The Gravitational Lensing Effect on the CMB Polarization Anisotropy in the Λ-LTB Model

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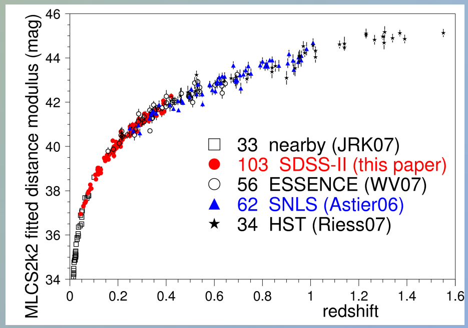
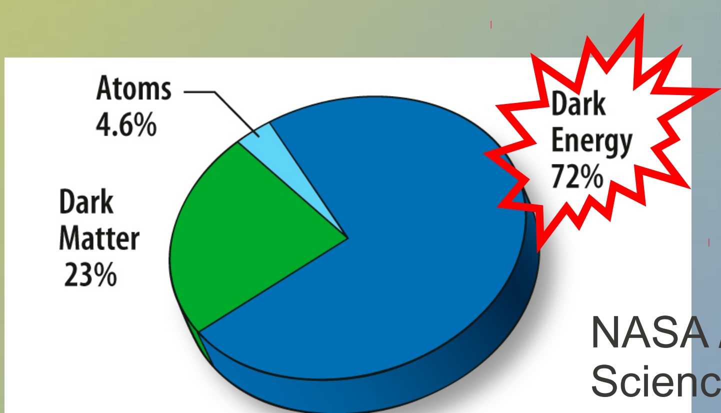
Type-Ia supernova observations



Kessler+'09

Assumption
Cosmological Principle

Conclusion
cosmic acceleration



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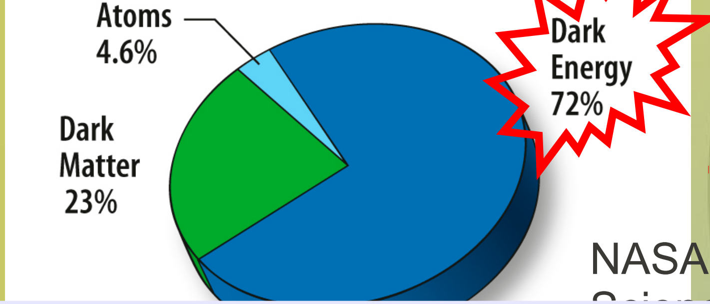
Other explanations
e.g.) local void model (LTB model)(Tomita'00)



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(or modified gravity)

How can they be distinguished?

- CMB temperature & polarization anisotropies
- Gravitational lensing effect → B-mode & EB(&TB) correlations, for an off-center observer in the local void model
- Expected to be characteristic observables

Contents

- Gravitational lensing effect in the Lemaître-Tolman-Bondi(LTB) model
  - Null geodesic in the LTB model
  - Shift vector $\delta\boldsymbol{\theta}_{\mathrm{obs}}$
- CMB polarization in the LTB model
  - Flux intensity tensor
  - Initial condition at the last scattering surface
  - Propagation after last scattering
  - Change in the polarization distribution

Gravitational lensing effect in the LTB model

Null geodesic in the LTB model

- Lemaître-Tolman-Bondi(LTB) metric

$$ds^2 = -dt^2 + S^2 d\chi^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$S = \frac{r'}{\xi} \qquad r = r(t,\chi) \qquad ' = \frac{\partial}{\partial\chi} \qquad \dot{} = \frac{\partial}{\partial t}$$

$$\xi = \sqrt{1 - k(\chi)\chi^2}$$ $k(\chi)$: curvature function

- Geodesic equation $dp^\mu/d\lambda = -\Gamma^\mu_{\nu\rho} p^\nu p^\rho$
- Spherical symmetry → equations of $\omega, \mu, p_\perp$

$$p^t = \omega \qquad p^\chi = \mu\omega/S \qquad p_\perp := r\sqrt{(p^\theta)^2 + (p^\varphi)^2 \sin^2\theta}$$

- 2-dim. problem → use the plane of $\varphi = 0, \pi$

Null geodesic in the LTB model

→ Set of ordinary differential equations for $\omega(t),\ \chi(t),\ \theta(t),\ \mu(t)$

