YITP
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Solving the Big Bang singularity using string theory

Bouncing-model perturbations and the Singularity problem

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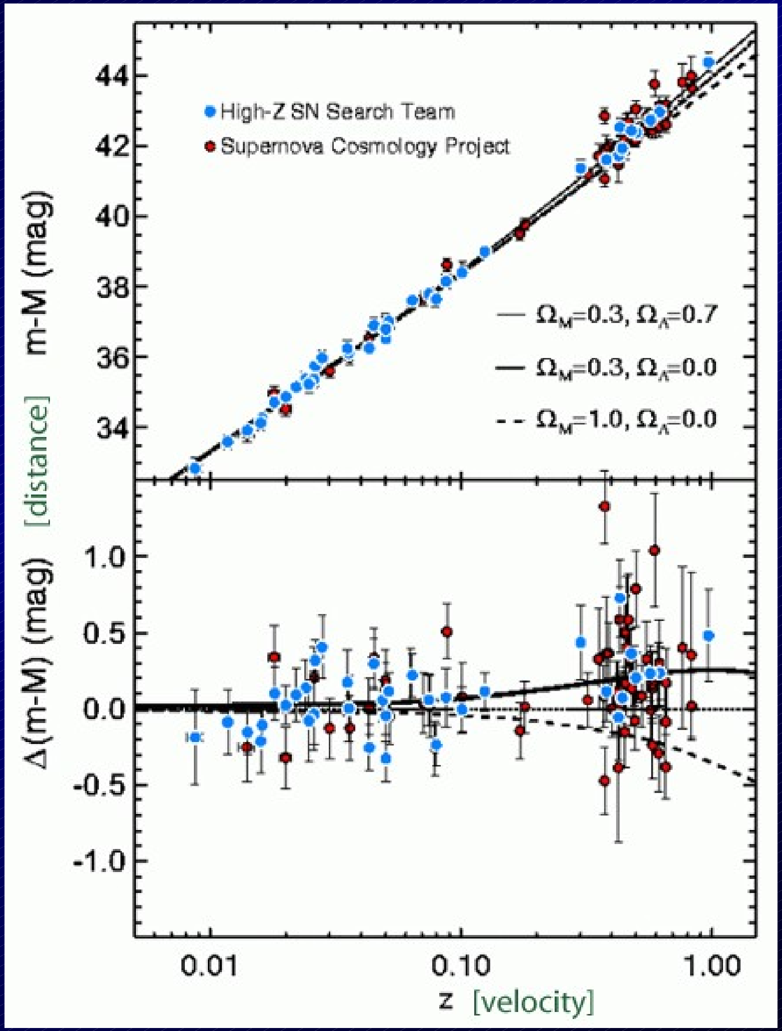
The Story of Λ

Supernova data



A single parameter (Λ) can fit the data perfectly; and it is already part of the theory!

The Story of Λ

One problem:

Why is got that tiny value?

This problem lead to many interesting alternatives (predictions/understandings):

Such as: quintessence, k-essence, inhomogeneous cosmologies, chameleons, quintom, phantom fields, frustrated domain walls, string landscape, LTB, f(R), DGP, cosmological casimir effect, chaplygin gas, holographic dark energy, XCDM, quartessence...

Early universe

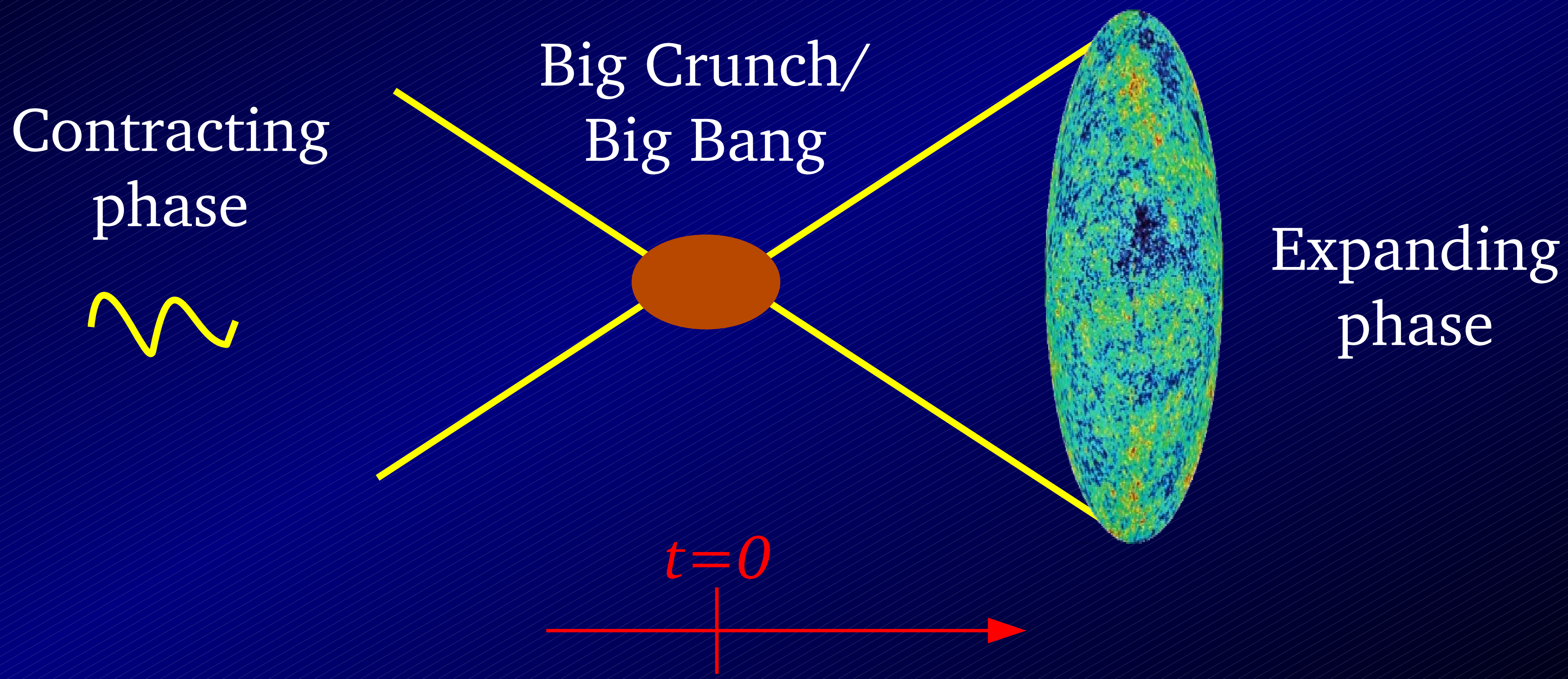
We know inflationary models fit the data, but also have certain problems...

Use same philosophy to study alternatives to inflation...

Consider, in particular, bouncing cosmologies

Bouncing models

Key: Pre-Big bang phase where CMB pertns. are created



Two flavours: singular & non-singular

Bouncing models

- Consider **singular** models
- Assume spectrum of scale invariant perturbations
- Need to understand how these perturbations **go through** the singularity

What do we know?

