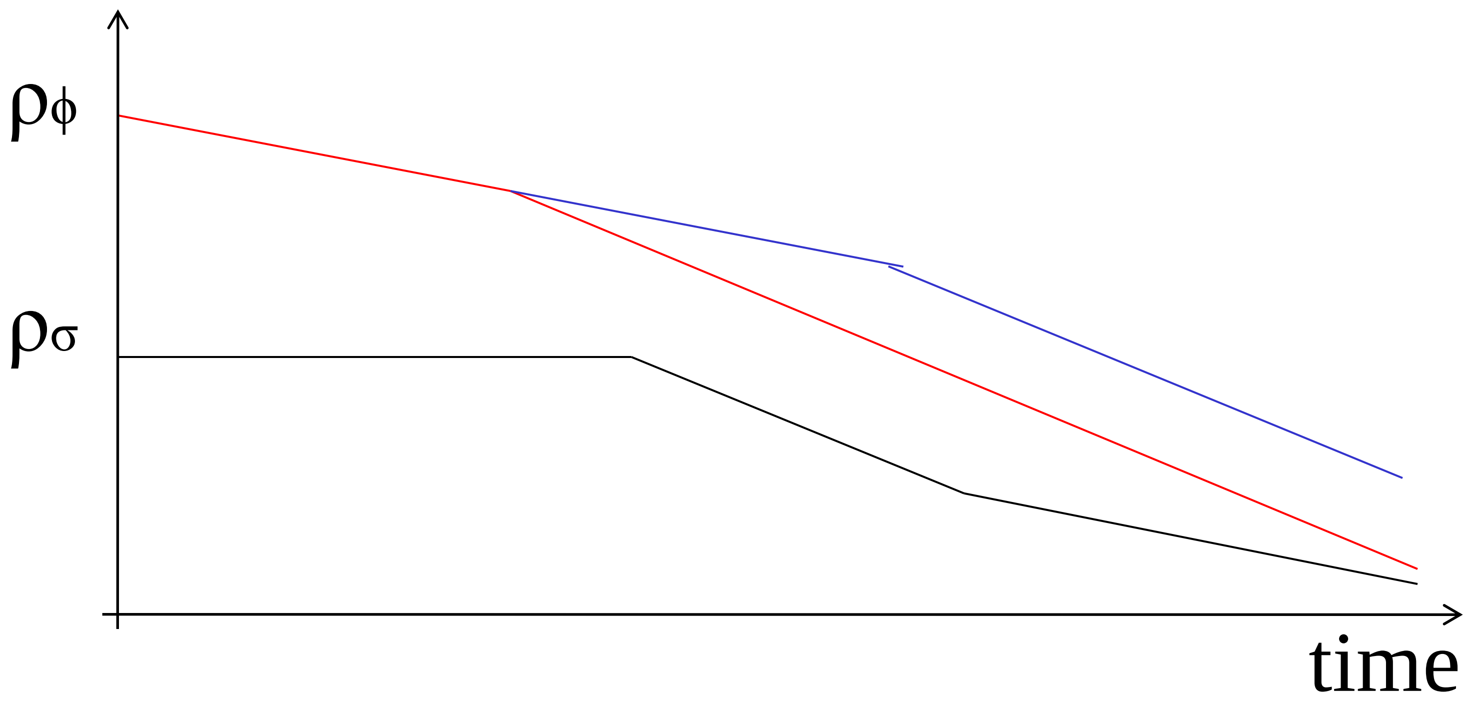
$$\sigma''_{2os} = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{\rm os}^2 - v^2)^{11/8}} \left(-\frac{9}{2}\sigma_{\rm os} - \frac{9}{4}(\sigma_{\rm os} - v) + \frac{297}{16}\frac{(\sigma_{\rm os} - v)\sigma_{\rm os}^2}{(3\sigma_{\rm os}^2 - v^2)}\right),$$

$$\sigma'''_{2os} = \frac{(m_\sigma/2\sqrt{\lambda})^{3/4}}{(3\sigma_{\rm os}^2 - v^2)^{11/8}} \left(-\frac{27}{4} + \frac{891}{16}\frac{\sigma_{\rm os}^2 + (\sigma_{\rm os} - v)\sigma_{\rm os}}{(3\sigma_{\rm os}^2 - v^2)} - \frac{16929}{64}\frac{(\sigma_{\rm os} - v)\sigma_{\rm os}^3}{(3\sigma_{\rm os}^2 - v^2)^2}\right).$$