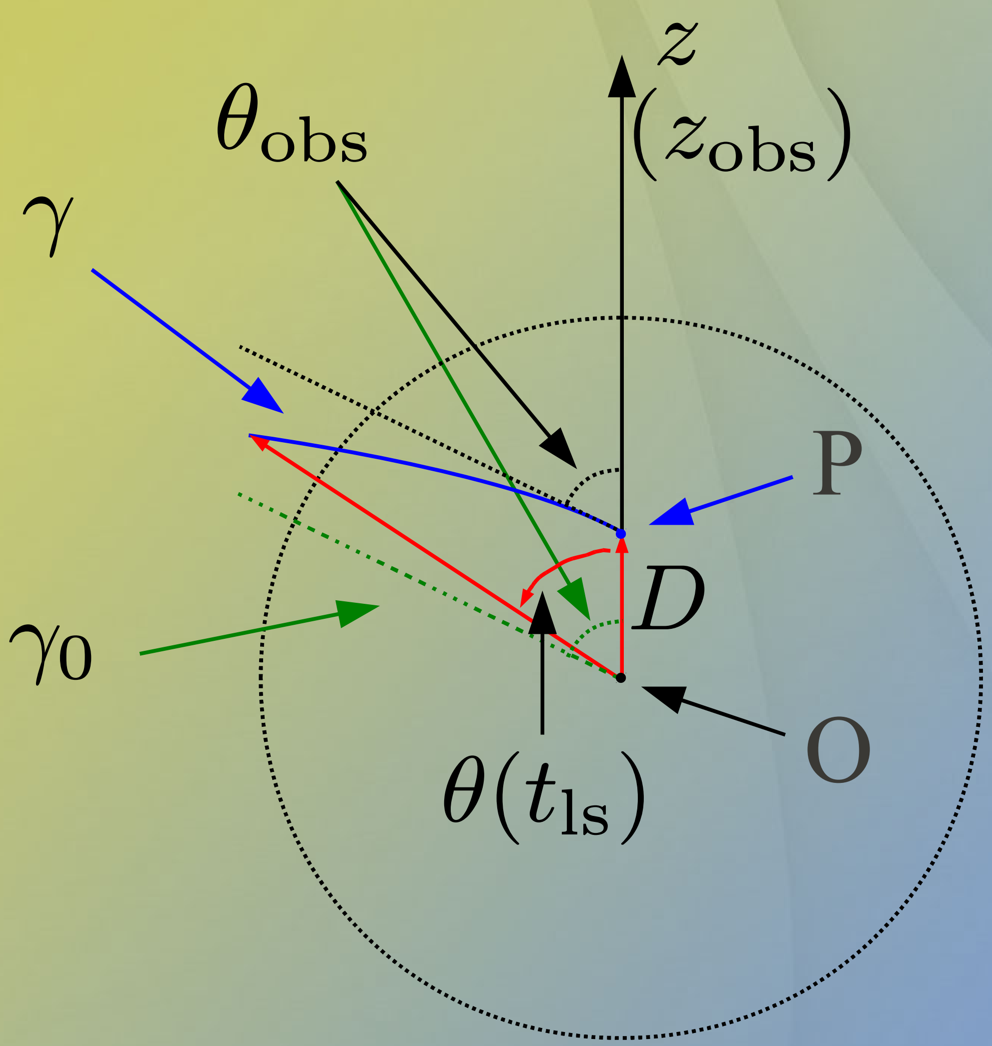
$$\frac{d}{dt}\ln\omega = -\frac{\dot{S}}{S}\mu^2 - (1-\mu^2)\frac{\dot{r}}{r}$$

$$\frac{d\chi}{dt} = \frac{\mu}{S}$$

$$\frac{d\theta}{dt} = \pm\frac{\sqrt{1-\mu^2}}{r}$$

$$\frac{1}{1-\mu^2}\frac{d\mu}{dt} = \frac{\xi}{r} + \mu\left(\frac{\dot{r}}{r} - \frac{\dot{S}}{S}\right)$$

Null geodesic in the LTB model



- Determine the geodesic γ perturbatively with respect to γ_0
- However μ cannot be treated perturbatively
- $\rightarrow$ Instead, we use

$$b := \chi\sqrt{1-\mu^2}$$

$$c := \chi\mu$$

- Solve the perturbation equations

Null geodesic in the LTB model

$$\theta(t_{\rm ls}) = -\int_{t_0}^{t_{\rm ls}} dt \frac{b}{\chi r} = \cdots\cdots$$

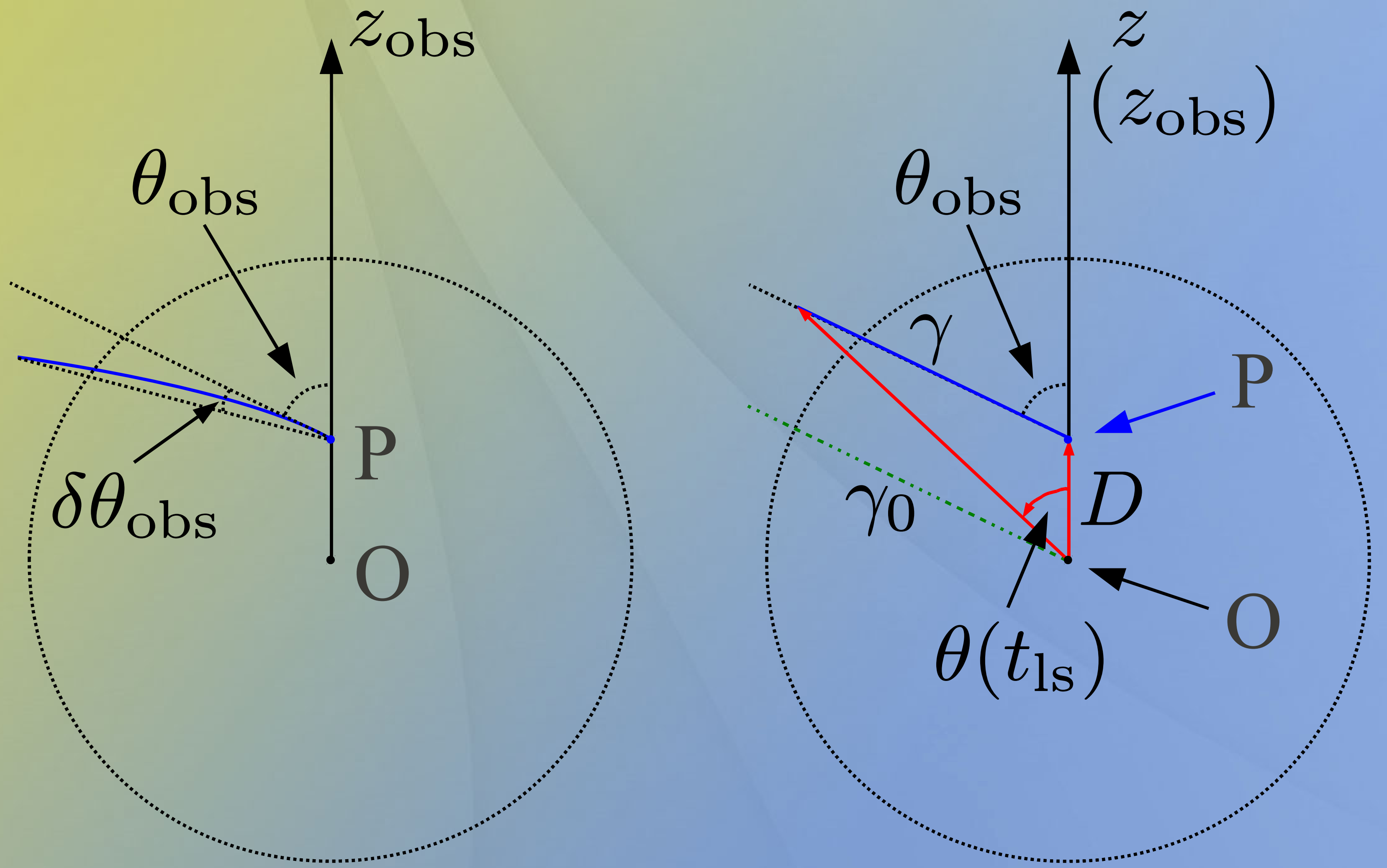
$$= \theta_{\rm obs} - \frac{b(t_0)}{\chi_{\rm ls}}$$