- GR breaks down near the singularity
- Quantum gravity should give an answer
  (e.g. string/M theory)

Toy model

- Cannot solve the problem in general

Study a simple model

$$ds^2 = a^2(t)\left(-dt^2 + \sum_{i=1}^{3}(dx^i)^2\right)$$

$$a(t) = |t|^{1/2}$$

Toy model

Higher dimensional motivation

5d

$$ds^2 = -dt^2 + t^2 d\theta^2 + \sum_{i=1}^{3} (dx^i)^2$$

Milne Universe

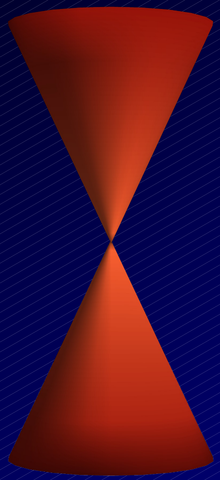
KK reduction

4d

$$ds^2 = |t| \left(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right)$$

Toy model

Higher dimensional motivation



$$ds^2 = -dt^2 + t^2 d\theta^2 + \sum_{i=1}^{3} (dx^i)^2$$

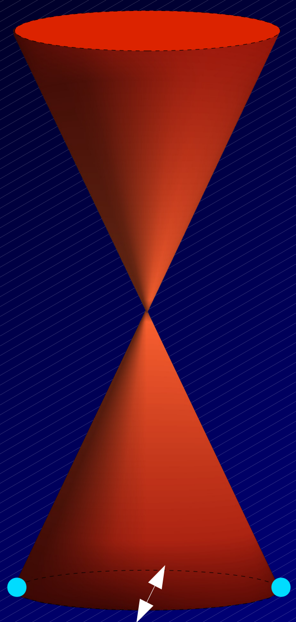
Milne Universe

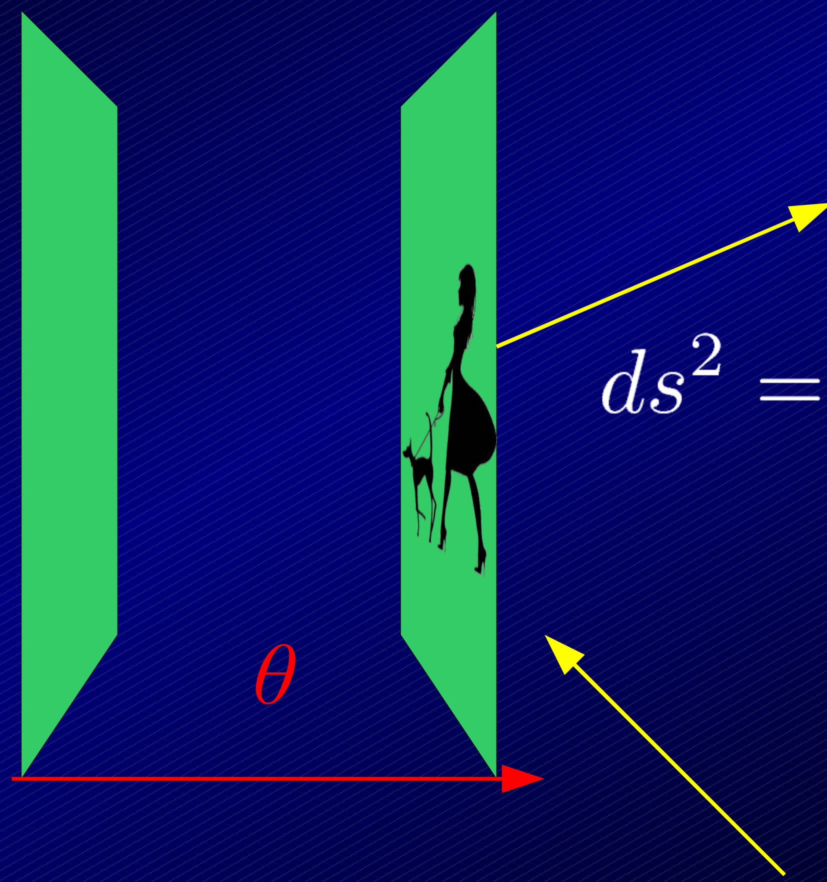
If $\theta \in (-\infty, \infty)$ **flat space in unusual coordinates**

If $\theta \in [0, 2\pi)$ **there indeed a conical singularity**

Toy model

Actually, if a Z_2 symmetry is assumed





4d Obsever

$$ds^2 = |t|(-dt^2 + (d\vec{x}^i)^2)$$

Relevant for
Ekpyrotic/Cyclic
(Khoury et al)

$$ds^2 = -dt^2 + t^2 d\theta^2 + \sum (dx^i)^2$$

Fields on Milne

Scalar field in 5d (near singularity)

$$ds^2 = -dt^2 + t^2 d\theta^2 + \sum (dx^i)^2$$

Get Bessel equation, and in the long-wavelength limit

$$\phi(t) = A + B \ln(t)$$

Diverges at *t=0*

However, can match across using analytic continuation of Bessel functions (Tolley & Turok, 2006)

String/M-theory analysis

Can we do better with **string/M-theory?**

Heterotic string effective action

$$S = \int d^{10}x\sqrt{-g}e^{-2\phi}\Big(R + 4(\partial\phi)^2 + \mathcal{O}(\alpha')\Big)$$

(Gross & Sloan, 1987)

$$\mathcal{O}(\alpha') = \frac{\alpha'}{8}\underbrace{(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2)}_{\text{Gauss-Bonnet}} + \cdots$$

String/M-theory analysis

Background

GN & Turok, 2007

$$a^2(t) = a_0^2(t) + \alpha' a_1^2(t) + \ldots \qquad \phi(t) = \phi_0(t) + \alpha' \phi_1(t) + \ldots$$

0th order

$$a_0^2(t) = t \qquad \phi_0(t) = A\ln(t) + B$$

1st, 2nd order

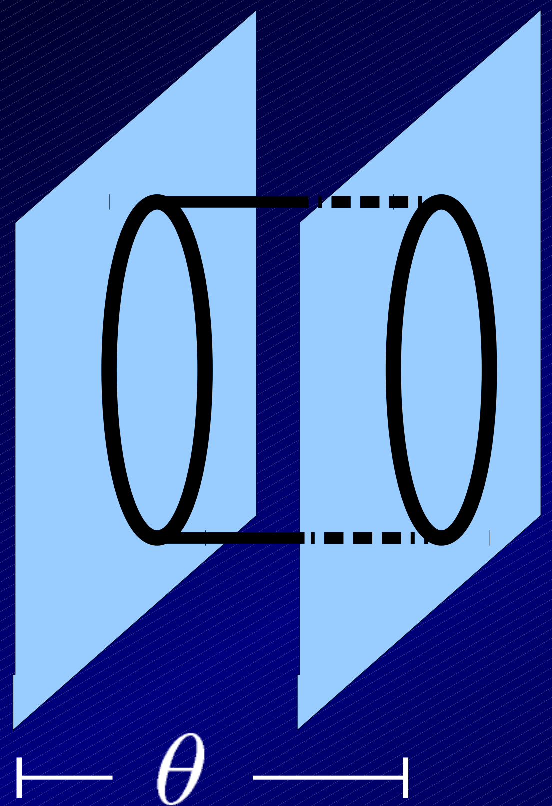
$$a^2(t) = t\left(1 - \frac{\alpha'}{8t^3} - \mathcal{O}\left(\frac{\alpha'^2}{t^6}\right)\right)$$

$$\phi(t) = A\ln(t) + B - \frac{3\alpha'}{16t^3} - \mathcal{O}\left(\frac{\alpha'^2}{t^6}\right)$$

Singularity is reached sooner, so theory breaks down!