- $\sigma_{\rm os} \to v$ limit, the formula is reduced to that of quadratic model’s

§§ Reheating temperature for a large initial amplitude

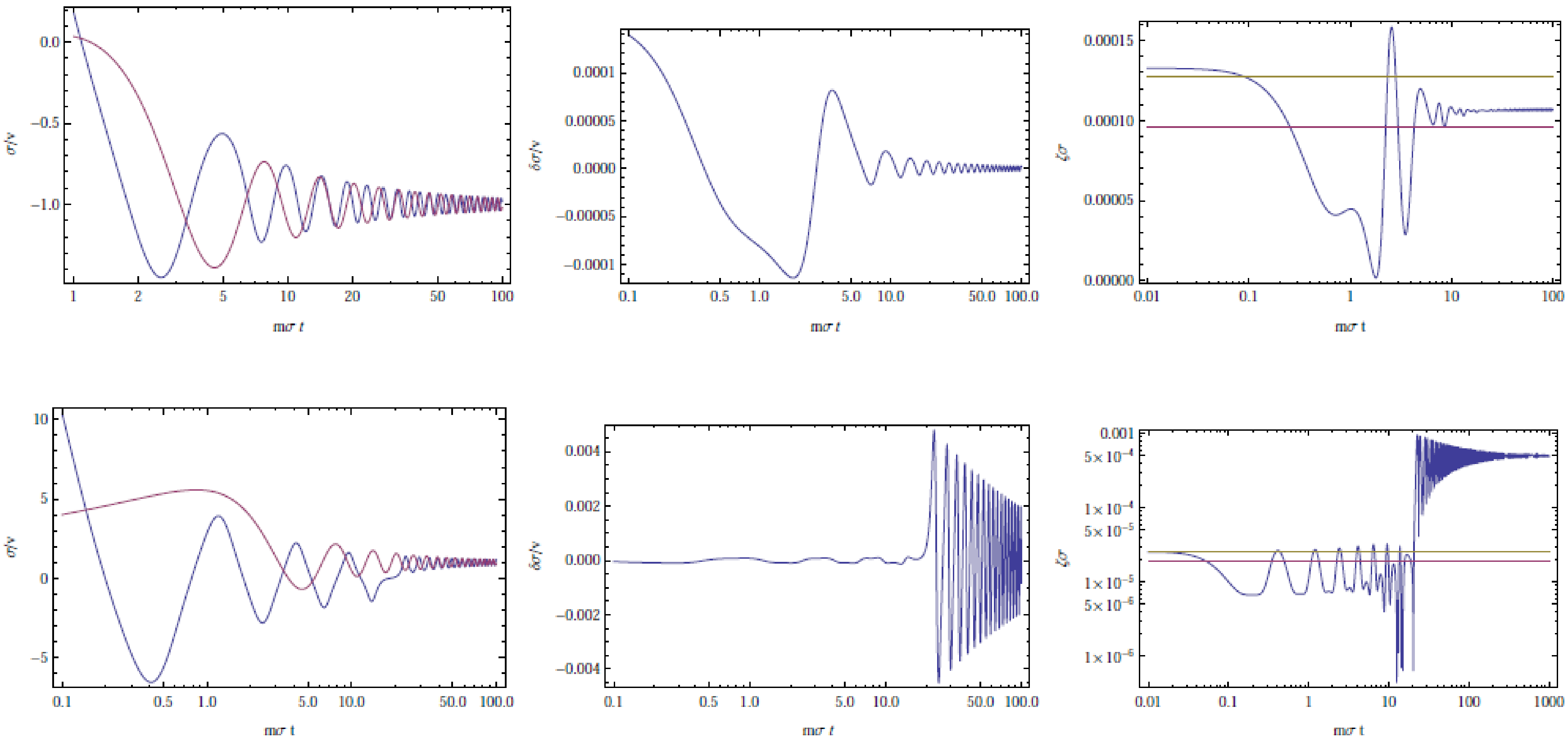
- Reheating temperature T_R (background evolution) affects parameters somewhat