$$-b(t_0)\int_{t_{\rm ls}}^{t_0} \frac{dt}{\chi^2}\left(\frac{1}{S} - \frac{\chi}{r}e^{Y(t,t_0)}\right)$$

where $Y(a,b) = \int_a^b dt \left\{\frac{\xi}{\chi}\left(\frac{1}{r'} - \frac{\chi}{r}\right) + \frac{\dot{r}}{r} - \frac{\dot{S}}{S}\right\}$

Shift vector $\delta\boldsymbol{\theta}_{\rm obs}$

Homogeneous limit



Shift vector $\delta\boldsymbol{\theta}_{\rm obs}$

- In the homogeneous limit, $\delta\boldsymbol{\theta}_{\rm obs} = 0$
- By eliminating the part that survives in the homogeneous limit,

$$\delta\theta_{\rm obs} = -D\sin\theta_{\rm obs} \cdot \int_{t_{\rm ls}}^{t_0} \frac{dt}{\chi^2}\left(\frac{\chi}{r}(1-\xi)+\frac{1}{S}-\frac{\chi}{r}e^{Y(t,t_0)}\right)$$

$$\delta\varphi_{\rm obs} = 0$$

- This is used later

CMB polarization in the LTB model

Flux intensity tensor

- Electric field

Polarization basis

$$\boldsymbol{E} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2}\sum_p (\boldsymbol{\epsilon}_p(\boldsymbol{k})a_p(\boldsymbol{k})e^{ik\cdot x} + \boldsymbol{\epsilon}_p^*(\boldsymbol{k})a_p(\boldsymbol{k})^\dagger e^{-ik\cdot x})$$

- Measured components of the electric field

$$\mathcal{E}_p = \int d^3\boldsymbol{x} W(\boldsymbol{x})\boldsymbol{\epsilon}_p^{\mathrm{o}} \cdot \boldsymbol{E}(t_0, \boldsymbol{x})$$

Sensitivity function

Detector polarization basis

Flux intensity tensor

- $\langle a_p(\boldsymbol{k})^\dagger a_q(\boldsymbol{k}')\rangle = 2(2\pi)^3 \rho_{pq}(\boldsymbol{k})\delta^3(\boldsymbol{k}-\boldsymbol{k}')$

  ρ_{pq} : flux polarization matrix
- Flux density tensor

$$\rho_{ij}(\boldsymbol{k}) = \sum_{p,q} \epsilon^*_{pi}(\boldsymbol{k})\epsilon_{qj}(\boldsymbol{k})\rho_{pq}(\boldsymbol{k})$$

- Observed correlation of the electric fields

$$\langle : \mathcal{E}_p \mathcal{E}_q : \rangle = \epsilon^{\mathrm{o}\,i}_p \epsilon^{\mathrm{o}\,j}_q \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} |\hat{W}(\boldsymbol{k})|^2 \rho_{(ij)}(\boldsymbol{k})$$

($\hat{W}(\boldsymbol{k})$ is the Fourier transform of $W(\boldsymbol{x})$)

Initial condition at the last scattering surface

- Boltzmann equation

ρ_{ij} generalized

$$\left(\frac{k^\mu}{k^0}\nabla_\mu + \frac{\dot{k}^i}{k^0}\partial_{k^i}\right)\rho^{\mu\nu}(x,\boldsymbol{k}) = C^{\mu\nu}(\rho)$$

- Up to the linear order in perturbations, $\delta\rho_{\mu\nu}$ on the last scattering surface can be Fourier decomposed into the contribution of each perturbation mode with the wave vector $\boldsymbol{K}$

$$\delta\rho_{\mu\nu}(t_{\rm ls},\boldsymbol{x},\boldsymbol{k}) = \int d^3\boldsymbol{K} e^{i\boldsymbol{K}\cdot\boldsymbol{x}}\rho^{(1)}_{\mu\nu}(\boldsymbol{K};\boldsymbol{k})$$

Initial condition at the last scattering surface

- For an appropriate, customary choice of $\boldsymbol{\epsilon}_p(k)$, $\rho^{(1)}_{pq} = \epsilon^{\mu *}_p \epsilon^{\nu}_q \rho^{(1)}_{\mu\nu}(\boldsymbol{K};\boldsymbol{k})$ depends only on ω and $\mu = \cos\theta = \hat{\boldsymbol{K}} \cdot \hat{\boldsymbol{k}}$
- Solve Boltzmann equation → initial condition at the last scattering surface