String/M-theory analysis

Are we missing something?



(Turok, Perry & Steinhardt, 2002)

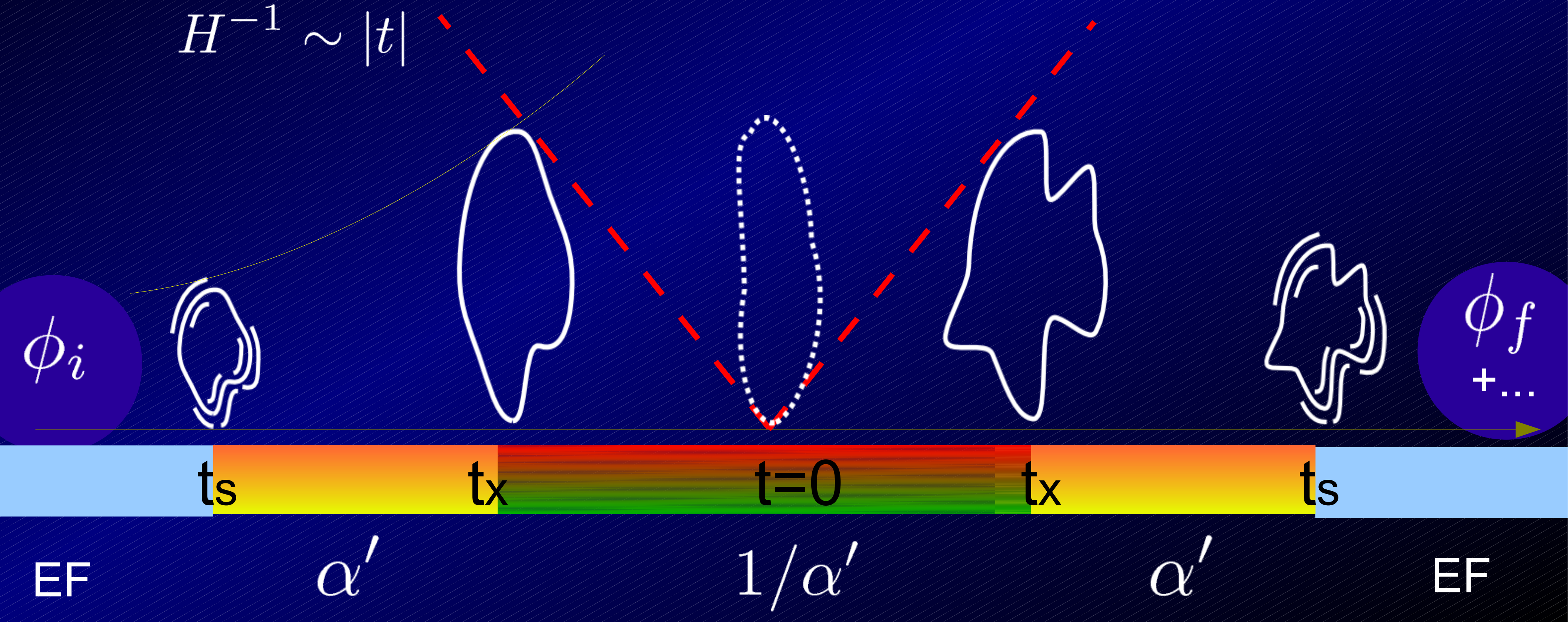
5d $\xrightarrow[\text{0-modes}]{\text{KK}}$ 4d

Membranes → Strings (IIA, Het)

Relevant for cosmological perturbations (Include graviton & dilaton)

Classical evolution

Solutions are regular across the singularity



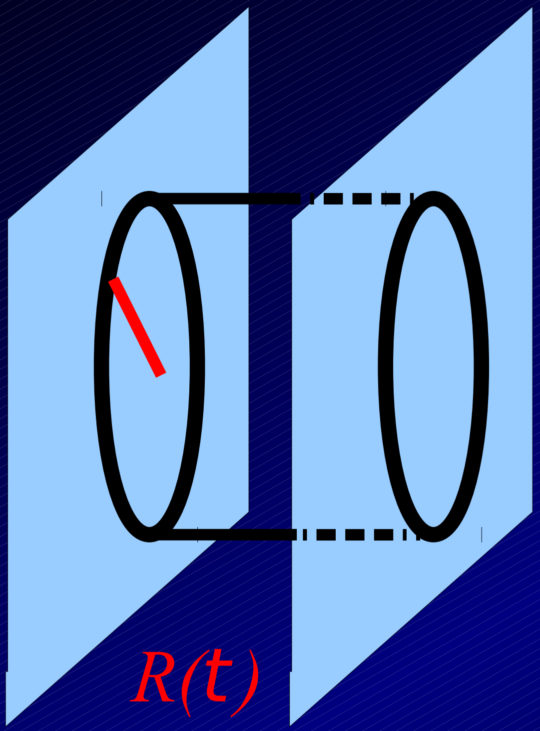
GN & Turok, 2006

Loop quantisation

In the case of circular loops,

Copeland, GN & Turok, 2010

the Hamiltonian constraint is:



$$H = -\pi_0^2 + \pi_R^2 + t^2 R^2 \simeq 0$$

$$\pi_0 = i\partial_t$$

$$\pi_R = i\partial_R$$

$$\hat{H}\Psi = \left(\partial_t^2 - \partial_R^2 + t^2 R^2\right)\Psi \simeq 0$$