§ § Large initial expectation 1/5

- Field fluctuation δσ

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + \lambda(3\sigma^2 - v^2)\delta\sigma = 0$$



§ § Large initial expectation 2/5

- Curvature perturbation ζ

$$\zeta = (1-R)\zeta_r + \frac{R}{2}\left(\frac{\delta\sigma_*}{\sigma_*}\right) + \frac{1}{8}\left(\frac{1}{R} - 2 - R\right) R^2 \left(\frac{\delta\sigma_*}{\sigma_*}\right)^2 + \frac{1}{48}\left(-\frac{1}{R^2} - \frac{6}{R} + 2 + 10R + 3R^2\right) R^3 \left(\frac{\delta\sigma_*}{\sigma_*}\right)^3, \qquad \text{for high } T_R,$$

$$\zeta = (1-R)\zeta_r + \frac{R}{3}\left(\frac{\delta\sigma_*}{\sigma_*}\right) + \frac{1}{18}\left(-2 - R\right) R^2 \left(\frac{\delta\sigma_*}{\sigma_*}\right)^2 + \frac{1}{162}\left(5 + 10R + 3R^2\right) R^3 \left(\frac{\delta\sigma_*}{\sigma_*}\right)^3, \qquad \text{for low } T_R,$$

§ § Large initial expectation 2/5

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Absence of inverse power of R

§ § Large initial expectation 3/5

- Resultant nonlinear parameters

$$f_{\rm NL} = \left(\frac{\tilde{r}}{1+\tilde{r}}\right)^2 \frac{5}{6}\left(\frac{1}{R} - 2 - R\right) > -2$$

$$g_{\rm NL} = \left(\frac{\tilde{r}}{1+\tilde{r}}\right)^3 \frac{25}{54}\left(-\frac{1}{R^2} - \frac{6}{R} + 2 + 10R + 3R^2\right), \qquad \text{for high } T_R,$$