Propagation after last scattering

- LTB spacetime $\tilde{g}_{\mu\nu}$
- Let us work in the synchronous gauge

$$ds^2 = -dt^2 + \tilde{g}_{ij}(t,x)dx^i dx^j$$

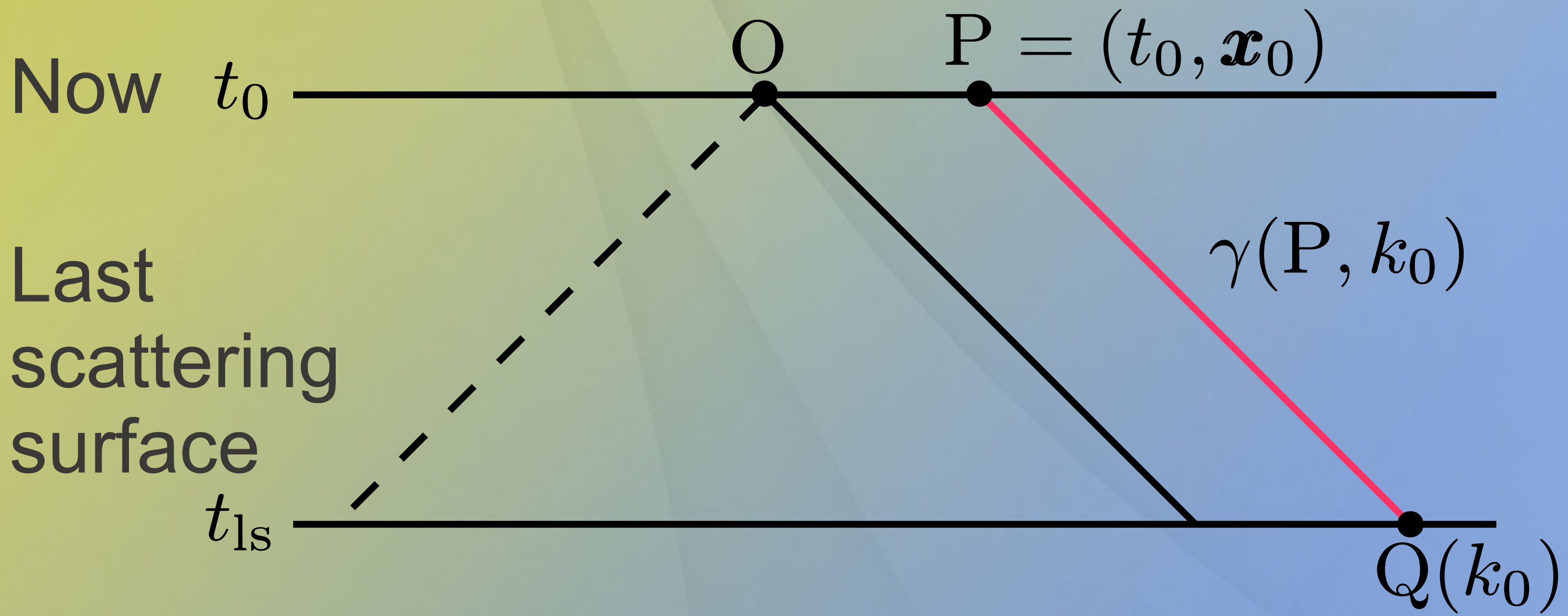
We take the constant-time surfaces so that the last scattering surface is represented by $t = t_{\rm ls}$

- $\tilde{g}_{ij}$ is close to a spatially homogeneous and isotropic metric g_{ij} in the early universe