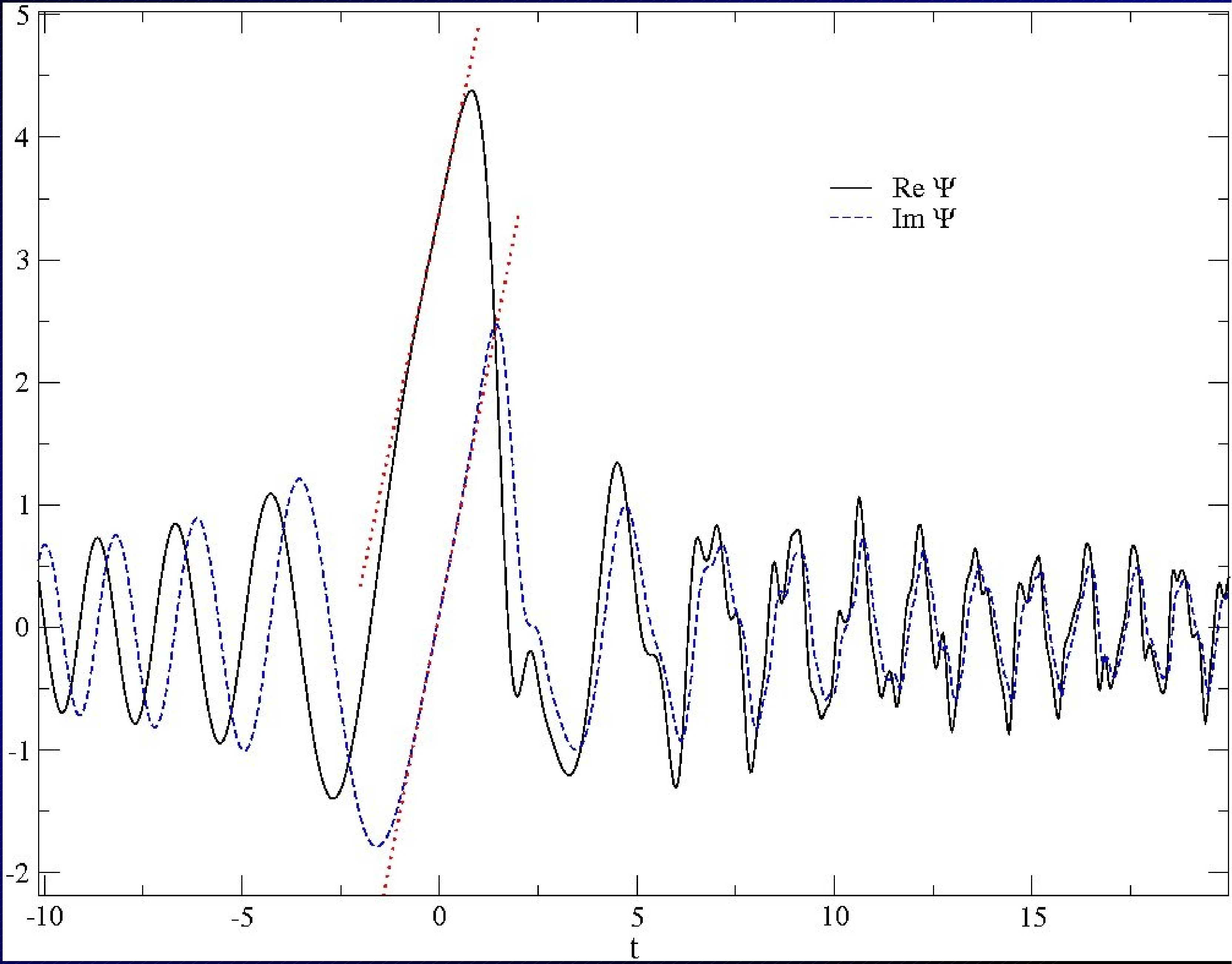
String wave function

$\Psi(t, R)$

c.f. Harmonic oscillator

String wave function

$$\Psi(t) \equiv \int_0^\infty \Psi(t,R)dR$$



Particle production

$$\Psi(t,R) = \sum_n \mathcal{A}_n(t) H_n(x) e^{-x^2/2}, \qquad x \equiv \sqrt{|t|}R$$

Hermite polynomials

for large |t|: $A(t) \to e^{\pm E_n |t|}, \qquad E_n \sim \sqrt{(2n+1)|t|}$

So can send IN vacuum (positive frequency mode) and read the OUT state in terms harmonic oscillator states.

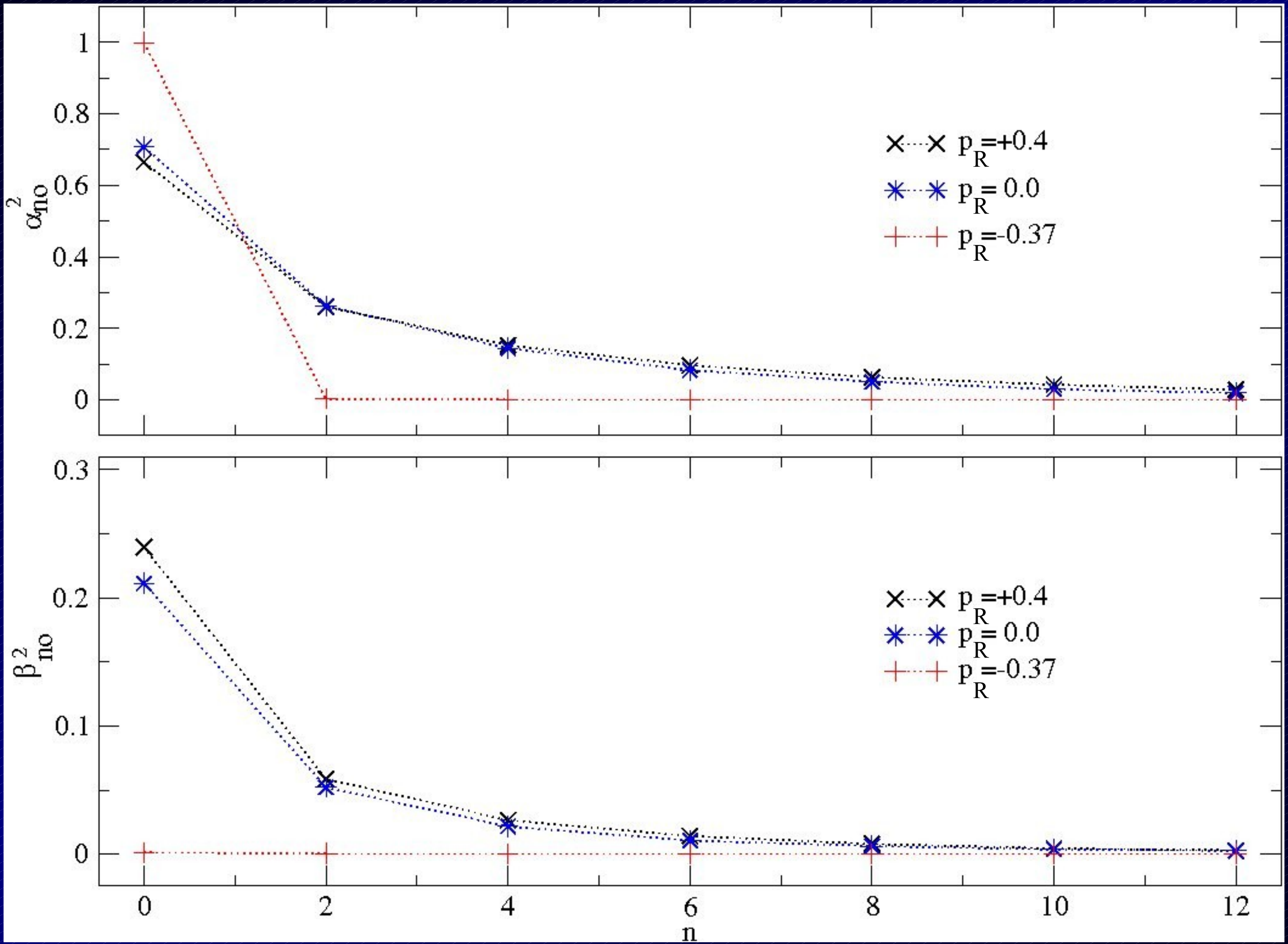
$$|\Psi_n\rangle_{out} = \sum_m \alpha_{nm}|\Psi_m^+\rangle + \beta_{nm}|\Psi_m^-\rangle$$

Bogoliubov coeffs.