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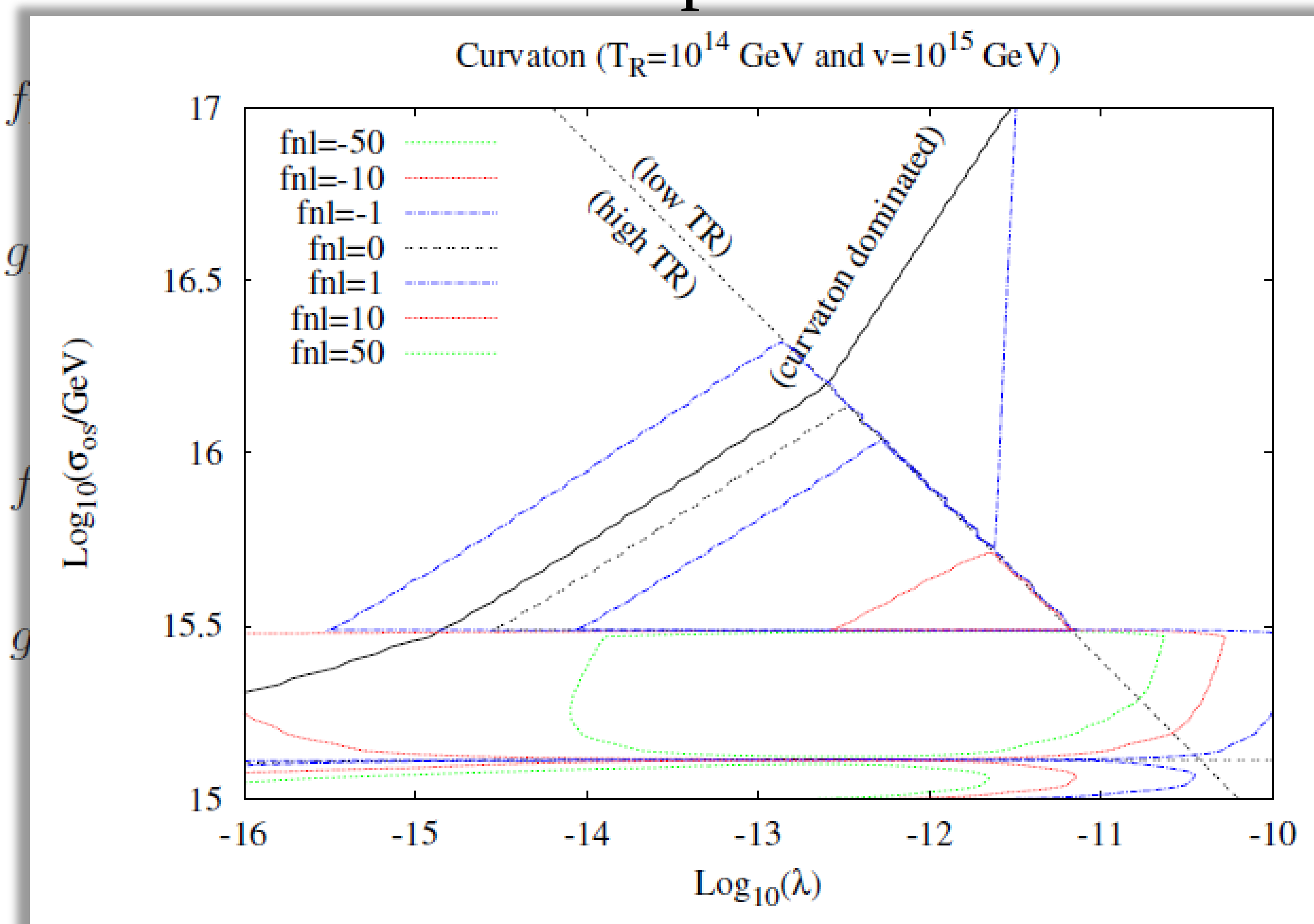
§ § Large initial expectation 3/5