$$\tilde{g}_{ij} = g_{ij} + \delta_{\rm LTB} g_{ij}$$

Propagation after last scattering

$$k_0 = (\omega(\boldsymbol{k}_0), \boldsymbol{k_0})$$

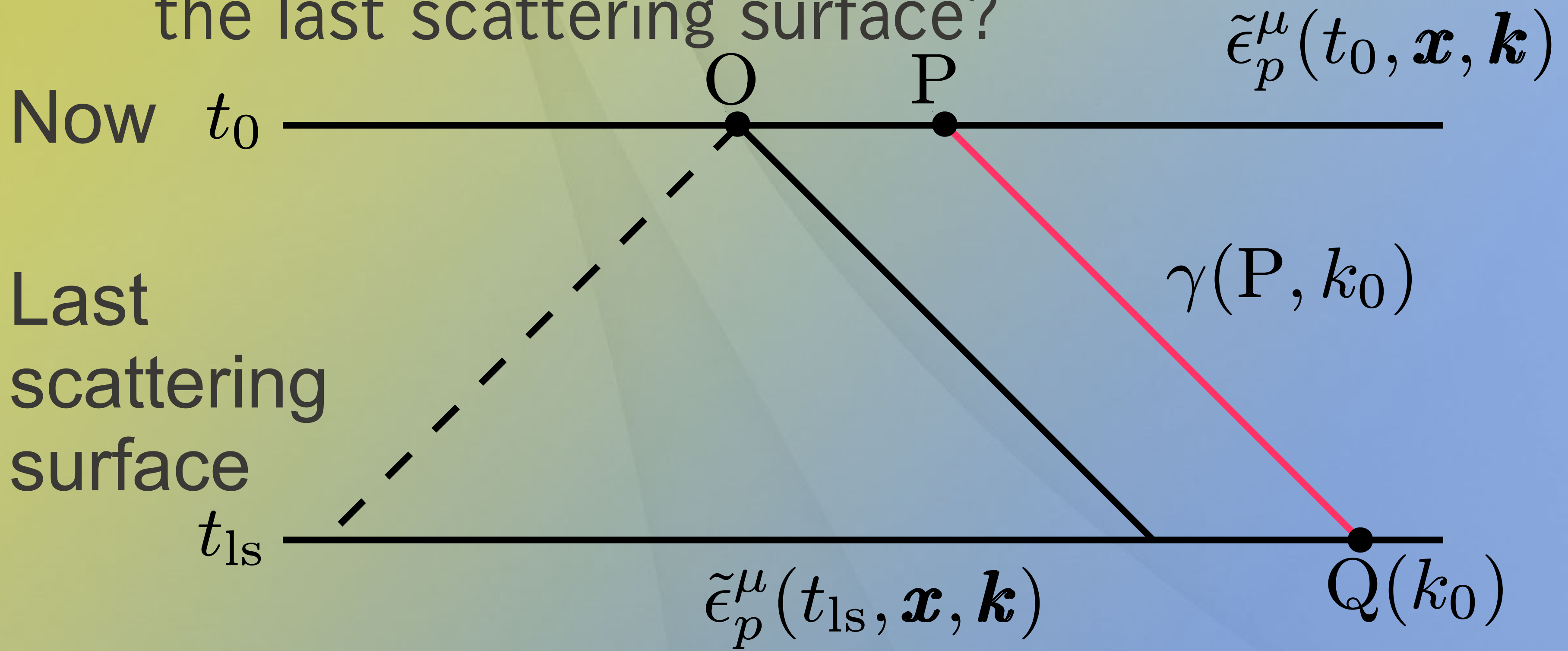


$$\boldsymbol{x} = \boldsymbol{x}(x_0, k_0)$$

$$k = k(x_0, k_0)$$

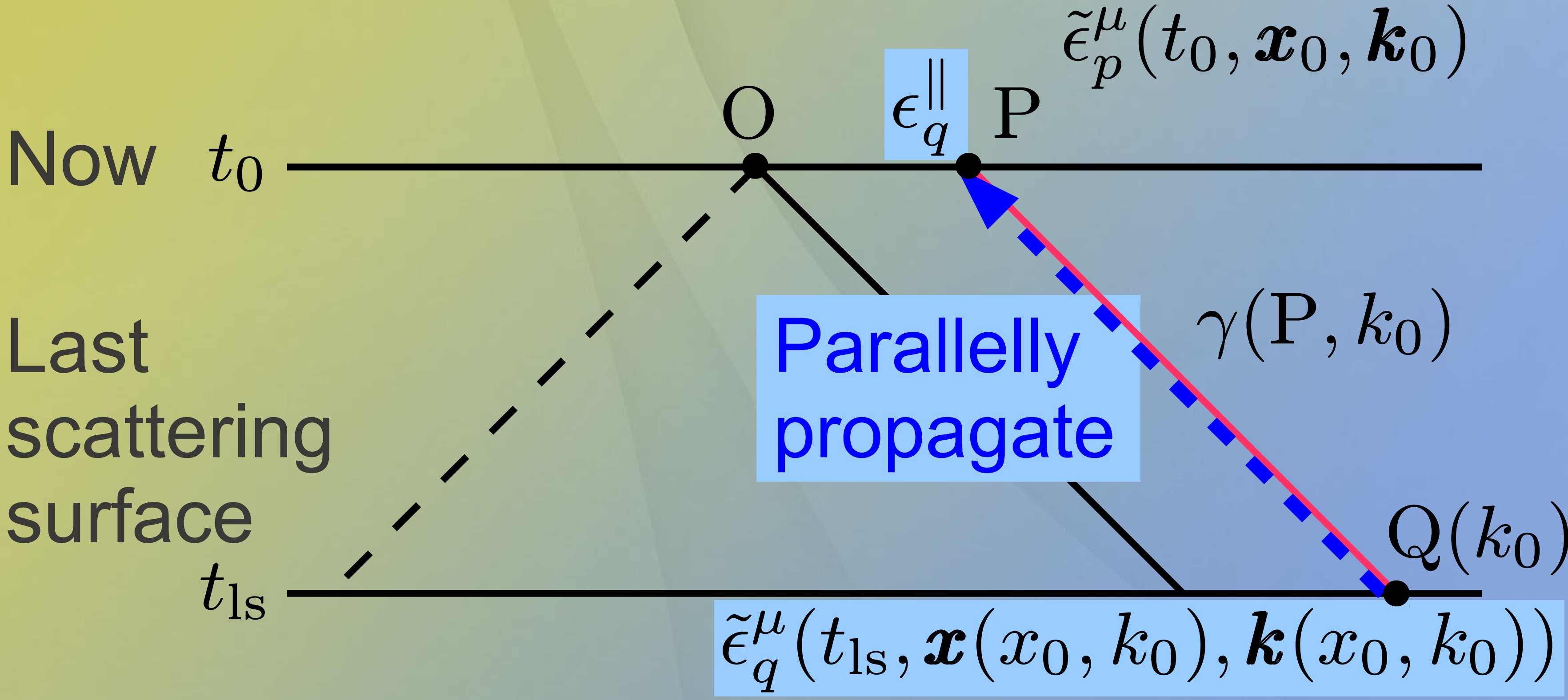
Propagation after last scattering

- How is the polarization now related to that at the last scattering surface?



$$\tilde{\rho}(\mathrm{P}, \boldsymbol{k}_0) = \tilde{C}\tilde{\rho}(t_{\mathrm{ls}}, \boldsymbol{x}, \boldsymbol{k})\tilde{C}^{\dagger}$$

Propagation after last scattering



$$\tilde{\rho}(\mathrm{P}, \boldsymbol{k}_0) = \tilde{C}\tilde{\rho}(t_{\mathrm{ls}}, \boldsymbol{x}, \boldsymbol{k})\tilde{C}^{\dagger}$$

$$\tilde{C}_{pq} = \tilde{\epsilon}_{p}(t_0, \boldsymbol{x}_0, \boldsymbol{k}_0) \cdot {\epsilon^{\parallel}_{q}}^{*}$$

Propagation after last scattering

- When the universe is well-described by the FLRW model until last scattering,

$$\tilde{\rho}_{pq}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k}) = \frac{1}{2}\delta_{pq} I\left(\frac{\omega}{T_{\rm ls}}\right) + \delta\rho_{pq}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$$