Particle production:

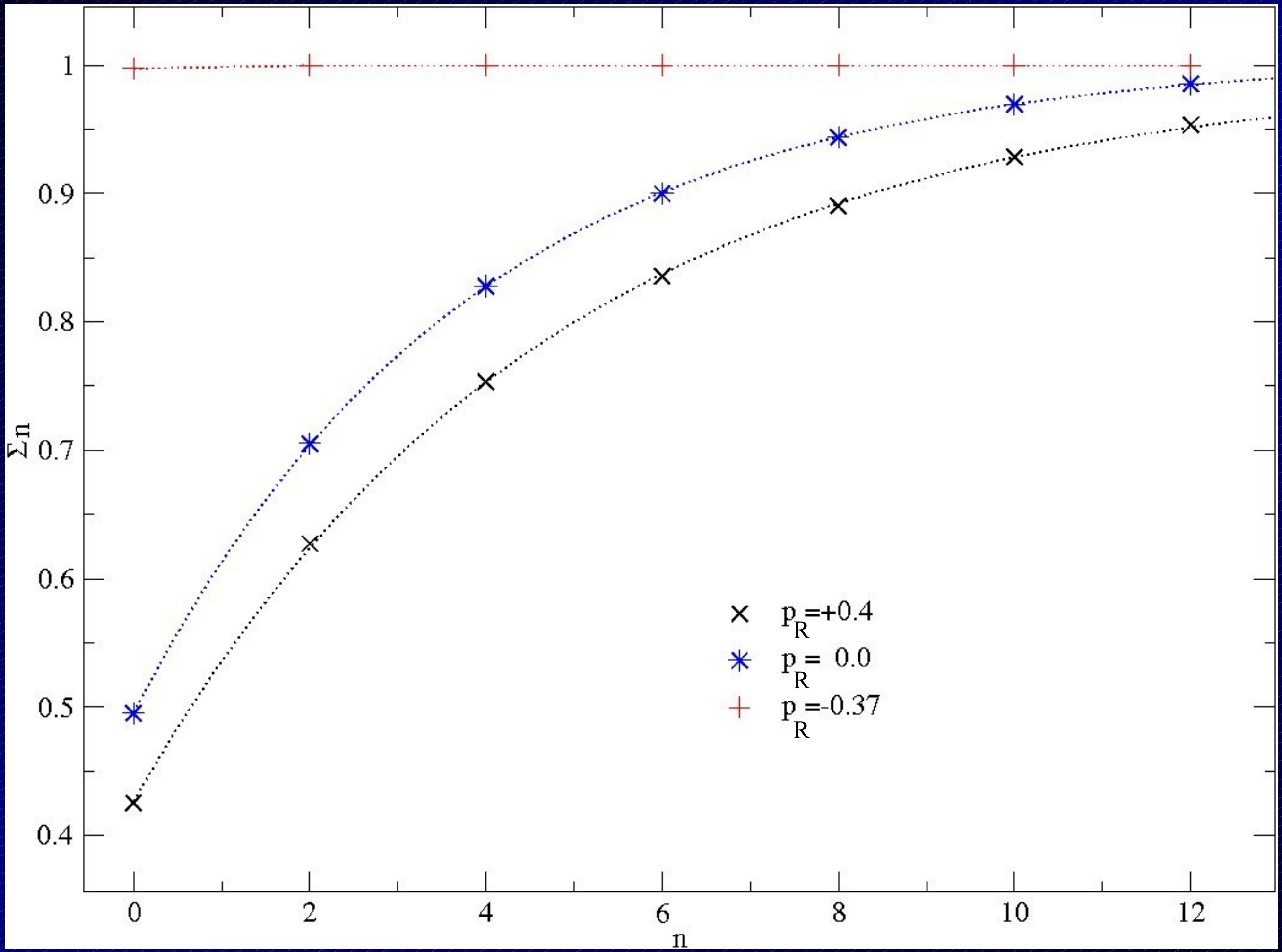
$$\langle N_n \rangle = \sum_m |\beta_{nm}|^2$$

Particle production



Finite Particle Production (decays exponentially with occupation number)

Unitary



Unitarity is preserved if: $$\sum_m |\alpha_{nm}|^2 - |\beta_{nm}|^2 = 1$$

conclusions

- Classical string (winding membranes) are regular across a Milne 5d singularity, featuring
  - String travels at the speed of light at $t=0$
  - Higher oscillation modes resolve the singularity
  - Ultralocal behaviour (string bits)
- Circular loops can be quantised and there is finite particle production
- Unitarity is preserved
- Can be extended to more complicated singularities (Kasner metrics, maybe to FRWL and some BH's)

M-theory model

Winding membranes (strings): Nambu-Goto action

$$S = -\mu_2 \int d^3\sigma \sqrt{-\mathrm{Det}(\partial_A X^M \partial_B X^N g_{MN})}$$

11d Milne

Winding membranes ⟷ field independent of: $\sigma^3 = y$

$$S = -\int \mu_2 \theta_0 |t| \, d^2\sigma \sqrt{-\mathrm{Det}(\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu})}$$

Efective Tension

DyNamics

Two equivalent descriptions

1

- String living on flat spacetime
- Tension: $\mu_1 = \mu_2 \theta_0 |t|$
- Tensionless at ***t=0***
- Like harmonic oscillator with a time-dependent frequency (quantum)

2

- String living on FRWL
  $$g_{\mu\nu} = |t/t_s| \eta_{\mu\nu}$$
- Fixed tension: $\mu_1 = \mu_2 \theta_0 |t_s|$
- Speed of light at ***t=0***
- Better to study classical behaviour

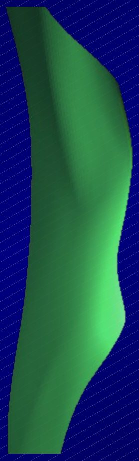
DyNamics

Bare in mind...

Quantum corrections:

- String interactions are suppressed ($g_s = |t/t_s|^{3/2}$)
- α'-corrections are under control GN & Turok.

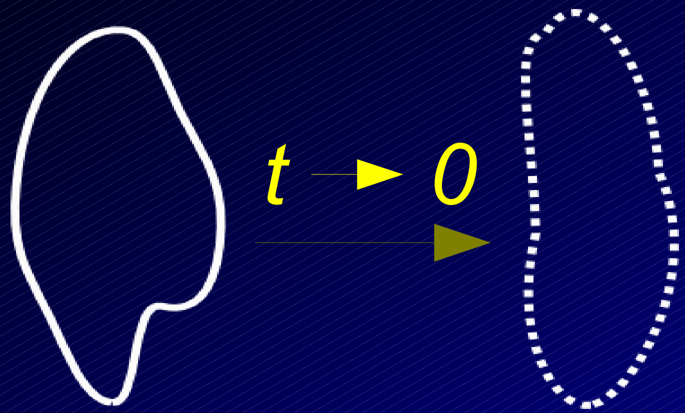
Small perturbations (ripples on the orbifolds) lead to 11d ***Kasner*** backgrounds:



$$ds^2 = -dt^2 + |\theta_0 t|^{2p_{10}} d\theta^2 + \sum_{i=0}^{9} |\theta_0 t|^{2p_i} (dx^i)^2$$

$$\sum_{n=1}^{10} p_n^2 = 1 = \sum_{n=1}^{10} p_n$$

Classical evolution



String breaks into bits!!