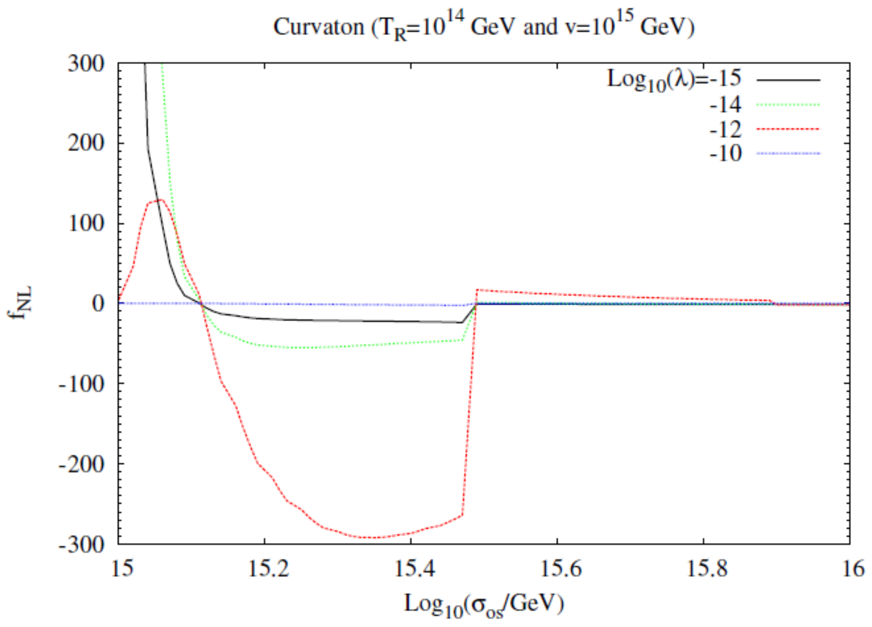
- Resultant nonlinear parameters



high T_R,

§ § Large initial expectation 4/5

- Another feature of nonlinear parameters



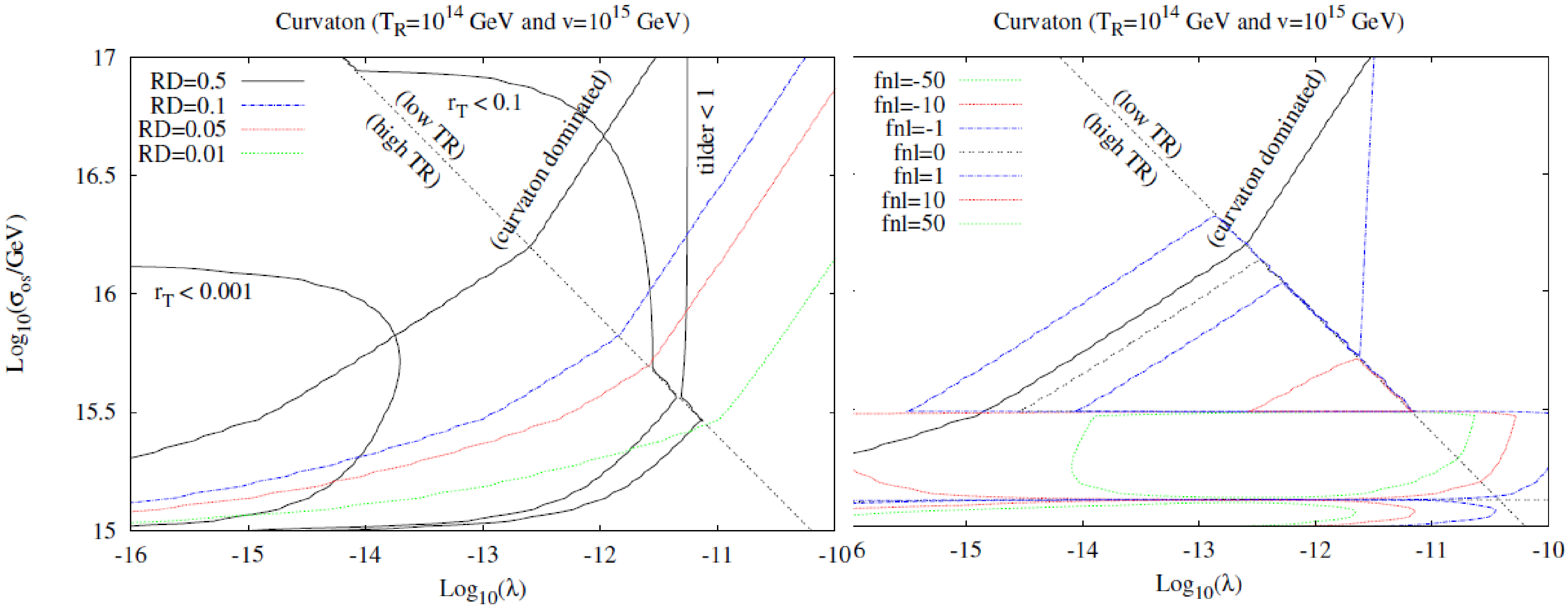
Observation

$$-10 < f_{\rm NL} < 74$$

- A negative f_{NL} is possible, unlike quadratic curvaton.

§ § Large initial expectation 5/5

- Tensor (gravitational wave) modes



§ Summary

We have studied curvaton model which has a double well potential

- **Possible tachyonic amplification**
- **For a low reheating temperature, f_{NL} can not be large because of accidental cancellations**

  The statement "a subdominant curvaton predict a large f_{NL}" is not ALWAYS ture
- **Nontrivial initial amplitude dependence of f_{NL}**