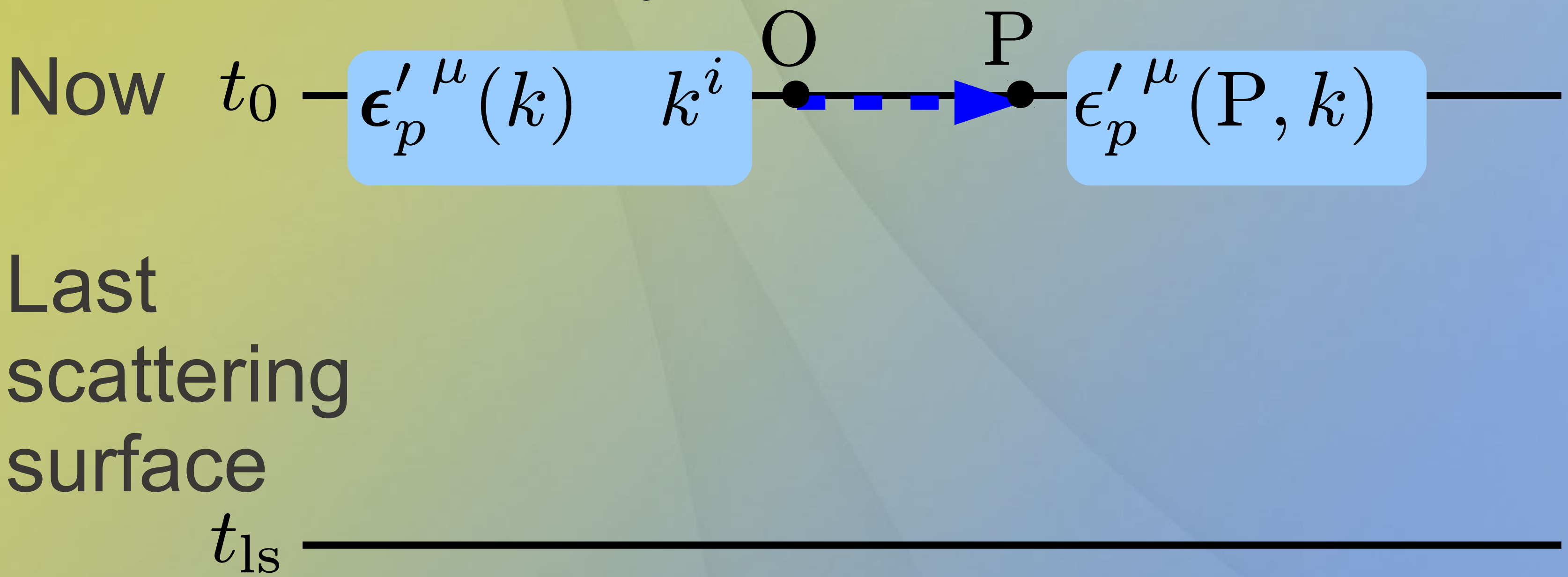
- Using $\sum_p \tilde{C}_{pq}\tilde{C}^*_{pr} = \delta_{qr}$,

$$\tilde{\rho}_{pq}(\mathrm{P}, \boldsymbol{k}_0) = \frac{1}{2}\delta_{pq} I\left(\frac{\omega}{T_{\rm ls}}\right) + \tilde{C}_{pr}\tilde{C}^*_{qs}\delta\rho_{rs}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$$

- It turns out that $\tilde{C}_{pq} \approx 1$ in an appropriate global polarization basis

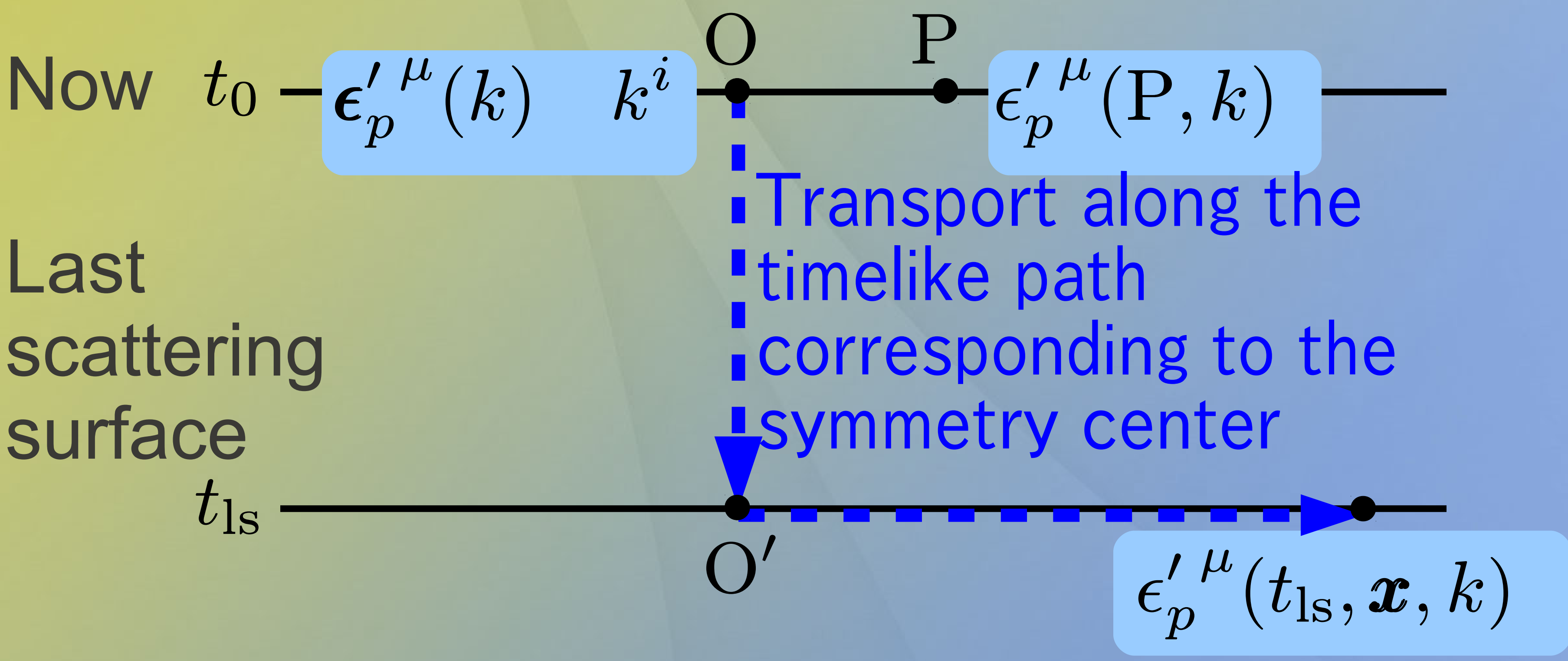
Propagation after last scattering

Transport along the radial line Ω=const. parallelly with respect to $g_{ij}(t_0, x)$



In this way, we can define a polarization basis everywhere on the hypersurface $t = t_0$

Propagation after last scattering



Extend to an arbitrary point by the parallel transport with respect to $g_{ij}(t_{\rm ls},x)$