Like ultra-locality
in BKL analysis

Consider Hamiltonian for winding membrane

$$H \sim \int d\sigma \left[\mathcal{A}(\Pi_\mu \Pi_\nu \eta^{\mu\nu} + \mu_2 \theta_0^2 t^2 X'^{\,\mu} X'^{\,\nu} \eta_{\mu\nu}) + \mathcal{A}^i \Pi^\mu X'_{\mu,i} \right]$$

Arbitrary function

Interaction term, coupling ~ $1/\alpha'$

- |t|>>1 → α' expansion
- |t|<<1 → $1/\alpha'$ expansion

Perry, Steinhardt and Turok, 2004

String Spectrum

Left/right-mover decomposition:

$$\vec{X}_R = \dot{\vec{X}} - \vec{X}' = \sum_{i=-\infty}^{+\infty} \frac{1}{2} \vec{\alpha}_n e^{in\xi_-}$$

$$\vec{X}_L = \dot{\vec{X}} + \vec{X}' = \sum_{i=-\infty}^{+\infty} \frac{1}{2} \tilde{\vec{\alpha}}_n e^{in\xi_+}$$

$$\xi_\pm = \tau \pm \sigma$$

$$\vec{\alpha}_n^\dagger = \vec{\alpha}_{-n}$$

$$\tilde{\vec{\alpha}}_n^\dagger = \tilde{\vec{\alpha}}_{-n}$$

Constraint $\vec{X}_R^2 = 1 = \vec{X}_L^2$ ⟶ curves in S^{d-1}

Kibble & Turok, 1982

String's massless bosonic sector: $\phi,\ h_{\mu\nu},\ B_{\mu\nu}$

spin: 0 2 1

String Spectrum

Left/right-mover decomposition:

$$\vec{X}_R = \dot{\vec{X}} - \vec{X}' = \sum_{i=-\infty}^{+\infty} \frac{1}{2} \vec{\alpha}_n e^{in\xi_-} \qquad \xi_\pm = \tau \pm \sigma$$

$$\vec{X}_L = \dot{\vec{X}} + \vec{X}' = \sum_{i=-\infty}^{+\infty} \frac{1}{2} \tilde{\vec{\alpha}}_n e$$

Consider dilaton mode only!

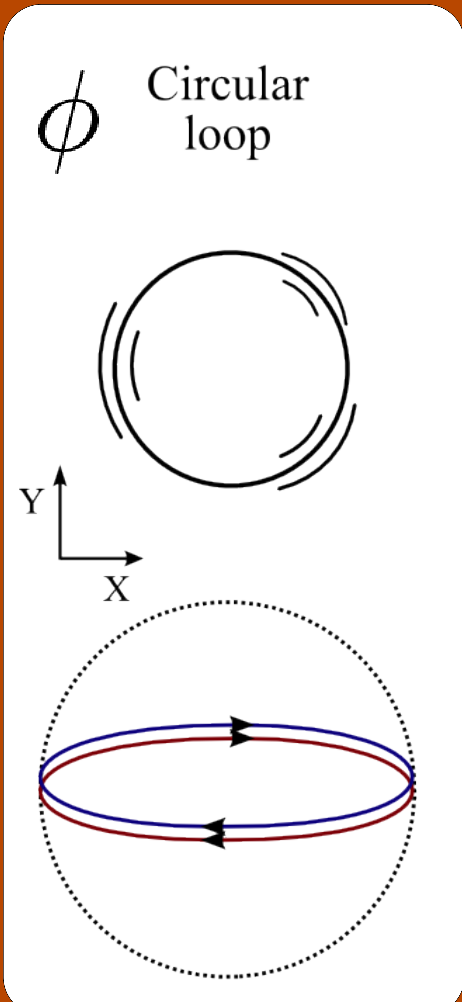
Constraint $\vec{X}_R^2 = 1 = \vec{X}_L^2$ S $^{d-1}$