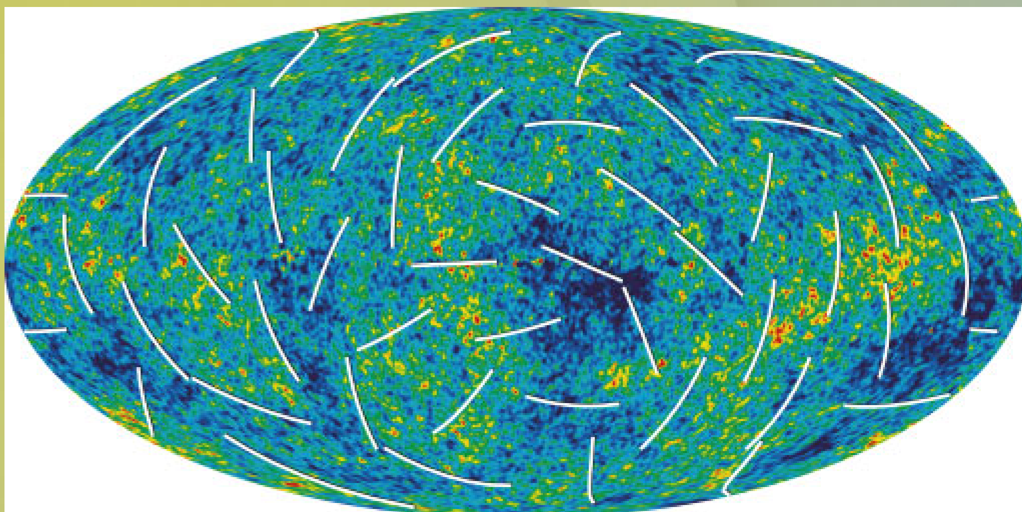
Propagation after last scattering

- For the null vector k parallel to γ_0 , the corresponding polarization basis $\epsilon'^{\mu}_{p}(t,k)$ at $t = t_0$ and $t = t_{\rm ls}$ are parallelly related
- Considering the geodesic deviation of γ from γ_0, we have found that we can set $\tilde{C}$ to be the unit matrix and

$$\tilde{\rho}_{pq}(t_0, \boldsymbol{x}_0, \boldsymbol{k}_0) = \frac{1}{2}\delta_{pq} I\left(\frac{\omega}{T_{\rm ls}}\right) + \delta\rho_{pq}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$$

- Thus, the expressions for the temperature and polarization can be calculated as in the FLRW-universe case

Change in the polarization distribution



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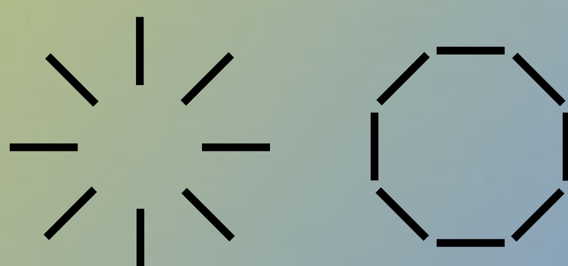
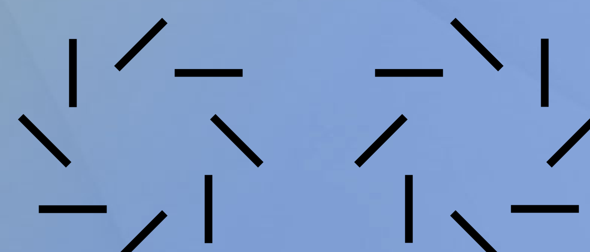
Polarization distribution on the celestial sphere consists of two patterns

'gradient'
E-mode



'curl'
B-mode



Change in the polarization distribution

- CMB polarization distribution (omitting "obs")

$$\rho_{ab}(\hat{\boldsymbol{n}}) = {}_{+2}A(\hat{\boldsymbol{n}})\bar{m}_a\bar{m}_b + {}_{-2}A(\hat{\boldsymbol{n}})m_a m_b$$

$$\hat{\boldsymbol{n}} = (\theta,\ \varphi) \qquad a, b \in \{\theta \equiv 1,\ \varphi \equiv 2\}$$

$$\boldsymbol{m} := \frac{1}{\sqrt{2}}(\boldsymbol{e}_\theta + i\boldsymbol{e}_\varphi) \qquad \bar{\boldsymbol{m}} := \frac{1}{\sqrt{2}}(\boldsymbol{e}_\theta - i\boldsymbol{e}_\varphi)$$

- ${}_{\pm 2}A(\hat{\boldsymbol{n}})$, spin-2 functions, are expanded with spin-weighted spherical harmonics

$$ {}_{\pm 2}A(\hat{\boldsymbol{n}}) = \sum_{\ell m} {}_{\pm 2}A_\ell^m \, {}_{\pm 2}Y_\ell^m(\hat{\boldsymbol{n}})$$

Change in the polarization distribution

- Rotationally invariant combination of ${}_{\pm 2}A_\ell^m$

$$E_\ell^m := \frac{1}{2}\left({}_{+2}A_\ell^m + {}_{-2}A_\ell^m\right)$$

curl-free (E-mode)

$$B_\ell^m := \frac{1}{2i}\left({}_{+2}A_\ell^m - {}_{-2}A_\ell^m\right)$$

gradient-free (B-mode)

- Power spectra $\mathrm{X}_1, \mathrm{X}_2 \in \{\mathrm{T}, \mathrm{E}, \mathrm{B}\}$

temperature

$$\langle {X_1}_\ell^{m*} {X_2}_{\ell'}^{m'} \rangle = C_\ell^{\mathrm{X}_1\mathrm{X}_2} \delta_{\ell\ell'} \delta_{mm'}$$

- If physics and the ensemble for averaging are invariant under a parity inversion,

$$C_\ell^{\mathrm{TB}} = C_\ell^{\mathrm{EB}} = 0$$

Change in the polarization distribution

- ρ_{ab} : polarization distribution without lensing
- ρ'_{ab} : polarization distribution with lensing
- $\hat{\boldsymbol{n}} := \hat{\boldsymbol{n}}' + \delta\boldsymbol{\theta}$