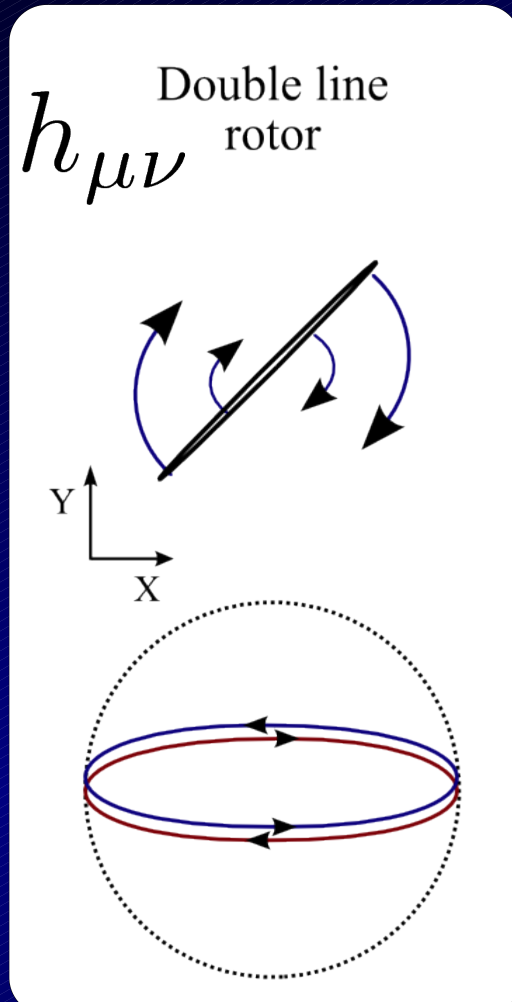
Kibble & Turok, 1982

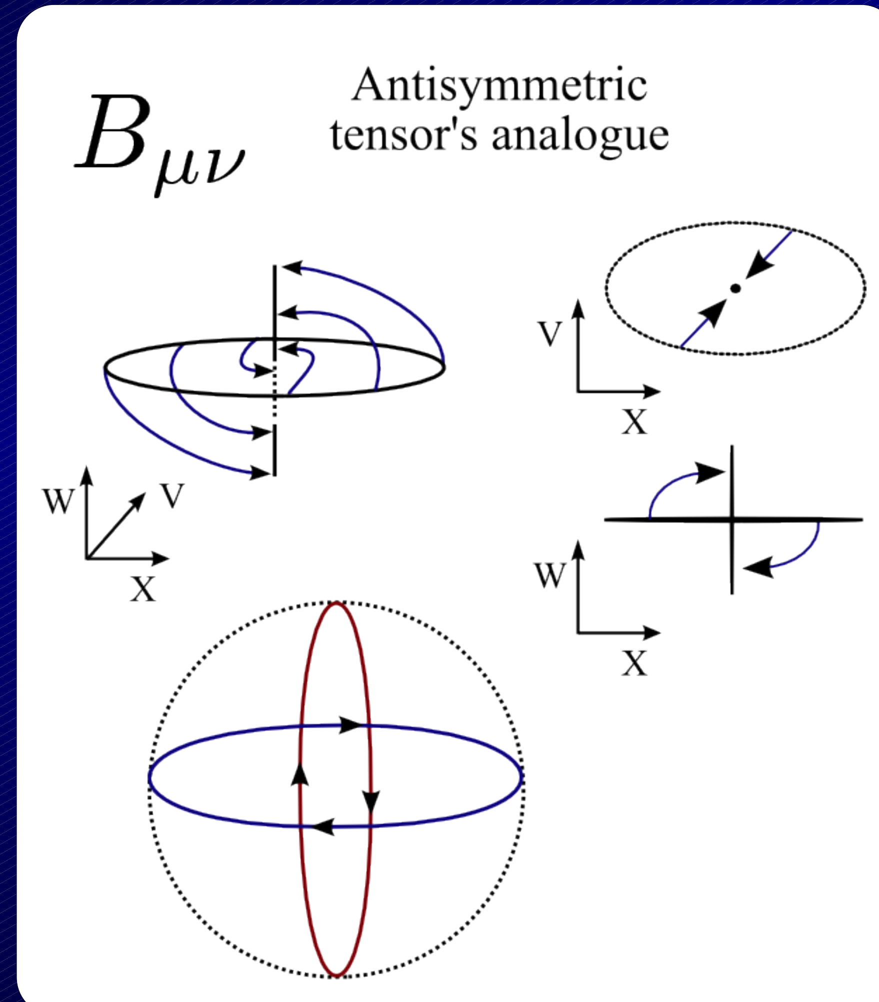
String's massless bosonic sector: ϕ, $h_{\mu\nu}$, $B_{\mu\nu}$

spin: 0 2 1

String Spectrum







0	maximum	in between	Ang. Mom.

Effective Action

Heterotic string

$$S = \int d^{10}x\sqrt{-g}e^{-2\phi}\Big(R + 4(\partial\phi)^2 + \mathcal{O}(\alpha')\Big)$$

(Gross & Sloan, 1987)

$$\mathcal{O}(\alpha') = \frac{\alpha'}{8}(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2) +$$ terms involving the dilaton

Gauss-Bonnet

Background

$$a^2(t) = a_0^2(t) + \alpha' a_1^2(t) + ...$$

$$\phi(t) = \phi_0(t) + \alpha' \phi_1(t) + ...$$

0th order

$$a_0^2(t) = t \qquad \phi_0(t) = \frac{3}{2}\ln(t)$$

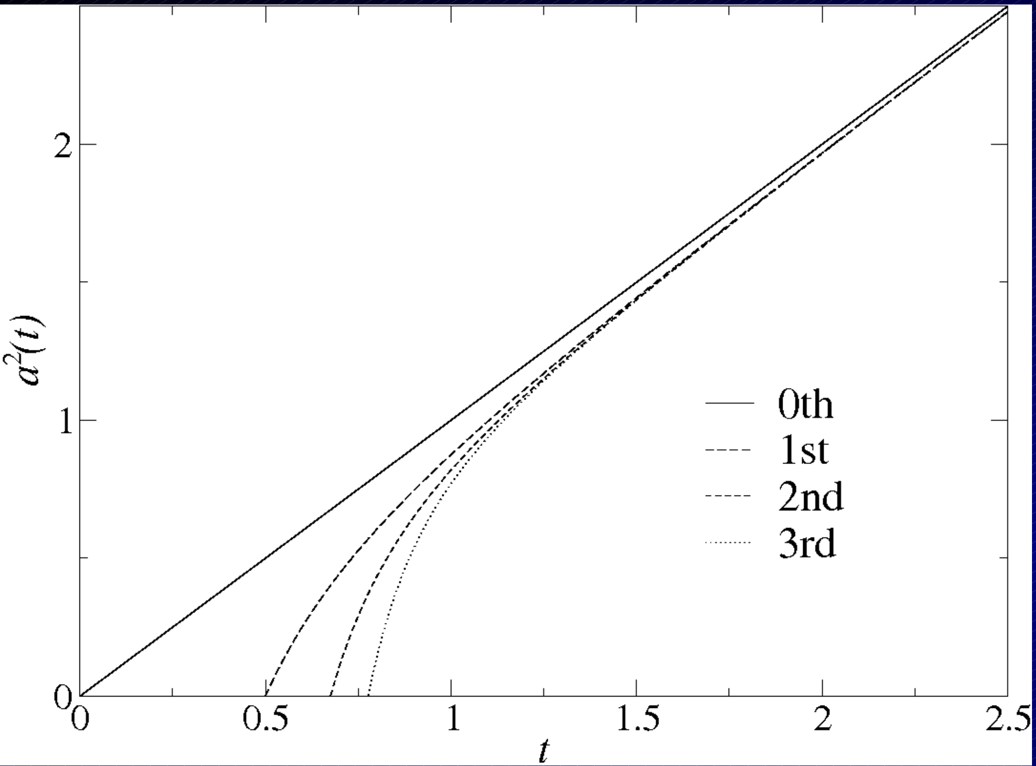
1st, 2nd order

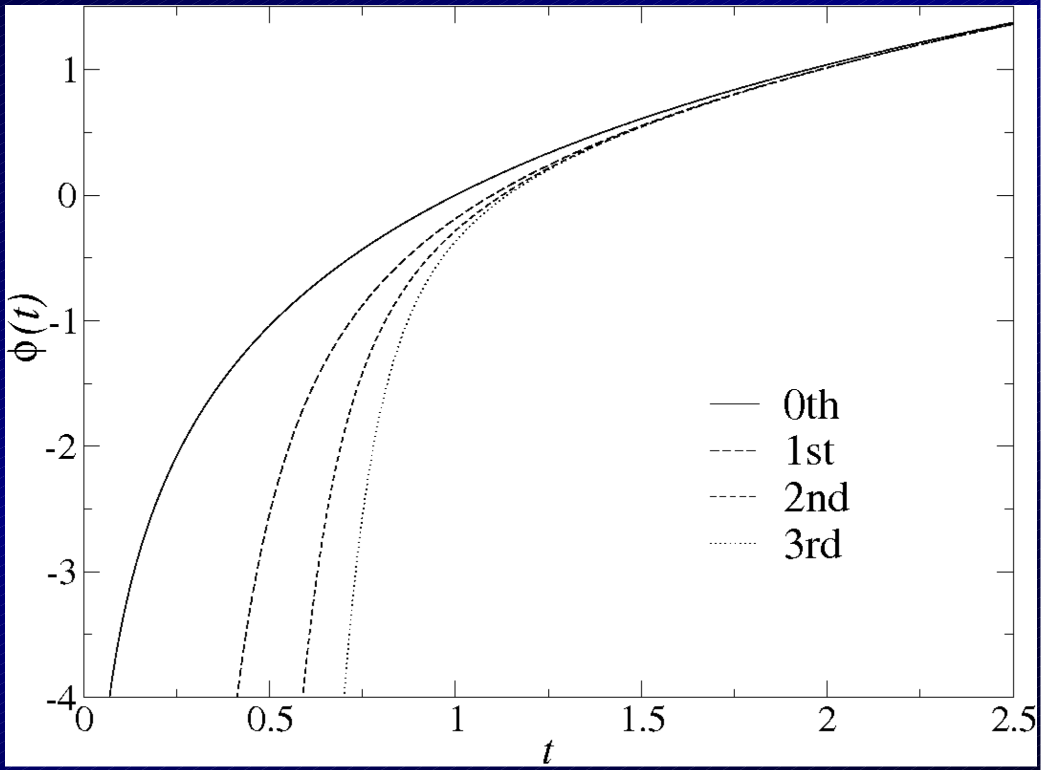
$$a^2(t) = t\left(1 - \frac{\alpha'}{8t^3} - \mathcal{O}\left(\frac{\alpha'^2}{t^6}\right)\right)$$

$$\phi(t) = \frac{3}{2}\ln(t) - \frac{3\alpha'}{16t^3} - \mathcal{O}\left(\frac{\alpha'^2}{t^6}\right)$$

Note that $a^2(t) = e^{2\phi/3}$

Background





- Singularity is reached sooner
- Corrections become important around *t=tx*

Perturbations

Tensor modes

$$ds^2 = a(t)^2\Big(-dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j \Big)$$

$$h_{ij}(t,\vec{x}) = e^{ik\cdot x}\hat{h}_{ij}(t)$$

$$h_{ij}(t) = h^0_{ij}(t) + \alpha' h^1_{ij}(t) + \ldots$$

0^{th} order

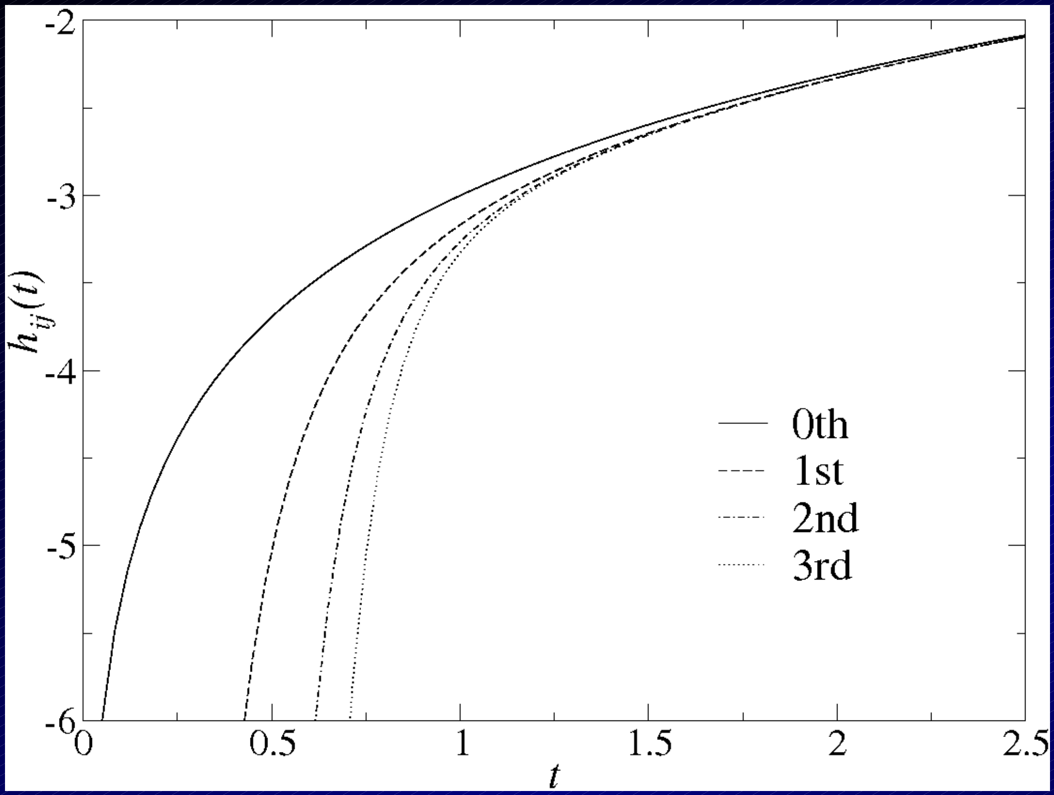
Bessel's equation. Consider $k \to 0$ case:

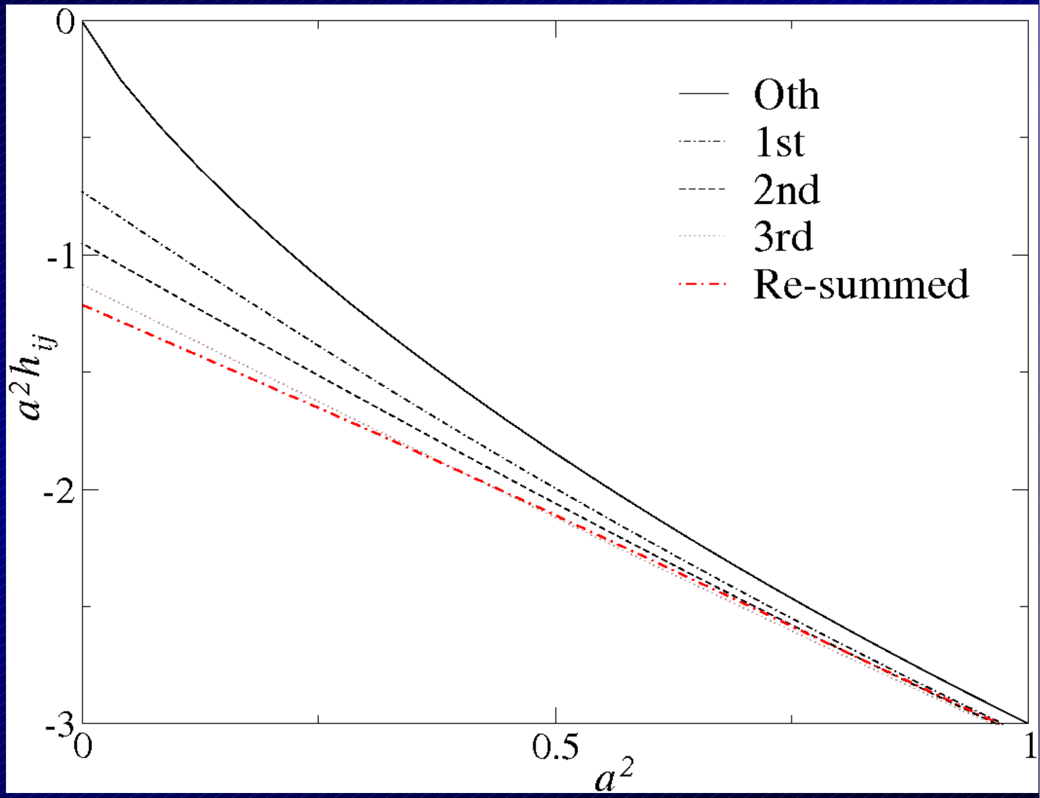
$$h^0_{ij}(t) = A_{ij} + B_{ij}\ln(t)$$

1^{st}, 2^{nd} order

$$h_{ij}(t) = A_{ij} + B_{ij}\ln(t) - \frac{\alpha' B_{ij}}{6t^3} - \mathcal{O}\left(\frac{\alpha'^2 B_{ij}}{t^6}\right)$$

Perturbations





Scalar perturbations + gauge choice = tensor perts.