  $\hat{\boldsymbol{n}}'$: direction in which the observer looks
- Gravitational lensing effect → $\rho'_{ab}(\hat{\boldsymbol{n}}') = \rho_{ab}(\hat{\boldsymbol{n}})$
- Finally,

$$\Theta'^m_\ell = \Theta^m_\ell - D\Gamma(\ell+2)\sqrt{\frac{(\ell+1)^2 - m^2}{4(\ell+1)^2 - 1}}\Theta^m_{\ell+1}$$

$$+D\Gamma(\ell-1)\sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}\Theta^m_{\ell-1} + \mathcal{O}(D^2)$$

Change in the polarization distribution

$$E'^{m}_{\ell} = E^{m}_{\ell}$$

$$-D\Gamma\sqrt{\frac{(((\ell+1)^2-m^2)(((\ell+1)^2-4)}{(\ell+1)^2(4(\ell+1)^2-1)}}(\ell+2)E^{m}_{\ell+1}$$

$$+D\Gamma\sqrt{\frac{(\ell^2-m^2)(\ell^2-4)}{\ell^2(4\ell^2-1)}}(\ell-1)E^{m}_{\ell-1}$$

$$+iD\Gamma\frac{2m}{\ell(\ell+1)}B^{m}_{\ell}+\mathcal{O}(D^2)$$

Change in the polarization distribution

$$
\begin{aligned}
B'^{m}_{\ell} &= B^{m}_{\ell} \\
&-D\Gamma\sqrt{\frac{(((\ell+1)^2-m^2)(((\ell+1)^2-4)}{(\ell+1)^2(4(\ell+1)^2-1)}}(\ell+2)B^{m}_{\ell+1} \\
&+D\Gamma\sqrt{\frac{(\ell^2-m^2)(\ell^2-4)}{\ell^2(4\ell^2-1)}}(\ell-1)B^{m}_{\ell-1} \\
&-iD\Gamma\frac{2m}{\ell(\ell+1)}E^{m}_{\ell}+\mathcal{O}(D^2)
\end{aligned}
$$

Change in the polarization distribution

- If $B = 0$ in the homogeneous case (the case without lensing),

$$B'^{m}_{\ell} \approx D\Gamma \frac{2m}{i\ell(\ell+1)} E^{m}_{\ell}$$

$$\Gamma := -\int_{t_{\rm ls}}^{t_0} \frac{dt}{\chi^2} \left[\frac{\chi}{r}(1-\xi(\chi)) + \frac{1}{S} - \frac{\chi}{r} \exp \int_t^{t_0} dt_1 \left\{ \frac{\xi(\chi)}{\chi} \left(\frac{1}{r'} - \frac{\chi}{r} \right) + \frac{\dot{r}}{r} - \frac{\dot{S}}{S} \right\}_{t_1} \right]$$

EB correlation (non-zero part only)

$$\langle E^{m*}_{\ell} B'^{m}_{\ell} \rangle_{\rm CMB} \approx D\Gamma \frac{2m}{i\ell(\ell+1)} C^{\rm EE}_{\ell}$$

(TB similarly)

Change in the polarization distribution

- If B=0 in the homogeneous case, correlations in the first order in D become nonzero only if

θ dependence of the shift vector	TT, TE, EE correlations	TB, EB correlations
$\sin\theta$	$\ell' = \ell \pm 1$	$\ell' = \ell$
Cf. $\sin^2\theta$	$\ell' = \ell \pm 2$	$\ell' = \ell \pm 1$

Cf. anisotropic inflation(Masa-aki Watanabe's talk)

Summary & discussion

- We developed a formulation to calculate the gravitational lensing effect on the CMB temperature and polarization for an off-center observer in a spherically symmetric void described by the LTB model
- We have found that for an off-center observer in the local void, there appear nonzero correlations between T and B and between E and B that are diagonal in the harmonic coefficient expression in the leading order with respect to the observer offset distance

Numerical estimation

- From the analytical results, $B \sim D\Gamma E$
- By numerical calculation, $\Gamma / H_0 \sim 1$
- Constraint on the off-center distance of the observer from the dipole anisotropy of the CMB temperature $H_0 D \sim 10^{-2} H_0 \chi_0$
- Hence $B \sim 10^{-3} E$

Table I. Numerical estimates of Γ for the AA model and the modified AA model.

Void radius $H_0\chi_0$	Wall width $H_0\Delta\chi$	Value of Γ/H_0 AA model	Value of Γ/H_0 Modified AA model
0.0235	0.020	-3.62	1.68
0.235	0.20	-0.09	0.30
0.470	0.40	0.125	0.303

Density contrast parameter $\alpha = 0.9$

Numerical estimation

- Quantities characterizing models
  - Curvature parameter function

$$\Omega_k(\chi) = -\frac{1}{H_0^2} k(\chi)$$

  - Density parameter function

$$\Omega_m(\chi) = \frac{2GM(\chi)}{H_0^2 \chi^3}$$

$M(\chi)$: Mass inside the sphere of the coodinate radius χ at present $t = t_0$

Numerical estimation

- AA model

Alnes-Amarzguioui'06, Alnes-Amarzguioui-Gron'06

$$\Omega_k(\chi) = \frac{\alpha}{2}\left(1 - \tanh\frac{\chi - \chi_0}{\Delta\chi}\right)$$

$$\Omega_m(\chi) = 1 - \Omega_k(\chi)$$

- Modified AA model

$$\Omega_k(\chi) = \frac{\alpha}{2}\left(1 - \tanh\frac{\chi^2 - \chi_0^2}{\Delta\chi^2}\right)$$

$$\Omega_m(\chi) = 1 - \Omega_k(\chi)$$

Numerical estimation

- These models approaches the Einstein-de Sitter model at infinity outside the void
- $H_\infty/H_0 = 2/(3H_0 t_0)$

  t_0 : age of the universe at the center
- AA model has a cusp singularity in the density and curvature at the center, therefore the integrand for Γ has a kind of δ -function type singularity at the center
- Modified AA model is smooth at the center, therefore the integrand is finite at the center